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THE ORIENTATION OF MAGNETIC AXES IN THE MAGNETIC VARIABLES

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ABSTRACT

The distribution of magnetic field over the surface of a star required to reproduce observed magnetic curves of Ap stars on the rigid-rotator model is discussed, and it is shown that the field distribution may be taken to be axisymmetric but is not necessarily antisymmetric to reflections through the magnetic equator. A statistical method for determining the angle β between the rotation and magnetic axes, devised by Preston, is extended to the class of models considered here, and it is shown that most magnetic stars have β near 90° independently of the precise field distribution over the surface which is assumed.

I. INTRODUCTION

In a recent paper, Preston (1967*c*) has argued on statistical grounds that if periodic magnetic variables can be described by the oblique-dipole-rotator model, then it is a general characteristic of these magnetic variables that their magnetic and rotation axes are approximately orthogonal. It is known that some of the magnetic stars have magnetic curves which cannot be described accurately with a dipole distribution of the magnetic field over the surface of the star, and it is the purpose of this paper to discuss the field distributions which satisfy the observations, and to show that Preston's conclusion is not dependent on his assumption of dipole-field distribution.

II. SURFACE DISTRIBUTION OF THE MAGNETIC FIELD

Consider first the distribution of the magnetic field over the stellar surface which must be assumed to reproduce the observations with the rigid-rotator model. Three investigations of this subject have been made. Stibbs (1950) used a dipolar-field distribution, and Böhm-Vitense (1965) assumed a field distribution rather like that of a dipole but with the greatest effective field strength at about 35° from the magnetic poles. In the case of such stars as α^2 CVn and 53 Cam, these models do not reproduce the observed magnetic curves at all well (Böhm-Vitense 1965, Figs. 7 and 12). Deutsch's (1958) harmonic analysis of HD 125248, on the other hand, is rather successful at reproducing the observed magnetic curve, but the inferred field distribution is complicated and does not lend itself readily to generalization.

We may infer from the observed magnetic curves some general properties which the postulated magnetic-field distributions must have to account for the observations. The situation in an oblique rotator is shown in Figure 1, where the vectors $\boldsymbol{m}$, $\boldsymbol{\omega}$, and $\boldsymbol{s}$ are, respectively, the axis of the magnetic field, the axis of rotation, and the line of sight. As the star rotates, $\boldsymbol{s}$ describes a cone about ω, intersecting the surface of the star along the circle *abcda*. For each point s at which $\boldsymbol{s}$ intersects the surface of the star, there is an effective magnetic field $H_e(s)$ seen by the observer, which is approximately the average over the visible hemisphere of the star of $\boldsymbol{H}\cdot\boldsymbol{s}$, weighted (by limb darkening) according to local brightness. Here $\boldsymbol{H}$ is the local magnetic-field strength. For convenience measure s from the axis $\boldsymbol{m}$ with spherical polar coordinates θ_m, ϕ_m. If $H_e(s)$ is cylindrically symmetric about the axis $\boldsymbol{m}$, and hence a function of θ_m only, the magnetic curve will be symmetric (even) under reflections about the vertical axes at the phases $2\pi t_a/P$ and $2\pi t_c/P$, corresponding to points a and c, half a cycle apart, as shown on the magnetic curve in Figure 1. From an examination of the observations, it is clear that all the magnetic stars for which a magnetic curve has been well established possess two axes of reflection symmetry separated by half a cycle, within the limits set by the scatter of the data. It is therefore possible to represent all the observations with cylindrically symmetric fields, and this paper will be restricted to a consideration of such fields. (It was kindly pointed out to the author by the referee that fields which are symmetric to reflection through the $(\boldsymbol{m}, \boldsymbol{\omega})$-plane but are not cylindrically symmetric also result in magnetic curves having the symmetry described above. As it is not necessary to invoke such fields to satisfy the observations dealt with in this paper, they will not be considered further here.)

It thus remains to choose a magnetic-field distribution $\boldsymbol{H}$ which is a function only of polar angle from $\boldsymbol{m}$, so that H_e will be a function of θ_m only. In hope of further simplifying matters, one may ask if it is consistent with observations to choose a field distribution

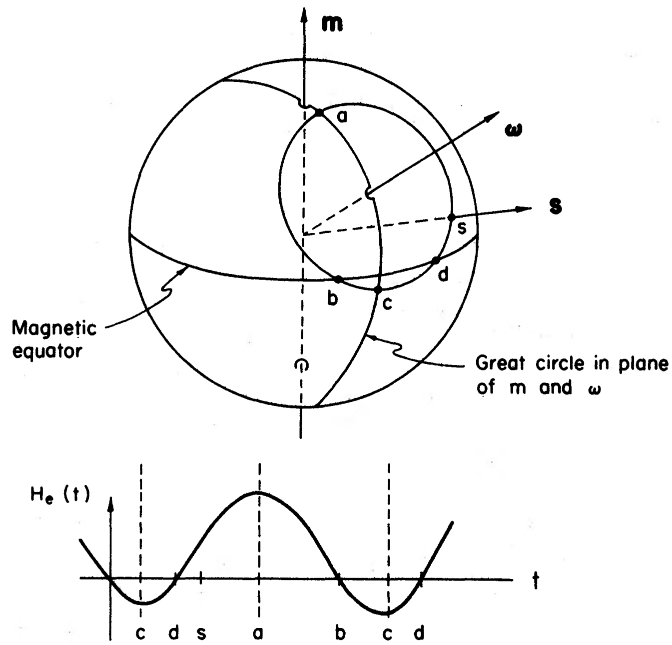

FIG. 1.—A magnetic rigid rotator and the resulting magnetic curve. Figure is described in the text.

such that H_e is antisymmetric to reflection through the magnetic equator, that is, $H_e(\theta_m) = -H_e(\pi - \theta_m)$. (The dipole distribution and the field distribution used by Böhm-Vitense (1965) are examples of such fields.) In this case, $H_e = 0$ when $\boldsymbol{s}$ passes through the magnetic equator. It is clear from Figure 1, if H_e is assumed to have constant sign on each hemisphere, that the vector $\boldsymbol{s}$ departs farther from the equator during the part of a cycle *dab*, and that this part of the cycle also lasts longer than the part *bcd*. Thus, the absolute value of the maximum H_e reached during the broader peak of the magnetic cycle must be at least as large as that reached during the narrower peak. This condition is fulfilled for most observed magnetic curves, but in the case of α^2 CVn and 53 Cam, as well as the Mount Wilson (but not the Lick) observations of HD 153882, this condition is violated. We therefore infer that not all magnetic stars can be described by a field H_e which is antisymmetric under reflections through the magnetic equator, so that at least some magnetic stars have one magnetic pole stronger than the other. This is physically quite interesting, for if it is true of most magnetic stars, it might lead to an explanation as to why events such as light and spectrum variation, which seem to be synchronized with magnetic variations, depend on the sign of H_e and not merely on its magnitude (cf. Ledoux and Renson 1966, p. 320). It is also worth noting that the models of Böhm-Vitense (1965), which are equatorially antisymmetric, are least successful for precisely those two stars, 53 Cam and α^2 CVn, for which the asymmetry seems to be most pronounced, as evidenced by a narrow peak in the magnetic curve, which has a height equal to or greater than that of the broad peak.

The most obvious axisymmetric field distribution with unequal pole strengths is that produced by a dipole field displaced along its axis of symmetry. Catalogs of magnetic curves were therefore constructed for various angles i (between the rotation axis and the line of sight) and β (between the rotation and magnetic axes) by using the prescription of Stibbs (1950) for the following four fields: an undisplaced dipole-field distribution, Böhm-Vitense's field distribution, and two fields produced by dipoles displaced by fractions a of the stellar radius of 0.345 and 0.668 along their axes of symmetry. Limb darkening is assumed to vary as $1 - 0.5(1 - \cos \phi)$, where ϕ is the colatitude measured from s. It is found that all known magnetic curves can be fitted satisfactorily, within the

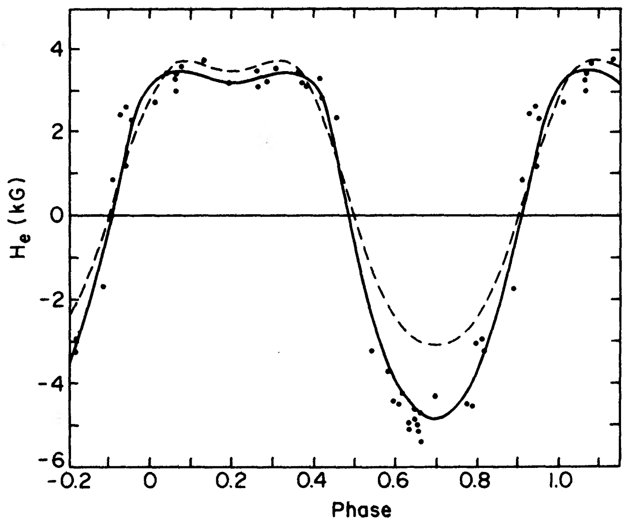

FIG. 2.—A comparison of the observed magnetic curve of 53 Cam with the theroetical curve (*solid line*) generated from the dipole displaced by $a = 0.668$, with $\beta = 80°$ and $i = 60°$. The best fit of Böhm-Vitense is shown as a dashed curve.

scatter of the data points on the magnetic curve, by at least one curve from the set generated as described above. A good fit to 53 Cam is shown in Figure 2.

Generally, several magnetic curves generated from different field distributions and values of i and β fit a given magnetic curve about equally well, so that one cannot usually infer i and β from the magnetic curve. However, if i is known to be small from the line widths and rotation period for a star with a reversing field, a lower limit to β may be found. The reason is geometrical: if i is small, one sees only values of H_e from a small cone surrounding the rotation axis; and if the star reverses polarity, the colatitude θ_m from $\boldsymbol{m}$ having $H_e = 0$ must pass through this cone. For all four models considered here, H_e is 0 for a value of θ_m within 10° of $\theta_m = 90°$, and hence the magnetic axis must lie roughly at right angles to the observed cone and to the rotation axis. The three stars for which a significant lower limit on β may be found in this way are listed in Table 1. In computing i from $v_e \sin i$ and v_e (where v_e is the equatorial velocity of the star), we determined the values of $v_e \sin i$ for HD 8441 and HD 10783 by using Babcock's (1958) "index of line width" w, together with the calibration for this quantity $v_e \sin i \simeq 50w$ (km sec^{-1}) recently derived by Preston (1969) from a comparison of calculated rotationally broadened line profiles with line profiles measured from coudé plates of a number of stars measured by Babcock. According to Preston, deviations from this calibration seem to be generally less than about ± 20 percent. The radii used in determining v_e were found by assuming the stellar radius to be the same as that of a main-sequence star of the same $B - V$. Since blanketing (Wolff 1967) and interstellar reddening both tend to redden Ap stars relative to normal main-sequence stars, and since Ap stars may be slightly evolved, this leads to an underestimate of the stellar radius and of v_e and hence to a slight overestimate of i. In the case of β CrB, $v_e \sin i$ was taken from Preston (1967*b*), and the deblanketed color index (Wolff 1967) and absolute visual magnitude from parallax measurements (Jenkins 1952) were used to find v_e. The value of β^* given in the last column of Table 1 is the lower limit to β, found by fitting various computed magnetic curves, with i less than or equal that given in Table 1, to the observed magnetic curves and then finding the smallest β which gives a satisfactory fit. As expected, β^* is fairly large for all three of these stars.

It is worth noting that Preston's calibration of Babcock's w leads to larger values of $v_e \sin i$ and hence to larger values of i than have usually been assumed for the magnetic

TABLE 1

STARS FOR WHICH A LOWER LIMIT TO β CAN BE DETERMINED

Star	$v_e \sin i$ (km sec^{-1})	v_e (km sec^{-1})	i	β^*
HD 8441.............	4	29	8°	80°
HD 10783............	15	24	40°	50°
β CrB................	3	6	30°	70°

stars. This change seems to be great enough to eliminate the problem of whether Ap stars are seen preferentially pole-on (cf. Preston 1967*a*); according to calculations of i for a sample of about a dozen Ap stars made as described above, the distribution of observed values of i now resembles the expected sin i distribution.

III. THE STATISTICAL DISTRIBUTION OF β

Since it is not possible with the available data to derive a unique value or a limited range of β for most individual magnetic stars, one must resort to statistical methods to obtain further information.

Preston (1967*c*) has shown that the distribution of the quantity r (the ratio of the value of H_e at the smaller extremum of the magnetic curve to that of the larger extremum) predicted by the inclined-*dipole*-rotator model depends strongly on the assumed value of β, in the sense that $\beta \sim 0°$ leads to a predominance of values of r near $r = +1$, while $\beta \sim 90°$ leads to a distribution of r peaked near -1. With the help of the magnetic curves calculated for the various field distributions described in the preceding section, one can easily find $r(i)$ for each nondipolar model and for various values of β; and then by assuming a probability distribution of i going as sin i, one can calculate the probability distribution of r. The functions $r(i)$ are shown in Figure 3 for the four models discussed here and $\beta = 20°$, 40°, 60°, and 80°, and the resulting probability distributions of r (which were calculated numerically for intervals $\Delta r = 0.1$) are shown in Figure 4. The probability distributions are normalized to equal areas under the four curves in each box. It is at once clear that the distribution of r is quite similar for the four fields considered.

The observed distribution of r for all known periodic variables is shown in Figure 5. Also included in the histogram, to supplement the statistics, are values of r for stars not known to be periodic but for which more than ten magnetic observations have been made. The values of r for the periodic variables (*unshaded bars*) are measured on the mean magnetic curve; for the other stars the value of r is the ratio of the extreme mea-

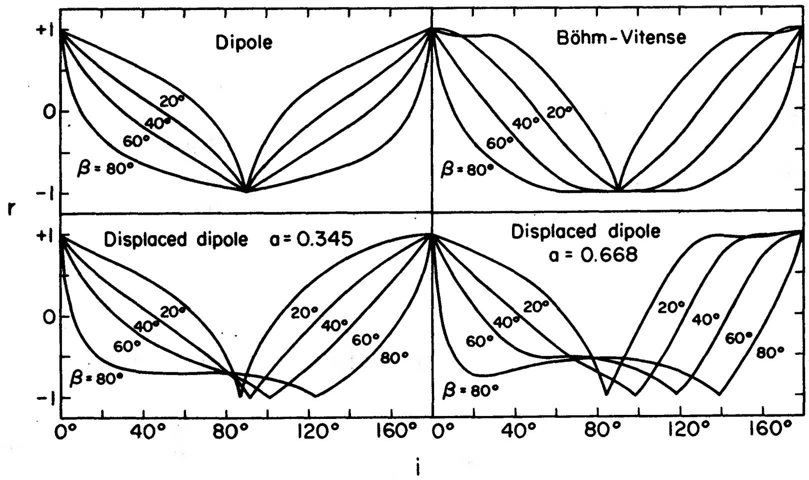

FIG. 3.—The function $r(i)$ for the four models considered, for $\beta = 20°, 40°, 60°$, and $80°$

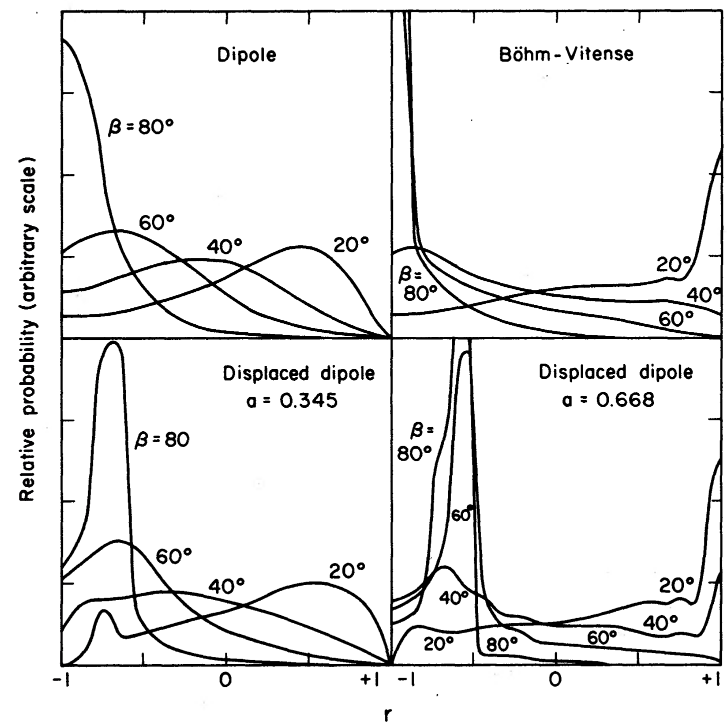

FIG. 4.—Probably distribution of r for $\beta = 20°, 40°, 60°$, and $80°$ for each of the four models considered

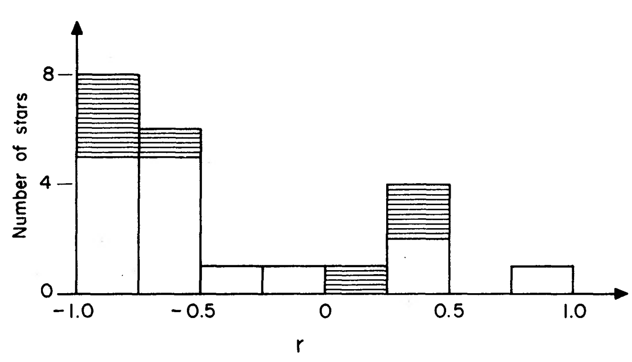

FIG. 5.—Distribution of r for all periodic magnetic variables with published magnetic curves (*unshaded bars*) and all other Ap stars for which at least ten measurements of H_e have been made (*dashed bars*).

sured H_e's given in Table V of Ledoux and Renson (1966). The values of r derived for these two groups of stars are not precisely comparable, as periodic variables all exhibit some scatter about their mean curves which is ignored in finding r for a mean curve but which clearly affects values of r derived from extreme values of H_e, especially for $r > 0$. In these cases the value of r may be determined primarily by scatter in the measurements rather than by the real magnetic curve, a method which would lead to anomalously small values of r.

It is clear from comparison of Figures 4 and 5 that most well-observed magnetic variables must have β greater than about 60° independently of the precise description of the magnetic-field distribution. However, there are enough stars with $r > 0$ so that one cannot conclude that almost all magnetic variables have β within a few degrees of 90°. An examination of the curves of Figure 4 will show that if all the stars observed have $\beta \gtrsim 80°$, then less than 5 percent of the total, or about one star, should have $r > -0.25$, whereas we find seven stars in this region. As four of these stars are periodic with well-defined mean curves, their locations in the distribution cannot be ascribed to

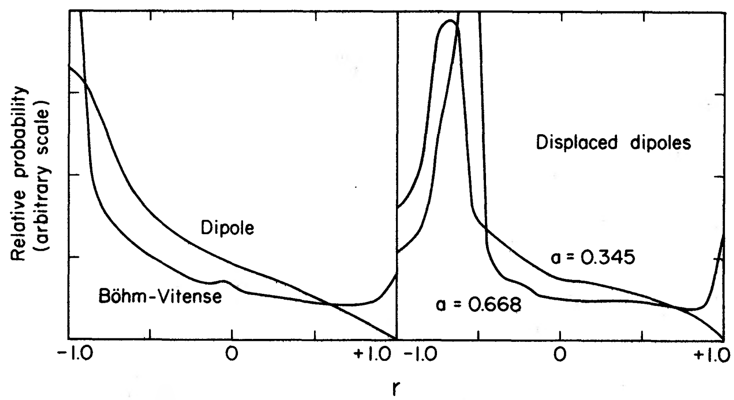

FIG. 6.—Approximate probability distribution of r for the four models considered, under the assumption that m has a random direction with respect to ω.

scatter in the data; it thus seems clear that some stars with β substantially different from 90° exist.

It has been shown above that Preston's conclusion that most magnetic variables have β near 90° does not depend on the precise field geometry, although "near 90°" has been weakened to mean "greater than about 60°." An examination of the curves of Figure 3 makes it clear also that this conclusion is not sensitive to the precise distribution of i, as long as it is not strongly peaked near $i = 0°$. Any other reasonable distribution of i still gives rise to probability distributions in r peaked near $r = -1$ if $\beta \sim 90°$ and near $r = +1$ if $\beta \sim 0°$.

One distribution of β which satisfies the requirement of being strongly peaked near $\beta = 90°$ which one would like to compare with observations is the distribution which would result from m being randomly oriented with respect to ω. The distribution of r resulting for this case is shown approximately in Figure 6 for the four fields considered. (The distribution shown is actually

$$p(r) = \sum_{i=1}^{4} \left[\int_{\theta_i}^{\theta_{i+1}} \sin\theta \, d\theta \right] p_i(r) ,$$

where the $p_i(r)$ for $i = 1, \ldots, 4$ are the probability distributions with $\beta = 20°$, 40°, 60°, and 80° shown in one panel of Figure 3, and the division points θ_i are 0°, 30°, 50°, 70°, and 90°.) It appears that the resulting distributions are in disagreement with the observed distribution of Figure 5 only in the region $-0.5 < r < 0.5$, where all the theoretical curves agree that there should be more stars with $r < 0$ than with $r > 0$, whereas the observations show more with $r > 0$. However, the number of stars in the observed sample with $r > -0.5$ (eight stars) is about the number predicted by the curves of Figure 6 (eight to ten stars), and the statistics in the subdivisions are poor, so that one cannot exclude the possibility that $\boldsymbol{m}$ has random orientation relative to $\boldsymbol{\omega}$, as it might well have if the fields of Ap stars are fossil fields.

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