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DIFFUSION PROCESSES IN PECULIAR A STARS

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ABSTRACT

It is suggested that diffusion processes are responsible for most of the peculiar abundances observed in Ap stars. If it is assumed that the atmosphere is stable enough for diffusion processes to be important, gravitational settling leads to the underabundances of He, Ne, and O in the stars where they are observed (that is, with the θ_{eff}, log g they are observed to have). Radiation pressure leads to the overabundances of Mn, Sr, Y, Zr, and the rare earths in the stars where they are observed. Silicon would be expected to be overabundant only if it has wide autoionization features. Phosphorus would be expected to be overabundant in stars with $\theta_{eff} \simeq 0.5$, but is observed to be overabundant in stars with $\theta_{eff} \simeq 0.4$.

The magnetic fields observed in Ap stars could bring to the atmosphere the stability needed for diffusion processes to be important. They would also guide diffusion into patches leading to the periodic variation of the observed overabundances.

I. INTRODUCTION

Sargent and Searle (1967*a*) have recently established that the abundance anomalies in Ap stars were closely related to the atmospheric parameters. It is found below that the same dependence on atmospheric parameters exists for abundance anomalies to be expected from diffusion processes. Diffusion would cause the observed abundance anomalies if the atmosphere is stable enough for diffusion to be important. The magnetic fields observed in Ap stars may give their atmosphere the needed stability.

One of the best-established characteristics of Ap stars (Sargent and Searle 1967*a*; Wolff 1967) seems to be the dependence of some of the abundance anomalies on the effective temperature of the star. Before the systematic studies of Sargent, Searle and co-workers, it could have been argued that the apparent dependence of the abundance anomalies on effective temperature was due to selection effects. However, by determining the abundances of certain elements (O, Mg, Si, Mn, . . .) in a large number of both normal and peculiar stars, Sargent and Searle (1967*a*) were able to establish that certain elements (O, Si, Mn) had unusual abundances only in peculiar stars with a given range of θ_{eff}. Outside that range they measured the abundances of the same elements and found them normal, or at least not so abnormal. They established that O was most underabundant in the cooler Ap stars ($0.43 \leq \theta_{eff} \leq 0.65$) (Sargent and Searle 1962) and that Si and Mn were most overabundant in the respective ranges $0.28 \leq \theta_{eff} \leq 0.45$ (Searle and Sargent 1964) and $0.35 \leq \theta_{eff} \leq 0.46$ (Searle, Lungershausen, and Sargent 1966). Others have established that Sr, Y, and Zr behaved differently with the effective temperatures of these stars and that heavy elements were generally overabundant (Hack 1968). The rare earths are more overabundant than Ba. Phosphorus and Be are overabundant in some of the Mn stars (Sargent and Searle 1967*a*).

Finally, the relationship between the atmospheric parameters and the peculiarities was strikingly confirmed when Sargent and Searle (1967*b*) found that the same peculiarities occurred at the same (θ_{eff}, log g) in Population II stars as they did in Ap stars.

Detailed studies of a few Ap stars have also been made. Two model-atmosphere

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studies of the same observational material for 53 Tau (Auer *et al.* 1966; Strom 1969) give results differing frequently by factors of 4. The differences come mainly from the different turbulent velocities adopted. Auer *et al.* (1966) took $v_T = 0.0$. Strom (1969) obtained turbulent velocities (one for the visible and one for the ultraviolet) by forcing strong and weak lines to give the same abundance. It has been shown by an analysis of line profiles (Hardorp, Bidelman, and Prolss 1968) that the turbulent velocity obtained in this way for 3 Cen A was inconsistent with the upper limit of the width observed for certain lines. Until the parameters entering model-atmosphere studies are better understood, it seems preferable to rely for abundance determination in Ap stars on the comparative curve-of-growth analyses of Sargent, Searle and co-workers. For their abundance analyses they chose lines that are relatively independent of the uncertain structure of the atmosphere.

Even when compared only with the well-established properties of Ap stars, the nuclear-physics models all run into serious difficulties. If the nuclear reactions occurred in the interior of stars (Fowler *et al.* 1965), there should be no Ap stars in young stellar clusters, a conclusion contrary to observations (Kraft 1967; Hyland 1967; Garrison 1967; Adelman 1968). It seems hard to imagine how the binary-star hypothesis (Fowler *et al.* 1965; Van Den Heuvel 1968*a*, *b*) could lead to any dependence of the abundances on the parameters of the atmosphere. Finally, the models that assume nuclear reactions in the atmosphere (Fowler, Burbidge, and Burbidge 1955; Brancazio and Cameron 1967) run into difficulty with element conservation (Searle and Sargent 1964; Michaud 1969). It then seems worthwhile to look for other explanations to the Ap stars.

We will discuss the possibility that the atmospheres of Ap stars are stable enough for diffusion to become important. This is reasonable since the Ap stars are known to be slow rotators and convection zones are not expected to be important in B and early A stars. The magnetic fields may also add to the stability of the atmosphere since magnetic fields are observed to reduce convection in sunspots. Also, the ability of the rigid-rotator hypothesis (Deutsch 1958) to explain a large fraction of the observed magnetic-field variations argues for their stability. We shall discuss (§ II) some general aspects of diffusion in A and B stars. In particular, it will be found that, in order not to sink, a heavy element must be pushed upward by a force at least as strong as gravity, that the diffused elements either float on the top of the atmosphere or lie at its bottom, and that the diffusion velocity is of the order of 10^{-3} cm sec^{-1}. The effect of magnetic fields will also be discussed.

The radiation force (§ III) transferred through bound-free and bound-bound transitions will be found to be frequently larger than the gravitational force. Autoionization levels may play an important role in transferring momentum from the radiation field to certain preferred elements.

The comparison (§ IV) of the calculated radiation force to the observed abundance anomalies will show that the radiation force on Mn, Sr, Y, Zr, and the rare earths should push them into the outer atmosphere in those stars in which they are overabundant. Oxygen should be underabundant in the cooler Ap stars, as is observed. The Si overabundance would be explained only if Si II has wide autoionization features. The P overabundance is expected in stars with $\theta_{\rm eff} \simeq 0.5$, but is observed in stars with $\theta_{\rm eff} \simeq 0.4$.

II. DIFFUSION PROCESSES

Diffusion processes in stars have been discussed by a number of authors. They studied the effects of gravitational settling on the abundances of the heavier elements in the atmosphere of the Sun (Aller and Chapman 1960) and of He in the old halo B stars (Greenstein, Truran, and Cameron 1967). Praderie (1968) discussed the possibility that the metallic-line stars could be *the normal* stars. Gravitational settling would have reduced the abundances of heavy elements in the other stars, including the Sun. Browne (1968) found that diffusion processes might explain the periodic variation of the over-

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abundances in magnetic stars. We will find that diffusion processes naturally lead to the abundance anomalies observed in the Ap stars and to their observed dependence on atmospheric parameters. But we must first discuss some general aspects of diffusion in the atmosphere of A and B stars and then (§ III) the effect of radiation pressure on diffusion.

First we will discuss the diffusion equation proposed by Aller and Chapman (1960). We shall then use it to calculate diffusion velocities and hence diffusion time scales in Ap stars. The time scale turns out to be of the order of 10^4 years—much smaller than the expected lifetimes of these stars. To oppose gravity, a force must be of the same order as the gravitational force. Diffusion due to a magnetic-field gradient will then be seen to be negligible (Babcock 1947, 1958*b*, 1963).

Starting from the diffusion equation for a binary mixture, Aller and Chapman (1960) argued for the following equation for a mixture of ionized hydrogen, electrons, and an ionized gas of charge Z and atomic mass A:

$$w_{12} = D_{12}\Big[-\frac{1}{c_2}\frac{\partial c_2}{\partial r} + \frac{1}{p}(2A - Z - 1)\frac{\partial p}{\partial r} + \frac{1}{T}[2.65Z^2 + 0.805(A - Z)]\frac{\partial T}{\partial r} + \frac{A(F_1 - F_2)}{kT(c_1 + c_2 A)}\Big], \tag{1}$$

where

c_1 = mass fraction of hydrogen,
Ac_2 = mass fraction of the gas (Z, A),
w_{12} = relative velocity of the gas (Z, A) with respect to hydrogen,
r = a measure of distance (where plane symmetry is assumed) in cm,
T = temperature in ° K,
p = pressure (cgs units),
F_1 = force on hydrogen (dynes),
F_2 = force, per unit atomic mass, on the gas (Z, A) (dynes),

$$\begin{aligned} D_{12} &= 1.947 \times 10^9 T^{5/2}[NZ^2 A_1(2)]^{-1}, \\ A_1(2) &= \log_e(1 + x_0^2), \\ x_0^2 &= 2.73 \times 10^8 T^3/(Z^2 N), \end{aligned} \tag{2}$$

N = number density of hydrogen (cm^{-3}).

To evaluate equation (1) for the stars of interest, we used the model atmospheres of Mihalas (1965). Before giving the numerical results, we will briefly discuss the relative importance of each term and obtain, as a by-product, the force needed to counter gravity and the distribution with height of diffused elements.

In the atmospheres of A and B stars the term containing $\partial T/\partial r$ is found numerically to be less than 10 percent of the term containing $\partial p/\partial r$; it may thus be neglected.

We assume that the gas is mainly made up of ionized hydrogen. Then:

$$\frac{1}{p}\frac{\partial p}{\partial r} \simeq \frac{1}{2}\frac{m_p g}{kT}$$

(note that in deriving eq. [1], it was assumed that H was completely ionized and that He was not present). For diffusion to occur toward the top of the atmosphere, one then requires (if $Ac_2 \ll c_1$, and $\partial c_2/\partial r = 0$)

$$Wm_p|F_1 - F_2| > \frac{2A - Z - 1}{2} m_p g = \left(A - \frac{Z}{2} - \frac{1}{2}\right) g m_p ; \tag{3}$$

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that is, if

$$F_1 \ll F_2 \quad \text{and} \quad (\tfrac{1}{2}Z + \tfrac{1}{2}) \ll A\,,$$

the upward force on (Z, A) has to be larger than the gravitational force on (Z, A) for diffusion to occur toward the top of the atmosphere.

We will now see that diffused elements will either be floating at the top of the stable zone or lying at its bottom. The terms

$$\left(\frac{1}{c_2}\frac{\partial c_2}{\partial r}\right)^{-1} \quad \text{and} \quad \left(\frac{1}{p}\frac{\partial p}{\partial r}\right)^{-1}$$

represent, respectively, the scale height of settling due to diffusion (R_D) and the scale height of pressure variations in the atmosphere. If diffusion is caused by gravity, it stops when

$$\frac{1}{c_2}\frac{\partial c_2}{\partial r} = \frac{2A - Z - 1}{p}\frac{\partial p}{\partial r} \simeq 3.5 \times 10^{-7}\ \text{cm}^{-1}\,,$$

$$R_D \simeq 3 \times 10^6\ \text{cm}\,.$$

A radiation force twice as large as the gravitational force would lead to the same R_D as the pressure-gradient term. For diffusion to cause observable effects, it has to take elements from the zone where lines are formed to a zone one cannot observe from the outside (or vice versa). This corresponds to a $\Delta\tau \gtrsim 1$. The corresponding ΔR is of the order of 10^8 cm in A stars. There are 30 diffusion scale heights over that distance. The diffused elements will be concentrated at the top or at the bottom of the atmosphere.

For diffusion to be of any importance, it is at least necessary that the diffusion time scale be shorter than the lifetime of the star. To determine the diffusion time scale, we have used equation (1) (with $\partial c_2/\partial r = 0.0$ and $F_1 = 0 = F_2$), along with Mihalas's (1965) models. The results are shown on Figure 1 for Mn III ions. One can use Figure 1 to calculate time scales for other elements and other ions by remembering that (eqs. [1] and [2])

$$t_0 \propto Z^2/A\,. \tag{4}$$

It is seen that the time scales are of the order of 10^4 years for the ions of interest to move from $\tau = 0.1$ to $\tau = 10.0$ under the influence of the gravitational force. If the radiation force on an element is twice the gravitational force, that element will diffuse toward the top of the atmosphere on the gravitational-settling time scale. These time scales are much shorter than the expected main-sequence lifetimes ($\simeq 10^8$ years) of A or B stars (Iben 1965 and 1966).

To determine how stable the atmosphere has to be for diffusion to be important, we have plotted the diffusion velocities as a function of τ for Mn III ions (Fig. 2).

For diffusion to be important, convection and rotational circulation currents would have to be slower than 10^{-3} cm sec^{-1} or so in the atmosphere of the star.

We have also calculated the diffusion speed for neutral O. The diffusion velocity in a gravitational field is approximately (Loeb 1934)

$$w \simeq \tfrac{1}{2}gt_A = 2 \times 10^{-6}\, g\,\frac{(AT)^{1/2}}{p\sigma}\,, \tag{5}$$

where t_A is the average time between two collisions (see Fig. 2). Since the diffusion velocity of singly ionized O is about the same as the diffusion velocity of doubly ionized Mn, the ionization of O is seen, from Figure 2, to reduce its diffusion velocity by a factor of about 10.

The atmospheres of early A and B stars might be very stable. In the Sun, convection

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leads to granules whose speed is observed to be 10^5 cm sec^{-1}. However, the Sun is known to have an important convection zone close to the surface, whereas late B and early A stars have only weak convection zones (Mihalas 1965). As mentioned in § I, we have not found any reliable estimates of turbulent velocities in Ap stars; however, the abundance anomalies appear in those stars in which convection becomes relatively unimportant and in which diffusion processes are therefore most likely to be important.

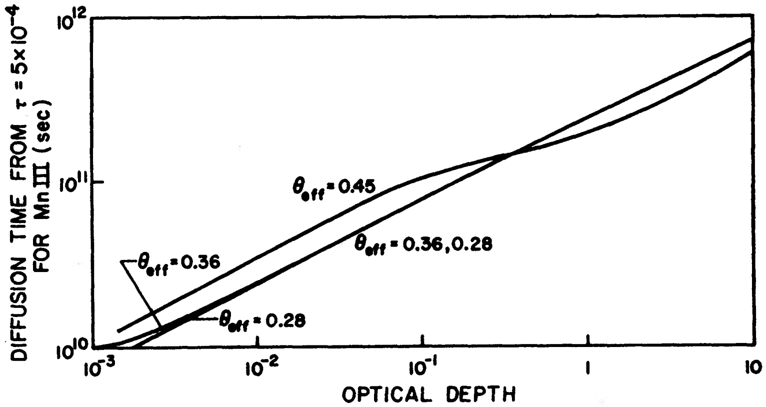

Fig. 1.—The time needed for Mn III ions to diffuse from $\tau = 10.0$ to $\tau = 0.2$ is seen to be of the order of 10^4 years, much shorter than the expected main-sequence lifetimes of A or B stars. If the atmosphere is stable enough, diffusion processes should then be important.

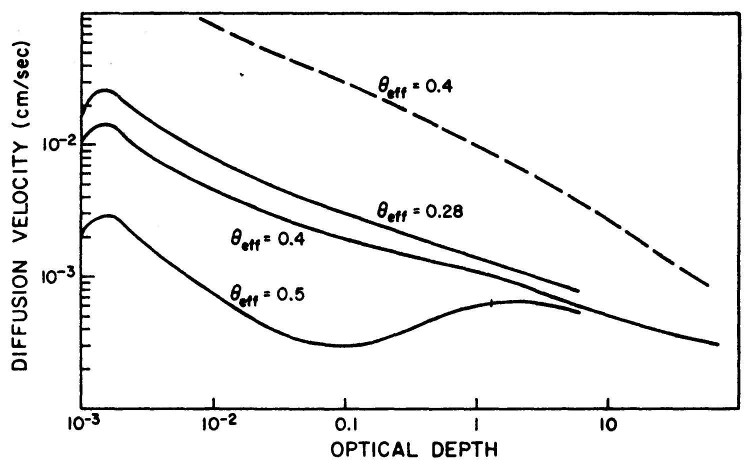

Fig. 2.—The diffusion velocity shows that there must be no currents faster than 10^{-3} cm sec^{-1} for the diffusion of ions (*solid lines*) to be important in the atmosphere of A or B stars. Neutral elements would diffuse 10 times faster (*dashed line*). Diffusion could not be important when there is convection.

Peculiar A stars have recently been shown to be slow rotators (Sargent and Searle 1967*a;* Abt 1967; Deutsch 1967). Their rotational velocity is about a quarter that of normal main-sequence stars. We know of no reliable estimates of the velocities of rotational circulation. Assuming a star rotates uniformly, Sweet (1950) has calculated the velocity of the rotational currents. We have used the 3 $M_\odot$ model of Iben (1965) to obtain ρ_c and T_c needed to calculate $v_{\rm rotation} \lesssim 10^{-3}$ cm sec^{-1} from Sweet's calculation, if the Ap stars have equatorial rotational velocities of 90 km sec^{-1} or less (Wolff 1968). The normal A stars, which rotate faster, would then have currents too rapid to allow diffusion. Mestel (1966) has shown that Sweet's calculations should not be applied that far out in the atmosphere, but Mestel neglects the influence of magnetic fields.

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a) *Magnetic Fields*

Magnetic fields may be important in increasing stability both against convection and against rotational currents. One can expect stability against convection if $\rho g|\nabla T|/T < \pi H^2/\lambda^2$ (Cowling 1953), where λ is the size of the instabilities. Here

$$\nabla T/T \simeq 10^{-10}\ \mathrm{cm}^{-1}\,, \qquad H \simeq 10^4\ \mathrm{gauss}\,,$$

so that

$$\lambda \leq 10^{11}\ \mathrm{cm}\,.$$

Since the radius is approximately 10^{11} cm (Iben 1965), one expects that the magnetic field will increase the stability against convection. Similarly, for stability against rotation one should replace $\rho g|\nabla T|/T$ by $\rho\Omega^2$. For rotational periods of $0^{\mathrm{d}}.5$, one obtains $\lambda \lesssim 10^{12}$ cm (if $\rho \simeq 10^{-9}$ g cm^{-3}, $H \simeq 10^4$ gauss).

Even though it cannot be shown that Ap stars have stable enough atmospheres for diffusion processes to take place, they are expected to be among the stars having the least convection and the smallest meridional circulation due to rotation. Furthermore, the magnetic fields observed in their atmospheres (Babcock 1958*a*) may very well reduce convection and rotational currents. It seems reasonable, then, to make the ***working hypothesis*** that the atmospheres of Ap stars are stable and to study what diffusion processes are likely to be important.

We discussed above the possible importance of magnetic fields in increasing the stability of the atmospheres of Ap stars. Magnetic fields might also be important in slowing the diffusion of ions, and their gradients may cause diffusion (Babcock 1947, 1958*b*, 1963).

The effect of a magnetic field on diffusion velocities may be approximated (Spitzer 1962; Chapman and Cowling 1953) by multiplying the component perpendicular to the magnetic field by $(1 + \omega_c^2 t_c^2)^{-1}$, where

$$\omega_c t_c = 3 \times 10^{-12} \frac{BT^{5/2}}{Z p_G}\,. \tag{6}$$

The magnetic field will start having appreciable effect in slowing down diffusion when $\omega_c t_c \sim 1$. If $\tau = 0.2$ in the atmosphere of a star with $\log g = 4.0$ and $\theta_{\mathrm{eff}} = 0.4$, one obtains for O II

$$B \simeq 4 \times 10^3\ \mathrm{gauss}\,.$$

Effective magnetic fields of 1000–6000 gauss are frequently observed in Ap stars (Babcock 1958*a*). They correspond to uniform dipolar fields H_p ($H_p = H_e/[0.303 \cos i]$) of 5000–30000 gauss in the star. Magnetic fields could be expected to slow down diffusion perpendicular to the $\boldsymbol{H}$ field by a factor from 2 to 50.

Babcock (1947, 1958*b*, 1963) has discussed the possible importance of magnetic-field gradients in causing diffusion in Ap stars. If H is assumed to be about 10^5 gauss and to vary over a distance of 10^7 cm, then $|\nabla \cdot \boldsymbol{H}| \simeq 10^{-2}$ gauss cm^{-1}, and the force on the ions is

$$\mu \nabla \cdot \boldsymbol{H} \simeq 5 \times 10^{-22}\mathrm{g\text{-}cm\ sec^{-2}} \tag{7}$$

(where μ is taken to be 5 μ_{B}, where μ_{B} is the Bohr magneton). This is 3 orders of magnitude smaller than the gravitational force and so could not cause diffusion upward.

III. RADIATION PRESSURE

We will now discuss the importance of the radiation field in causing diffusion. We first show that in spite of its discrete nature, the radiation force can be treated in the

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same way as the gravitational force. We then calculate the momentum transferred to the ions of interest by bound-free transitions (continuum) and bound-bound ones (lines). The force of radiation pressure is frequently larger than the force of gravity. It can then push elements outward and cause overabundances in the outer part of the atmosphere.

One might argue that the momentum received by an ion from the radiation field is rapidly transferred through collisions to all species present and therefore cannot cause diffusion. Under the hypothesis that the time scale for this transfer is approximately the same as the time for an ion to be scattered through 90°, t_c, it will be seen that a given radiation force causes diffusion velocities equal to those produced by the same gravitational force.

For simplicity, suppose that all photons are streaming in the same direction. (In general, one should consider only the anisotropic part of the radiation field, i.e., the radiation flux). Let t_p be the average time between the absorption of two photons. The momentum transferred from the radiation field per second (the force) is then $h\nu/(t_p c)$. The distance traveled per second $h\nu t_c/(2t_p cM)$ (where the factor $\frac{1}{2}t_c$ is introduced, since the push is remembered for a time t_c, but over the time $\frac{1}{2}t_c$ the strength of the push is reduced by a factor 0.5). Here, M is the mass of the ion. Similarly, the distance traveled per second under the influence of gravity is approximately 0.5 gt_c. When the two expressions are compared, it is easily seen that a radiation force as large as a gravitational force causes diffusion through the same distance as the gravitational force. The direction in which the ions will diffuse will be determined by the relative size of the gravitational and radiation forces, independently of t_c. In the diffusion equation, one can introduce the radiation force in the same way as a continuous force, in spite of the discrete nature of the photons. The collision frequency will affect only the diffusion velocity.

We now calculate radiation forces from bound-free and bound-bound transitions in the usual way and expect that, when the radiation force is larger than the gravitational force, the ions will diffuse toward the top of the atmosphere. One then observes overabundances.

a) Continuous Absorption

The force exerted on an atom by a radiation flux ϕ_ν (physical flux) in a wavelength interval $d\lambda$, through absorption of photons by the level n, when the fraction N_n/N_{tot} of the given element are in the level n, is given by (Pecker and Schatzman 1959)

$$F_\lambda d\lambda = 10^8 \frac{N_n}{N_{\text{tot}}} \sigma_n \phi_\nu \frac{d\lambda}{\lambda^2} \text{ (cgs)} , \tag{8}$$

where σ_n is the cross-section for absorption of photons by level n, in the wavelength interval $d\lambda$ (λ in Å).

We estimated the contribution of the continuum by calculating the cross-section for the bound-free transition in the classical hydrogenic approximation

$$\sigma_n^{(c)} = 7.9 \frac{n}{Z_n^2} \frac{1}{Y^3} \times 10^{-18} \text{ cm}^2 , \tag{9}$$

where Z_n is the charge of the ion after ionization, n is the principal quantum number of the level, and $Y = h\nu/\text{I.E.}$ (I.E. = ionization energy of the level n).

For photoionization, equation (8) has to be slightly modified since not all the momentum of the ionizing photon is transferred to the ion. Some of it is given to the electron. Far away from the atom, the distribution of electrons after photoionization is given by (Sommerfeld 1939)

$$J \propto \sin^2 \theta \cos^2 \phi \, (1 + 4\beta \cos \theta) , \tag{10}$$

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where θ is the angle between the direction of the light quantum and that of the electron the photon, and the ion, and the plane containing the photon and its polarization vector ($\beta \equiv v/c$). The fraction F of the momentum given to the electron is then

$$F = \frac{\int mv \cos\theta \, J \, d\Omega}{\int (h\nu/c) J \, d\Omega} = \tfrac{8}{5} K \,, \tag{11}$$

where

$$K = \frac{\frac{1}{2}mv^2}{h\nu} = \frac{h\nu - \text{I.E.}}{h\nu} \,. \tag{12}$$

As $K \rightarrow 1$, the electrons go away with more momentum than the photons brought in; the ions are then pushed back by the photoionization. However, in the cases of interest to us $K \lesssim 15$ percent, so that most of the momentum is transferred to the ion and not to the ejected electron. In our calculations we subtracted the momentum transferred to the electron by multiplying the right-hand side of equation (8) by $(1 - 1.6\,K)$.

Equation (10) was obtained by Sommerfeld for the ionization of the ground state of the hydrogen atom. To our knowledge, J has not been calculated for any case except hydrogen when K is appreciably smaller than 1. When K is close to 1, the effect becomes measurable. For krypton the anisotropy has been measured and, on the average, less momentum would be transferred to the electron than in the case of hydrogen (Krause 1969). It seems impossible, however, to observe the $(4\beta \cos\theta)$-term at low energy. It would then be useful to have J determined theoretically for cases other than hydrogen.

We have calculated for a number of model atmospheres the force transferred through photoionization from the radiation field to different elements of interest. The results for the force at $\tau = 0.1$ are shown in Figures 3 and 8 as a function of θ_{eff}, $\log g = 4$. The special cases of O and Si will be discussed below in a section on autoionization.

To calculate the forces, we used Mihalas's (1965) models. The main feature in his calculated outgoing flux, insofar as our calculations are concerned, is the Lyman continuum. Below the Lyman continuum, hardly any flux is coming out. *Photoionization will only occur, then, for those levels in which the ionization potential is less than 13.6 eV.*

The elements with ionization potentials larger than 10.5 eV but smaller than 13.6 eV will be pushed out by large radiation forces in the cooler peculiar stars (C, P, Cl, Ca, Sc, As, Br, Sr, Y, Zr, Xe, Rn, and the rare earths fall in this category). The elements with ionization potentials smaller than 10 eV or greater than 18 eV will not absorb enough radiation to counter gravity in the Ap stars (He, Li, Be, B, Ne, Na, Al, K, Ni, Cu, Ga, Se, Rb, In, Sb, Te, and Cs fall in this category). Those with an ionization potential between 13.6 and 18 eV may or may not be pushed outward by the radiation field in the warmer Ap stars. Whether or not they are pushed out depends on the details of the atomic structure. Compare Mg, Mn, Fe, and Cr.

In Figure 4 is plotted the variation with τ of the force transferred through photoionization on Si, Mn, P, and Cl for the model atmopshere $\theta_{\text{eff}} = 0.4$ and $\log g = 4$. The behavior with τ depends on the excitation energy of the levels from which photoionization occurs. The special case of Si will be discussed at greater length below. The radiation force on Si does not decrease as rapidly when τ increases as that on Cl or P.

One may wonder why hydrogen, the lightest element, should not be thrown out. The answer is simple. There is either too much H or not enough light. The radiation pressure in these stars is only a few percent of the total pressure (Mihalas 1965). The flux at the proper wavelength to push hydrogen up (below the Lyman continuum) is very small. Whereas the radiation force transferred through photoionization on Mn is 4 times the gravitational force, that on H is only 1 percent of the gravitational force.

b) Line Absorption

The possible influence of lines in pushing elements outward can first be seen by comparing with the gravitational force the momentum per second which a line would

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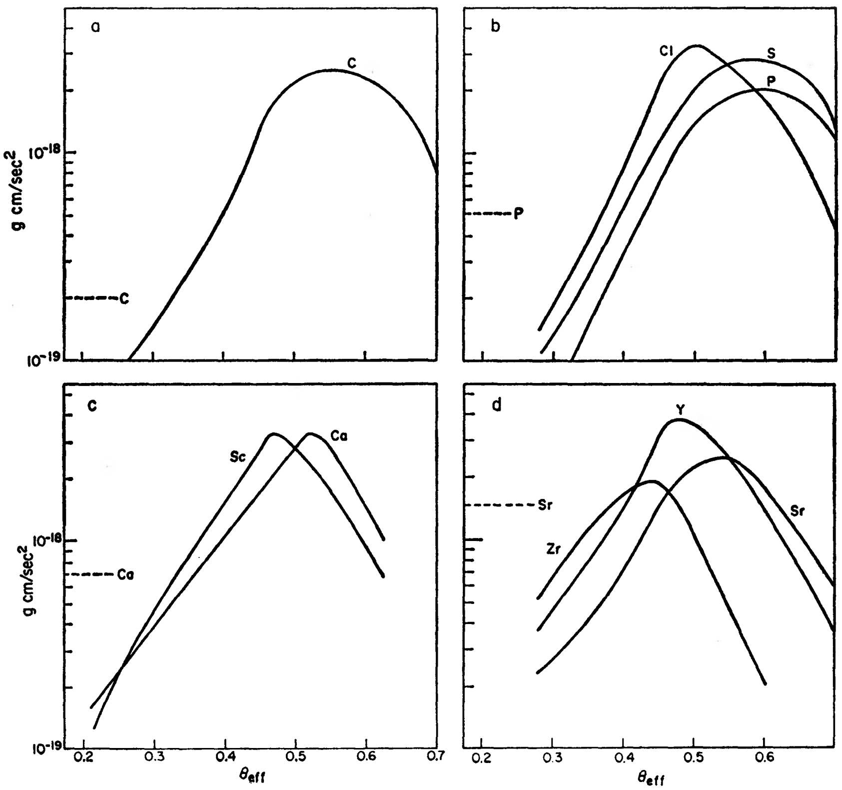

FIG. 3.—Radiation force transferred through photoionization to a number of elements. All calculations were done at $\tau = 0.1$ in atmospheres with log $g = 4.0$. *Dashed lines*, gravitational forces. Note that because of the relatively large abundance of C in normal stars, the radiation field could support overabundances of C by a factor of at most 5.

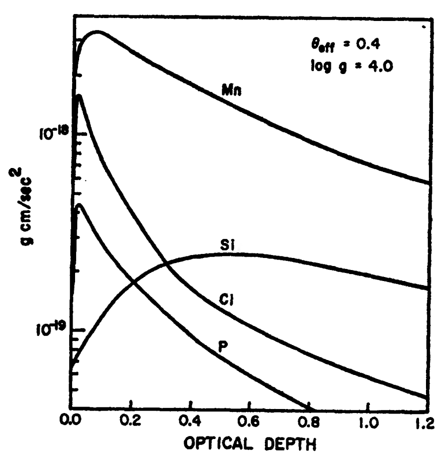

FIG. 4.—Variation with τ of the force transferred through photoionization to certain elements in an atmosphere with $\theta_{\rm eff} = 0.4$, log $g = 4.0$. Notice that the different behavior with τ of the force from element to element may have important effects in causing overabundances.

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transfer from the radiation field to the element of interest. For one line, $\int \sigma_n d\nu = (\pi e^2/mc)f$. Equation (12) then becomes (with ϕ_ν kept constant over the width of the line; N_{tot} is the number density of the element of interest)

$$F_{\text{rad}} = \frac{\pi e^2}{mc^2}\frac{N_n}{N_{\text{tot}}}\phi_\nu f \simeq 3 \times 10^{-16}\frac{N_n}{N_{\text{tot}}} f \text{ (g-cm sec}^{-2}) \tag{13}$$

if one assumes $\phi_\nu = 3 \times 10^{-3}$ (cgs) (as is typical in the atmospheres of A and B stars, Mihalas 1965). This force compares with 1.7×10^{-20} A (g-cm sec^{-2}), the gravitational force on an element with atomic mass A, if $\log g = 4$. Saturation could reduce the flux by a factor of $2 \times 10^4/A$, and the radiation force from one line could still be as strong as the gravitational force. If the flux in the line can be approximated by

$$\phi_{\text{line}} = \phi_{\nu_{\text{cont}}}(\kappa_\nu/\kappa_{\nu_{nm}}) , \tag{14}$$

saturation can be estimated by comparing $\kappa_{\nu_{nm}}$ in the line with κ_ν in the continuum. In the line

$$\kappa_{\nu_{nm}} = \frac{\pi e^2 \lambda}{m_e c^2 n_{\text{H}}}\left(\frac{f_{nm} N_n \lambda}{N \Delta\lambda}\right) = 10^7 \left(\frac{f_{nm} N_n}{N}\frac{\lambda}{\Delta\lambda}\right). \tag{15}$$

We require, in order to take saturation into account,

$$F_{\text{rad}}\frac{\kappa_\nu}{\kappa_{\nu_{nm}}} = F'_{\text{rad}} = F_{\text{grav}} . \tag{16}$$

This leads to

$$\text{mass fraction that one line can support} = 0.1\ \phi_\nu \kappa_\nu/\lambda \tag{17}$$

if $\Delta\lambda/\lambda = (1/c)(2kT/M)^{1/2} = 10^{-5}$, as would be the case for an ion with $A = 100$ at a temperature of 10^4 ° K. We calculated, for different values of λ, $0.1\ \phi_\nu\kappa_\nu/\lambda$ at $\tau = 0.2$, in a model atmosphere with $\log g = 4$ and $\theta_{\text{eff}} = 0.4$. The results are shown in Figure 5, *a*. Depending on where it is in the spectrum, a line of thermal Doppler width can support a mass fraction anywhere from 10^{-7} to 10^{-6}.

The line width could be defined as the width over which the product $\phi_{\nu_{nm}}\kappa_{\nu_{nm}}$ is independent of $\kappa_{\nu_{nm}}$. If $\kappa_{\nu_{nm}}/\kappa_\nu \simeq 200$ in the center of a line, and if $\kappa_{\nu_{nm}} \propto \exp\{-[(\lambda - \lambda_0)/\Delta\lambda_0]^2\}$, where $\Delta\lambda_0$ is the thermal Doppler width, then $\exp\{-[(\lambda - \lambda_0)/\Delta\lambda_0]^2\} \simeq 0.005$ for $|\lambda - \lambda_0|/\Delta\lambda_0 \simeq 2.5$. The total width of the line is then approximately $5\Delta\lambda_0$. If a given element has a resonant line at 4000 Å, that line alone could support a mass fraction of 10^{-6}. We now compare the above results for the mass fraction that a line can support, with the results one obtains by supposing LTE in the formation of the line (Milne 1927). We then calculate in a few model atmospheres how the mass fraction supported by one line varies with τ, λ. Comparing the calculated with the observed mass fractions, we determine which element can be supported by the radiation pressure transferred through lines to an element.

According to Milne (1927), when $\exp(-\tau_{\nu_{nm}})$ is small, one line of width $\Delta\nu$ would transfer $4\pi\Delta\nu(dB_\nu/d\tau)\Delta\tau/(3c)$ from the radiation pressure to the pressure supporting one element in a shell of thickness $\Delta\tau$. The force one line exerts on one atom is then

$$F_{\text{line}} = \frac{4\pi}{3c}\Delta\nu\frac{dB_\nu}{dT}\frac{A m_p \bar{\kappa}}{X(A)}\frac{dT}{d\tau} . \tag{18}$$

Equating F_{line} to the gravitational force, one obtains the mass fraction $X(A)$ of a given element one line can support:

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$$X(A) = 4 \times 10^4 \frac{\Delta\lambda}{\lambda^2} \frac{h\nu}{kT} \frac{\exp(h\nu/kT)}{1 - \exp(h\nu/kT)} B_\nu(T)\bar{\kappa} \frac{dT/T}{d\tau} \tag{19}$$

(cgs units except for λ in Å). Using Mihalas's (1965) models, we calculated for a number of cases the mass fraction a line of thermal Doppler width can support (Fig. 5, *b*, *c*, *d*). The Doppler width is the appropriate one for an element with $A \simeq 30$. Since $\Delta\lambda_0 \propto A^{-1/2}$, the plotted results can be extended to an element with any A.

Listed in Table 1 are the number of "average" lines of thermal Doppler width needed to support normal abundances of a few elements of interest. Each line is assumed to

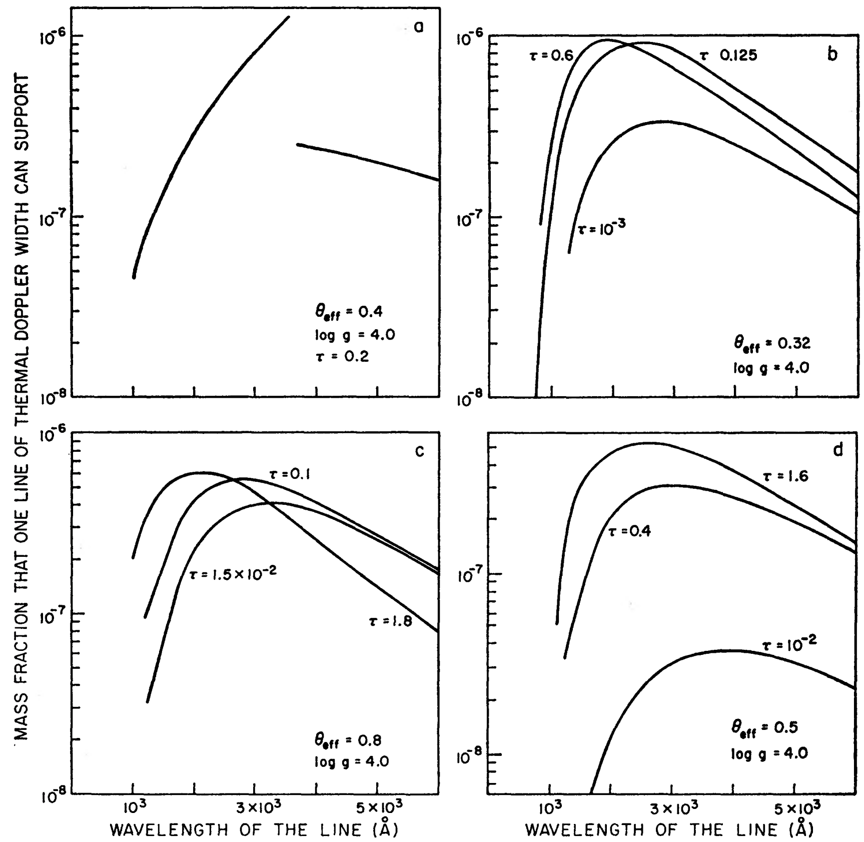

FIG. 5.—On the average, one line of thermal Doppler width can support a mass fraction of 3×10^{-3}. (*a*) Calculated from eq. (17) for an atmosphere with $\theta_{eff} = 0.4$, $\log g = 4$. (*b*), (*c*), and (*d*) Calculated with Milne's approximation for a number of cases. Note that on the average the force transferred through lines does not decrease as τ increases, contrary to that transferred through photoionization.

support a mass fraction of 3×10^{-7}. Note from Table 1 that overabundances of elements heavier than the iron peak can be supported by line absorption. However, large overabundances of the iron-peak elements or of the lighter metals (except Li, Be, and B) have to be supported by continuum absorption.

c) Autoionization Levels

The force transferred from the radiation field to elements might be increased for two elements of interest, O and Si, by an unusual type of levels, the autoionization levels. Those of interest here are levels in which two electrons are excited. The total excitation energy of the two electrons is greater than the energy needed to ionize the ground state.

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At the same energy, then, there exists a continuum of states. There is a finite probability that, after the two electrons have been excited, the system will ionize without collision or absorption of a photon; hence the name given to these states.

If the parity and the quantum numbers J, L, and S of the excited states are the same as those of the continuum for one value of the emitted electron, the probability for autoionization is large. The levels are then much wider than the usual excited levels. For instance, autoionizing levels as wide as 0.07 eV have been observed in Ca (Newson 1966; Ditchburn and Hudson 1960), and as wide as 0.3 eV in Cd and Hg (Berkowitz and Lifshitz 1968).

TABLE 1

EFFECT OF LINES

Element	Number of Lines Required*	Abundance
O	5×10^4	2.5×10^7
Ne	1.5×10^3	8×10^5
Mg	2×10^3	9.1×10^5
Si	3×10^3	10^6
Cl	10	2.6×10^3
Ca	250	5×10^4
Cr	40	7.8×10^3
Mn	35	6.9×10^3
Fe	500	8.5×10^4
Ga	0.1	9
Sr	0.5	61
Y	0.1	8.9
Zr	0.1	14.2
Ba	0.05	2.6
La	0.01	0.5
Eu	0.002	0.115

* Number of lines required to counter gravity for the observed normal abundances of some elements of interest. Each line was taken as supporting a mass fraction of 3×10^{-7}. The abundances were taken from Bashkin (1963). The number density of silicon is normalized to 10^6. The number density of hydrogen is then 3.0×10^{10}.

Calculation of what mass fraction a line related to such a level can push upward shows how important it might be. In an atmosphere with $\theta_{\text{eff}} = 0.32$ and $\log g = 4$, a line related to a level 0.3 eV wide at 1000 Å can support a mass fraction

$$X = \phi_\nu \Delta\nu/(c p_G) \simeq 0.6 \text{ percent} ,$$

where p_G and ϕ_ν were obtained from Mihalas's (1965) models.

One single line ending at such a level could support nearly 1 percent of the mass (an overabundance of Si by a factor of 10) of the elements above $\tau = 0.1$.

Most elements are not strongly affected by autoionization levels. When an element has an ionization potential smaller than 13.6 eV, it can be ionized from the ground state by continuum radiation above the Lyman continuum. The integrated value of the photoionization cross-section should not then be changed by an order of magnitude by the autoionization levels, and the same holds true for elements having numerous excited levels contributing to the photoionization opacity. The autoionization levels cannot contribute substantially here to the transfer of momentum to ions like Mg II which have only one electron outside a closed shell; the excitation of two electrons, then, always involves more energy than is available above the Lyman continuum.

Both O and Si are special cases. There is only one level of O which is properly placed

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to transfer an appreciable amount of momentum from the continuum: the $2p^4\ ^1D$ level, which is 2.0 eV above the ground state (ionization from the ground state would require radiation from below the Lyman continuum). However, it is a ^1D term, and, in Russell-Saunders coupling, the $\Delta S = 0$ selection rule does not allow it to be photoionized to the ground state of O II, a ^4S term. Furthermore, in such a light element as O, Russell-Saunders coupling holds, as exemplified by the small difference in energy between terms of different J in a multiplet. Transitions to autoionization levels might then be the main source of absorption. However, in the case of O, either transitions from the ^1D level have to go to ^3P or ^3F levels and thus are expected to be weak transitions (forbidden by the $\Delta S = 0$ selection rule) or they go to ^1P and ^1F levels and are not expected to be broadened by autoionization because the available continuum of states has $S = 3$. The experimental results of Rudd and Smith (1968) agree with the preceding discussion: they have found no evidence of the autoionization of ^1D or of ^1S levels to the continuum involving the ground state of O II (a ^4S level). The force transferred from the radiation field to O is then calculated to be smaller than 10 percent of the gravitational force on O. It could not impede gravitational settling.

The case of silicon is different, mainly because of the breakdown of Russell-Saunders coupling. Singly ionized silicon has two excited levels that are properly placed: a $3p^2\ ^4P$ level and a $3p^2\ ^2D$ level. The $3p^2\ ^2D$ level is perturbed by a $3d\ ^2D$ level. Froese and Underhill (1966) calculated that the wave function for the level of interest was actually

$$0.84\ 3s3p^2\ ^2D + 0.54\ 3s^23d\ ^2D\,.$$

The photoionization of the $3s3p^2\ ^2D$ configuration to the ground state of Si II is prohibited, but that of the $3s^23d\ ^2D$ configuration is not. Furthermore, from the latter level there are permitted transitions to levels which are known to be strongly autoionizing (Shenstone 1961), although their width is unknown.

The photoionization of the $3p^2\ ^4P$ level to the ground state of Si III is forbidden. It is a two-electron jump. However, this level has permitted transitions to quadruplet levels which, in spite of the $\Delta S = 0$ selection rule, show autoionization effects to the continuum involving the ground state of Si III (Shenstone 1961). These autoionization levels would not be expected to be very wide, but they are further examples of the breakdown of Russell-Saunders coupling. Transitions to the doublet terms may then not be strongly forbidden.

The available information on Si II does not allow us, then, to say whether or not autoionization will be important in transferring momentum from the radiation field to Si. However, it is quite possibly important. An upper limit on the transferred momentum is obtained by assuming that an autoionization level is 0.5 eV wide and is connected to the levels of interest by lines with $f \simeq 1$. The radiation force on Si is then found to be 20 times the gravitational force in an atmosphere with $\theta_{\rm eff} = 0.32$, $\log g \simeq 4$, and with saturation taken into consideration, for an overabundance of Si by a factor of 20.

Whether radiation pressure is transferred to Si through photoexcitation to autoionization levels or through photoionization, the lower levels are always the same and the radiation comes from the same part of the spectrum. Both processes then give the same dependence of the radiation force on the effective temperature of the star. Plotted in Figure 8 are the results for photoionization in the hydrogenic approximation. This could be an overestimate if Russell-Saunders coupling is a good approximation. It would be an underestimate if the autoionization levels which are known to exist were wide enough, $\Gamma \gtrsim 0.05$ eV, and if they were connected to the levels of interest by strong lines.

IV. COMPARISON WITH OBSERVATIONS

We will now discuss the abundance anomalies that can be expected from diffusion processes in Ap stars, and we will compare these with the observed abundance anomalies.

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Given the diffusion hypothesis, most well-established abundance anomalies would be expected. The most important difficulty comes from the Si overabundance. Its explanation requires the autoionization levels known to exist at the proper energy in Si II to be unusually wide features.

If the atmosphere of the star is stable, abundance anomalies should manifest themselves on a time scale of 10^4 years. Those elements not contributing enough to the opacity in the outer atmosphere of the star will settle down. For large underabundances to be observed, elements have to settle over the whole surface of the star. For instance, for an underabundance by a factor of 100 to be observed, the elements must have settled down over 99 percent of the surface. Ionized elements will settle down 10 times more slowly than the un-ionized ones. Furthermore, when they have to cross magnetic-field lines they will be slowed down by another factor of 10. An element is then expected to be most underabundant when it is not ionized.

Diffusion upward will also be slower for ionized elements than for un-ionized ones. However, the magnetic field is now expected to channel elements into patches and not to slow appreciably the flow of elements upward when the magnetic field is parallel to the gravitational field. In those stars with magnetic fields, overabundances of ionized elements should be expected to occur in patches and to vary with a period related to that of the magnetic field.

The transfer of momentum through bound-bound transitions (lines) is expected to be efficient in pushing outward the less abundant elements. Each line of thermal Doppler width can, on the average, support a mass fraction of 3×10^{-7}. Since the mass fraction of a typical normally abundant rare earth is only 10^{-8} to 10^{-9}, lines could easily cause overabundances of rare earths. As seen from Figure 5, the force exerted by lines is not dependent on τ as long as the element remains in an ionization state where there are lines, i.e., does not take the configuration of a rare gas. Lines cannot only maintain overabundances in the upper part of the atmosphere, they can also push elements from below.

Lines cannot support overabundances of the more abundant elements, like Si, in the upper part of the atmosphere. Overabundances of the light elements are possible only when the radiation force transferred through photoionization or through autoionization levels is larger than the gravitational force. However, lines can still push elements from below to fill the upper part of the atmosphere, where the continuum takes over. Note that an overabundance by a factor of 10 requires only that the elements from above $\tau = 1$ be pushed up.

The diffusion hypothesis predicts that the abundant, light, un-ionized elements, which have no properly placed continuum and which do not have enough lines to be kept up by lines, should be most underabundant. This is confirmed by the results of Sargent and Searle (1962). When O and Mg are both singly ionized, one expects both to have settled down by the same amount. On the average, this happens around $\theta_{\mathrm{eff}} = 0.4$ (see Fig. 6). When O is not ionized, one expects O to be much more underabundant than Mg. This is confirmed at $\theta_{\mathrm{eff}} \gtrsim 0.45$. When Mg is doubly ionized and O is singly ionized, one would expect O to be slightly more underabundant than Mg. This would occur for $\theta_{\mathrm{eff}} \lesssim 0.36$. The data are not good enough yet to confirm this trend; in any case, the difference should be relatively small. Whereas O not being ionized increases its diffusion velocity by a factor of about 100, Mg being doubly ionized changes the diffusion velocity by a factor of only 4.

Similarly, Ne and He would be expected to be most underabundant when they are not ionized. The data are not as conclusive for Ne as for O (see Fig. 7). Note that Ne can be observed only when it is starting to be ionized. We have compared Ne with C because, for a given abundance, the observed lines of Ne and C behave similarly with θ_{eff} (Sargent *et al.* 1969). Helium seems to be underabundant in the stars in which it is not ionized in the upper atmosphere as would be expected.

Silicon will be pushed up only if there exist wide autoionization states in Si II, as

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mentioned above. If they do exist, the radiation force on Si will be largest in the temperature range in which the Si stars are observed.

The elements P, S, Cl, Ca, and Sc can be expected to be overabundant in the cooler Ap stars, but neither Ar nor K should be. Sargent and Searle have made no systematic studies on these elements. The observational data are not conclusive except for P. Compare the different values obtained for the abundance of Ca in 53 Tau. The overabundances of P observed in 3 Cen A and a few Mn stars, for instance κ Cnc, are not

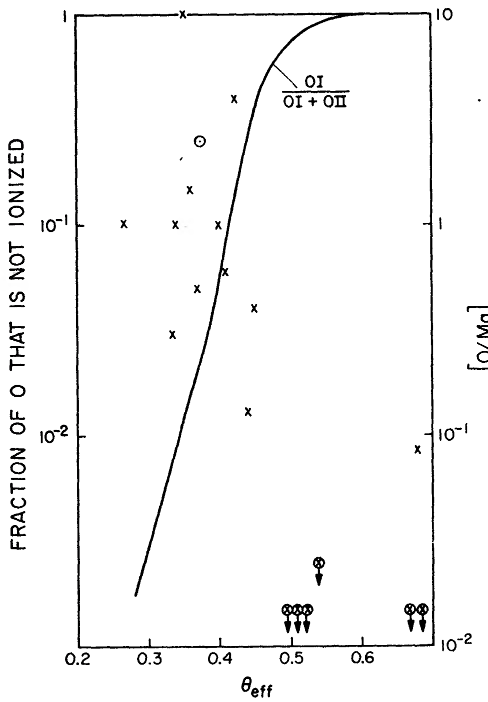

FIG. 6

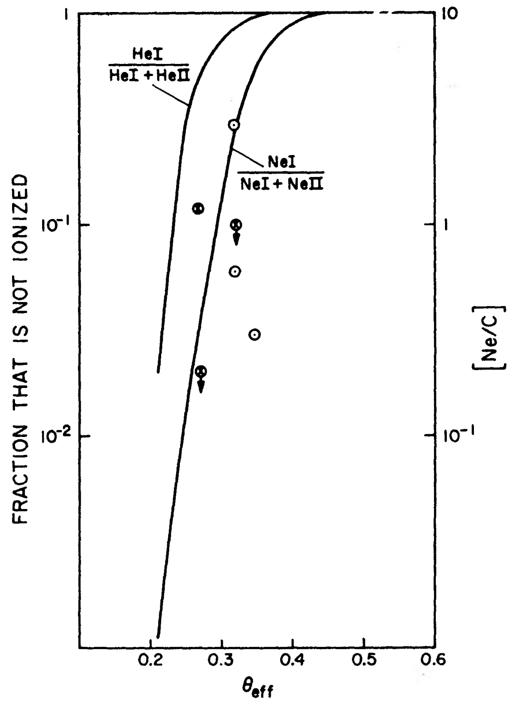

FIG. 7

FIG. 6.—*Solid line,* fraction of O that is not ionized at $\tau = 0.3$ in model atmospheres with $\log g = 4.0$ and different θ_{eff}. Points show the abundance of O in a number of stars. *Open circle,* a helium-weak star; *crosses,* Ap stars (from Sargent and Searle 1962). The greater underabundances of O occur only in stars where O is not ionized in the outer atmosphere, as would be expected from the diffusion hypothesis.

FIG. 7.—*Solid lines,* fractions of He and Ne that are not ionized at $\tau = 0.3$ in model atmospheres with $\log g = 4.0$ and different θ_{eff}. Points show abundance of Ne in a number of stars (from Sargent *et al.* 1969). *Open circles,* He-weak stars; *crosses,* Ap stars. Data are not as conclusive as for O, since Ne has been observed only in stars where it is getting ionized in the atmosphere.

explained here. In κ Cnc the ratio of O to Mg does not seem to follow the same trend as in the other Ap stars.

In the iron peak, Figure 8 shows that the diffusion hypothesis explains naturally the overabundance of Mn. The occurrence of Mn stars in the (θ_{eff}, $\log g$)-plane is also explained naturally. There is excellent agreement between the observations and the overabundances predicted by the diffusion hypothesis. Line absorption could explain slight overabundances of the other elements of the iron peak. Titanium and Fe may also be affected by continuum radiation, but not as much as Mn. One would not expect from diffusion large overabundances of Cr. In fact Searle, Lungershausen, and Sargent (1966) conclude that Cr is not as overabundant as is generally believed.

Lines can be expected to cause overabundances of most elements heavier than the iron peak. They are so underabundant that they are generally observed to have more

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lines than are necessary to push them upward. They are generally observed to be overabundant. When Figure 4, *d*, is compared with the résumé of Hack (1968), it seems that Sr, Y, and Zr are most overabundant in those stars where the radiation pressure transferred to them through the continuum is largest.

The rare earths are observed to be overabundant, as expected. Barium is not expected to be as overabundant as the rare earths, since in the atmosphere of the Ap stars it is mainly in the form of Ba III which has the same atomic configuration as the preceding noble gas. It must have very few lines, if any.

However, Sr will also take the configuration of the preceding noble gas in the atmospheres of the Ap stars, and it is observed to be overabundant. It should first be mentioned that the maximum of the radiation-pressure force on Sr is in stars with $\theta_{\mathrm{eff}} \simeq 0.55$. The maximum on Ba would occur in stars with $\theta_{\mathrm{eff}} \simeq 0.61$, because the ionization potential of Ba II is smaller than that of Sr II. At that temperature, the convection zone

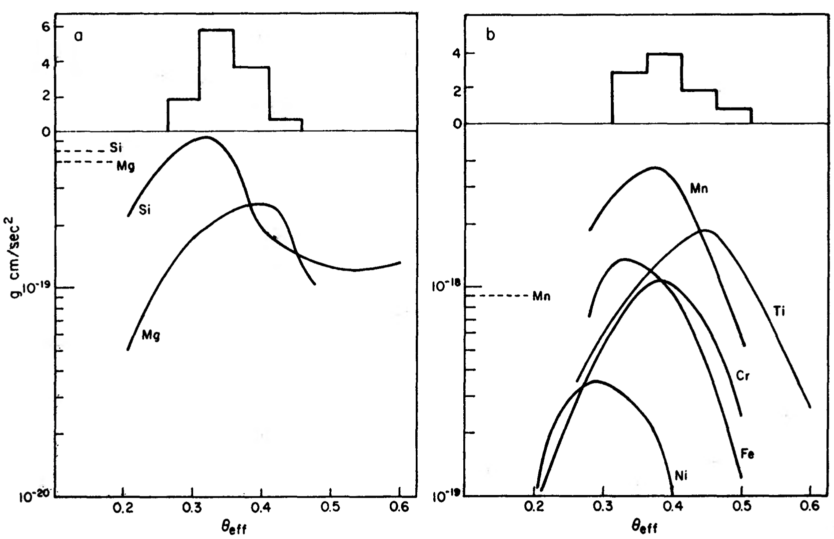

FIG. 8.—Radiation force transferred through photoionization to Mg, Si, and iron-peak elements at $\tau = 0.1$ in atmospheres with $\log g = 4.0$. Histograms show number of Si stars (*a*) and Mn stars (*b*) as a function of θ_{eff} (Sargent and Searle 1967*a*). Radiation force from the continuum appears related to the Mn overabundance in Ap stars. Wide autoionization levels would have to exist in Si II for the radiation force to be able to push Si into the outer atmosphere. *Dashed lines*, gravitational forces.

is becoming more important and diffusion processes are probably unimportant. Similar remarks apply to the contribution of lines. Strontium is then expected to be more overabundant than Ba; however, the observed difference between the two is perhaps bigger than would be expected.

Thus the diffusion hypothesis predicts the better-established abundance anomalies in the Ap stars. In particular, it predicts the dependence of the anomalies on θ_{eff}, unlike any nuclear model. It predicts that, in a given star, Sr, Y, and Zr should behave differently, which again no nuclear model predicts. It also predicts the existence of patches leading to a periodic variation in the overabundances. However, it is difficult to explain the overabundance of P and the existence of both Si and Mn stars at the same θ_{eff}.

In our calculation, we had no free parameter to play with. However, if we suppose that the atmospheres are not stable in their uppermost part but only below a given value of τ, which may vary from star to star, we would expect to have both Si and Mn stars at the same θ_{eff}. We see on Figure 4 that the radiation force on Si does not decrease

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as rapidly when τ increases as does that on Mn. If the relevant τ for a given star were 1.0 or 2.0, it might become a Si star, whereas it would be a Mn star if the relevant τ were 0.1. Some of the characteristics of the metallic-line stars can probably be explained in the same way. However, it is not clear how one could explain the underabundances of Sc and Ca observed in some metallic-line stars. Because of the uncertainty as to the value of τ one should use, we have not pursued the matter any further.

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