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ISOCHRONES, AGES, CURVES OF EVOLUTIONARY DEVIATION, AND THE COMPOSITE C-M DIAGRAM FOR OLD GALACTIC CLUSTERS

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ABSTRACT

Isochrones in the H-R diagram are computed in the range $4.5 \geq M_{\mathrm{bol}} \geq 2$ by using models due to Aizenman, Demarque, and Miller, and to Iben. The curves apply for chemical compositions of $X = 0.67$, $Z = 0.03$ for the first set of models, and $X = 0.708$, $Z = 0.020$ for the second.

The phase of hydrogen exhaustion on the evolving main sequence, and the shallow departure from the zero-age main sequence of tracks with $\mathfrak{M}/\mathfrak{M}_{\odot} \gtrsim 1.3$ cause adjacent isochrones to intersect one another near $M_{\mathrm{bol}} \simeq +3$. Observational data shown suggest that the predicted crossover effect may occur.

New curves of evolutionary deviation are calculated from the isochrones. Distance moduli for M67 and NGC 188, derived with these curves, are used to assign absolute luminosities to the cluster stars. Comparison of the cluster sequences, converted to M_{bol} and $\log T_e$, with the isochrones, with account taken of the chemical composition, gives ages of $(5.5 \pm 0.5) \times 10^9$ years for M67 and $(8\text{–}10) \times 10^9$ years for NGC 188.

A composite C-M diagram for six galactic clusters shows several new features, notable of which is the funneling of all giant branches into a small region of the H-R diagram which may be defined by the Hayashi fully convective track.

I. INTRODUCTION

The ages of M67 and NGC 188 are important in establishing the timing of events in the old galactic disk. NGC 188 is the oldest disk cluster found to date, and its age may be close to that of the disk itself (Sandage 1962*a*, Fig. 9).

A new discussion of ages is possible because of (1) more accurate photometric data and better distance moduli for M67 and NGC 188 (Eggen and Sandage 1964, 1969), and (2) new sets of evolutionary tracks (cf. Demarque and Larson 1964; Iben 1967; Aizenman, Demarque, and Miller 1969, hereinafter called ADM) for stars in the mass interval $1.5 \geq \mathfrak{M}/\mathfrak{M}_{\odot} \geq 0.8$ which covers the necessary range of the two clusters.

Isochrones in the H-R diagram can now be calculated by taking into account the difference in the shape of the track as a function of mass. Demarque and Larson (1964) were the first to do this for NGC 188 by using a series of models in the range $1.03 \geq \mathfrak{M}/\mathfrak{M}_{\odot} \geq 0.8$. Models for two sets of chemical compositions ($X = 0.67, Z = 0.03$; $X = 0.76$, $Z = 0.01$) were explored by these authors, who used opacities due to Keller and Meyerott (1955) and the rates of energy generation of Reeves (1965). Ages between 9×10^9 and 13×10^9 years were obtained by fitting the calculations to the older observations. The new observations (Eggen and Sandage 1969) place NGC 188 brighter by 0.10 mag in the $[M_V, (B - V)_{0,c}]$-diagram, which will decrease these ages by about 8 percent for the given chemical composition.

Iben (1967) calculated three evolutionary tracks for masses of 1.0, 1.25, and 1.5 $\mathfrak{M}_{\odot}$ by using the composition $X = 0.708$, $Z = 0.020$. The change of track shape with mass is particularly evident and appears to explain the observed difference in slope between the subgiant sequences of M67 and NGC 188. From his own fit of the observations to a calculated zero-age main sequence, Iben obtained $T = (11 \pm 2) \times 10^9$ years for NGC 188, and $(5.5 \pm 1) \times 10^9$ years for M67.

Neither of these sets of models predicted a gap on the evolving main sequence of NGC 188. However, new calculations by ADM, who used the opacities of Cox (1965), do give a gap in the range of absolute magnitude required by the NGC 188 observations. The predicted gap occurred near $M_{\rm bol} \simeq 4.4$ for all chemical compositions considered ($0.57 \leq X \leq 0.77$ for $Z = 0.03$; $0.64 \leq X \leq 0.74$ for $Z = 0.06$).

ADM gave two models for the same composition ($X = 0.67$, $Z = 0.03$) but with different masses (1.05 and 0.98 $\mathfrak{M}_\odot$), and, of course, with different luminosities at the start of the main-sequence phase ($M_{\rm bol} = 4.57$ and $M_{\rm bol} = 4.93$). The tracks had a different shape, as in Iben's models, and each showed the phenomenon of central hydrogen exhaustion. From their correlations of gap width with chemical composition, ADM give $Y = 0.38 \pm 0.02$ for M67, which, with $Z = 0.03$, leads them to 3.5×10^9 years for M67. For NGC 188 these authors quote 4.6×10^9 years.

We needed the curve of evolutionary deviation to analyze the observational data for M67 and NGC 188. The previous curve for old clusters (Sandage 1962*b*, Table 9) had been calculated by using only one evolutionary track, spread with the same shape over the ($M_{\rm bol}$, log T_e)-plane. The effect of different track shapes can now be taken into account, and we have constructed two sets of isochrones over the H-R diagram in the range $4.5 \geq M_{\rm bol} \geq 2$ from the models of Iben and ADM. A by-product is a new age determination of M67 and NGC 188. The effect of chemical composition on the estimates is discussed (§ IV).

II. CALCULATION OF THE ISOCHRONES AND NEW CURVES OF EVOLUTIONARY DEVIATION

Each of the evolutionary tracks was attached to the empirical zero-age main sequence (plotted in $M_{\rm bol}$, log T_e) by horizontal shifts, Δ log T_e, with the calculated luminosity, $L_{\rm bol}$, kept constant. The adopted main sequence is the compromise between that given by Sandage (1962*b*, Table 3) and that given by Morton and Adams (1968). The two agree quite well over the relevant range. Justification for shifting the model tracks from the calculated T_e to the adopted main sequence comes from Demarque's (1968, Figs. 10 and 11) demonstration that ages are insensitive to differences in model radii, provided that $L_{\rm bol}$ is given correctly.

Three models ($\mathfrak{M} = 1.0$, 1.25, and 1.5 $\mathfrak{M}_\odot$) are available from Iben's (1967) calculations, while two models with the same composition could be used from ADM. Figure 1 shows Iben's three models as dashed lines attached to the zero-age main sequence at $M_{\rm bol} = 2.93$, 3.85, and 5.10—values which follow by adopting $M_{\rm bol\odot} = 4.77$ (Morton and Adams 1968) and using Iben's (1967) values of $L/L_\odot$ read from his Figure 1.

Graphical interpolation between the three computed models permitted the shapes of twenty-eight additional tracks to be found. Such tracks were attached to the main sequence in Figure 1 at the appropriate $M_{\rm bol}$ at intervals of 0.1 mag between $6.40 \geq M_{\rm bol} \geq 2.93$. To avoid confusion, we have not shown these tracks.

An elaborate interpolation for the ages along each of the thirty-one tracks was done as follows. The tabulated ages for each of Iben's (1967, Table 1) three tracks were multiplied by $L/\mathfrak{M}$ for each track and plotted as the ordinate against the magnitude difference, $\Delta M_{\rm bol}$, from the starting main-sequence luminosities as the abscissa, to form part of an interpolation diagram. Absence of strict homology between the models appears in this diagram by the different shapes and position of the three model lines. Use of the $L/\mathfrak{M}$ factor reduces the large variation in age so greatly that the nonhomologous age differences can be easily managed. Adoption of a mass-luminosity relation which connects the three calculated models ($M_{\rm bol} = 2.93$, 3.85, and 5.10, respectively, for $\mathfrak{M} = 1.5$, 1.25, and 1.0 $\mathfrak{M}_\odot$) gives $L/\mathfrak{M}$ for each of the twenty-eight interpolated tracks. For any given track, the $TL/\mathfrak{M}$ value is found for selected $\Delta M_{\rm bol}$ values from the interpolation diagram, and multiplication by the appropriate $\mathfrak{M}/L$ gives ages at the selected $\Delta M_{\rm bol}$ on all tracks. These can be connected to produce the isochrones which are shown as heavy lines in Figure 1. These curves represent the predicted shapes of C-M diagrams in the (M_{bol}, log T_e)-plane.

An interesting new effect appears in Figure 1 in the range $2.5 \leq M_{\mathrm{bol}} \leq 3.5$. Because of (1) the much shallower departure of the higher-mass tracks from the main sequence, and (2) the sharper return after the depletion of hydrogen in the core for such tracks,

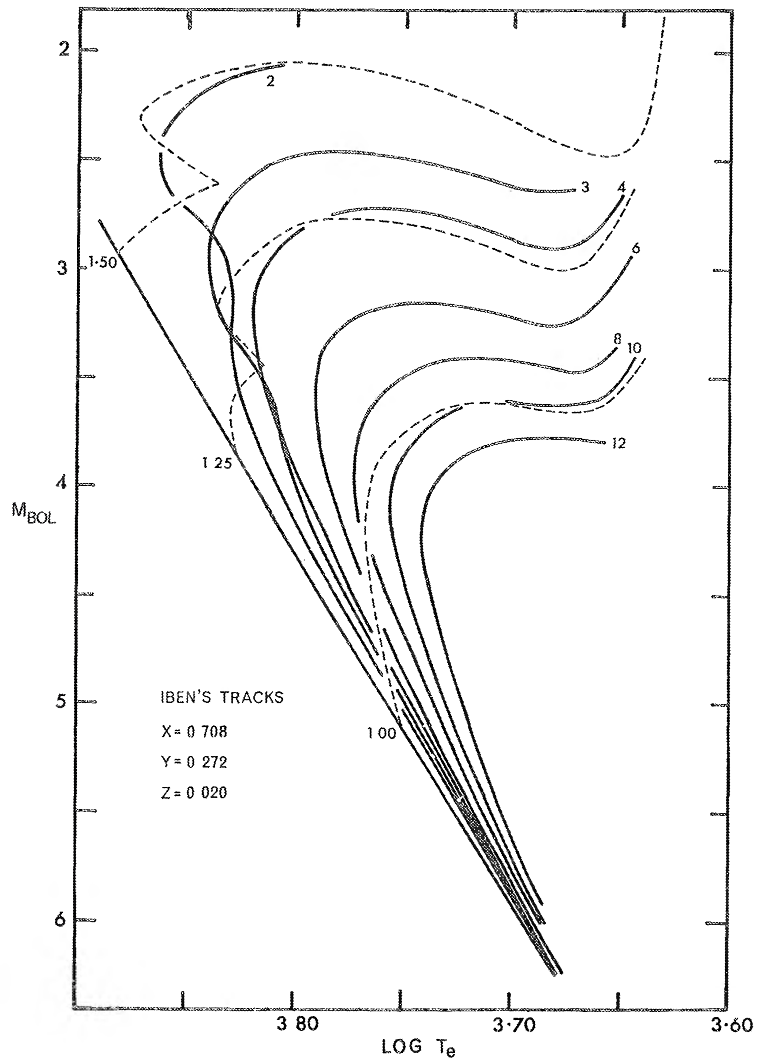

FIG. 1.—Isochrones (*heavy lines*) calculated from Iben's (1967) evolutionary tracks (*dashed lines*) for masses of 1, 1.25, and 1.5 $\mathfrak{M}_\odot$. The observed zeio-age main sequence is shown as transformed from the (M_V, $B - V$)-diagram by the relations given elsewhere (Sandage 1962*b*). The phase of core hydrogen exhaustion, evident in the 1.5 and 1 25 $\mathfrak{M}_\odot$ tracks, causes the isochrones for $T = 2$, 3, and 4×10^9 years to cross each other near the knees at the termination of their main sequence. Ages are in units of 10^9 years.

the H-R diagrams *cross over* near the knee at the termination of the main sequence. Such an effect will confuse the monotonic progression of evolving main-sequence slopes with age for evolving sequences just below the knee—an effect which might be observable in star clusters of intermediate age.

For an observational test, the Hyades Cluster and extended group (Eggen 1964*a*, 1970), NGC 2477 (Eggen and Stoy 1961), and NGC 7789 (Burbidge and Sandage 1958) are plotted in Figure 2. Data for the normal points and distance moduli, corrected for reddening and absorption, are taken from the summary in § V. A crossover effect of the form predicted in Figure 1 may be present. However, we are cautious of the result because observations of both NGC 2477 and NGC 7789 depend on photographic data, which, although calibrated from photoelectric measurements, may be subject to systematic errors. New photoelectric observations of these and other star clusters of intermediate age are needed to confirm the effect.

The isochrones of Figure 1, together with a similar set from the ADM models, were used to derive new curves of evolutionary deviation. Because the shape of the isochrones is a function of M_{bol}, the shape of the deviation curves will also be a slow function of age. Three curves were computed from ADM's models and two from Iben's models as follows. Along a chosen isochrone, the magnitude difference (ΔM_{bol}) is read from the

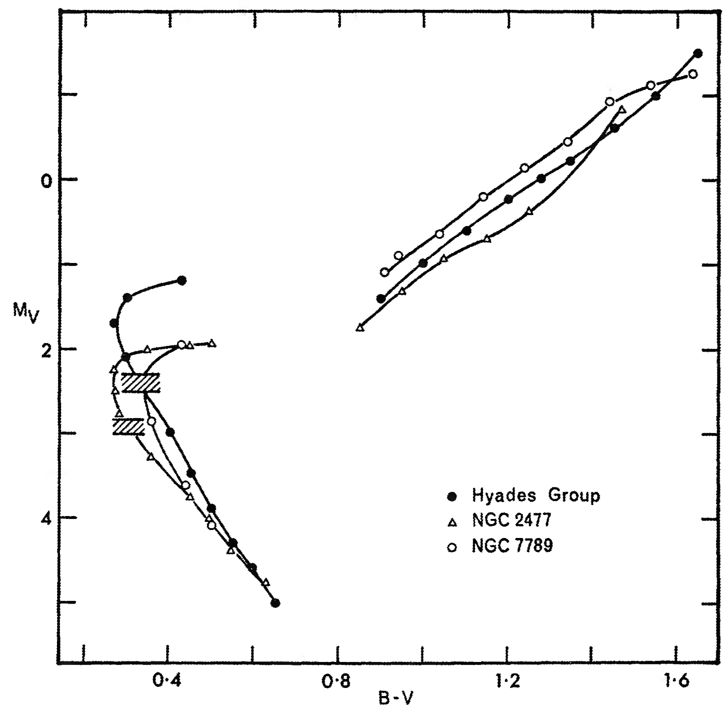

FIG. 2.—Observed C-M diagrams for three star clusters of intermediate age taken from data in Table 3. The crossover effect may be present near the knee at the termination of the main sequence. Gaps on the evolving main sequence for the Hyades and NGC 2477 are indicated.

main sequence at constant log T_e for various values of M_{bol} (main sequence). The main-sequence values of M_{bol} are converted to $M_V{}^0$ (main sequence) values by the relation $\Delta M_{\mathrm{bol}} = f(T_e)$. Since $\Delta M_{\mathrm{bol}} = \Delta M_V$ (i.e., the magnitude differences between the isochrones and the main sequence are read at constant T_e), the deviation curve follows from the $M_V{}^0$, ΔM_V number pairs. Figure 3 shows two typical curves plotted from the more complete listing in Table 1. Use of the curves to obtain distance moduli of clusters is described elsewhere (Johnson 1960; Sandage 1962*b*).

Figure 3 shows that the shape is a function of age. The curve for 3×10^9 years shows a dip near $\Delta M_V = 0.5$ at $M_V{}^0 \simeq +4$, caused by the phase of hydrogen exhaustion in the core. Although improvement of Table 1 can be expected when more extensive models are availble which show the dip at the luminosity of the NGC 188 gap, use of the present curves should not introduce systematic errors in photometric moduli determinations because the curves for all models, including the initial model of Hoyle (1959), are so similar near the limit $\Delta M_V = 0$.

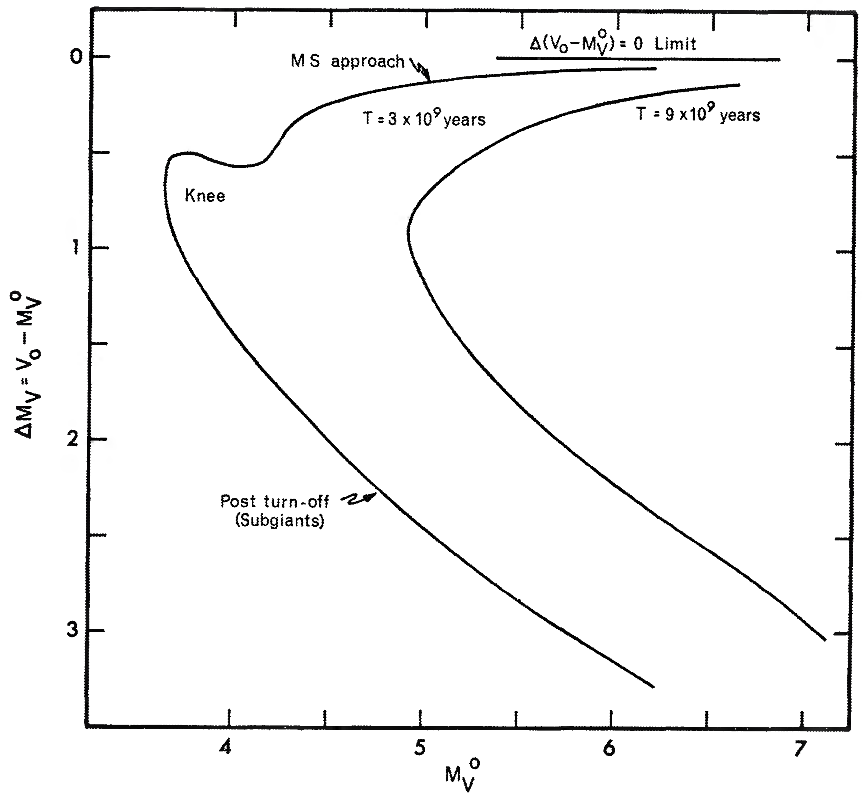

FIG. 3.—Two curves of evolutionary deviation calculated from Fig. 1 for Iben's models. The dip near $\Delta M_V = 0.5$ for the $T = 3 \times 10^9$ years is caused by the evolutionary phase of core hydrogen exhaustion.

TABLE 1

THEORETICAL CURVES OF EVOLUTIONARY DEVIATION

AIZENMAN, DEMARQUE, AND MILLER MODELS						IBEN'S MODELS				HOYLE TYPE I	
$T=4.5\times10^9$, $M_B\simeq2.8$		$T=7\times10^9$, $M_B\simeq3.5$		$T=12\times10^9$, $M_B\simeq4.2$		$T=3\times10^9$, $M_B\simeq2.6$		$T=9\times10^9$, $M_B\simeq3.5$		$4\geq M_B\geq3$	
$M_V{}^0$	ΔM_V	$M_V{}^0$	ΔM_V	$M_V{}^0$	ΔM_V	$M_V{}^0$	ΔM_V	$M_V{}^0$	ΔM_V	$M_V{}^0$	ΔM_V
5.19	2.21	6.45	2.56	6.91	2.11	6.21	3.28	7.13	3.01	5.08	2.18
4.84	1.84	5.99	2.22	6.45	1.74	5.78	3.02	6.68	2.67	4.54	1.49
4.67	1.64	5.58	1.84	6.21	1.54	5.37	2.74	6.21	2.38	4.33	1.08
4.50	1.36	5.38	1.64	5.99	1.32	5.02	2.46	5.78	2.06	4.29	0.85
4.36	1.03	5.20	1.38	5.78	0.98	4.67	2.16	5.58	1.88	4.33	0.60
4.50	0.64	5.02	1.04	5.91	0.68	4.32	1.82	5.37	1.69	4.54	0.39
4.67	0.54	4.98	0.72	6.04	0.58	3.97	1.40	5.20	1.49	5.08	0.24
4.84	0.38	5.12	0.55	6.45	0.34	3.80	1.14	5.02	1.22	5.30	0.20
5.19	0.23	5.30	0.40	6.90	0.24	3.66	0.86	4.94	1.01	5.64	0.16
5.38	0.20	5.66	0.28	7.42	0.19	3.65	0.65	4.92	0.89	5.87	0.12
5.78	0.16	6.32	0.18	...	...	3.66	0.55	4.95	0.78	6.24	0.09
6.59	0.09	6.68	0.14	...	...	3.70	0.50	5.05	0.67	6.91	0.08
7.39	0.07	7.46	0.10	...	...	3.97	0.55	5.16	0.56	7.46	0.06
7.47	0.06	...	...	...	...	4.16	0.54	5.49	0.36	...	...
...	...	...	...	...	...	4.32	0.32	5.68	0.30	...	...
...	...	...	...	...	...	4.67	0.20	6.07	0.20	...	...
...	...	...	...	...	...	4.82	0.16	6.64	0.14	...	...
...	...	...	...	...	...	5.44	0.08	...	...	...	...
...	...	...	...	...	...	6.21	0.04	...	...	...	...

III. AGES OF M67 AND NGC 188 FOR FIXED CHEMICAL COMPOSITION

The C-M diagrams of M67 and NGC 188 can be compared with the two sets of isochrones by transforming the observational data into the (M_{bol}, log T_e)-plane. To be consistent with the calculations in § II, we must adopt the *same* $\Delta M_{\text{bol}} = f(B - V)$ and log $T_e = g(B - V)$ functions which were used to transform the adopted age-zero main sequence from (M_V, $B - V$) to (M_{bol}, log T_e) (Sandage 1962*b*, Table 3). This is because the observational data themselves were originally fitted to this sequence.

The reddening-free and absorption-free normal points [V_0, $(B - V)_0$] for M67 and NGC 188 are listed in the summary table given in § V (Table 3). To correct the reddening-free colors to the line-strength system of the Hyades, we have *added* 0.03 mag to

TABLE 2

NORMAL M_{bol}, log T_e POINTS FOR M67 AND NGC 188

M67 $(m-M)_0=9.38$		NGC 188 $(m-M)_0=10.85$	
M_{bol}*	log T_e	M_{bol}*	log T_e
2.57	3.675	3.37	3.670
3.14	3.694	3.54	3.678
3.13	3.706	3.61	3.700
3.01	3.740	3.58	3.725
2.98	3.775	3.66	3.740
3.13	3.789	3.74	3.756
{3.28	3.789}	3.91	3.765
{3.48	3.785}	4.10	3.760
3.63	3.785	...	...
3.93	3.785	{4.24	3.753}
4.22	3.780	{4.43	3.750}
4.58	3.760	4.54	3.750
5.21	3.727	4.96	3.740
5.70	3.702	5.26	3.720
6.14	3.680	5.79	3.695
6.46	3.662	6.00	3.679
..	...	6.32	3.660
. .	..	6.72	3.641

* Based on the empirical values for the zero-age main sequence. If the chemical compositions differ from $X = 0.70$, $Y = 0.27$, and $Z = 0.03$ which is adopted to apply to this sequence, the tabulated values of M_{bol} should be changed by equation (7).

$(B - V)_0$ for both clusters based on the ultraviolet excess discussed elsewhere (Eggen and Sandage 1964, 1969). These corrected colors and the V_0 magnitudes have been converted to values of M_{bol} and log T_e by using (1) moduli of $(m - M)_0 = 9.38$ for M67 and $(m - M)_0 = 10.85$ for NGC 188 derived previously and (2) the adopted color, ΔM_{bol}, and temperature relations. The results for subgiants and stars near the main sequence are listed in Table 2. The gap regions for both clusters are enclosed in braces in this table.

Figure 4 shows the resulting (M_{bol}, log T_e)-diagrams superposed on the isochrones from the ADM models. Figure 5 shows the same for Iben's tracks. In general, the cluster diagrams agree well with the shapes of the isochrones. Iben's tracks produce the downward slope of the M67 subgiants, while the ADM isochrones agree with the shape of NGC 188 subgiants.

Ages for the clusters depend slightly on where the isochrones are read. The ages from Figure 4 are $T \simeq (5.0 \pm 0.5) \times 10^9$ years for M67 and $T = 7.7 \times 10^9$ years for NGC

188. Figure 5 (Iben's tracks) gives $T \simeq (5.3 \pm 0.5) \times 10^9$ years for M67 and $T \simeq (9.5 \pm 0.5) \times 10^9$ years for NGC 188. The quoted errors refer only to the interval over which the various segments of the subgiant branches intersect the isochrones. They do not include errors in our assignment of absolute luminosities, nor systematic uncertainties in the models.

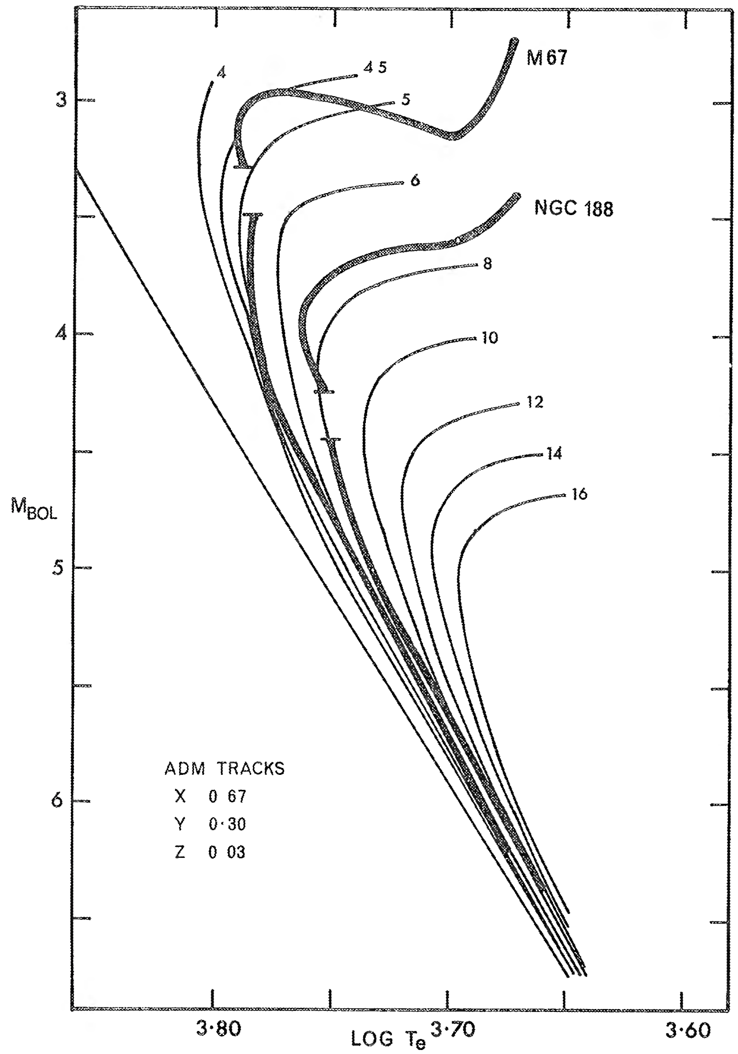

FIG. 4.—Observed H-R diagrams for M67 and NGC 188 from Table 2 superposed on the isochrones from models due to Aizenman, Demarque, and Miller.

The ages from the two sets of models agree for M67, but not for NGC 188. This is because the spacings of the isochrones differ between Figures 4 and 5 for unexplained reasons. Approximate equations of $T = f(L)$ can be fitted graphically to the diagrams by reading the subgiant luminosities of each isochrone at a point $\Delta \log T_e = 0.04$ redward of the bluest $\log T_e$ reached at the knee at the termination of the main sequence. Figure 4 for the ADM models gives

$$\log T(\text{years}) = 0.315 M_{\text{bol}} + 8.727 \quad \text{for} \quad 4.7 \geq M_{\text{bol}} \geq 2.9$$

or

$$\log T(\text{years}) = 0.250 M_V + 8.910 \quad \text{for} \quad 4.7 \geq M_V \geq 2.8\,, \tag{1}$$

with $X = 0.67$, $Y = 0.30$, $Z = 0.03$, while Figure 5 for the Iben models gives

$$\log T(\text{years}) = 0.455 M_{\text{bol}} + 8.344 \quad \text{for} \quad 3.8 \geq M_{\text{bol}} \geq 2.1$$

or

$$\log T(\text{years}) = 0.406 M_V + 8.461 \quad \text{for} \quad 4.0 \geq M_V \geq 2.1 \,, \tag{2}$$

for $X = 0.708$, $Y = 0.272$, $Z = 0.02$.

The two sets of isochrones coincide in age near $M_{\text{bol}} \simeq 2.8$ or $T = 4 \times 10^9$ years, but differ by a factor of 1.5 at $M_{\text{bol}} = +4$. We are uncertain which of the models corresponds more closely with reality. On the one hand, the ADM models produce the NGC 188 gap while Iben's models do not. On the other hand, Iben's models cover the entire relevant range of the H-R diagram, but we were forced to extrapolate the ADM models upward into the region occupied by M67. Because of this uncertainty we shall discuss both sets of ages in the next section.

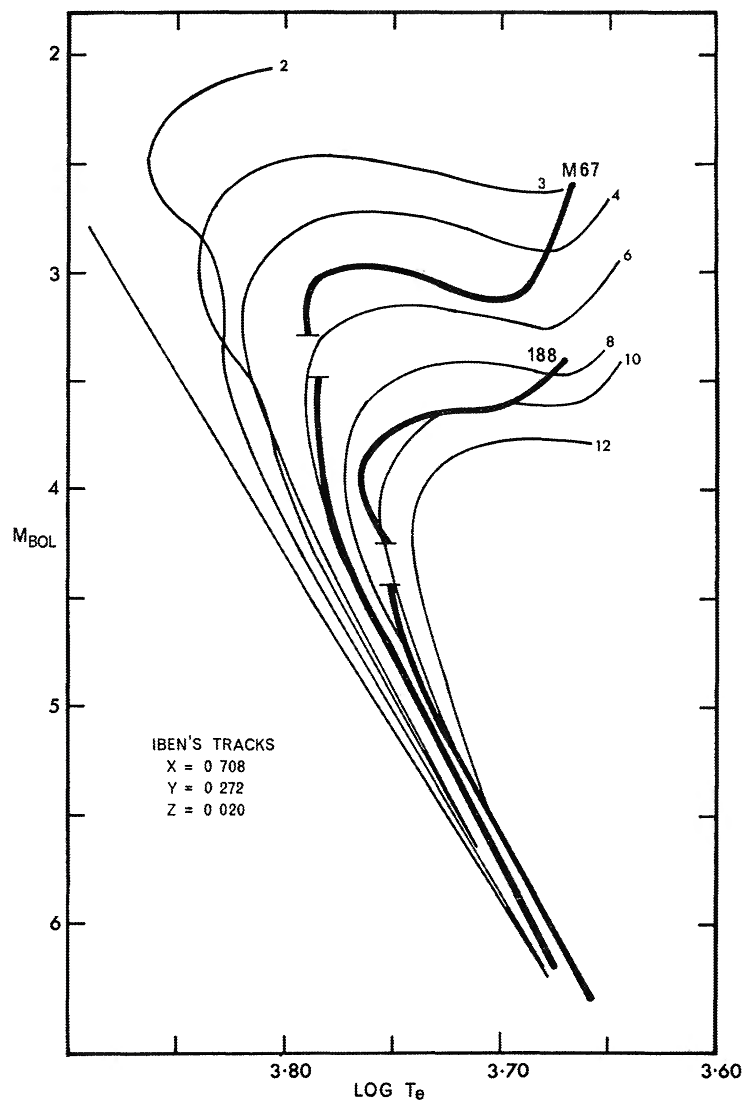

FIG. 5.—Same as Fig. 4 for Iben's models

IV. THE EFFECT OF CHEMICAL COMPOSITION ON THE AGES

a) The Problem

The ages in § III refer to the fixed chemical compositions of $X = 0.67$, $Z = 0.03$ for the ADM models and $X = 0.708$, $Z = 0.02$ for Iben's models. It is well known that age estimates depend sensitively on the composition for at least two reasons. (1) The ages along evolutionary tracks which pass through a given (M_{bol}, log T_e)-point of the H-R diagram but which have different values of X and Z are not the same owing to differences in the models. This can be called the theoretician's problem. (2) For clusters whose distances are found by the method of photometric parallax, the modulus depends on the position of the adopted main sequence, which is itself a function of X and Z (for early results from dimensional analysis, cf. Strömgren 1952, eq. [12]; Sandage and Eggen 1959, eqs. [12] and [17]; Sandage and Gratton 1963, eqs. [1.35] and [1.39]; and others; for detailed model calculations, cf. Faulkner and Griffiths 1963; Kelsall and Strömgren 1965; Demarque 1967; Faulkner 1967; Demarque and Schlesinger 1969; and others). Therefore, the observer's assignment of M_V for cluster stars depends on X and Z. This can be called the observer's problem.[1]

Consider the theoretician's problem first. It arises because the mass at a given luminosity is a function of X and Z through its dependence on the mean molecular weight. The time for a star to reach a given evolutionary state at some specified L will depend on the amount of mass burned, and therefore on X and Z.

An approximate functional dependence can be estimated from homology considerations such as first given by Hoyle (1964). Neglecting the Z dependence, Hoyle's table shows that the age ratio between two evolved stars which have the same observed M_{bol} but different values of X is

$$T_1/T_2 = (X_1/X_2)^{1\ 7} . \qquad (3)$$

Similar considerations using Faulkner's quasi-homology relations, which are based on calculated main sequences (Faulkner and Griffiths 1963), suggest

$$T \propto X^{1.84}(Z + 0.012)^{0.2} \qquad (4)$$

at constant L. These expressions can be only approximate due to the second-order effect of a change in the shape of tracks for different values of X and Z.

The most reliable estimate of the second-order effect would come from calculated models which are carried into the subgiant region. Sufficient models with different X and Z do not exist, although certain of the ADM tracks have been carried far enough to make crude estimates. Reading the ages along each of the ADM tracks (which start at the same main-sequence M_{bol} value) at a point $\Delta M_{\mathrm{bol}} = 0.8$ brighter than the starting point gives

$$T \propto X^{1.70}Z^{0.22} . \qquad (5)$$

Or, more realistically, one can allow for a lower limit of the Z-dependence caused by the dominance of the bound-free opacity of hydrogen and helium for sufficiently low Z by

$$T \propto X^{1.70}(Z + 0.012)^{0.28} \quad \text{for} \quad 1.3 \leq \mathfrak{M}/\mathfrak{M}_{\odot} \leq 0.9 . \qquad (6)$$

But the problem is not yet solved satisfactorily. Equation (6) is tentative and is subject to revision when new models are available.

[1] For those special cases where M_V is known by any nonphotometric method, such as trigonometric or moving-group parallaxes, the problem is not relevant and need not be considered.

b) Application to M67 and NGC 188

To solve the observer's problem, we must know the chemical composition of M67 and NGC 188, and the adopted zero-age main sequence (ZAMS). Morton and Adams (1968) have compared the theoretical main-sequence models of Kelsall and Strömgren (1965) with the empirical zero-age main sequence adopted here. They conclude that $X = 0.70$, $Y = 0.27$, and $Z = 0.03$ gives the best fit, but they comment that "neither the observations nor our transformations are accurate enough to eliminate the $Z = 0.02$ case" for the ZAMS. We adopt $Z = 0.03$ on the following basis.

It is known that the Sun has a slight ultraviolet excess relative to the ZAMS by an amount which requires $\log (\mathrm{Fe/H})_{\mathrm{ZAMS}} - \log (\mathrm{Fe/H})_{\odot} \simeq +0.2$, or $Z_{\mathrm{ZAMS}}/Z_{\odot} = 1.6$ (Eggen 1964*b*, Fig. 19). Evolving models fitted to the Sun give $X = 0.71$, $Z = 0.02$ (Sears 1964), $X = 0.70$, $Z = 0.02$ (Demarque and Percy 1964), and $X = 0.72$, $Z = 0.02$ (Weymann and Sears 1965). Furthermore, Gaustad (1964) finds $X = 0.72$, $Z = 0.02$ from a study of solar cosmic rays. The four quoted solar values are all $Z = 0.02$, which, when combined with $Z_{\mathrm{ZAMS}}/Z_{\odot} = 1.6$, gives $Z_{\mathrm{ZAMS}} = 0.03$, a value in agreement with Morton and Adams (1968) and Morton (1968). We therefore accept $X = 0.70$, $Y = 0.27$, and $Z = 0.03$ for the adopted position of the ZAMS.

The adoption of this main sequence and its composition as the *standard* with which clusters with different compositions are to be compared requires that the times along the isochrones in Figures 4 and 5 be changed according to equation (6). The ages marked in Figure 4 should be increased by 7 percent because of the change from $X = 0.67$ to $X = 0.70$. The ages in Figure 5 should be increased by 6 percent because of the change of X from 0.708 to 0.70 and of Z from 0.02 to 0.03. Therefore, if M67 and NGC 188 had $X = 0.70$, $Y = 0.27$, $Z = 0.03$, our derived distance moduli would be correct because these compositions agree with those of the ZAMS. The cluster ages would be $(5.3 \pm 0.5) \times 10^9$ and $(8.3 \pm 0.5) \times 10^9$ years, respectively, from Figure 4, and $(5.6 \pm 0.5) \times 10^9$ and $(10 \pm 0.5) \times 10^9$ years from Figure 5 if equation (6) is used to correct for the composition differences.

But the correct ages depend on the *actual* values of X, Y, and Z for the clusters. The Z for M67 and NGC 188 can be estimated from the observed ultraviolet excess. The observations show that $\Delta(B - V)_{\mathrm{bl}} = 0.03$ for both clusters (Eggen and Sandage 1964, 1969), which requires that $\log (\mathrm{Fe/H})_{\mathrm{MS}} - \log (\mathrm{Fe/H})_{\mathrm{cl}} = +0.14$ by the previous calibration (Eggen 1964*b*, Fig. 19). Therefore, $Z_{67,188} = 0.022$.

The effect on the position of the ZAMS of decreasing Z from 0.03 to 0.022 while keeping X fixed is small. Faulkner's (1967) approximation for the H-R diagram is

$$L \propto (X + 0.4)^{2.67}(Z + 0.010)^{0.455} f(T_e) \,. \tag{7}$$

Consequently, the correct main sequence for M67 and NGC 188 will be only 0.11 mag fainter than the adopted ZAMS if Z alone is varied. Our adopted absolute magnitudes in Table 2 would then be too bright by this amount if only the Z effect applies. If the M_V values are fainter by 0.11 mag in Figures 4 and 5, the ages will be *increased* by about 8 percent (cf. eqs. [1] and [2]) by this observer's problem, but the difference will be nearly compensated for by a 6 percent *decrease* in ages by the theoretician's problem of equation (6) with Z decreased from 0.03 to 0.022.

The same type of compensation holds for variations in X if, again, the theoretician's and observer's effects are combined. Suppose, for example, that $X = 0.5$ rather than 0.7, and that Z is kept the same. Equation (7) shows that the correct main sequence would be 0.58 mag fainter than our adopted ZAMS. All cluster stars would then be fainter by this amount. Equations (1) and (2) make the ages *longer* by a factor of ~ 1.5 because of the observer's problem, while the theoretician's equation (6) makes the ages *shorter* by a factor $\sim (0.7/0.5)^{1.7} = 1.7$, which nearly compensates. The curious conclusion is, therefore, that if equations (1), (2), (6), and (7) are combined, ages that are obtained in a first trial by using the position and composition of the adopted ZAMS are nearly independent of small changes in the chemical composition.[2] We do not expect the effect to apply to globular clusters in exactly the same manner because of the larger range of the Z-variation and the possible difference in the sign of the Z-dependence in equation (6) (Simoda and Iben 1968).

It would appear from these considerations that the values $T \simeq 5.5 \times 10^9$ years for M67 and $T \simeq (8\text{–}10) \times 10^9$ years for NGC 188 follow from the data as interpreted with the present models, and that the values are relatively insensitive to chemical composition. The ages agree well with those obtained previously by Demarque and Larson (1964) and by Iben (1967). They are greater than those obtained by ADM, but some of the difference is due to the much larger reddening corrections used by these authors than by us.

It should be noted that we recognize in connection with the theoretician's problem that, although models with the adopted composition reproduce the Hyades main sequence brighter than the Sun, they do not reproduce the observed mass-luminosity relation for fainter Hyades stars (Eggen 1969*b*). Morton and Adams have already noted that, although the Kelsall-Strömgren and the Iben models agree in reproducing the observed zero-age main sequence, the mass-luminosity relations differ up to 0.25 mag.

The foregoing discussion represents a change of philosophy from procedures used by all observers, including ourselves, during the past ten years in which the ZAMS position has not been permitted to change with changing X and Z. Justification for the older procedure seemed to be provided by the correction of the low-Z subdwarfs to the adopted ZAMS after blanketing corrections (Eggen and Sandage 1962). We now believe, with others, that the ZAMS must be permitted to vary with X and Z, as shown primarily by the results of Cayrel (1968) and Strom, Cohen, and Strom (1967), together with the observational result that Y is high ($\sim$30 percent) for Population II stars with low Z (Christy 1966; Sandage 1969). The "observed" high value of Y is inconsistent with the theoretical demand (cf. eq. [7]) for low values of Y in stars of low Z if our previous results (Eggen and Sandage 1962) had been correct. We hope to rediscuss the problem, using *UBVRI* data for all extreme subdwarfs with large trigonometric parallaxes (including several new ones), to see if these points of view can be reconciled.

V. A NEW COMPOSITE C-M DIAGRAM FOR CLUSTERS OLDER THAN M11

Enough new information has been obtained for galactic clusters to extend an earlier version (Sandage 1957) of the composite color-magnitude diagram. New distance moduli have been obtained by using the curves in Table 1 for M11 (Johnson, Sandage, and Wahlquist 1956), NGC 2477 (Eggen and Stoy 1961), NGC 7789 (Burbidge and Sandage 1958), and NGC 3680 (Eggen 1969*a*). Normal points for these clusters, together with data for the extended Hyades group (Eggen 1970) and for M67 and NGC 188, are listed in Table 3. The adopted moduli and the reddening are given for each cluster in the heading to the table. Where gaps on the evolving main sequence have been observed, the data are enclosed in braces at the end of each column.

Figure 6 shows the data using $(m - M)_0 = 9.38$ for M67, 10.85 for NGC 188, and 9.50 for NGC 3680, which are values obtained if line *weakening* of $\Delta(B - V)_{\rm bl} = 0.03$ is used in the analysis. The principal new features of Figure 6 are (1) the extension of the giant branches of M67 and NGC 188 to brighter absolute magnitudes than previously given, (2) the extension of the giant branch of the Hyades extended group, (3) the general coalescence of all giant branches toward what may be a single region defined by the Hayashi convective track, (4) the possible existence of the crossover effect at the knee at the termination of the main sequence near $M_V \simeq +2.5$ to $+3$, (5) the existence of gaps

[2] Again, this is not true for stars whose M_V are known from trigonometric parallaxes, because no use is made of the standard main sequence. Equation (7) is then not germane; only equation (6) applies.

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TABLE 3

REDDENING-FREE NORMAL POINTS FOR SEVEN GALACTIC CLUSTERS

M11 $E(B-V)=0.42$ $(m-M)_o=11.25$		Hyades Group $E(B-V)=0.00$ $(m-M)_o=$ Var		NGC 2477 $E(B-V)=0.25$ $(m-M)_o=10.00$		NGC 7789 $E(B-V)=0.26$ $(m-M)_o=11.34$		NGC 3680 $E(B-V)=0.04$		M67 $E(B-V)=0.06$ $(m-M)_o$ depends on blanketing values*		NGC 188 $E(B-V)=0.09$	
V_o	$(B-V)_o$	M_V^o	$(B-V)_o$	V_o	$(B-V)_o$	V_o	$(B-V)_o$	V_o	$(B-V)_o$	V_o	$(B-V)_o$	V_o	$(B-V)_o$
9.76	1.56	-1.50	1.65	9.15	1.47	10.07	1.64	11.70	1.00	8.65	1.50	9.43	1.44
9.94	1.38	-1.00	1.55	10.37	1.25	10.22	1.54	11.87	0.90	9.02	1.40	9.73	1.41
10.09	1.28	-0.60	1.45	10.70	1.15	10.40	1.44	11.87	0.80	9.51	1.30	10.33	1.35
10.24	1.18	-0.20	1.35	10.93	1.05	10.87	1.34	11.87	0.72	10.10	1.20	10.73	1.31
10.44	1.08	0.00	1.28	11.30	0.95	11.22	1.24	11.70	0.52	10.70	1.10	11.33	1.25
10.57	0.98	0.22	1.20	11.75	0.85	11.54	1.14	11.85	0.45	11.40	1.00	11.70	1.21
10.06	0.02	0.60	1.10	11.95	0.50	12.02	1.04	12.25	0.45	11.95	0.96	12.33	1.15
10.54	0.00	1.00	1.00	11.95	0.45	12.24	0.94	13.00	0.46	12.30	0.92	12.83	1.11
11.03	-0.04	1.41	0.90	12.00	0.35	12.32	0.91	13.50	0.50	12.77	0.84	13.33	1.07
11.47	-0.05	1.20	0.43	12.05	0.30	13.25	0.44	14.00	0.57	12.70	0.78	13.73	1.03
11.98	-0.03	1.40	0.30	12.25	0.27	13.34	0.39	14.50	0.64	12.50	0.66	14.23	0.99
12.50	0.00	1.70	0.27	12.50	0.27	13.72	0.34	15.00	0.72	12.42	0.55	14.63	0.95
12.86	0.04	2.10	0.30	12.75	0.28	14.22	0.36	{12.50	0.47}	12.55	0.51	14.73	0.91
13.10	0.09	3.00	0.40	13.25	0.36	14.97	0.44	12.70	0.45	13.05	0.52	14.68	0.81
13.35	0.14	3.50	0.45	13.75	0.45	15.42	0.50	.....	....	13.35	0.52	14.63	0.71
13.54	0.15	{2.30	0.31}	14.00	0.50	.....	....	.....	....	13.65	0.53	14.63	0.66
13.74	0.18	2.50	0.33	14.35	0.55	.....	....	.....	....	14.04	0.60	14.67	0.61
.....	....	....	....	14.75	0.63	.....	....	.....	....	14.73	0.70	14.83	0.59
.....	....	....	....	{12.85	0.29}	.....	....	.....	....	15.29	0.80	15.03	0.60
.....	....	....	....	13.00	0.33	.....	....	.....	....	15.84	0.90	15.49	0.63
.....	....	....	....	.....	....	.....	....	.....	....	16.29	1.00	15.93	0.66
.....	....	....	....	.....	....	.....	....	.....	....	{12.70	0.51}	16.27	0.73
.....	....	....	....	.....	....	.....	....	.....	....	12.90	0.52	16.88	0.83
.....	....	....	....	.....	....	.....	....	.....	....	.....	....	17.18	0.91
.....	....	....	....	.....	....	.....	....	.....	....	.....	....	17.63	1.01
.....	....	....	....	.....	....	.....	....	.....	....	.....	....	18.21	1.12
.....	....	....	....	.....	....	.....	....	.....	....	.....	....	18.62	1.24
.....	....	.....	....	.....	....	.....	....	.....	....	.....	....	{15.18	0.62}
.....	....	.....	....	.....	....	.....	....	.....	....	.....	....	15.38	0.63

* For line blanketing of $\Delta(B-V)_{bl} = 0.03$ (weaker lines), the moduli are $(m-M)_o = 9.50$ for NGC 3680, 9.38 for M67, and 10.85 for NGC 188.

on the evolving main sequence, and (6) the narrowing of the Hertzsprung gap as M_V becomes fainter, reaching zero width at about the position of M67. Ages from equations (1) and (2) are shown along the right-hand border.

Spinrad and Taylor (1967) have suggested that members of M67 and NGC 188, as well as some old stars in the field, have metal abundances greater than those of Hyades

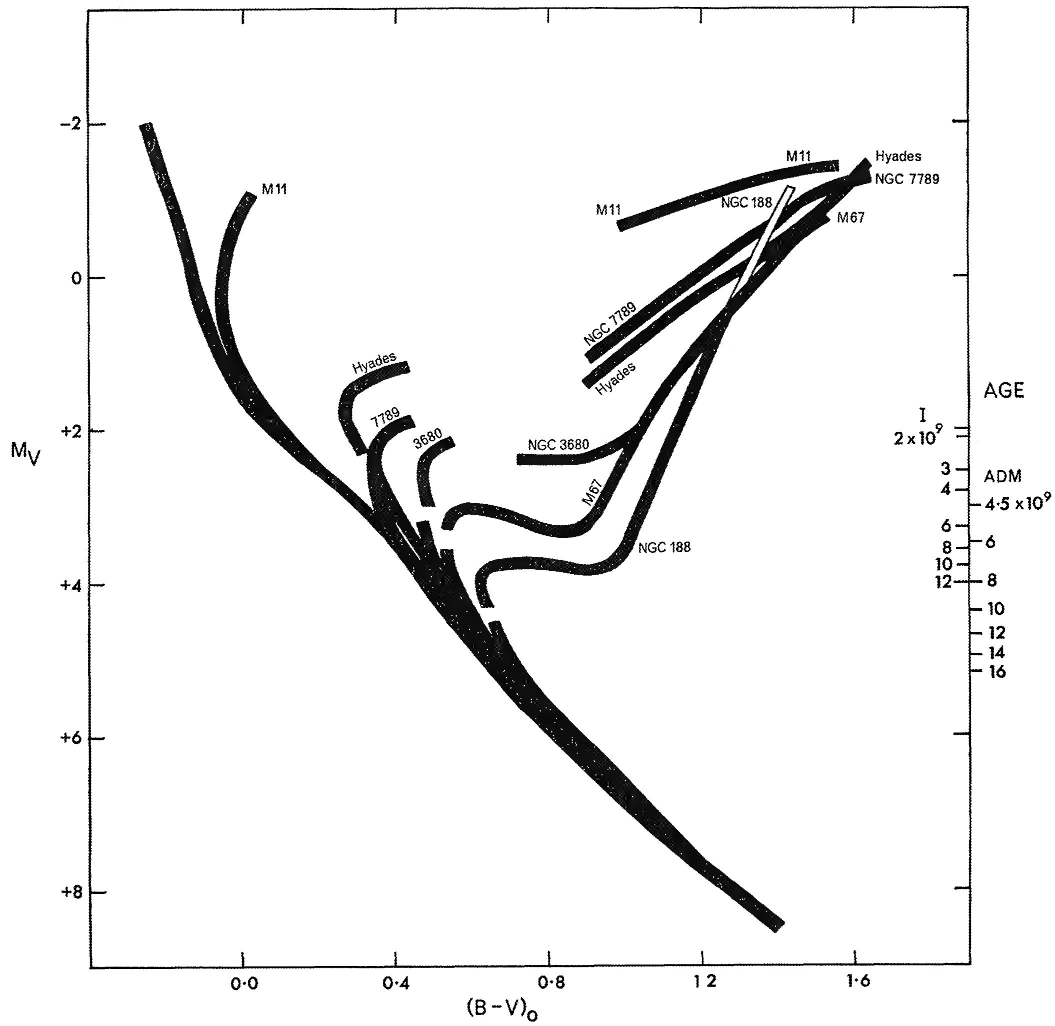

Fig. 6.—Composite C-M diagram for six galactic clusters fitted to the zero-age main sequence by the method of photometric moduli. The ages calculated from equations (1) and (2) are shown on the right-hand border from the models of ADM and of Iben (1967). The extension of the giant branch of NGC 188 depends on only a few stars and is shown as an open region.

stars. Under the assumption that the greater abundances apply to main-sequence stars, a new version of Figure 6 was prepared, but it is not reproduced here. The main result obtained by assuming $\Delta(B - V)_{\text{bl}} = -0.04$ (stronger lines) is that the moduli become $(m = M)_0 = 9.90$ for NGC 3680, 9.80 for M67, and 11.20 for NGC 188. Stars in these clusters become intrinsically brighter than in Figure 6, and have correspondingly shorter ages (eqs. [1] and [2]). This interpretation is, however, suspect by the arguments given in the preceding paper (Eggen and Sandage 1969).

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