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A STUDY OF THE ECLIPSING BINARY BETA AURIGAE

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ABSTRACT

The present study has led to the determination of the average effective temperature and surface gravity of the two components of the eclipsing binary system β Aur. By use of photoelectric spectrum scans in conjunction with detailed abundance analyses, an effective temperature $T_{\text{eff}} = 8750^\circ$ K and surface gravity $\log g = 3.7$ were derived. The abundance analyses indicate similar chemical compositions for both components. Both members of this system appear to show abundance anomalies reminiscent of mild Am stars.

I. INTRODUCTION

The eclipsing binary system β Aur provides the opportunity to derive the physical parameters, effective temperature, and surface gravity of a star by two independent methods. The system, a pair of A2 IV ($V = 1.90$, $B - V = +0.04$) stars (Popper 1959), has been the subject of several investigations (Stebbins 1911; Shapley 1915; Nekrasova 1936; Piotrowski 1948; Smith 1948). Most recently, Popper (1959), using a light curve measured by Stebbins (1911), an orbital solution by Piotrowski (1948), and a parallax by Jenkins (1952), derived an average effective temperature for the two components of β Aur of 10500° K and a surface gravity of $\log g = 4.0$. Because of the shallow eclipses and the consequent uncertainty of the orbit, and the uncertainty of the parallax, Popper concluded that his determinations could be subject to significant error. Further, the effective temperature derived for β Aur is quite inconsistent with the effective temperatures determined for Vega (9500° K) and Sirius (10300° K) by Hanbury Brown *et al.* (1967) from interferometric measurements. Because Vega is classified as A0 V with $B - V = 0.00$ and Sirius is classified as A1 V with $B - V = -0.01$, the higher T_{eff} derived for β Aur would seem to be inconsistent with the T_{eff} deduced for these stars. The possible error in the effective-temperature determination of β Aur may arise from any of the sources of error outlined above.

The present study to determine the physical parameters of β Aur was undertaken with the use of data of a different nature from those employed by Popper. From a comparison of photoelectric-spectrum scans of the star with stellar-atmosphere models, the possible range of effective temperatures and surface gravities was determined. Next, high-dispersion photographic spectra were used to obtain equivalent widths for a detailed abundance determination. The effective temperature and surface gravity used in the analysis were then confirmed by examination of the ionization equilibrium and plots of abundance versus excitation potential for iron, computed at various effective temperatures and surface gravities. A check of the surface gravity was also made by comparison of observed and computed hydrogen-line profiles (Olson 1968).

II. EFFECTIVE-TEMPERATURE AND SURFACE-GRAVITY DETERMINATIONS

To establish the effective temperature and surface gravity for the stars, we compared the average of seven photoelectric-spectrum scans of β Aur with the fluxes predicted from a grid of model stellar atmospheres. The scans, taken during noneclipse portions of the period, were made with an f/5 photoelectric-spectrum scanner (Code and Liller 1962) and a 20 Å band pass used at the Newtonian focus of the 61-inch Wyeth reflector of Harvard College Observatory. The scans were reduced with the use of individual extinction curves for each night's observations relative to β Tau using Bahner's (1963) magnitudes. An additional set of photoelectric scans was also made by B. Taylor, who used the Wampler scanner at the Lick Observatory Crossley reflector. The results of both sets of observations are presented in Table 1, with magnitudes per frequency interval given relative to the magnitude of 5560 Å. By comparing these scans with the hydrogen-line-blanketed models of Mihalas (1966) and using the absolute calibration of Vega proposed by Wolff, Kuhi, and Hayes (1968), we derived $T_{\rm eff} = 8750° \pm 400°$ K and $\log g = 3.6 \pm 0.2$. The surface gravity was confirmed by Olson (1968), who obtained a value of $\log g = 3.65$ for both components by using observed Hγ profiles compared with theoretically computed profiles based on the semiempirical approach of Edmonds, Schlüter, and Wells (1967) from calculations of Peterson (1967).

TABLE 1

BETA AURIGAE SCANNER MAGNITUDES*

Wavelength (Å)	Magnitude: Toy	Taylor
3400	+1.21	+1.17
3650........	+1.15	+1.09
4190.......	−0.24	−0.23
4590........	−0.19	−0.17
5060........	−0.08	−0.09
5560	0.00	0.00

* $m(\lambda)/m(5560$ Å$)$.

III. ABUNDANCE DETERMINATION

The abundance analysis was based on equivalent widths determined in the regions λλ3900–4500 from two high-dispersion (3 Å mm^{-1}) photographic spectra taken at the coudé focus of the 100-inch Mount Wilson reflector by Struve (Struve and Driscoll 1953). The David Mann microdensitometer of the Harvard College Observatory Shock Tube Laboratory was used to reduce the spectra. Output appeared in density in both analogue and punched-card forms. To minimize errors in the reduction from density to log intensity caused by nonuniformities in the plates, we took calibration measurements for every 10-mm (30 Å) region of the plates. We obtained equivalent widths by iteratively fitting theoretical line contours to the observed points in log intensity by employing a least-squares technique, and then by numerically integrating the contours in intensity. Our line-fitting technique was necessitated by the numerous blended profiles in the spectrum caused both by the small difference between the radial velocities of the two components and by the fact that the line profiles for each component are rotationally broadened. The least-squares solution as coded for the CDC 6400 digital computer is described elsewhere (Grasdalen and Toy 1969).

We estimate the errors in the calculated equivalent widths as $0.15\ W_\lambda$ for $W_\lambda > 150$ mÅ, and $0.30\ W_\lambda$ for $W_\lambda < 150$ mÅ. The derivation of the equivalent widths for the two stars from the combined spectrum was facilitated by the assumption that the two stars have equal luminosities at all wavelengths. This assumption is supported by the distribution of points in Figure 1, where the difference between equivalent widths for the same lines in each component is plotted against wavelength.

The abundance determination was performed by the method described by Strom, Gingerich, and Strom (1966). In this procedure a model atmosphere and assumed abundance are used to calculate for each line the line opacity for a series of optical depths. A theoretical profile is computed, and the equivalent width is then calculated. The abundance is varied until the observed equivalent width is bracketed by theoretical values.

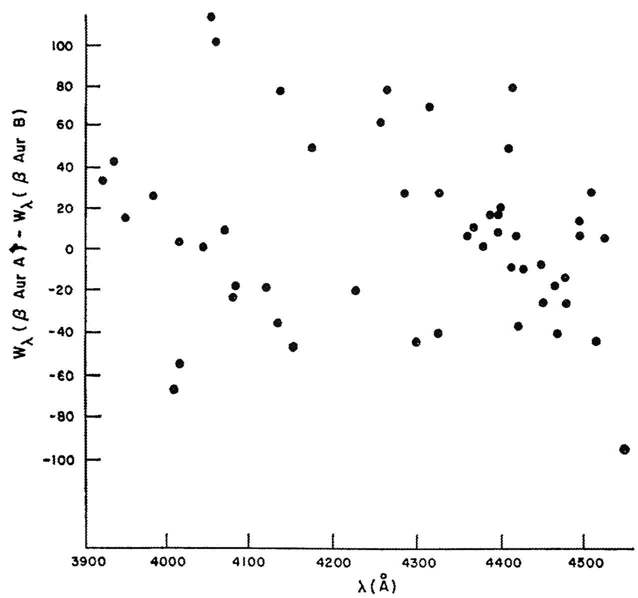

FIG. 1.—Comparison of equivalent widths (in Å) of lines measured in both β Aur *A* and *B* as a function of wavelength.

TABLE 2

IONIZATION EQUILIBRIA PREDICTED FOR IRON FOR VARIOUS VALUES OF T_{eff} AND log *g*

		ABUNDANCES*			
		β Aur *A*		β Aur *B*	
T_{eff}(°K)	log *g*	Fe I	Fe II	Fe I	Fe II
8750. . .	3.7	6.60	6.59	6.56	6.64
8500. . .	3.7	6.40	6.51	6.37	6.56
9000. . .	3.7	6.78	6.65	6.75	6.71
8750. . .	3.3	6.68	6.62	6.65	6.56
8750.	4.0	6.55	6.65	6.51	6.71

* These abundances for Fe I and Fe II are given as log (abundance), with H normalized at 12.

IV. RESULTS

For the abundance analysis a model atmosphere with the parameters $T_{\rm eff} = 8750°$ K and log $g = 3.7$ was chosen for both components from the analysis of the spectral scans. The accuracy of this choice of atmospheric parameters was further checked by examination of plots of the abundances deduced for individual Fe I lines against the lower excitation potential of the lines and of the ionization equilibrium for Fe I and Fe II as computed for trial effective temperatures and surface gravities in the range suggested by the spectral scans. Table 2 gives the ionization-equilibrium values calculated for effective temperatures and surface gravities bracketing the chosen values. The estimated error in $T_{\rm eff}$ is $\pm 250°$ K, and that in log g is ± 0.2.

We obtained a value for the microturbulent-velocity parameter v_t for each component by plotting the abundances deduced for each Fe I line against its measured equivalent width for various test values of v_t. In both cases, the value of v_t yielding the smallest slope in this relation was 8 km sec^{-1}. An error of ± 1 km sec^{-1} is possible in this choice of v_t.

The equivalent-width and abundance calculations for the lines observed in β Aur *A* and *B* are given in Table 3, followed by remarks concerning the choice of *gf*-values. Table 4 gives the average abundances for each element in each star and compares these values with those derived for normal A stars by Conti and Strom (1968*a*, *b*). The abundances are logarithmic relative to H = 12.00. A summary of the various temperature and surface-gravity determinations is given in Table 5.

The deduced abundances indicate the following anomalies with respect to the normal A stars: (*a*) slight underabundances of scandium and calcium (0.3 dex); (*b*) slight overabundance of zirconium (0.3 dex); (*c*) strong overabundance of barium (1.0 dex); (*d*) overabundance of nickel (0.6 dex); (*e*) underabundance of titanium (0.4 dex).

Aside from (*e*), these abundance anomalies are reminiscent of those associated with the classical Am and early-type analogues of Am stars, including a similarity to Sirius (Strom *et al.* 1966). Moreover, as is the case for most Am stars, the turbulent velocity deduced for each component of β Aur is large.

It is perhaps not surprising, however, that this system shows Am characteristics, since close A-type binary systems with slowly rotating components ($v \sim 30$ km sec^{-1} for both components of β Aur) are prime candidates for Am stars or early-type analogues of these stars.

V. CONCLUSION

The methods of determination of effective temperature and surface gravity made in this study indicate a self-consistent set of atmospheric parameters for β Aur. The values of $T_{\rm eff}$, we find, are consistent with the scale of $T_{\rm eff}$ for early A stars suggested by the values derived for Sirius and Vega. In view of these results, we suggest that further photometric study of the properties of this important system is definitely necessary.

I am indebted to Dr. Stephen Strom for suggesting the problem, for the model-atmosphere computer program, and for many useful discussions. The assistance of Mr. Gary Grasdalen in many phases of data reduction is also gratefully acknowledged, as is the observational assistance of Dr. William Liller and Mrs. Nancy Morrison. The Crossley scans were graciously made by Mr. Benjamin Taylor. Some of the computation time was supplied by the University of California Computer Center. This work was also supported in part by contract NGR 22-024-001 with the National Aeronautics and Space Administration.

TABLE 3

EQUIVALENT-WIDTH AND ABUNDANCE DETERMINATIONS FOR INDIVIDUAL LINES FOR BETA AURIGAE A AND B

Wavelength	Element and Ion	Excitation Potential	Multiplet Number	log gf	Ref*	β Aur A		β Aur B	
						W_λ	Abundance	W_λ	Abundance
4167 27	Mg I	4.33	15	-1 00	KO	34	7 13	37	7 15
4390 58	Mg II	9.96	10	-0 56	KO	100	7 79	85	7.67
4433 99	Mg II	9.96	9	-0 93	KO	69	7 92	106	8 21
3944 01	Al I	0	1	-0.82	CB	159	5 74	142	5 66
3961.52	Al I	0 01	1	-0 51	CB	213	5 69	187	5 56
4128 05	Si II	9.79	3	0 22	G	180	7 71	105	7 33
4130 88	Si II	9 80	3	0 40	G	141	7 38		
†4226 73	Ca I	0	2	0 17	KO	150	5 22	88	4 89
4283 01	Ca I	1.87	5	-0 37	CB	15	6 11		
4454 78	Ca I	1.89	4	0 36	KO	13	5 40		
4246.83	Sc II	0.31	7	0 09	CB	204	2 78	125	2 42
4320.74	Sc II	0 60	15	-0 32	CB	51	2 54	22	2 32
4374 46	Sc II	0 62	14	-0.55	CB	16	2 54	12	2 52
3913 46	Ti II	1 12	34	-0 24	WA	129	3 66	238	4 20
4012 37	Ti II	0 57	11	-1 68	WA	282	5 39	279	5 38
4053 81	Ti II	1 89	87	-0 88	WA			183	5 05
4163 64	Ti II	2 59	105	0 20	WA	183	4 42	136	4 20
4287 89	Ti II	1 08	20	-1 56	WA	174	5 15		
4290 23	Ti II	1 16	41	-0.79	WA			259	4 78
4294 10	Ti II	1.08	20	-0 90	WA	196	4 58	238	4.78
4301 93	Ti II	1 16	41	-1.11	WA			129	4 54
4312 86	Ti II	1.18	41	-1 06	WA	144	4.57		
4314.98	Ti II	1 16	41	-1 02	WA			192	4 74
4337 92	Ti II	1 08	20	-0.90	WA	103	4 14		
4367 66	Ti II	2 59	104	-0 39	WA	46	4 21		
4386 86	Ti II	2 60	104	-0.46	WA			121	5 16
4394 06	Ti II	1 22	51	-1 47	WA	22	4.22		
4395 03	Ti II	1 08	19	-0 50	WA	226	4 32	202	4 20
4395 85	Ti II	1.24	61	-1.53	WA			70	4 66
4399 77	Ti II	1 24	51	-1 06	WA	105	4 41		
4411 08	Ti II	3 09	115	-0 07	WA	63	4.35	12	4 00
4417.72	Ti II	1.16	40	-1 18	WA	22	3 90	30	3 94
4443 80	Ti II	1 08	19	-0 74	WA			72	3 77
4450.49	Ti II	1.08	19	-1 41	WA	31	4 13	54	4 30
4464 46	Ti II	1 17	40	-1 66	WA	60	4 66	72	4 75
4468 49	Ti II	1 13	31	-0 65	WA	204	4 39	217	4 45
4488 32	Ti II	3 12	115	0.01	WA	60	4 26	43	4 13
4501 27	Ti II	1 12	31	-0 79	WA	114	4.10	83	3 92
3916.42	V II	1 43	10	-0 85	WA	160	4 23		
3997 13	V II	1 48	9	-1 03	WA			114	4.21
4254 35	Cr I	0	1	-0 27	CB	100	5 47	70	5 27
4274 80	Cr I	0	1	-0 39	CB			135	5 77
4242 38	Cr II	3.87	31	-0.77	WA	93	5 11	112	5 13
4261.92	Cr II	3.86	31	-0 96	WA			123	5 37
4269.28	Cr II	3 85	31	-1 81	WA	27	5.54		
4275 57	Cr II	3.86	31	-1 08	WA			123	5 49
4285.21	Cr II	3 85	31	-1 39	WA			124	5.80
4055 54	Mn I	2 14	5	0 47	CB	43	5 14		

TABLE 3 (Cont.)

Wavelength	Element and Ion	Excitation Potential	Multiplet Number	log gf	Ref.*	β Aur A		β Aur B	
						W_λ	Abundance	W_λ	Abundance
3920.26	Fe I	0.12	4	-1 48	CW			70	6 51
3922.91	Fe I	0.05	4	-1 44	CW	97	6 60	61	6 35
3927 92	Fe I	0.11	4	-1 30	CW	262	7.25	220	7.11
3930.30	Fe I	0.09	4	-1.31	CW	63	6.26		
3997.39	Fe I	2.73	278	0.38	CW			97	6 52
4000.47	Fe I	2.99	426	-0 74	CW			64	7 58
4005.25	Fe I	1.56	43	-0.09	CW	252	6 95	319	7 33
4009.71	Fe I	2.22	72	-0.43	CW			18	6 40
4021.87	Fe I	2.76	278	-0 12	CW	94	7.02		
4045.82	Fe I	1.48	43	0.66	CW	367	6 64	202	5.94
4063.60	Fe I	1.56	43	0 44	CW	276	6 51	178	6 10
4066.98	Fe I	2.83	358	-0.23	CW	145	7.44	135	7 39
4067.98	Fe I	3.21	559	0.24	CW			72	6 79
4071.74	Fe I	1.61	43	0 42	CW	112	5 84	135	5.95
†4118.55	Fe I	3.57	801	1 21	CW	57	5 93	76	6 08
4143.42	Fe I	3.05	523	0.61	CW			37	6 03
4143 87	Fe I	1.56	43	-0.12	CW	148	6 52	186	6 69
4147 67	Fe I	1 48	42	-1.50	CW	102	7 61		
†4153.91	Fe I	3 40	695	0 47	CW	60	6.60		
†4154.81	Fe I	3.37	694	0 37	CW	43	6 53		
4181.76	Fe I	2 83	354	0 41	CW	76	6 41		
4235 94	Fe I	2.42	152	0 31	CW			130	6.56
†4238 82	Fe I	3.40	693	0 47	CW			37	6.40
†4247.43	Fe I	3 37	693	0 52	CW	30	6 27		
4260.48	Fe I	2 40	152	0 63	CW			177	6 44
4282.41	Fe I	2.18	71	-0.16	CW	94	6 67		
4307 91	Fe I	1 56	42	0.32	CW	297	6.72	229	6.45
4325 64	Fe I	1 61	42	0 36	CW	135	6 01	174	6 19
4383.55	Fe I	1.48	41	0 51	CW	225	6 18		
4404 75	Fe I	1.56	41	0 25	CW	189	6.33	112	5 96
4415 12	Fe I	1.61	41	-0.13	CW	105	6 33		
4447.72	Fe I	2.22	68	-0.58	CW	6	6.53	12	6 55
4459.12	Fe I	2.18	68	-0 50	CW	54	6.72	70	6 85
4476 02	Fe I	2.84	350	0 14	CW	57	6 53	82	6.72
†4485 68	Fe I	3.68	830	-0 20	CW	22	7.16	12	7 11
†4044 01	Fe II	5 55	172	-1 97	G	75	7 78	74	7 78
†4122.64	Fe II	2.57	28	-2.55	G	135	6 83	136	6 84
4173 45	Fe II	2 58	27	-2 01	WA			175	6.49
4178.60	Fe II	2.58	28	-2 00	WA	208	6 63		
4258.16	Fe II	2.70	28	-2 59	WA	94	6 73		
4296 57	Fe II	2.70	28	-2 36	WA	99	6.53	196	7 01
4303 18	Fe II	2.70	27	-2.00	WA			259	6 92
4351.77	Fe II	2.70	27	-1.76	WA	202	6.44	183	6 35
†4369.40	Fe II	2.77	28	-2.66	G			25	6.34
4416.82	Fe II	2.78	27	-2 09	WA	120	6.43	107	6.35
†4472.92	Fe II	2.83	37	-2 65	G	51	6 56	87	6 83
4489.18	Fe II	2.83	37	-2.23	WA	63	6.24		
4491 40	Fe II	2.85	37	-2 09	WA	187	6.80	75	6 20
4508.28	Fe II	2.85	38	-1 76	WA	109	6.09	150	6 29
4515.34	Fe II	2.85	37	-1 91	WA	123	6 30	114	6 26
4520.22	Fe II	2.81	37	-1.87	WA	127	6.27		
4359.58	Ni I	3.40	86	-0.09	C	43	6.67	30	6 56
4015.50	Ni II	4.03	12	-0.95	WA	200	5 98	256	6 21

TABLE 3 (Cont.)

Wavelength	Element and Ion	Excitation Potential	Multiplet Number	log gf	Ref.*	β Aur A		β Aur B	
						W_λ	Abundance	W_λ	Abundance
†4077 71	Sr II	0.	1	0.16	G	208	1 88	226	1 97
4374.94	Y II	0.40	13	-0.14	CB	114	5.17	97	5.07
4050.32	Zr II	0.71	43	-0.96	CB			202	4 48
4149 22	Zr II	0.79	41	-0.13	CB	115	3.18	163	3 51
4379 78	Zr II	1.52	88	-0.19	CB	50	3 37	30	3.21
4554.03	Ba II	0.	1	-0.55	CB	422	4 19	508	4 38
4238 38	La II	0.40	41	-0 82	CB	30	2.89		

*References for gf values
C Corliss (1965)
CB Corliss and Bozman (1962)
CW Corliss and Warner (1964)
G Groth (1961)
KO Kohl (1964)
WA Warner (1967)

†Corrections to gf values in Table 3
Lines marked with a dagger had a choice of possible gf values or were modified in value. The author thanks Mr. Gary Grasdalen for a valuable discussion that led to these choices.

Wavelength	Element	log gf	Ref.*	Remarks
4226 73	Ca I			KO was chosen because of possible errors in CB caused by self-absorption in resonance line Cf. Katterbach, K. 1964, Internal Report, Max Plank Institute MPI/PA/27/64
4118 55	Fe I	+0 31	to CW	Corrections to these lines were made as suggested by Huber, M., and Tobey, F. L. (1968, Ap. J., 152, 609), because of inappropriate normalization of gf values dependent on upper excitation potentials
4153 91	Fe I	+0 14	to CW	
4154.81	Fe I	+0.12	to CW	
4238 82	Fe I	+0.10	to CW	
4247.43	Fe I	+0.07	to CW	
4485.68	Fe I	+0.20	to CW	
4044.01	Fe II	+0.65	to G to convert to CW	Corrections vary because of differing lower excitation potentials. Cf. WA.
4122.64	Fe II	+0.96	to G to convert to CW	
4369.40	Fe II	+0.96	to G to convert to CW	
4472.92	Fe II	+0.96	to G to convert to CW	
4077.71	Sr II			G computed from Bates-Damgaard method chosen over CB because of possible errors caused by self-absorption in resonance line.

TABLE 4

ABUNDANCE VALUES OF BETA AURIGAE *A* AND *B* COMPARED WITH THOSE OF NORMAL A STARS

Element and Ion	Number Lines β Aur *A*	Number Lines β Aur *B*	β Aur *A*	β Aur *A*	Normal A Stars (Conti and Strom 1968*a*, *b*)
Mg I	1	1	7.13	7.15	7.4
Mg II	2	2	7.86±0.05	7.94±0.19	
Al I	2	2	5.71±0.02	5.61±0.04	5.7
Si II	2	1	7.58±0.15	7.33	7.8
Ca I	3	1	5.58±0.17	4.89	5.8
Sc II	3	3	2.62±0.06	2.42±0.03	2.8
Ti II	18	19	4.10±0.07	4.47±0.08	4.7
V II	1	1	4.23	4.21	3.5
Cr I	1	2	5.47	4.52±0.17	5.4
Cr II	2	4	5.28±0.16	5.45±0.10	
Mn I	1	..	5.14	..	5.2
Fe I	26	25	6.60±0.06	6.56±0.06	6.6
Fe II	13	12	6.59±0.08	6.64±0.09	
Ni I	1	1	6.67	6.56	6.0
Ni II	1	1	5.98	6.21	5.2
Sr II	1	1	1.88	1.97	2.1*
Y II	1	1	2.60	2.51	2.5
Zr II	2	3	3.32±0.03	3.73±0.28	3.0
Ba II	1	1	4.19	4.38	3.1
La II	1	..	2.89	..	3.0

* Scaled by *gf*-value difference between Corliss and Bozman (1962) and Groth (1961)

TABLE 5

COMPARISON OF T_{eff} AND log *g* DETERMINATIONS FOR BETA AURIGAE

Source	T_{eff}(°K)	log *g*
Popper (1959) (eclipse solution)	10500	4.0
Scanner	8700±400	3.6±0.2
Hβ and Hγ profiles (Olson 1968)	...	3.65
Ionization equilibrium	8750	3.7
Spectral type–luminosity class–absolute magnitude	8900	3.75
Final choice	8750±250	3.7±0.2

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