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THE ORIGIN OF MAGNETIC FIELDS*

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I. INTRODUCTION

Magnetic fields in the astrophysical universe have been a subject of increasing concern and observational study over the past two decades. One appreciates today that magnetic forces—like gravitation and nuclear forces—link together such diverse phenomena as the interstellar medium, cosmic rays, galactic structure, solar flares, the internal rotation of the Sun, and supernovae.

The weight of unanswered questions concerning magnetic fields probably still exceeds our accumulated knowledge of them. But observations have furnished a sufficiently broad base of information that theory is now able to make *some* general statements on the origin of the magnetic fields in the Universe. I propose to review for you this evening the present understanding of their origin.

Wherever we look about us in the Universe, there are magnetic fields, fields of sufficient strength as to have interesting effects. Earth itself has a field of half a gauss at its surface, and 10^2 gauss in its core. The external field of Earth is basically a dipole, declining outward for some 10^5 km to a value of 10^{-4} gauss, where the extended field of the Sun takes over. The presence of the geomagnetic field is essential for such "unconventional" effects as the aurora, the Van Allen radiation belts, and atmospheric whistlers.

The Sun shows a mottled magnetic face, with general fields of 1–2 gauss over most of the surface, and fields concentrated to densities of 10–10^3 gauss in places on the surface to form the so-called active regions (see review of Howard 1967). The weaker fields of the Sun are extended by the solar wind, filling the entire solar system to distances of 30 a.u. or more. It should be borne in mind that both Earth and the Sun are opaque bodies, so that we can only infer from the fields at their surfaces what fields they may contain in their interiors. The field of Earth presumably extends out from the liquid metal core. The field of the Sun presumably extends out from the convective zone.

There is some observational evidence, and every theoretical reason to expect, that most other stars have magnetic fields. Indeed, the fields of some stars are so strong as to be directly observable. So far the fields of about 10^2 stars—mostly class A—have been observed directly (Babcock 1958, 1960*a;* see review by Preston 1967). To be observable, it is necessary that the mean field over the hemisphere facing the observer be 10^2 gauss or more and that the axis of rotation of the star point at the observer so that the lines are not broadened by rotation. Thus the magnetic fields of the Sun would be completely unobservable if we were too far away to resolve small portions of the disk. The fact that 100 stars have observable fields attests to the commonness of strong stellar magnetic fields. I never cease to marvel at HD 215441 with its mean field of 34 kilogauss over the observable hemisphere (discovered by Babcock 1960*b*). The stresses in such a field are 100 times those in the densest sunspot field at the surface of the Sun. The pressure is 4×10^7 dynes cm^{-2}, or 40 bars, 550 pounds inch^{-2}. We can only guess at what the peak fields in the star might be.

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And while speaking of strong fields we should not fail to note that the pulsar phenomenon looks more and more like a rotating neutron star (Gold 1969) with a surface field of 10^{10}–10^{12} gauss (Pacini 1968; Gunn and Ostriker 1969). Presumably so strong a field arises from the collapse of a star with a field of 1–10^2 gauss to the 10-km size of the neutron star. Supernova remnants such as the Crab Nebula are a related phenomenon, presumably with fields from the supernova and pulsar which created and maintain them.

The interstellar spaces in our Galaxy contain magnetic fields which, on the average, are oriented parallel to the disk and more or less in the azimuthal direction (Hiltner 1949, 1951, 1956; Davis and Greenstein 1951; Smith 1956; Behr 1959; Gardner and Davies 1966; Berge and Seielstad 1967; Davis and Berge 1968). The rms fluctuations in the field are comparable to the field itself (Hiltner 1956; Berge and Seielstad 1967; Jokipii and Lerche 1969; Jokipii, Lerche, and Schommer 1969). There still exists a wide range of observational speculation on the strength of the field in the disk of the Galaxy (van de Hulst 1967; Verschuur 1969). The existence of a quasi-steady background of cosmic rays, originating from sources within the disk of the Galaxy and retained by the fields in the disk for some 10^6 years before escaping from the Galaxy, indicates that the galactic field has a mean strength in the neighborhood of 3–5 microgauss (Parker 1969*a*). This is more or less in agreement with the integrated field densities obtained from Faraday-rotation measures (Morris and Berge 1964).

What the magnetic fields are in intergalactic space we have no idea. Strong fields, of 10^{-3} gauss and more, are often associated with extragalactic radio sources because much of the emission appears to be synchrotron radiation, but these are special regions in special galaxies. It is usually assumed that in the space between the galaxies the field is small compared to 1 microgauss.

Magnetic fields transfer stresses, and their time variations induce electric fields, so that they are responsible for much of the nonclassical activity of the outer atmospheres of the planets, the Sun, stars, and the Galaxy. For instance, in the absence of magnetic fields the Sun would be without active regions, prominences, plages, flares, and sunspots. The solar corona and the solar wind would be minimal, because magnetic fields are responsible for most of the heating of the solar corona. There would be no fast particles from the Sun, no cosmic rays to fill the Galaxy, etc., no Van Allen belts, and no aurora. There would be no radiation hazard in space. Radio emission would be reduced to the thermal level. Altogether, the Universe would be a more orderly place, but certainly far less interesting than the bag of tricks in which we find ourselves.

Now let us inquire as to *why* there are magnetic fields in the astronomical Universe. This question had its beginnings (in Western history) with Gilbert's suggestion that Earth is a lodestone. The question was expanded in scope with Hale's (1908, 1913) observation of magnetic fields in sunspots, leading to Larmor's (1919) suggestion that the sunspot fields are produced by the swirl of the gases in the Sun. Cowling (1934) took up the question in the thirties, producing his theorem on the impossibility of stationary dynamos with axial symmetry. Alfvén, at the end of the thirties, made two notable points, that the magnetic lines of force are carried bodily with the fluid and that low-frequency disturbances propagate as waves in the field (Alfvén 1950). In 1945 Elsasser began a study which introduced the modern concepts of the generation of astronomical magnetic fields in homogeneous dynamos.

But there is a more fundamental question that should be considered first. Extensive magnetic fields are possible in the Universe only because of the general abundance of free electric charges and the general absence of magnetic charges, or monopoles. An abundance of free magnetic monopoles would permit large currents of monopoles in the magnetic fields, in the same way that the existing abundance of free electrons neutralizes electric fields. Free electrons and ions generally limit electric potential differences to the thermal energy of the background plasma. Presumably an abundance of monopoles would similarly limit magnetic fields.

The observed lack of magnetic monopoles in the terrestrial laboratory has been noted as a curious fact for many decades. It has been known for a century that electric and magnetic fields appear with complete symmetry in Maxwell's equations, so why do not electric and magnetic charges appear equally in nature? One answer is that the structure of electrons and monopoles lies outside the range of Maxwell's equations, so electric-magnetic symmetry in Maxwell's equations does not compel a belief in the symmetry of electrons and monopoles. But the idea is tantalizing, and there have been determined efforts to find, or to place an upper limit on the abundance of, magnetic monopoles.

The idea of electric-magnetic symmetry would suggest that the magnetic charge g is just equal to the electric charge e. If this is correct, it would make the identification of a monopole extremely difficult. On the other hand, arguments based on quantum electrodynamics (Dirac 1931, 1948; Schwinger 1968) suggest that g might perhaps be 137/2 or 137 times e.[1] Though it has been argued that quantum electrodynamics can also give $g = e$ (Cabibbo and Ferrari 1962; see discussion of the problem by Wentzel 1966), efforts to find monopoles have been based largely on the optimistic view that, if monopoles exist, they have a charge $g = 137e$.

The point I want to make here is that attempts at direct detection of monopoles show that monopoles are rare, if they exist at all, but the upper limit on the abundance of monopoles placed by experiment does not rule out important dissipation and distortion of astrophysical fields by magnetic monopoles. It is customary to ignore magnetic monopoles completely in theoretical discussions of astrophysical magnetic fields. Lacking any evidence for their existence, it is the only reasonable course. But we should be aware that experiments do not yet force this hypothesis upon us. The problem is of such importance that a brief elaboration is in order.

If free monopoles exist in the tenuous gases in space, they would be accelerated to very high energies by the magnetic fields. For instance, if $g = e$, a typical sunspot field of 10^3 gauss over 10^9 cm would accelerate a monopole to 3×10^{14} eV, the geomagnetic field to 5×10^{10} eV, and the galactic field (say, 10^{-6} gauss over 5 kpc) to 10^{18} eV. This, together with the possibility that monopoles may be created by very high-energy particle interactions, has led investigators to search for monopoles among cosmic rays and among the collision products of cosmic rays. One may look for monopoles either in massive solid objects that have been exposed to monopoles and/or cosmic rays (Goto, Kolm, and Ford 1963; Fleischer, Jacobs, Schwarz, and Price 1969) or directly in the secondary cosmic-ray particles in the terrestrial atmosphere (Carithers, Stefanski, and Adair 1966). If $g \geq 137e/2$, and if present theoretical estimates of the attachment of monopoles to matter are correct, then the experiments indicate an upper limit of 10^{-13} monopoles $(\text{cm}^2\ \text{sec})^{-1}$ incident on the ground. This implies a density of $3 \times 10^{-24}\ \text{cm}^{-3}$ in space, or one per 10^{23} nucleons. Studies of meteoritic material suggest even fewer monopoles, an upper limit of 10^{-15} monopoles $(\text{cm}^2\ \text{sec})^{-1}$. Or, to state the results in a different way, examination of about 1 kg of meteoritic material failed to detect any monopoles. If this means that there were no monopoles in the 1 kg, the relative abundance is less than one monopole per 10^{27} nucleons.

More recently it has been established (Fleischer, Price, and Woods 1969; Fleischer, Hart, Jacobs, Price, Schwarz, and Aumento 1969) that the flux of monopoles with $g \geq 400e$ penetrating into terrestrial rocks is less than 10^{-19} $(\text{cm}^2\ \text{sterad sec})^{-1}$ (based on the absence of monopole tracks in samples of mica and obsidian) and the flux of monopoles with $g \geq 137e$ penetrating into the deep oceans is less than $4 \times 10^{-18}\ \text{cm}^{-2}$ sec based on unsuccessful attempts to extract monopoles from manganese nodules. These limits appear to rule out the possibility, suggested by Porter (1960), that the very high-

[1] If g does not have a value $137e$, etc., then there arise ambiguities in the phase differences of wave functions around closed paths, and a Hamiltonian formulation of quantum mechanics is not possible. Note that the requirement that $g = 137e$ is sufficient to provide a bound state of an electron in the field of a monopole, i.e., the de Broglie wavelength is equal to 2π times the cyclotron radius.

energy cosmic rays, above 10^{17} eV, may be monopoles. The experiments failed to find any monopoles in about 8 kg of manganese nodules, and, if this means that none were present, it places an upper limit of about one per 10^{28} nucleons in the nodules. But the experimenters have been careful to point out that their failure to find monopoles means either that monopoles are exceedingly rare or that their charge g is small ($\sim e$) or that they attach more firmly than calculations have indicated, etc. So caution must be exercised in applying the tentative limits on monopole abundance.

It is interesting, then, to inquire what abundance of free magnetic monopoles would neutralize or dissipate the existing magnetic fields. If it is assumed that a magnetic field does not extend to infinity, the distortion of the usual solenoidal magnetic field $\boldsymbol{B}_T$ ($\nabla\cdot \boldsymbol{B}_T = 0$) by a longitudinal field $\boldsymbol{B}_L$ produced by monopoles is related to the monopole number density n by

$$\nabla\cdot \boldsymbol{B}_L = 4\pi n g \, .$$

Thus, a field B_T over a scale L can be grossly distorted ($B_L \cong B_T$) by a net monopole density

$$n = \frac{B_T}{4\pi g L} \, .$$

For $g = 137e$, this requires one monopole per 10^3 cm^3 to produce a longitudinal field of the same strength and scale as the geomagnetic field. Within the solid body of Earth, this amounts to about one monopole per 10^{27} nucleons. Gross distortion of the general solar field would require one monopole per 10^5 cm^3, or one per 10^{25} nucleons in the convective zone. Distortion of the galactic field would require one per 10^{21} cm^3 or 10^{21} nucleons.

The solenoidal or transverse component $\boldsymbol{B}_T$ of a magnetic field, produced by an electric current density $\boldsymbol{j} = c\nabla \times \boldsymbol{B}_T/4\pi$ is dissipated by magnetic current $\boldsymbol{J} = g n \boldsymbol{u}$, where $\boldsymbol{u}$ is the drift velocity of the monopoles.[2] The rate of dissipation of field energy per unit volume is

$$d\mathfrak{E}/dt = \boldsymbol{J}\cdot \boldsymbol{B} \, ,$$

so that the characteristic dissipation time τ_D is

$$\tau_D \cong B^2/8\pi n g \boldsymbol{u}\cdot \boldsymbol{B} \, .$$

As already noted, free monopoles are accelerated to high energies in the fields in tenuous gases, so it is not unreasonable to suppose that $|\boldsymbol{u}| \cong c$. If the particles have been accelerated by the local field, then $\boldsymbol{u}$ is more or less parallel to $\boldsymbol{B}$ and $\tau_D \cong B/8\pi n c$. If this dissipation time is less than the growth time τ_G of the field, then the dissipation inhibits or prevents the growth of the field. Thus, the existence of fields in tenuous gases suggests that the number density n of monopoles is bounded by

$$n < B/8\pi g c \tau_G \, .$$

The growth time of a large-scale field on the Sun is at least as large as 10^6 sec, so with $B = 1$ gauss we have $n < 10^{-11}$ cm^{-3} in the solar corona, or less than one monopole per 10^{19} nucleons. In interstellar space where $\tau_G \cong 10^8$ years and $B \cong 10^{-6}$ gauss, we must have $n < 10^{-26}$ cm^{-3} or less than one per 10^{26} nucleons. If instead we compare the monopole dissipation time with the dissipation time of the interstellar field by ambipolar diffusion, say 10^{10} years, then one monopole per 10^{28} nucleons would significantly alter the rate of dissipation.

[2] Maxwell's equations are then written $4\pi \boldsymbol{j} + \partial \boldsymbol{E}/\partial t = c\nabla \times \boldsymbol{B}$ and $4\pi \boldsymbol{J} + \partial \boldsymbol{B}/\partial t = -c\nabla \times \boldsymbol{E}$, of course, so that the energy equation is $(\partial/\partial t)(E^2 + B^2)/8\pi = -\boldsymbol{j}\cdot \boldsymbol{E} - \boldsymbol{J}\cdot \boldsymbol{B} + \nabla\cdot (c\boldsymbol{E} \times \boldsymbol{B}/4\pi)$. Since $\nabla\cdot \boldsymbol{B}$ is not zero, one may not employ the vector potential to represent the magnetic field (Dirac 1931; Wentzel 1966).

Altogether, then, we conclude that unless magnetic monopoles (with $g = 137e$) are *much less* abundant than about one per 10^{27} nucleons in the solid body of Earth, one per 10^{26}–10^{28} nucleons in interstellar space, and one per 10^{25} nucleons in the solar convective zone, they may introduce serious distortion and/or dissipation of the magnetic fields observed there. If $g = e$, these limits are 10^{25}, 10^{24}–10^{26}, and 10^{23}, respectively. Experiments aimed at direct detection of monopoles as yet give no limit on the abundance of monopoles if g is as small[3] as e, and give only $g \lesssim 10^{27}$–10^{28} nucleons in meteorites and deep-ocean nodules if $g \gtrsim 137/2$. Hence the experiments do not exclude the possibility that magnetic monopoles play a role in distorting and dissipating the fields which we observe in space, and whose origin and behavior we are endeavoring to understand with the assumption that monopoles are absent.

I call this problem to your attention because I think it is an important gap in the present fabric of astrophysical knowledge and deserves further theoretical and experimental inquiry.

It is amusing to note that in keeping with the spirit of present speculation on sectors of antimatter in the Universe—the speculations are motivated by a philosophical conviction of particle-antiparticle symmetry—we should postulate that portions of the Universe are dominated by magnetic monopoles and free of electric charges in order to preserve the basic electric-magnetic symmetry of Maxwell's equations. In such a Universe each electric effect of our own world would have its complementary magnetic effect and vice versa. Magnetic fields there would be shorted out as effectively as electric fields are here. If we suppose that $g = e$, we point out that stars made of such magnetic material are indistinguishable, at a distance, from ordinary stars. And if one objects that the quantum-mechanical arguments require instead that $g = 137e$, in which case magnetic material would appear very strange indeed, we need only reply that without a doubt there is at this very moment a magnetic theoretician in that magnetic corner of the Universe arguing that electric monopoles, if they exist, must have $e = 137g$, so that a hypothetical universe composed of electrical particles (electrons and protons) would be clearly distinguishable from *real* magnetic particles.

But speculations of this general type, while amusing, do not appear fruitful.

II. BASIC EQUATIONS

Magnetic fields in our monopole-free Universe appear in association with electric currents. Under the usual circumstance that the particle motions are slow compared with the speed of light, the current density j and the magnetic field are related by

$$4\pi \boldsymbol{j} = c\nabla \times \boldsymbol{B} \tag{1}$$

(Alfvén 1950; Elsasser 1954), where $|\boldsymbol{j}|$ is in esu and $|\boldsymbol{B}|$ is in gauss. Except for the special case of the small currents produced by the thermal effects and/or inertial effects (Biermann 1950), the current is driven by the time derivative of the magnetic field via the induced electric fields $\boldsymbol{E}$,

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}\,. \tag{2}$$

The magnetic field is related to the motion $\boldsymbol{v}(\ll c)$ of the gases in the Universe by noting that essentially everywhere in the Universe (except in such odd places as cold planetary atmospheres) there are so many free low-energy electrons that the electric fields $\boldsymbol{E}'$ in the gas (on a scale larger than the Debye length, typically 10 m or less) are very weak, sufficient only to drive the weak currents required by equation (1). If we choose to work

[3] The quantum interferometer is the only device to date sufficiently sensitive to detect such monopoles (Vant Hull 1968). I am indebted to R. L. Fleischer for pointing out this work.

with a coordinate system relative to which the gas has a velocity $\boldsymbol{v}$, then a Lorentz transformation tells us that the field $\boldsymbol{E}$ in the coordinate system is

$$\boldsymbol{E} = \boldsymbol{E}' - \boldsymbol{v} \times \boldsymbol{B}/c \tag{3}$$

with terms of $O(v^2/c^2)$ neglected. The magnetic fields in the two frames are equal, to $O(v^2/c^2)$, and need not be distinguished, $\boldsymbol{B} = \boldsymbol{B}'$. Since the free electric charges neutralize $\boldsymbol{E}'$, equations (2) and (3) together give

$$\partial \boldsymbol{B}/\partial t = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) . \tag{4}$$

If the gas offers some resistance to the passage of the electric current produced by $\boldsymbol{B}$ (eq. [1]), then $\boldsymbol{E}'$ is not precisely zero and an additional term appears in equation (4) to represent the dissipation of the magnetic field. In the simple case that the current is related to $\boldsymbol{E}'$ by the scalar form of Ohm's law, $\boldsymbol{j} = \sigma \boldsymbol{E}'$, the dissipation takes the form of a diffusion term,[4]

$$\partial \boldsymbol{B}/\partial t = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B} , \tag{5}$$

where $\eta = c^2/4\pi\sigma$. In gases which are only partially ionized and/or of low density the dissipation term may be rather more complicated (see discussion in Spitzer 1956; Cowling 1957*a*). Equations (4) and (5) will be sufficient for the purposes of this exposition.

As Alfvén (1950) first pointed out, the magnetic lines of force are transported bodily by the fluid velocity $\boldsymbol{v}$ (eq. [4]) permitting graphic representation of the changing field. Only if there is significant dissipation do the lines slip relative to the fluid or reconnect among themselves (see examples in Parker and Krook 1956). In fields with scale L the ratio of the dissipation term $\eta\nabla^2\boldsymbol{B}$ to the term $\nabla \times (\boldsymbol{v} \times \boldsymbol{B})$ is equal to the reciprocal of the magnetic Reynolds number $\mathfrak{R}_M = vL/\eta$. In the astrophysical universe η is typically 10^5 cm^2 sec^{-1} or less and the magnetic Reynolds number 10^6 or more, so for considering the gross behavior of magnetic fields equation (4) suffices in most cases. Thus the magnetic lines of force, which are the instantaneous solutions of

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z} , \tag{6}$$

move with the fluid and, because of their graphic representation of the field $\boldsymbol{B}$, occupy the center of our attention.

The stresses in the magnetic field $\boldsymbol{B}$, or B_i, are described by the Maxwell stress tensor

$$M_{ij} = -\delta_{ij}\frac{B^2}{8\pi} + \frac{B_i B_j}{4\pi} . \tag{7}$$

The first term represents an isotropic pressure $B^2/8\pi$, and the second term a tension $B_iB_j/4\pi$ along the lines of force. We will not be concerned directly with dynamical problems here, but it must be borne in mind that in circumstances where the magnetic-energy density $B^2/8\pi$ is comparable to the kinetic-energy density $\frac{1}{2}\rho v^2$ of the gas, the Maxwell stresses M_{ij} are comparable to the Reynolds stresses

$$\mathfrak{R}_{ij} = -\rho v_i v_j , \tag{8}$$

and the magnetic field significantly affects the fluid motion.

III. THE ROLE OF PRIMORDIAL FIELDS

When we come to consider the origin of the magnetic fields presently existing in the universe, there are a number of considerations that must be weighted before we can

[4] More precisely, $\nabla \times (\eta\nabla \times \boldsymbol{B})$, which reduces to $\eta\nabla^2\boldsymbol{B}$ if $\nabla\cdot\boldsymbol{B} = 0$ and η is independent of position.

form an opinion. The science of astronomy gives us no more than a snapshot of the magnetic fields, which have lives of 10^8 years or more. There are a number of ideas that are consistent with the snapshot and that can be evaluated only upon careful consideration of all their implications.

Consider the role of primordial fields—the initial magnetic conditions in the Universe. Cowling (1945, 1953) has pointed out that the size and electrical conductivity of astronomical bodies is so large that primordial fields entrapped in the bodies would not decay significantly in the life of the Universe. Putting $v = 0$ in equation (5) leads to a diffusion equation, indicating a relaxation time $L^2/4\eta$ for magnetic fields of dimensions L. The decay time for a field permeating the Sun is 10^{10} years, and about 10^{10}–10^{12} years for a field in the disk of the Galaxy (including ambipolar diffusion). On this simple basis one might argue that the present fields are simply the primordial fields perhaps considerably distorted by $\nabla \times (\boldsymbol{v} \times \boldsymbol{B})$, but generally preserved by the large value of L^2/η.

To pursue this further, the formation of galaxies from primordial gases, as well as the formation of individual stars from interstellar gas within galaxies, is largely a process of condensation and compression, which would have the effect of enormously increasing the strength of fields entrapped in the gas. Thus, suppose, for instance, that the gas of which the Galaxy is composed was once part of the intergalactic medium, which today may have a density of $10^{-5\pm1}$ atom cm^{-3}. The mean density in the Galaxy is some 10^5 times greater, so that if the uncondensed intergalactic material has a magnetic field of 10^{-9} gauss and a density of 10^{-5} atom cm^{-3} today, we expect the material in the Galaxy, where the mean density is 1 atom cm^{-3} or more, to have a field of more than 10^{-6} gauss in interstellar space. This is comparable to the observed interstellar fields. Note further that 10^{-6} gauss in interstellar space where the density is 1 atom cm^{-3} suggests more than 10^6 gauss upon condensation into a star, leaving plenty of room for losses and still accounting easily for the observed stellar fields.

Now there is no objection at the present time to intergalactic fields of 10^{-9} gauss. The energy density is, and has always been, negligible compared with the radiation density. Isotropic expansion of a volume of gas of radius R reduces the magnetic field in the gas in proportion to $1/R^2$, with the result that a magnetic-energy density declines with the expanding Universe in direct proportion to the primordial radiation density. It is believed that the so-called 3° K blackbody radiation represents the primordial radiation field. The 3° K radiation-energy density is 10^{-12} erg cm^{-3}, which is equivalent to the energy density of a field of 5×10^{-6} gauss. Thus an extragalactic field of, say, 10^{-9} gauss has an energy density of 4×10^{-20} erg cm^{-3}, which is negligible compared with the radiation density today, and hence negligible at all times in the past.

It is very tempting, therefore, to theorize that the present magnetic fields of the Galaxy and of the stars are simply a concentration in the dense objects of a primordial magnetic field. In the Galaxy such an explanation is adequate (in view of present ignorance of both galactic and intergalactic fields), and for stars such an explanation is rather more than adequate. But, unfortunately, closer examination of the problem demonstrates that there are other and more powerful effects of work which render the question of primordial fields almost completely irrelevant.

The magnetic field of our own planet is a good place to start the discussion. It has a weak field of half a gauss at its surface, easily accounted for by the compression of interstellar fields. But it turns out that Earth is so small that the decay time of the geomagnetic field is only about 4×10^4 years (Elsasser 1950*b*, 1956*a*). Any primordial fields would have long since disappeared. What is more, it is now known from fossil magnetism that the geomagnetic field is more or less steady in time except that it suddenly reverses at irregular intervals of 10^6 years (Runcorn 1955; Wilson 1966, 1967; Cox and Dalrymple 1967). Something is going on in the Earth today that is responsible for today's geomagnetic field. The characteristic generation time is apparently 10^4 years or less, so that the field today is independent of what occurred more than 10^4 years ago.

The field of the Sun is another example of active magnetic generation today, wiping out all traces of earlier fields in periods of only 10^2 years. Within about 35° of the poles the magnetic field of the Sun resembles a magnetic dipole, with a strength of 1–2 gauss directed in at the north pole and out at the south pole, just like Earth (Babcock and Babcock 1955; Babcock 1959; Babcock 1963). But prior to 1958 the polar fields had the opposite sense. And it is expected that the present polar fields will reverse by the end of 1970 (in association with the present sunspot maximum [Waldmeier 1960]) to take up the earlier orientation in association with the 22-year sunspot cycle.

Toward the solar equator the fields are migratory and, at the visible surface, much more irregular. The sunspots evidently arise from toroidal bands of field beneath the photosphere with opposite signs north and south of the equator. The toroidal bands migrate from latitudes of about 45° to the equator in each 11-year half of the sunspot cycle, with successive bands of opposite sign. Some process obviously builds up the fields and destroys and reverses them every 11 years.

It is evident from these two examples that the maintenance of magnetic fields in planets and stars, if not in interstellar space, is an active process from which all traces of primordial fields have long since vanished. The term $\nabla \times (\boldsymbol{v} \times \boldsymbol{B})$ in equation (5) plays the dominant role and cannot be neglected. It is these active processes for the rapid creation and destruction of magnetic fields that are of central interest for understanding the observed fields.

IV. GENERATION OF FIELDS BY FLUID MOTIONS

If we conclude that the magnetic fields in the Universe today result from contemporary generation of fields, then we must inquire into the effect of the induction term on the right-hand side of equation (5).

First of all, consider the magnetic-field structure of the Sun and of the Galaxy. It appears that the largest scales of the fields are determined directly by the large-scale fluid motions in which the field is generated and have little to do with the small scales on which the generation of field takes place. Thus the extension of the magnetic lines of force of the general solar field outward through the solar system is a direct result of the solar wind. The solar field extends as far as the wind blows, presumably 30–1000 a.u. The interstellar magnetic field, i.e., the galactic field, is stretched out in the azimuthal direction around the galactic disk by the nonuniform rotation of the Galaxy, so that its scale is evidently very much in excess of 1 kpc, even though the small-scale structure across the disk is 100 pc or less (see summary and analysis of the observations in Jokipii and Lerche 1969). Our primary concern here is with the generation of the magnetic lines of force, and only incidentally with the simple stretching of the lines of force to the large scales on which we observe them.

a) *Turbulent-Velocity Fields*

Much of the fluid motion in interstellar space, the solar photosphere and convection zone, etc., appears to be chaotic, or turbulent. So consider that problem first. Historically there are two basic points of view on the effect of turbulent fluid motions on a magnetic field. Alfvén (1947; Biermann and Schlüter 1951; Biermann 1953; Chandrasekhar 1955) suggested that the close coupling of the magnetic field and the velocity field leads to equipartition of energy between the two systems in the final state of dynamical equilibrium. Write the equation of motion as

$$\frac{\partial}{\partial t}\,\rho v_i = -\frac{\partial}{\partial x_i}\left(p + \frac{B^2}{8\pi}\right) + \frac{\partial}{\partial x_j}\left(\frac{B_iB_j}{4\pi} - \rho v_i v_j\right), \qquad (9)$$

and write equation (4) as

$$\frac{\partial B_i}{\partial t} = \frac{\partial}{\partial x_j}\,(v_iB_j - v_jB_i) \qquad (10)$$

for an incompressible, inviscid, and infinitely conducting fluid. One might expect that two stress fields—the Reynolds and Maxwell stresses—so closely and symmetrically coupled, as indicated by equation (10) and by the last term on the right-hand side of equation (9), would share the energy equally between them after some significant period of interaction. On this basis one expects equipartition of energy between the chaotic fluid motions and the chaotic magnetic field which they create:

$$\langle B^2/8\pi \rangle \cong \langle \tfrac{1}{2}\rho v^2 \rangle \, . \tag{11}$$

To put the idea in its most seductive form, we note, after Elsasser (1950*b*), that the equations can be symmetrized by introducing the variables

$$U_i = v_i + B_i/(4\pi\rho)^{1/2} \, , \qquad V_i = v_i - B_i/(4\pi\rho)^{1/2} \, ,$$

whereupon equations (9) and (10) can be written

$$\rho\left(\frac{\partial U_i}{\partial t} + V_j \frac{\partial U_i}{\partial x_j}\right) = -\frac{\partial}{\partial x_i}\left(p + \frac{B^2}{8\pi}\right),$$

$$\rho\left(\frac{\partial V_i}{\partial t} + U_j \frac{\partial V_i}{\partial x_j}\right) = -\frac{\partial}{\partial x_i}\left(p + \frac{B^2}{8\pi}\right).$$

The equations treat U_i and V_i, and hence v_i and B_i, with complete symmetry, except for the pressure term on the right-hand side in which $B^2/8\pi$ appears directly, but p appears in place of $\frac{1}{2}\rho v^2$. We can, if we choose, make the initial conditions and the boundary conditions symmetric in U_i and V_i. So it is not unreasonable to expect equipartition of energy between U_i and V_i, and between the Reynolds and Maxwell stress systems of v_i and B_i.

On the other hand, there is an equally plausible point of view based on the fact that the equation for the vorticity ω_i in a hydrodynamic fluid can be written

$$\frac{\partial \omega_i}{\partial t} = \frac{\partial}{\partial x_j}(v_i\omega_j - v_j\omega_i) \, , \tag{12}$$

in precise analogy to equation (10) (Batchelor 1950; Chandrasekhar 1950, 1951). The magnetic lines of force are carried with the fluid exactly as are the vortex lines. Hence one might expect that a magnetic field is transformed by a given velocity field in exactly the same way as the vorticity. We know from studies of hydrodynamic turbulence that the vorticity is large only in the small eddies, from which one would expect that the magnetic field builds up only in the small eddies (provided, of course, that the conductivity of the fluid is sufficiently high that the small eddies are able to carry the field with them). On this basis, then, one expects equipartition of energy only in the small eddies, leading, in place of equation (11), to

$$\langle B^2/8\pi \rangle \cong \tfrac{1}{2}\rho \langle v^2 \rangle_{\text{small eddies}} \ll \tfrac{1}{2}\rho \langle v^2 \rangle \, . \tag{13}$$

Both ideas have been restated in a variety of contexts by various authors. Equipartition is a popular point of view in the astronomical literature, particularly when one is dealing with little-known circumstances. The field-vorticity analogy has been developed from several different physical models (Moffat 1961; Parker 1963; Pao 1963; Saffman 1964).

Unfortunately, observations have been too limited to settle the question of which, if either, point of view is the correct one. Turbulent magnetic fields must be resolved to be measured, so that the best information available is from the Sun, where fields of 1–3500 gauss are observed on scales of 10^3 km and larger in the photosphere. The kinetic-energy density in the photosphere is 10^2–10^3 ergs cm^{-3} corresponding to the energy density of fields of the order of 10^2 gauss. This would appear to support neither theoretical view.

Recently Kraichnan and Nagarajan (1967) have examined the complete hydromagnetic equations in detail, and showed that the dynamical interaction of the Fourier transforms of v_i and B_i involve a variety of terms whose relative values depend upon the relative phases of the various interacting Fourier components. They identify which terms must be retained and which suppressed to arrive at the field-velocity analogy, and likewise for the field-vorticity analogy. They conclude that the problem is too subtle to be settled by the heuristic arguments that have been employed so far, and a formal calculation is the only resolution of the dilemma. They point out, too, the difficulty of an adequate formal calculation.

There is another approach, of interest for a very restricted aspect of the problem, which employs the theory of random functions (Parker 1969*a*). If the magnetic field is weak and if one imagines a turbulent-velocity field with a correlation time very short compared with the dimensions of its eddies divided by the velocity of the eddies, then the velocity and magnetic fields are uncorrelated random functions, and it is possible to show that equation (10) leads to a growth of the Fourier components of the field at all wavenumbers. Stopping the turbulence leads to decay of the field at large wavenumbers, with the small wavenumbers surviving. Hence, alternate periods of turbulence and quiet would appear under these hypothetical conditions, to build up fields on a large scale (small wavenumbers). We have suggested the possibility (Parker 1969*b*) that the magnetic lines of force of which the galactic field is composed have been generated in this way, and have then been stretched out and compressed to the present orientation and strength by the nonuniform rotation of the Galaxy. But there must be something more than a physical analogy between the turbulence in interstellar space and the hypothetical turbulence with a short correlation time before we can say that turbulence is, in fact, the origin of the present galactic field.

Altogether, then, it seems that the question of two decades' standing, as to the generation of magnetic field by random turbulence, is still unanswered. Nor do we know the origin of the galactic fields. Our fondest theories are still only ideas without a solid foundation.

V. GENERATION OF FIELDS BY ORDERED FLUID MOTIONS

If we have failed so far to understand the magnetic effects of disordered turbulent motions, there remains the question of the magnetic effects of suitably ordered motions, the so-called hydromagnetic dynamo. The vague notion of the generation of magnetic fields by ordered fluid motions goes back to Larmor's suggestion that the swirling pattern in the photosphere around a sunspot represents the fluid motions which generate the field of the spot (Larmor 1919). Cowling (1934, 1945, 1957*a*, *b*, 1965*a*, *b*, 1968), Elsasser (1955), Backus and Chandrasekhar (1956), Lortz (1968), and Jayanthan (1968) looked into the question of the generation of magnetic fields by the motions in a homogeneous body of conducting fluid and demonstrated the fundamental theorem that fluid motions with axial symmetry cannot maintain a steady magnetic field.

The next important step was taken by Elsasser (1945, 1946, 1947, 1950*a*, *b*, 1956*a*, *b*). Confining his attention to the field of Earth as the one example most clearly defined by observation at that time, he pointed out first that none of the effects of ferromagnetism, magnetostriction, thermoelectricity, etc., to which the geomagnetic field was commonly attributed at that time, were tenable in light of modern knowledge of the properties of matter. And therefore the only possible explanation of the field lay in the convective motions in the liquid core of our planet. It was known from seismic studies that Earth has a liquid-metal core with a radius of half of the radius of Earth. The slow variation of the small-scale inhomogeneities in the geomagnetic field over the past couple of centuries indicates that the core is convecting, with velocities of the order of 10^{-2} cm sec^{-1}. There is some seismic information that there may be a small solid core in the center of the liquid core, but the question is not important here. The electrical conductivity of the liquid-metal core is of the order of 10^{15} sec^{-1} (esu) (about the same as ionized hydrogen at 10^6 ° K), so that the resistive diffusivity of the core is $\eta = c^2/4\pi\sigma = 10^5$ cm^2 sec^{-1}. The relaxation time for a field extending over the radius a of the core is $a^2/\eta \cong 10^{12}$ sec $= 3 \times 10^4$ years. But the convective velocities v in the core traverse the radius a in a time $a/v \cong 10^{10}$ sec or 3×10^2 years, so that they should carry the field with them to a large degree (the magnetic Reynolds number $\Re_M \equiv av/\eta = (a^2/\eta)(v/a) \cong 3 \times 10^2$). Thus there was a good impedance match between the liquid motion and the field, and the idea that the motions are responsible for the field was not implausible.

Elsasser then pointed out that, in view of the rapid rotation of Earth, the Coriolis forces on the convective motions in the core should produce a nonuniform rotation of the core, with the outer equatorial regions rotating more slowly than the inner regions near the axis. The expectation is reinforced by the observed slow westward drift, of a few millimeters per second, of the identifiable inhomogeneities in the field. A nonuniform rotation shears the dipole field in the core and draws out the lines of force into an azimuthal field. With the regions farther from the axis rotating more slowly ($d\omega/d\varpi < 0$,

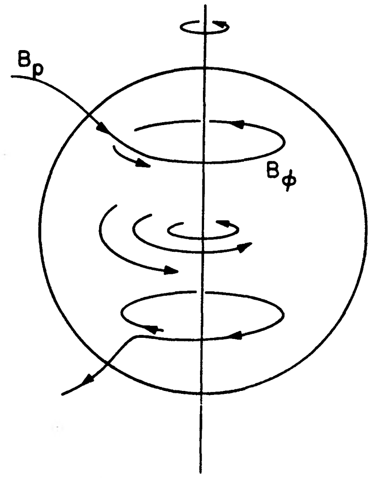

FIG. 1.—Sketch of the toroidal field B_ϕ in the core of Earth produced from the poloidal field B_p by the nonuniform rotation.

where ϖ is the distance from the axis), the toroidal field is eastward in the northern hemisphere and westward in the southern hemisphere, as sketched in Fig. 1. The shear does not affect the axisymmetric part of the dipole field. The strong Coriolis forces suggest that the forces driving the nonuniform rotation are strong and the toroidal field is strong, perhaps as much as 50–400 gauss (Bullard 1949*a*), with the magnetic stresses balancing the Coriolis forces. This is to be compared with the dipole field of about 5 gauss in the core. The mathematics describing the generation of the toroidal field from the poloidal (dipole) field follows immediately from equation (5) as

$$\left[\frac{\partial}{\partial t} - \eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)\right]B_\phi = B_\varpi\left(\frac{dv_\phi}{d\varpi} - \frac{v_\phi}{\varpi}\right) \qquad (14)$$

for the simple case of axial symmetry, and v_ϕ is a function only of the distance from the axis of rotation. The quantity $dv_\phi/d\varpi - v_\phi/\varpi$ is the nonuniform part of the rotational velocity. Presumably B_ϕ grows to the point where the Maxwell stresses $B_\phi B_z/4\pi$ and $B_\phi B_\varpi/4\pi$ slow down the nonuniform rotation enough to strike a balance between shear and diffusion.

The next question concerns the regeneration of the poloidal, or dipole, field from the toroidal field. Without some way to regenerate the poloidal field the whole magnetic system decays in a time a^2/η regardless of the generation of the toroidal field. We know from Cowling's theorem that the regeneration cannot involve axial symmetry, so Elsasser pointed out that the general radial convection throughout the core must be the link which produces the poloidal field. There are some ten to twenty identifiable inhomogeneities in the geomagnetic field at the surface of Earth, suggesting a comparable number of convective cells in the core. Elsasser developed the mathematics for treating the interaction of the convection with the field by expanding both the magnetic field and the velocity field in the magnetic modes of a conducting sphere. A thorough exploration of the regeneration question was carried out for stationary conditions within this mathematical framework (Bullard 1949*a*, *b*, 1955; Bullard and Gellman 1954; Takeuchi and Elsasser 1954; Takeuchi and Shimazu 1954; Elsasser and Takeuchi 1955; Rikitake 1958, 1966). The expansion in terms of the modes of a sphere proved nonconvergent when applied to the simplest examples of fluid motion for the regeneration of the poloidal field.

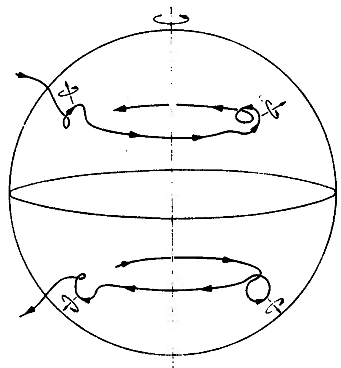

FIG. 2.—Sketch of the meridional loops of flux produced by the interaction of rising cyclonic convective cells with the toroidal field B_ϕ. Note that the projections of the loops on the meridional planes have the same sense as the dipole field (from which B_ϕ is produced by the nonuniform rotation).

Some individuals (see, for instance, Bullard 1950) began to wonder at the time if the failure (nonconvergence) of the mathematics to find a means for steady regeneration of the poloidal field was not a consequence of some yet undiscovered "generalized Cowling's theorem" which forbade stationary dynamos whether with axial symmetry or not. The conjecture was wrong, as shown later, but profitable nonetheless, because it suggested to me that the solution to the problems must lie in the nonstationary dynamo. The dipole field of Earth is quasi-stationary at the surface of Earth, but its generation in the core need not be if it is remembered that the 1000-year diffusion time of the core field gives considerable smoothing of any rapid fluctuation that must be present.

The convection in the core of Earth gives every appearance, from the meager two centuries of magnetic data that are available, of being nonsteady. Presumably the individual convective cells grow and die, perhaps much like the convective cells in the terrestrial atmosphere. The rapid rotation of Earth and the strong Coriolis forces suggest that the convective cells in the core of Earth must be cyclonic, much like their atmospheric counterparts. If one then inquires what the effect is of a localized cyclonic convective cell, it follows at once that it raises and rotates the lines of force of the toroidal field to form loops of magnetic flux with a nonvanishing projection on the meridional plane, as sketched in Figure 2. A large number of cyclonic convective cells leads to a large number of loops, which upon coalescence, in periods of a few thousand years, give a large poloidal loop of flux. If the loop has the same sense as the poloidal field, it reinforces the poloidal field; if it has the opposite sense, it reduces the poloidal field. If we suppose that there are rising convective cells in the liquid core of Earth, the Coriolis force on the converging flow at the bottom of each cell causes the cell to rotate more rapidly than the rest of the core. The resulting loop of magnetic flux proves to be regenerative (see Fig. 2). A number of mathematical examples were worked out to illustrate the general process of the formation of loops by cyclonic convective cells (Parker 1955*b*). So it follows that cyclonic convective cells can regenerate the poloidal field, giving a complete dynamo in two stages, poloidal and toroidal fields, each regenerated from the other. In both stages the Coriolis force is responsible for the proper ordering of the motion. Sufficient conditions for a regenerative dynamo are (*a*) convection and (*b*) rotation. The convection is the source of energy. The rotation produces the nonuniform rotation and the cyclonic motions.

In order to treat the situation mathematically we took advantage of the fact, pointed out by Elsasser, that the toroidal field in the core is considerably stronger than the poloidal field. Hence the principal interaction of the cyclonic convective motions is with the toroidal field. Their interaction with the poloidal field produces weaker loops of flux which coalesce to generate a higher toroidal mode, with a rapid decay time. Hence, apart from a small effect on the dominant toroidal field, the cyclonic convective motions serve only to regenerate the poloidal field from the toroidal field. We introduced one further approximation in order to simplify the mathematics, and that was to assume that each individual cyclonic cell was vigorous but short lived, so that it was sufficient to use equation (4), rather than equation (5), during the life of the cell. Following the short life of the cell, the coalescence of its field with the general poloidal field is described by the diffusion equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \eta \nabla^2 \boldsymbol{B} \, . \tag{15}$$

If it is assumed that the individual cells are small compared with the dimensions of the core, but numerous, i.e., many cells appear in the core in the decay time a^2/η of the general field, then the generation and diffusion can be combined to give a mean production of poloidal field. If we write the poloidal field $\boldsymbol{B}_p$ in terms of the azimuthal vector potential $\boldsymbol{A}_\phi$ and integrate equation (5) once, we obtain

$$\left(\frac{\partial}{\partial t} - \eta \nabla^2\right) \boldsymbol{e}_\phi A_\phi = (\boldsymbol{v} \times \boldsymbol{B})_\phi \, ,$$

where $\boldsymbol{e}_\phi$ is a unit vector in the azimuthal direction and where $(\boldsymbol{v} \times \boldsymbol{B})_\phi$ is the rate of generation of $\boldsymbol{A}_\phi$ by the cyclonic convective motions. If now we smooth out the contributions from individual convective cells by averaging over periods of time and volumes which are small compared with a^2/η and a, respectively, but large enough for the occurrence of many individual cells, and if we assume that the mean rate of occurrence of the cells is independent of azimuth (longitude) in the core, then A_ϕ is independent of ϕ, and

$$\left[\frac{\partial}{\partial t} - \eta \left(\nabla^2 - \frac{1}{\varpi^2}\right)\right] A_\phi = N \Delta A_\phi \, ,$$

where N is the number of cyclonic cells appearing per second per unit volume and ΔA_ϕ is the contribution of each cell. For a given convective cell velocity $\boldsymbol{u}$, the contribution of each cell ΔA_ϕ is proportional to the toroidal field with which the cell interacts, and the proportionality constant depends upon the form and duration of $\boldsymbol{u}$, computed from equation (4) during the life of the cell (see mathematical examples in Parker 1955*b*). Thus one writes

$$\left[\frac{\partial}{\partial t} - \eta \left(\nabla^2 - \frac{1}{\varpi^2}\right)\right] A_\phi = \Gamma B_\phi \, , \tag{16}$$

where Γ is a measure of the mean rate and strength of the cyclonic motions. Γ may be a function of position, and it may also be a function of B_ϕ if we wish to take account of the reaction of the field on the convective motions (as in Leighton's [1969] recent model of the solar dynamo).

Together, equations (14) and (16) constitute a complete set of equations, the ***dynamo equations*** (Parker 1955*b*). The conditions appropriate for the core of Earth are $\sigma \cong 0$, $\eta \cong \infty$ outside the core, so that B_ϕ vanishes on the surface (and everywhere outside the core), and $\nabla \times \boldsymbol{B}_p = 0$ outside the core, so that the radial component of $\nabla \times \boldsymbol{B}_p$ vanishes at the surface of the core.

We worked out a particularly simple stationary solution, in which Γ was chosen so that ΓB_ϕ was a constant within the core, to see what external poloidal field results. Elsasser's representation of the field in terms of the modes of a conducting sphere is particularly convenient[5] for treating the mean field and gives rapid convergence because of the rapid decay of the higher modes (at a rate proportional to the square of the order of the mode). Thus the poloidal field outside the core is dominated by the lowest mode, the dipole, in agreement with observation. Inside the core the field is, of course, complicated by the inhomogeneities of the individual convective cells. But in diffusing out through the core to the outside, the poloidal field is smoothed in both space and time to give a more or less steady dipole at the surface of Earth, $r \cong 2a$. The interested reader is referred to the review articles by Elsasser (1956*a*, *b*) and Cowling (1957*a*).

VI. DISCUSSION

It seemed at this point that we had the explanation for the existence of the dipole field of Earth. The convection of the core is not unexpected and is strongly suggested by the past two centuries of magnetic observations. The Coriolis forces were established by Foucault in 1851. Together the two effects are sufficient. But if the explanation is correct, then it is only the beginning of the inquiry into the origin of magnetic fields in the astrophysical Universe. And if the explanation is correct, then it needs much more detailed investigation—for instance, a straightforward explanation for the abrupt reversal of the geomagnetic field at irregular intervals of the order of 10^6 years (Runcorn 1955; Cox and Doell 1964; Doell and Cox 1965; Doell, Dalrymple, and Cox 1966; Wilson 1966, 1967; Cox and Dalrymple 1967) as determined from fossil magnetism. A formal example of the motions in a convecting rotating sphere of liquid heated from within is needed to show that the convective cells in the core of Earth really have the cyclonic rotation that our simple arguments ascribe to them. What is the source of heat that drives the convection? Can we give more complete solutions to the dynamo equations (14) and (16), to illustrate the dynamo more effectively than the single example of ΓB_ϕ = constant? Can we construct a better approximation than ΓB_ϕ for the effect of the cyclonic convective cells? In fact, can we be sure that our heuristic derivation of the dynamo equation (16), in which the formation of meridional loops of flux was demonstrated by formal calculation of the fields of various idealized convective cells and represented by the mean rate or production ΓB_ϕ, does not overlook some subtle consideration which vitiates our plausible result? That is, equation (16) is not a formal rigorous mathematical proof of the existence of a dynamo. Is a formal proof tractable? The physical ideas on which the dynamo equations were originally based are perfectly sound, but the active human mind can always find questions to worry about—which is often the source of further progress and broader understanding.

Finally, we must not overlook the question of what other fluid motions, besides nonuniform rotation and cyclonic convective motions, will generate fields. This is a general

[5] The expansion becomes inconvenient if we attempt to handle the fields of the individual cyclonic convective cells, which, because of their small size, involve high modes. It is for this reason that the earlier attempts to handle the complete solution of equation (5) by expanding in the modes were not sufficiently convergent.

question in physics. We cannot say that we really understand the origin of fields in astronomical bodies, hidden as they are in the opaque interiors of the bodies, unless we have some idea of the other possibilities and their limitations.

There has been considerable progress in all of these directions in the fourteen years since the cyclonic convective dynamo was first proposed. We do not have time to go into great detail, but the basic points are of general interest.

Backus (1958) worked out a rigorous proof of the existence of a homogeneous hydromagnetic dynamo. He treated a situation in which idealized velocity fields are switched on for a brief period of time and then switched off for a longer period during which the field relaxes in the manner described by the diffusion equation (15) much as in the treatment of the cyclonic convective cells leading to the dynamo equation (16). But Backus was able to construct and carry through an expansion of the field rigorously, obtaining rapid convergence of the expansion from the fact that the higher modes decay much more rapidly. Thus Backus established by formal calculation the existence of a nonstationary dynamo.

At about the same time Herzenberg (1958) showed by formal solution of equation (5) that two small conducting spheres, steadily rotating about axes lying in a plane but inclined with respect to each other and surrounded by conducting material at rest, will regenerate a magnetic field which at some distance is predominantly dipole in character. Herzenberg's formal expansion of the fields was made to converge rapidly by separating the two small spheres by a distance equal to several of their radii so that the lowest mode of each dominated at the position of the other. Thus Herzenberg established by formal calculation the existence of a *stationary* dynamo.

Between Backus's rigorous mathematical proof of the existence of a regenerative dynamo based on intermittent fluid motions and Herzenberg's rigorous proof of the existence of a stationary dynamo, Cowling's theorem on the nonexistence of a stationary dynamo with axial symmetry was circumscribed: Dynamos do not have axial symmetry, but there is certainly a variety of circumstances which can regenerate a field. And there is no reason to suspect anything wrong with the cyclonic convective dynamo proposed for Earth.

More recently Braginskii (1964*a*, *b*, 1965) has derived the dynamo equation equivalent to equation (16) for the case in which the nonuniform rotation is strong, with magnetic Reynolds number $\mathfrak{R}_M$ (the toroidal field is again the dominant field), but the convective motions are weak $O(\mathfrak{R}_M^{1/2})$ and slowly varying in time, though not necessarily of small scale. This is in contrast to the sudden, small-scale convective motions considered in the derivation of equation (16). Thus in Braginskii's model of slow convection, the distortion of the toroidal field by the convective motions goes on simultaneously with the diffusion of the distortions to form the poloidal field. As one would expect, the mathematics becomes very complicated, but Braginskii was able to carry it through to the remarkable result that, in terms of an effective potential $A_e \equiv A_\phi + wB_\phi$ in place of A_ϕ [where w is a small number $O(\mathfrak{R}_M^{-1})$], the resulting equation reduces to the form (16). The toroidal and poloidal fields become mixed in the dynamo equation (16), then, because of the slow convection, but the form of the equation is unaltered. It is most interesting to see that such different physical circumstances lead again to the form (16).

Braginskii goes on to work out a number of solutions of the dynamo equation to illustrate the generation of the geomagnetic field in the core of the planet (see review of Braginskii's work by Roberts 1967).

Other authors have attacked the dynamo problem by direct solution of equation (5) for a given velocity field $\boldsymbol{v}$, without the intermediate step of averaging and smoothing the fields generated by the convective motions as done by Parker and Braginskii. Their work is valuable in that it gives a broader background in which to view the special Coriolis-convective dynamo that we propose for Earth (and the Sun). For instance, Lortz (1968) has demonstrated a dynamo involving helical motions. Roberts (1968,

1969) has demonstrated two dynamos consisting of steady periodic motions in an infinite space which are able to regenerate magnetic fields. He considers two cases, in which the x-, y-, and z-components of the velocity are proportional to sin $(ky + lz)$, sin $2lz$, sin $2ky$. The mathematics involves solving an enormous sequence of difference equations, so Roberts resorted to numerical methods.

Roberts finds regeneration if the motions are sufficiently strong ($\Re_M$ sufficiently large). That is, the regeneration may be inefficient, but is generally present. Indeed, he suggests that motions without some regenerative dynamo action are uncommon! (See remarks in Roberts 1967.) It will be most interesting to see how far this general statement can be extended. It is the opposite view to the earlier dark premonitions that there was a "super Cowling's theorem" that homogeneous dynamos are impossible.

Childress (1967*a*, *b*, *c*) has developed a formal mathematical method for solving the hydromagnetic equation (5). He divides the fields into small- and large-scale components and carries out a formal expansion in terms of the ratio ϵ of their scales. He does not make a corresponding decomposition of the velocity field, but considers only a single velocity of small scale and of large magnitude $O(\epsilon^{-1/2})$. Systematic expansion of the hydromagnetic equation is then possible, and Childress gives a complete mathematical development of the equations. He produces some stationary solutions as examples, for the special case that the vorticity is parallel to the velocity, but no direct application to Earth or the Sun was attempted.

Altogether it would appear that there is a large variety of both stationary and nonstationary motions which regenerate magnetic fields, and the mathematical approaches that have been developed in the past few years have opened the way to exploring this vast question.

With the current interest in dynamos we have worked out a formal derivation of the dynamo equation (16) from equation (5) recently, putting the physical arguments on which the equation was originally based (Parker 1955*b*) into formal mathematical terms (Parker 1970). We obtain just equation (16) in the limit of large nonuniform rotation. Γ is again calculable from the small-scale sudden convective motions when they are specified. When the nonuniform rotation is not large, there are additional small source terms on the right-hand side, but the additional modes which they produce are not of physical interest so far as we can see at the present time.

VII. SOLAR AND STELLAR MAGNETIC FIELDS

Now consider the generation of magnetic fields in stars. Essentially all stars, so far as we are aware, have rotation. If, therefore, a star has internal convective motions, it follows that there is dynamo activity. There should be nonuniform rotation and cyclonic convective cells, which are sufficient to form a dynamo. But there is a greater variety of possibilities than in the incompressible convecting core of Earth.

Consider first the nonuniform rotation. In the core of Earth we believe that the nonuniform rotation is purely a matter of Coriolis forces (conservation of angular momentum) in the convection, with the result that the angular velocity ω (considered a positive quantity) decreases outward, $d\omega/d\varpi < 0$. The same may occur in a star. But there is the additional possibility that $d\omega/d\varpi > 0$, as is suggested by the rapid rotation of solar equatorial regions. The question of whether the rotation increases or decreases outward is presently debated for the Sun. Some observational evidence (Maunder 1907; Adams 1908; Minnaert 1946; Livingston 1969) on the forward tilt of sunspot fields, the rate of rotation of the chromosphere, and the differential rotation of the solar photosphere suggest that $d\omega/d\varpi > 0$. On the other hand, Dicke (1964) and Haurwitz (1968) argue that the deep interior of the Sun still maintains much of the initial rapid rotation of the Sun, while the outer layers have been decelerated to their present slow rate of rotation by the solar wind (Brandt 1966, 1967; Weber and Davis 1967), in which case $d\omega/d\varpi < 0$. More recently Leighton (1969*a*) has pointed out that if the cyclonic convective motions are to be identified with the emergence and tilt of bipolar magnetic regions, then the magnetic behavior of the Sun requires $d\omega/d\varpi < 0$. The question of the sign of $d\omega/d\varpi$ is clearly open at the present time, and of fundamental importance.

Next consider the sense of rotation of the convective motions. The convective zone of the Sun occupies the outer 10^5 km, with the most rapid convection just beneath the surface. The convection is driven by heat from the interior of the Sun. Babcock (1961) and Leighton (1969*a*) suggest that the buoyancy of the magnetic fields themselves (Parker 1955*a*) may be the dominant factor in the vertical cyclonic displacement of the toroidal field. In either case, we expect the central region of a convective cell to move upward, with the return flow spread more broadly around the region outside. Thus we expect the fluid motion to converge at the bottom of the cell to form the vertical flow, and to disperse at the top. On this basis alone we would expect the upward-moving fluid to rotate faster than the Sun as a whole (as in Fig. 1 for Earth), so that in the northern hemisphere the cyclonic rotation is counterclockwise, as viewed from above. But there is an additional effect which complicates the picture. We must remember that the scale height of the gas density in the Sun is only a few hundred kilometers, and any upward motion over distances of a scale height or more is basically a diverging flow because of the rapid expansion of the upward-moving gas. In this case the cyclonic rotation is produced mainly by the diverging flow, so that the rotation relative to the surroundings is opposite to that of the Sun (Steenbeck, Krause, and Radler 1966). The cyclonic motion is clockwise in the northern hemisphere. It is interesting to note that this is the sense of rotation of bipolar regions in the Sun. We might guess that it is the sense of rotation of upwellings of gas and field in all stars.

Now consider the possibilities for a dynamo. If $d\omega/d\varpi < 0$ and the cyclonic convective cells are of small enough vertical scale to have a sense of rotation the same as the star (as in Fig. 1), then there is the possibility that the star has a stationary field, presumably a dipole as does the Earth. Or if $d\omega/d\varpi > 0$ and the cyclonic convective cells are of large enough vertical scale to have the opposite sense of rotation, the star may have a stationary field. In either case the field would be roughly aligned with the axis of rotation of the star, just as the field of Earth is within about 11° of the axis of rotation. On the other hand, the opposite combination of $d\omega/d\varpi < 0$ with large-scale vertical convection rotating opposite to the star, or $d\omega/d\varpi > 0$ with small-scale vertical convection rotating in the same direction as the star, is degenerative rather than regenerative, and there is no possibility for a general *stationary* magnetic field. So we wonder what other solutions there might be to the dynamo equations besides the stationary dipole field. To treat the simplest case, note that the convection in the Sun and in many other stars is confined to a relatively thin layer of the photosphere, rather than throughout a sphere, or thick spherical shell, as in the core of Earth. The curvature of a thin shell is unimportant, suggesting that we consider a rectangular, rather than spherical, geometry in which z represents the vertical direction and y the direction of the shear velocity (nonuniform rotation). Then equations (14) and (16) reduce to

$$\left(\frac{\partial}{\partial t} - \eta\nabla^2\right) B_y = B_z \frac{dV}{dz}, \quad \left(\frac{\partial}{\partial t} - \eta\nabla^2\right) A_y = \Gamma B_y ,$$

where the poloidal field is represented by

$$B_x = -\frac{\partial A_y}{\partial z}, \quad B_z = +\frac{\partial A_y}{\partial x},$$

and the toroidal field is B_y. The fields are independent of y, as they were independent of ϕ in the spherical geometry. In the simplest case we suppose that Γ and dV/dz are constants. Then if we write

$$B_y = C_1 \exp\,(t/\tau + ik_x x + ik_z z) ,$$

$$A_y = C_2 \exp\,(t/\tau + ik_x x + ik_z z) ,$$

the dispersion relation follows as

$$\left[\frac{1}{\tau} + \eta(k_x^2 + k_z^2)\right]^2 - ik_x\Gamma\frac{dV}{dz} = 0 \tag{17}$$

so that

$$\frac{1}{\tau} = -\eta(k_x^2 + k_z^2) \pm \left(\tfrac{1}{2}k_x\Gamma\frac{dV}{dz}\right)^{1/2}(1+i)\,. \tag{18}$$

For given values of k_x and k_z there is a regenerative and a degenerative mode, depending upon the $\pm$. If

$$\left|\tfrac{1}{2}k_x\Gamma\frac{dV}{dz}\right|^{1/2} > \eta(k_x^2 + k_z^2)\,,$$

then the generation of field exceeds the dissipation and the amplitude of the regenerative mode grows exponentially with time. Note that there is a regenerative mode for either sign of $\Gamma dV/dz$, unlike the stationary dynamo in spherical geometry. The regenerative mode propagates in the negative x-direction if $\Gamma dV/dz > 0$, and in the positive x-direction if $\Gamma dv/dz < 0$. The velocity of propagation is proportional to $[(\Gamma/2\ k_x)dV/dz]^{1/2}$. We have referred to the process as a migratory dynamo and to the fields as *dynamo waves* (Parker 1955*b*, 1957). We suggested at that time that the magnetic fields of the Sun are dynamo waves. The toroidal field B_y is the field from which the sunspots are formed. The poloidal field is a little harder to identify, being weaker and more disordered by local disturbances. Babcock (1961) and Leighton (1969*a*) both suggest that the poloidal field is to be identified with the fields of bipolar magnetic regions and sunspots, in which the leading region is invariably closer to the equator than the following region, thereby giving a poloidal (north-south) component to the field. The solar fields tend to migrate toward the equator. This follows from the dynamo equations if we suppose, after Leighton, that the cyclonic rotation is that represented by the north-south tilt of the bipolar regions (a clockwise rotation in the northern hemisphere) together with $d\omega/d\varpi < 0$. Thus, if $d\omega/d\varpi > 0$, (see discussion above) we could not understand the migration of the solar fields without giving up the identification of the poloidal field with bipolar magnetic regions. The implication that $d\omega/d\varpi < 0$ has important consequences for ideas on the evolution of the Sun over its lifetime and on the present state of rotation of the deep interior (Dicke 1964; see also the counterarguments of Goldreich and Schubert 1969). The actual fitting of solutions of the dynamo equations into a fixed and finite geometry is a more difficult problem than the unbounded problem employed here to illustrate the dynamo waves. It is necessary to solve the quartic dispersion relation for k_x for a given τ.

Leighton (1969*a*) has recently undertaken an extensive exploration of the migratory dynamo of the Sun. He sets up the dynamo equations for a thin spherical shell, representing the solar photosphere and the region of active convection beneath. He assumes that the upwelling of the toroidal field is principally the result of magnetic buoyancy, so that Γ is a function of B_ϕ. To keep the calculations tractable he supposes that Γ is zero for B_ϕ below a certain value B_c and has a finite fixed value for $B_\phi > B_c$. He introduces the idea that the principal loss to the toroidal field is through magnetic buoyancy, so that the toroidal field eventually floats up through the photosphere and is lost to the solar dynamo. The implications of this important idea need to be examined both theoretically and observationally. Another important point which he introduces (Leighton 1964) is the diffusion of the vertical field by the random walk of the supergranule motions. His observational estimates for the dispersal of the field by a random walk suggest an effective diffusion time T_D of 20 years, equivalent to a diffusivity $\eta \cong R_\odot^2/T_D \cong 10^{13}$ cm^2 sec^{-1}, or some 10^6 times larger than resistive diffusivity $c^2/4\pi\sigma$. (The other dissipation terms, of the form B_ϕ/T, in Leighton's equations are arbitrarily introduced to damp out the effects of the initial conditions, and thereby save computer time; they have little effect on the final long-term operation of the dynamo [Leighton 1969*b*].)

These two dissipation mechanisms are of fundamental importance, because without them there is no migratory solution of the dynamo equation inside fixed boundaries: If there were nothing but resistive diffusivity, then $\eta \cong 0$ and there is only one value of k_x which satisfies equation (17) or (18) for a given τ; one value of k_x is not enough to fit the four boundary conditions.

Leighton solves the dynamo equation numerically for a variety of forms and strengths of the nonuniform rotation, critical field B_c, and rate of sunspot tilt. Not only does he succeed in duplicating Spörer's and Maunder's butterfly diagram for the distribution and migration of sunspots, but at the same time the polar fields appear in about the right strength, reversing at about the peak of the sunspot cycle in agreement with observation. No less interesting is the variety of different dynamo behavior that turns up for other less realistic values of the parameters, such as a mode which begins with B_ϕ an odd function of latitude, as is normally observed, slowly evolving into an even function. The radial field changes from dipole to quadrupole. This solution is particularly interesting in view of the strong asymmetries that exist between the northern and southern hemispheres of the Sun during some sunspot periods. And further it suggests the large number of possibilities that exist for the Sun in other epochs and for other stars with the complex field variations. I strongly recommend a careful reading of Leighton's numerical results to the astronomer interested in stellar fields. The outstanding question is the actual sign of $d\omega/d\varpi$ beneath the surface of the Sun.

Altogether, it is my impression that we may now understand the physical origin of the magnetic fields of both Earth and the Sun, the latter subject to clarification of the sign of $d\omega/d\varpi$ by independent means. The geomagnetic field is a stationary dynamo, and the solar field is a migratory dynamo. Both arise from the combined effects of Coriolis forces and convection. Both are actively generated at the present time, and the field today is essentially independent of what magnetic fields were present 10^4 years ago in Earth and 10^2 years ago in the Sun. There lies ahead of us the task of working out the more detailed questions, such as the effective diffusivity, the upwelling and cyclonic rotation of the toroidal field in the Sun, the occasional reversal of the geomagnetic field, etc. And there is a lot of work yet to be done in generalizing the dynamo equation (16), and in exploring the whole range of fluid motions that can regenerate a magnetic field regardless of applicability to stars and planets. I think we can tackle these questions with confidence in the correctness of our basic picture of the origin of the fields, and it remains for us to muster enough energy and imagination to see how the questions still outstanding are to be fitted onto the basic picture.

So far as the fields of other stars are concerned, I think that we can proceed only a posteriori. When enough observational information becomes available to give a unique observational model for the field of a star, then, to the degree that the model is complete, we can hope to work out what combination of $d\omega/d\varpi$ and cyclonic convective motions would give such a field. We cannot hope to identify the convective motions with detailed phenomena (bipolar sunspots and magnetic regions) as we can on the Sun. One question that arises immediately concerns the difference between stationary and migratory stellar dynamos. For instance, how are we to distinguish a migratory dynamo with a period of 100 years from a quasi-stationary dynamo? It is evident that enough observational information to give a clear and unique picture of the magnetic field and internal dynamo and of any star but the Sun will be an enormous undertaking, requiring ingenuity, vigor, and patience. The assumptions that one might make, in setting up a dynamo to duplicate a particular stellar field, will involve the magnitude and sign of the product of $d\omega/d\varpi$ and the sense of cyclonic rotation, questions which are not unrelated to the past evolution and present state of the interior of the star. It is interesting to note that if magnetic buoyancy (Parker 1955*a*) is primarily responsible for the cyclonic convective motion in the solar dynamo, as suggested by Babcock (1961) and Leighton (1969*a*), then there is the possibility that strong nonuniform rotation alone is a *sufficient* condition for dynamo activity, without a naturally occurring convective zone. The nonuniform rotation generates a toroidal field from whatever poloidal field is present. The toroidal field then "bootstraps" itself to form a weak poloidal field. The convective zone of the Sun may be of only secondary importance, enhancing η but not driving the system. The basic source of energy is the nonuniform rotation. The common occurrence of strong fields in A stars would seem to support this general idea.

The fields of other planets in the solar system are of prime interest today. So far we know that Mars, Venus, and the Moon have little, if any, field generated in their interiors, presumably because they do not have convecting liquid cores (Mars and the Moon), or because their rate of rotation is so slow (Venus). Jupiter promises to be much more interesting, its radio emission suggesting a strong dipole field of 10 gauss or more at the surface of the planet. Such a field is not unexpected in view of the rapid rotation of the planet and the possibility of a conducting fluid core (see, for instance, Smoluchowski 1967).

Finally, in closing, let me remark that the origin of the galactic field, which we discussed only very briefly in § IV, is quite a different question from the origin of the fields of planets and stars. Whatever the origin of the magnetic flux that makes up the galactic field, its present orientation in the azimuthal direction is the result of the nonuniform rotation of the Galaxy, and its present strength is controlled by several factors, involving the hydrostatic balance between cosmic-ray pressure and the gravitational acceleration perpendicular to the disk of the Galaxy (Parker 1969*a*). There is no observational evidence for ordered motions of the gas in the disk of the Galaxy which might make up a suitable dynamo to generate the magnetic flux. We have speculated that the flux was generated by random turbulence (Parker 1969*b*), based on investigation of an idealized form of turbulence. But the formal theory for the magnetic field in real turbulence is not worked out yet. So for the present it is my opinion that we do not really know the origin of the galactic field. Observations are only beginning to grope their way toward unambiguous measurements of its strength and dynamical properties.

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