THE ASTROPHYSICAL JOURNAL, Vol. 157, August 1969

THE REDDENING, AGE DIFFERENCE, AND HELIUM ABUNDANCE OF THE GLOBULAR CLUSTERS M3, M13, M15, AND M92

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Received January 13, 1969

ABSTRACT

Reddening values of $E(B - V) = 0.00, 0.03, 0.12$, and 0.02 mag are obtained for M3, M13, M15, and M92, respectively, from new photometry of horizontal-branch stars via the two-color diagram. Justification for the method comes from Mihalas' calculation of $I(\lambda)$ for line-blanketed stars over the relevant range of log g.

The $B - V$ main-sequence-turnoff colors agree to within 0.01 mag in all four clusters when corrections are applied for differential line blanketing The result requires that M3 and M13, which have similar Z-values, have an age spread Δt of less than $\Delta t/t \simeq 0.03$ The same result applies separately to M15 and M92. To connect Δt for the low- and high-Z clusters requires knowledge of the turnoff temperature as a function of metal abundance.

The blue-boundary colors of the RR Lyrae instability strip in M3, M15, and M92 agree to within $\Delta(B - V)_{\mathrm{BE}}{}^{o\ c} = 0\ 025$ mag when reddening and blanketing corrections are applied. The difference in mean period between the Oosterhoff-Sawyer groups cannot then be due to a temperature difference, but may arise from a difference in luminosity, as previously supposed.

From Christy's models, the dependence of the blue boundary of the strip on Y, M_V, $\mathfrak{M}/\mathfrak{M}_\odot$, and $B - V$ is approximated by a linear equation which reproduces Christy's Y-values to within 0 004 ± 0.018 (A D.) over a stated parameter range. Using the assumptions in the text, a mean helium abundance of $Y = 0.32 \pm 0\ 09$ (total range) is derived using $(B - V)_{\mathrm{BE}}{}^{o\ c} = 0.175 \pm 0\ 02$, $\mathfrak{M}/\mathfrak{M}_\odot = 0\ 55 \pm 0.15$, and $M_V = +0\ 42 \pm 0\ 2$. The Y-value agrees with a previous result by Christy. The helium abundances for M3, M15, and M92 are the same to within the observational error, despite the large difference in metal abundance.

I. INTRODUCTION

Reddening values for globular clusters are important for (1) proper fitting of main sequences to obtain distances, absolute magnitudes, and ages, and (2) determination of intrinsic colors of the RR Lyrae instability strip in clusters of different metal abundances. Solution of the second problem would provide answers to several central questions.

a) Differences in the mean period of RR Lyrae stars in clusters of Oosterhoff-Sawyer groups I and II have been interpreted as due to different absolute luminosities for variables in the two groups (Sandage 1958; Christy 1966, 1968*a*, *b*), the temperature boundaries of the strip being fixed. This explanation requires $\Delta\langle M_V\rangle \simeq 0.24$ mag between clusters of group I ($\langle P\rangle_{a,b} \simeq 0^{\mathrm{d}}.55$) and group II ($\langle P\rangle_{a,b} \simeq 0^{\mathrm{d}}.65$), such that horizontal branches of group II are brighter. Alternatively, a shift of $\Delta \log T_e \simeq .025$ [or $\Delta(B - V) \simeq 0.14$] in the strip boundaries, keeping $\langle M_V\rangle$ the same, would have the same effect (Schwarzschild's comment at the Stellar Populations Semaine d'Étude 1958). Comparison of intrinsic colors of the strip boundaries would provide a test of the second possibility.

b) Christy (1966, 1968*a*) has shown that the position of the high-temperature boundary of the strip is sensitive to the helium abundance Y. In view of the cosmological significance of this abundance in clusters of widely different metal abundance, it is important to establish the variation (if any) of T_e for the blue edge of the strip among such clusters.

c) In addition to the reddening, knowledge of Y and Z, and their variation from cluster to cluster, must be available before we can obtain proper photometric distance moduli, main-sequence-turnoff luminosities, and ages of globular clusters.

II. NEW HORIZONTAL-BRANCH PHOTOMETRY

Photoelectric photometry of horizontal-branch stars in M3, M13, M15, and M92 has been obtained during the past 10 years with the 200-inch reflector at times of excellent seeing such that a small (7".6 diameter) diaphragm could be used. The program stars were chosen from previous photographic studies of M3 (Sandage 1953), M13 (Arp and Johnson 1955; Savedoff 1956), M15 (Arp 1955; Sandage, Katem, and Kristian 1968), and M92 (Arp, Baum, and Sandage 1952; Sandage and Walker 1966) in such a way that the gap boundaries would be well defined.

Special photometric procedures were required because the fields are generally crowded and the gradient of background light is steep, especially for stars close to the cluster center. Although preference was given to stars near the cluster periphery, it was necessary to observe the bluest stars on the horizontal branch to obtain adequate $E(B - V)$ values from the two-color diagram. These are generally faint ($18 > V > 15$), and some are close enough to the center to require extensive photoelectric mapping of the adjacent background light. From internal tests we found the accuracy for most final values to be about ± 0.02 mag in both $U - B$ and $B - V$, although somewhat larger errors may be present for the more difficult cases.

The photoelectric data in M3 and M15 were supplemented by photographic data of high weight for stars closest to the gap boundaries. These stars had previously been used as calibration standards in studies of *UBV* light curves for RR Lyrae variables in M3 (Roberts and Sandage 1955, Table III; Baker and Baker 1956) and M15 (Sandage, Sandage, Katem, and Breuckel [unpublished]). The photographic colors in M3 are based on twenty-five, twenty-six, and thirty-eight plates in V, B, and U, respectively; those in M15, by sixty-three, sixty-one, and thirty-nine plates in V, B, and U. There is no evidence for a systematic difference larger than 0.01 mag in either color relative to the photoelectric scale.

The photoelectric data for M3 are listed in Table 1, where the stars observed by Johnson (Johnson and Sandage 1956) are also given. Listed in Table 2 are photographic values for stars observed by Roberts and Sandage (1955) on the ($m_{\rm pg}$, $m_{\rm pv}$) system, but transformed from their Table III to B and V by

$$\left.\begin{aligned} V &= m_{\rm pv} - 0.065(m_{\rm pg} - m_{\rm pv}) - 0.006 \\ B - V &= 1.057(m_{\rm pg} - m_{\rm pv}) + 0.18 \end{aligned}\right\} 0.5 > (B - V) > -0.2\,, \qquad (1)$$

as determined from stars common to Tables 1 and 2. The U-magnitudes in Table 2 are those determined by Baker and Baker (1956) relative to a less complete version of Table 1 available in 1956. However, no use of the photographic $U - B$ data is made later in this paper to obtain $E(B - V)$ for M3.

Photoelectric data for M13 are given in Table 3. Observations by Johnson (Arp and Johnson 1955) are also listed. The stars are identified on various charts listed at the bottom of this table.

The photoelectric data for M15 are listed in Table 4. The notation is that used in the reconnaissance study of Sandage, Katem, and Kristian (1968). These stars will be identified on charts to be published when these complete data are discussed. Photographic data are given in Table 5. The numbering for stars designated by sectors in this table is that given by Arp (1955).

The data for M92 are given in Table 6. The stars are identified in Figure 1 of Sandage and Walker (1966). New photometry for many stars listed in their Table 1 has been obtained, and these data have been averaged with the previous values as listed in Table 6. Such stars are designated by SW in the "Remarks" column.

TABLE 1

Photoelectric Data for Horizontal Branch Stars In M3

Star	V	B-V	U-B	n	Star	V	B-V	U-B	n
182	15.70	+0.08	+0.11	25	I-II-45	15.64	+0.12	+0.10	1
235	15.76	+0.08	+0.13	1J	I-II-57	14.94	-0.30	-1.11	1J,2S
I-I-27	16.14	-0.02	+0.01	2	I-III-6	15.71	+0.08	+0.09	1
I-I-30	15.82	+0.01	+0.01	2	I-III-9	15.61	+0.48	-0.01	5
I-I-40	15.68	+0.39	0.00	1	I-III-12	15.71	+0.08	+0.10	1
I-I-42	15.58	+0.44	+0.02	1	I-III-14	15.76	+0.08	+0.11	1
I-I-47	16.37	-0.10	-0.22	1	I-III-19	15.46	+0.42	0.00	1
I-I-51	15.82	+0.07	+0.13	1	I-III-22	15.62	+0.12	+0.05	1
I-I-56	16.28	-0.07	-0.16	1	I-III-51	15.46	+0.50	-0.01	1
I-I-58	17.10:	-0.12:	-0.55:	1	I-III-57	16.95:	-0.21:	-0.47:	1
I-II-4	15.86	+0.08	-0.03	1J	I-IV-18*	15.73	+0.12	+0.05	1S
I-II-6	15.61	+0.08	+0.14	1J	I-VI-18	15.70	+0.16	+0.05	1J
I-II-11	15.68	+0.42	-0.03	1J	I-VI-26	15.76	+0.01	+0.05	1
I-II-13	15.76	+0.09	+0.16	1	I-VI-29	15.68	+0.42	+0.03	1
I-II-15	15.61	+0.45	-0.03	1	I-VI-48	15.90	+0.02	+0.06	
I-II-33	15.57	+0.14	+0.08	1					

* One observation by Johnson gives 15.95 -0.01 -0.02

TABLE 2

Photographic Data for Additional M3 Horizontal Branch Stars[+]

Star	V	B-V	U-B	n V B U	Star	V	B-V	U-B	n V B U
I-I-77	15.56	+0.43	-0.01	25,26,38	I-III-72	15.48	+0.17	+0.10	25,26,38
I-I-91	15.62	+0.17	+0.10	"	I-IV-33	15.67	+0.44	-0.02	"
I-I-97	15.79	+0.03	+0.06	"	I-IV-50	15.58	+0.46	-0.01	"
I-I-103	15.55	+0.13	+0.10	"	I-IV-56	15.67	+0.14	+0.11	"
I-II-11*	15.68	+0.43	-0.06	"	I-IV-86	15.63	+0.17	+0.12	"
I-II-15*	15.62	+0.45	-0.05	"	I-IV-92	15.63	+0.17	+0.10	"
I-II-45*	15.63	+0.14	+0.09	"	I-IV-98	15.61	+0.43	+0.01	"
I-II-63	15.74	+0.06	+0.12	"	I-V-75	15.68	+0.10	+0.12	"
I-III-9*	15.61	+0.48	-0.04	"	I-VI-18*	15.72	+0.15	+0.10	"
I-III-19*	15.46	+0.46	-0.04	"	I-VI-74	15.42	+0.15	+0.10	"
I-III-51*	15.56	+0.43	-0.01	"	I-VI-85	15.45	+0.10	+0.14	"

+ V and B-V transformed from Roberts and Sandage (1955) Table III by equations in the text. U from Baker and Baker (1956) Table IV.

* Stars common to Table 1.

1969ApJ...157..515S

TABLE 3

Photoelectric Data for Horizontal Branch Stars in M13

Star	V	B-V	U-B	n	Chart*	Star	V	B-V	U-B	n	Chart*
Barn29	13.14	-0.16	-0.80	3S	1	S-A404	15.58	-0.03	-0.13	1S	2
S-A531	14.94	+0.17	+0 12	2S	2	S-A581	15 61	-0.02	-0.07	2S	2
S-A16	14.97	+0.05	+0 09	1S	2	S-A342	15.64	-0 13	-0.53	1S	2
J11	15.00	+0.17	+0.18	3J	3	S-A113	15.69	+0.02	-0.14	1S	2
J52	15.08	+0.10	+0.16	2J	3	ARP-I-75	16.02	-0.15	-0.42	1S	5
S-A59	15.12	+0.07	+0.11	1S	2	S-B708	16.09	-0.07	-0.40	2S	2
S-A368	15.14	+0.06	+0.09	1S	2	J63	16.48	-0 19	-0.48	3J	3
S-A477	15.16	+0.09	+0.13	1S	2	J43	16.63	-0 16	-0 60	4J	3
J24	15.30	+0.04	+0.10	3J	3	S-A469	16.67	-0.13	-0 51	1S	2
S-A18	15.37	0.00	+0.01	1S	2	S-A197	16 97	-0.15	-0.61	1S	2
{J81	15.39	-0.16	-0.50	1J	3	S-A19	17.06:	-0.30	-1.09	2S	2
{J81	15.39	-0.14	-0.61	5S	3	S-A577	17.09	-0.17	-0.71	2S	2
J43a	15.41	+0 04	-0.01	2J	3	S-A360	17.17	-0 21	-0.71	1S	2
J7	15.47	-0.04	-0.05	2J	3	AJ-105	17.32	-0.16	-0.76	1S	3
J62	15.49	+0.06	-0.10	1J	3	AJ-110	17.33	-0 20	-0.76	1S	3
A11	15.45	-0.01	-0.04	8S	4	AJ-122	17.43	-0 24	-0.72	1S	3
S-A471	15.54	-0.02	-0 12	1S	2	S-A29	17.43	-0.16	-0.72	1S	2

* Chart references :

(1) E.E. Barnard, Pub.Yerkes Obs., 6, 62, 1931
(2) M P. Savedoff, A.J., 61, 254, 1956
(3) H.C. Arp and H.L. Johnson, Ap.J., 122, 171, 1955
(4) A. Sandage, 1969 : to be published
(5) H.C. Arp, A.J., 60, 317, 1955

TABLE 4

Photoelectric Data for Horizontal Branch Stars In M15

Star	V	B-V	U-B	n	Star	V	B-V	U-B	n
IV-108K	15.87	+0.22	+0.13	1	III-156K	16.28	+0.09	-0.02	1
I-135K	15.91	+0.18	+0.12	1	I-95K	16.59	0.00	-0.32	2
IV-19K	15.93	+0.17	+0.15	1	I-198K	16.64	-0.01	-0.32	2
IV-68K	15.95	+0.23	+0.22	1	III-W2-3	16.67	+0.01	-0.12	3
P4	15.98	+0.17	+0.16	4	I-73K	16.76	-0.04	-0.28	1
I-50K	16.05	+0.12	+0.25	1	P9	16.84	-0.01	-0.29	8
IV-25K	16.06	+0.10	+0.18	1	I-W5-4	17.21	-0.04	-0.50	4
IV-17K	16.06	+0.11	+0.11	1	I-112K	17.24	-0.05	-0.42	2
I-20K	16.08	+0.17	+0.23	1	AD	17.46	-0.08	-0.50	1
IV-4K	16.11	+0.12	+0.20	1	III-W3-3	18.24	-0.07	-0.67	2

TABLE 5

Photographic Data for Additional Horizontal Branch Stars in M15

Star	V	B-V	U-B	n V B U	Star	V	B-V	U-B	n V B U
S2	15.86	+0.18	+0.13	63,61,39	II-73	15.89	+0.19	+0.18	63,61,39
S10	15.92	+0.17	+0.19	"	74	15.95	+0.15	+0.13	"
S16	15.96	+0.14	+0.26	"	III-15	15.87	+0.17	+0.16	"
S18	16.84	-0.09	-0.60	"	28	15.68	+0.58	+0.09	"
S21	16.11	+0.12	+0.18	"	43	15.83	+0.21	+0.20	"
S31	16.11	+0.07	+0.06	"	52	15.82	+0.22	+0.21	"
I-1	15.97	+0.11	+0.07	"	67	15.93	+0.17	+0.20	"
4	15.87	+0.21	+0.18	"	71	15.51	+0.65	+0.09	"
7	15.65	+0.55	+0.07	"	IV- 2	15.84	+0.21	+0.19	"
9	15.93	+0.16	+0.12	"	44	15.79	+0.25	+0.18	"
11	15.89	+0.17	+0.14	"	45	15.64	+0.63	+0.06	"
14	16.02	+0.12	+0.09	"	63	16.06	+0.08	+0.09	"
42	16.52	-0.04	-0.19	"	66	15.95	+0.15	+0.15	"
51	15.80	+0.27	+0.22	"	68	15.77	+0.24	+0.17	"
54	15.86	+0.22	+0.18	"	Anon	15.75	+0.49	-	10 -
58	15.88	+0.20	+0.20	"	"	15.74	+0.50	-	10 -
II-10	16.06	+0.10	-0.09	"	"	15.78	+0.53	-	10 -
11	15.81	+0.20	+0.16	"	"	15.56	+0.54	-	10 -
23	15.78	+0.22	+0.15	"	"	15.62	+0.54	-	10 -
24	15.62	+0.57	+0.04	"	"	15.59	+0.55	-	10 -
36	15.95	+0.13	+0.12	"	"	15.65	+0.56	-	10 -
54	15.83	+0.23	+0.17	"	"	15.69	+0.58	-	10 -
59	15.87	+0.21	+0.17	"	"	15.74	+0.58	-	10 -

TABLE 6

Photoelectric Data for Horizontal Branch Stars in M92

Star	V	B-V	U-B	n	Remarks	Star	V	B-V	U-B	n	Remarks
I-10	15.48	+0.01	+0.02	2	SW	VII-46	15.86	-0.04	-0.16	2	
II-23	15.27	+0.08	+0.11	1		VIII-17	15.24	+0.09	+0.12	1	
II-40	15.86	-0.08	-0.29	5	SW	VIII-18	16.19	-0.08	-0.38	2	
II-66	15.00:	+0.10:	+0.12:	1	Cr?	VIII-19	15.31	+0.04	+0.04	1	
III-27	16.14	-0.10	-0.38	5	SW	X-5	15.48	+0.01	0.00	1	
IV-17	15.50	+0.02	+0.01	15	SW,B13	X-22	15.16	+0.17	+0.08	1	
IV-27	15.19	+0.17	+0.12	5	SW	X-23	15.38	+0.06	+0.07	1	
V-23	15.89	-0.03	-0.19	1	SW	XI-1	16.12	-0.09	-0.35	3	SW
V-36	15.60	-0.05	-0.17	2	SW	XI-11	16.02	-0.13	-0.31	3	SW, reject
V-55	15.30	+0.16	+0.11	1	SW	XI-23	15.93	-0.10	-0.14	1	
VI-3	15.32	+0.08	+0.08	2	SW	XII-1	15.11	+0.19	+0.07	3	SW
VI-10	15.24	+0.13	+0.09	4	SW	XII-6	15.92	-0.10:	-0.14	3	SW, reject
VI-55	15.18:	-0.02:	-0.04:	3	SW,Cr?	XII-9	15.09	+0.15	+0.09	1	
VI-61	15.31	+0.14	+0.12	1	SW	XII-10	15.27	+0.07	+0.05	1	
VI-67	15.19	+0.18	+0.09	1		XII-24	15.93	-0.06	-0.20	3	SW
VII-36	15.39	+0.02	+0.07	1		XII-26	15.25	+0.15	+0.12	1	

TABLE 7

Observed and Corrected RR Lyrae Boundaries in M3, M15, and M92

Cluster	ΔS	$(B-V)_o$ Blue Boundary		$(B-V)_o$ Red Boundary	
		Observed	Corrected To $\Delta S=6$	Observed	Corrected To $\Delta S=6$
M3	6	$0^m.175$	$0^m.175$	$0^m.425$	$0^m.425$
M15	10	0.150	0.170	0.380	~0.430
M92	10	0.175	0.195	0.420	~0.470

III. REDDENING VALUES

Two of the three methods used previously (Sandage 1964*b*) to derive reddenings for the four clusters depend on photometry of individual nonvariable cluster stars. The methods discussed here are (*a*) the two-color diagram of horizontal-branch stars, and (*b*) the observed color of the points of turnoff from the main sequence compared with the bluest nearby subdwarfs.

a) *The Two-Color Diagram*

Figure 1 shows the two-color diagrams using data from Tables 1, 3, 4, and 6. The intrinsic relation for stars of luminosity class V is shown as a solid line (Eggen 1965,

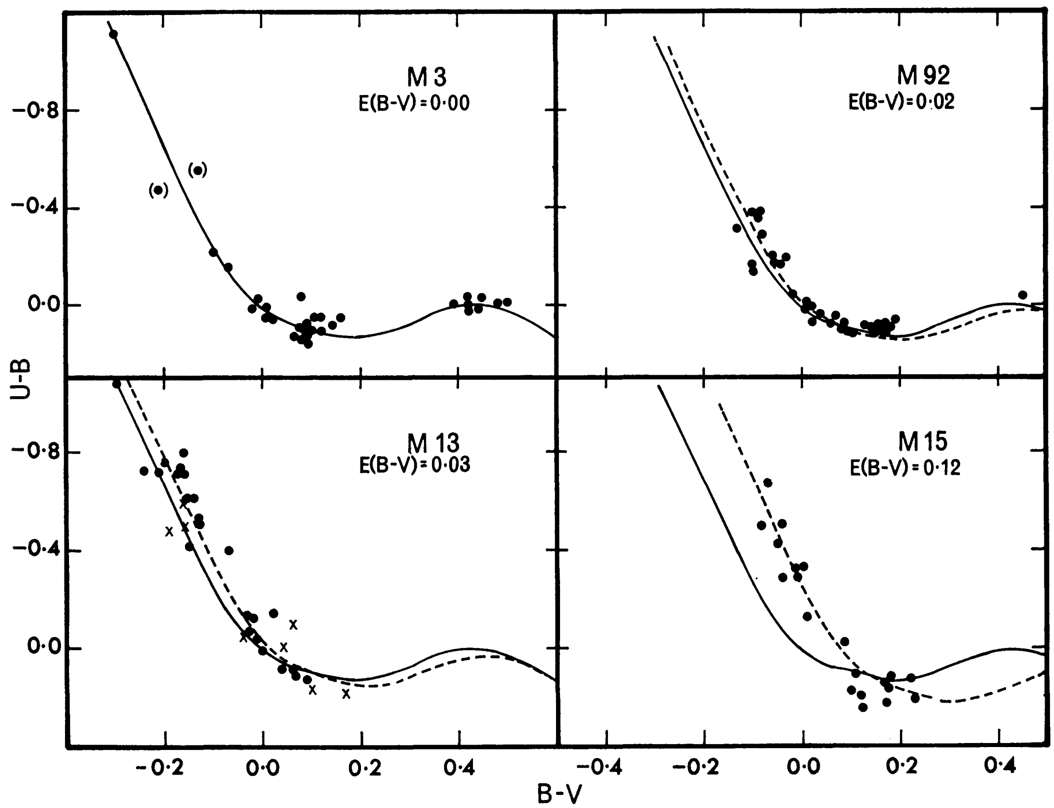

FIG. 1.—Two-color diagrams for horizontal-branch stars in four globular clusters. Photoelectric data from Tables 1, 3, 4, and 6 are plotted. For M13, stars observed by Johnson are shown as crosses. The intrinsic line for stars of luminosity class V has been translated along a reddening path of slope 0.72 to give the dashed lines.

Table 1). Justification for the use of this line for horizontal-branch stars comes from Mihalas' (1966, Table 8) calculated colors for hydrogen-line-blanketed models. These results show that the theoretical $(U - B, B - V)$ color-color lines for $4 \geq \log g \geq 3$ agree with the observed class V relation to better than $\Delta(B - V) = 0.02$ mag for $B - V$ bluer than -0.02. Other horizontal-branch stars in this color range are expected to lie within this range of log g. Newell, Rodgers, and Searle (1969) have measured log g for such stars in NGC 6397 and find no stars outside this interval. Other clusters are expected to cover the same interval in log g because of the similarity of horizontal-branch positions to within ~0.3 mag in the H-R diagram (see, e.g., Sandage and Smith

1966, Fig. 4). Therefore, unless horizontal-branch stars have anomalous energy distributions, contrary to Mihalas' models, $E(B - V)$ values can be derived from the $(U - B, B - V)$ diagrams to within 0.02 mag for $B - V$ bluer than -0.02.

The data in Figure 1 require $E(B - V) = 0.00$ for M3, 0.03 for M13, 0.12 for M15, and 0.02 for M92. These are similar to values reported earlier (Sandage 1964*b*), but are considerably smaller for M3, M13, and M92 than values given by van den Bergh (1967), Sturch (1967), and Arp (1962), where methods were used based either on some form of a bluest-object fit of observed colors to a reddening-free second parameter, such as Q obtained from integrated colors of globular clusters, or ΔS, or by the assumption of a dispersionless cosecant law.

One feature of Figure 1 is important in another connection. In M3, seven horizontal-branch stars are shown with $(B - V)_0$ redder than 0.40. One such star has been measured in M92, and five similar stars with photographically measured $U - B$ colors in M15 are listed in Table 5. None of the thirteen show a large ultraviolet excess relative to the Hyades intrinsic line, although all are undoubtedly metal poor. Presumably these stars lie close to the Hyades (high metal abundance) line owing to a combination of (1) the surface-gravity effect, which depresses the intrinsic line for stars of luminosity class III below the intrinsic line shown in Figure 1 for class V stars with $(B - V)_0 > 0.4$ (Johnson 1963; Eggen and Sandage 1964, Figs. 4 and 5; Sandage and Walker 1966) and (2) the abundance effect, which moves the stars along a blanketing line toward bluer $B - V$ and $U - B$ values. Apparently the two effects nearly cancel in M3, M15, and M92, causing the stars to lie fortuitously along the Hyades line. The same effect has been observed in RR Lyrae stars of low ΔS (Preston 1961*a*, Fig. 5) relative to variables of lower metal abundance; therefore, contrary to the situation for subdwarfs, the ultraviolet excess cannot be used to discover red horizontal-branch stars in the general field.

b) *Color of the Main-Sequence Termination Point*

For theoretical reasons this method is less certain than method *a* because it assumes that log T_e of the main-sequence turnoff is nearly the same for all clusters. If $E(B - V)$ is to be determined to within 0.01 mag, the requirement is that $\Delta(B - V)^{o,c} \leq 0.01$ mag, where $(B - V)^{o,c}$ is the intrinsic turnoff color corrected for differential line blanketing. This stringent requirement will be satisfied only if the age spread, Δt, between clusters is less than $\Delta t/t \simeq 0.03$, as discussed below, and if there is no log T_e (T.O.) $= f(Z)$ dependence (but see Simoda and Iben 1968 for a contrary prediction). Such a small age spread would follow from the model of galaxy collapse of Eggen, Lynden-Bell, and Sandage (1962), where $\Delta t/t \simeq 0.02$, but would not necessarily follow for all globular clusters in the collapse model of Rood and Iben (1968).

Adopting the stringent assumptions, the observed colors of points of turnoff from the main sequence are compared with the bluest $B - V$ color of field subdwarfs. After correcting for differential line blanketing, values of $E(B - V)$ can be obtained.

The intrinsic colors of field subdwarfs can be derived from the well-established fact that $B - V$ has a sharp cutoff for such stars near $B - V \simeq 0.36$ owing to the finite time of formation of Population II stars in the distant past. Evolution off the main sequence has wiped away bluer stars. Indeed, the sharpness of the cutoff shows that few, if any, main-sequence stars similar to those of the globular clusters were formed after a certain ancient epoch.

Work by Dixon (1963, 1966) gives an observed cutoff at $B - V \simeq 0.40$. Sandage (1964*a*) obtained 0.38; Sandage and Luyten (1967, 1969) find 0.40 and 0.37; Eggen (1968, 1969) gives 0.38; and Sandage and Kowal obtained 0.36 from an unpublished study of 2000 stars of high proper motion from the Lowell Proper Motion Survey. The data from this survey for stars bluer than $B - V = 0.80$ are shown in Figure 2.

The mean cutoff color from all quoted surveys is $B - V = 0.39$, uncorrected for reddening. The reddening cannot be zero for all field subdwarfs in these studies, but it must be small. If a Paranago-type reddening of the form

$$E(B - V) = 0.057 \csc b(1 - \exp D \sin b/187) \quad (2)$$

(Abt and Golson 1962) is applied to each star in Figure 2, where the distance D is obtained by fitting the $B - V$ colors after blanketing corrections to the zero-age main sequence, eight stars are bluer than a limit at $(B - V)_0 = 0.36$. However, five of these are within 30° of the direction to the center or anticenter of the Galaxy, where Eggen (1963, Fig. 4) has shown the reddening to be abnormally small to distances of at least 120 pc. Hence, our statistical reddening corrections are probably too large. Furthermore, because the bluest observed stars in any sample will themselves have the smallest mean reddening in a distribution, it is probable that equation (2) will always overestimate the reddening for stars near the blue boundary. These results suggest that $(B - V)_0 \simeq 0.36$ should represent the true cutoff color to within perhaps ± 0.02 mag.

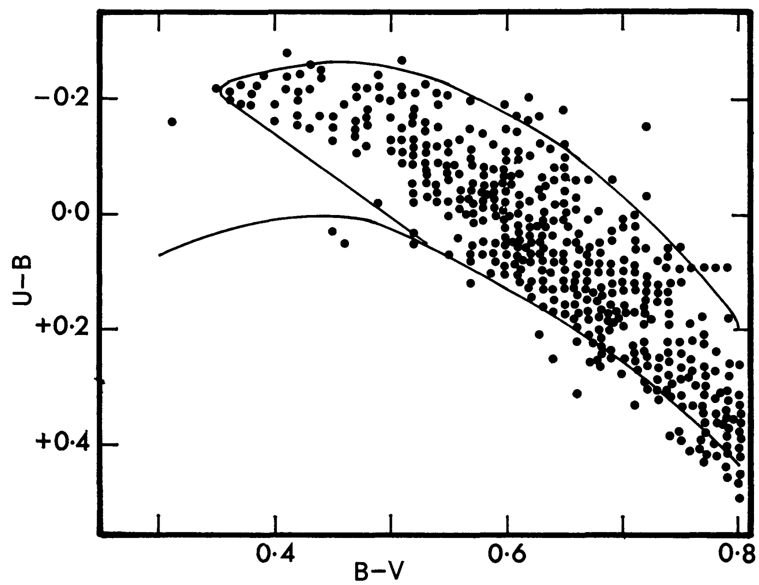

FIG. 2.—Two-color diagram for the bluest field subdwarfs observed by Sandage and Kowal from the Lowell Proper Motion Survey. The Hyades intrinsic relation is shown, together with envelope lines which enclose most of the subdwarfs.

Figure 3 summarizes photometric results for the main sequences of the four globular clusters under discussion. Large dots are photoelectric values, smoothed by photographic photometry on about ten plates in each color for M3, M15, and M92, and four plates in each color for M13. Small dots are photographically interpolated values for additional stars, measured on ten plates in each color. Figure 3 shows segments of more complete color-magnitude diagrams published elsewhere for M3 and M92 (Sandage 1968) and M13 (Baum *et al.* 1959).

The observed color of the turnoff is indicated on each plot. Reddenings are found by comparing this color with $(B - V)_0 = 0.36$ for M15 and M92, which have the smallest metal abundance and the largest deblanketing in $B - V$, and which therefore should be similar to the bluest stars in Figure 2. This comparison gives $E(B - V) = 0.12$ for M15 and 0.02 for M92, in agreement with values obtained from method *a*. M3 and M13 have considerably higher metal abundances. Therefore, differential blanketing corrections, $\Delta(B - V)$, must be applied to reduce the color system to that of M15 and M92.

An estimate of $\Delta(B - V)$ is available from the abundance analysis of M13 and M92 by Helfer, Wallerstein, and Greenstein (1959) where A, which is the logarithm of the hydrogen-to-metal ratio relative to the solar abundance, is near 2.2 for M92 and 1.4 for M13. These values confirm the Morgan (1959) and Deutsch (Kinman 1959*b*) result that M92 has a considerably lower metal abundance than M13.

Eggen's (1964, Fig. 19) rediscussion of Wallerstein's blanketing calibration for G stars shows that $\Delta(B - V) \simeq 0.04$ between M13 and M92. Consequently, the observed

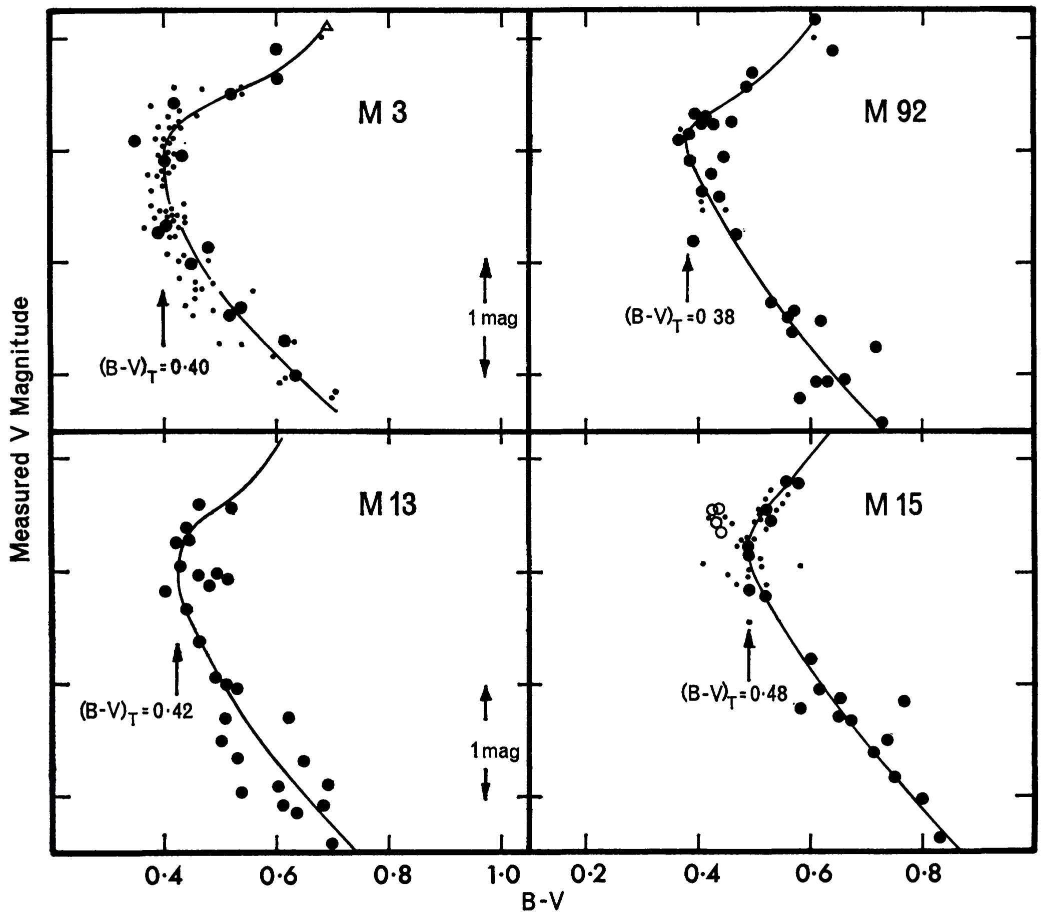

FIG. 3.—Observed C-M diagrams near the main-sequence termination points in four globular clusters. *Large circles*, photoelectric data; *small points*, stars measured by photographic interpolation. Four open circles in M15 may represent blue stragglers, similar to those previously known in M3 (Sandage 1953).

turnoff colors of M13 and of M3 (which has about the same metal abundance as M13) should each be made bluer by 0.04 mag to compare with the blue cutoff of field subdwarfs. The corrected color, when compared with $(B - V)_{o,T} = 0.36$, gives $E(B - V) = 0.00$ for M3, and 0.02 for M13, again in agreement with method *a* for M3 and differing by 0.01 mag for M13.

The result can be used in two ways. (1) If the assumptions of method *b* are accepted, the excellent agreement of $E(B - V)$ between the two methods gives added confidence in the reddening values. (2) If the assumptions of method *b* are held suspect, one can reverse the argument by applying the $E(B - V)$ values from method *a* to the blanketing-corrected turnoff colors, and thereby show that log T_e (T.O.) is the same for all four clusters to within Δ log $T_e = 0.004$ (corresponding to $\Delta(B - V) = 0.01$ mag).

The conclusion is of interest because Rood and Iben (1968) show that age is related to turnoff temperature by

$$\ln t_9 = 9.49 \log T_e + \text{const.} \quad (3)$$

in the range $2.5 > \ln t_9 > 1.8$ for clusters with fixed X and Z abundance values. This transforms to

$$\ln t_9 = 3.27(B - V) + \text{const.}\,, \quad (4)$$

using Morton and Adams' (1968) slope of the main-sequence color-temperature relation. Consequently, constancy of the blanketing-corrected turnoff color to 0.01 mag requires $\Delta t/t \leq 0.03$, which is closer to the Eggen *et al.* (1962) model of galactic collapse than to that of Rood and Iben, at least for these clusters. However, the conclusion that $\Delta t/t \leq 0.03$ applies to both the M3, M13 and the M92, M15 metal groups is not certain because of possible effects of variable Z on the turnoff T_e, following Simoda and Iben (1968). What can be stated is that M3 and M13 satisfy this time ratio, as do M15 and M92 separately, but to tie the two groups together requires knowledge of $\Delta \log T_e = f(Z)$. Therefore, we cannot at this time claim preference for one or the other collapse-time scales from the globular cluster data alone.

IV. INTRINSIC COLOR BOUNDARIES OF THE RR LYRAE GAP

Figure 4 shows the reddening-corrected C-M diagram of the horizontal branches of M3, M15, and M92. The photoelectric data are shown as closed circles, and the high-weight photographic data are plotted as open circles. Open triangles show the RR Lyrae stars in M3 and M15 plotted at intensity mean light and color. The photometry of Roberts and Sandage (1955) for M3 was transformed to V and $B - V$ by equation (1). Resulting values differ by less than 0.02 mag from results given previously (Sandage 1959). Data for the M15 variables were obtained directly on the UBV system (Sandage Sandage, Katem, and Breuckel [unpublished]). The data for M13 are not plotted because the instability strip is not well defined.

RR Lyrae stars are confined exclusively to a region which is nearly devoid of constant stars. Schwarzschild's (1940) discovery of the phenomenon has been subsequently confirmed in all adequately studied clusters. The boundaries of the strip are sharply defined, and occur in M3 at $(B - V)_0 = 0.175 \pm 0.005$ and $(B - V)_0 = 0.420 \pm 0.005$. Systematic errors in these colors should be smaller than 0.02 mag. The strip boundaries in M15 appear to be somewhat bluer at $(B - V)_0 = 0.150$ and 0.380.

Part of the difference between M3 and M15 is due to differential line blanketing caused by a metallic-abundance ratio of about a factor of 10 between the clusters. Preston's (1961*a*, Table 5) study of blanketing differences between RR Lyrae stars with $\Delta S =$ 6 and 10 shows that $\Delta(B - V)_{\Delta S=6,10} = 0.03$. The ΔS for M3 variables should be similar to that for RR Lyrae itself (Eggen and Sandage 1959, Fig. 4) at $\Delta S = 6$ (Preston 1959). Consequently, $\Delta(B - V)_{\text{blanketing}}$ near the middle of the RR Lyrae gap will be about 0.03 mag between M15 and M3. Stars near the blue edge of the strip will suffer less blanketing, while those on the red side will show more because of the increase of line strength with decreasing temperature. Correcting the M15 data for $\Delta(B - V) = 0.02$ and 0.05 mag at the blue and red boundaries of the strip to the $\Delta S = 6$ system gives $(B - V)^{o,c} =$ 0.170 and 0.430, in good agreement with the M3 values.

The gap position in M92 is less certain because an adequate study of the RR Lyrae stars has not been made. However, the strip boundaries cannot be bluer than $(B - V)^o =$ 0.175 nor redder than 0.420 as defined by the nonvariable stars. Again, correcting these colors to $\Delta S = 6$ gives $0.195 \leq (B - V)^{o,c} \leq 0.47$ for M92, both of which are redder than the M15 values. Table 7 summarizes the results.

These data appear to justify the common practice of assuming universal color boundaries for the instability strip to obtain estimates of $E(B - V)$. This method has been used in studies such as those of Arp (1955), Sandage and Smith (1966), Sandage and Wildey (1967), Sandage and Katem (1968), and others.

V. THE HELIUM ABUNDANCE DETERMINED FROM THE BLUE BOUNDARY OF THE GAP

Christy's (1966, 1968*a*) calculations show that the position of the blue boundary of the strip depends on the helium abundance, the absolute luminosity, and the mass. Adopting this theory permits Y to be determined once T_e (blue edge), M_V, and $\mathfrak{M}/\mathfrak{M}_\odot$

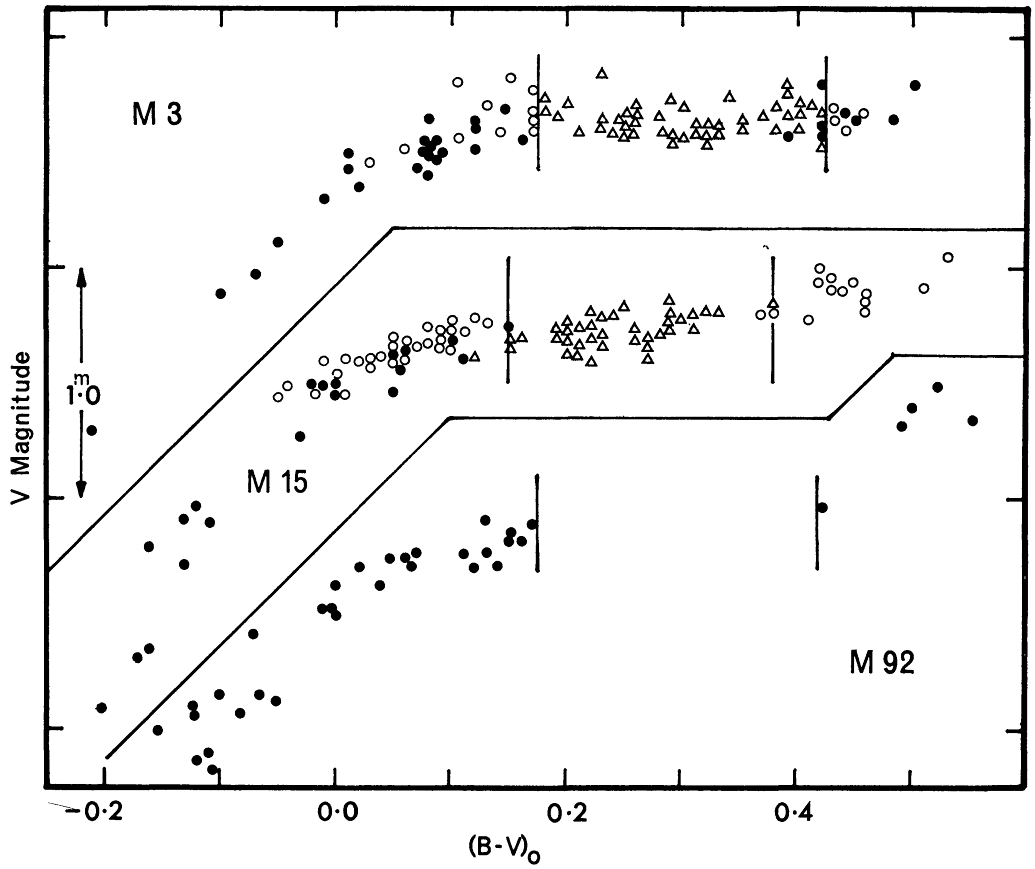

FIG. 4.—Enlarged segment of the horizontal branch in three globular clusters. *Closed circles*, photoelectric data from Tables 1, 4, and 6; *open circles*, photographic data from Tables 2 and 5; *open triangles*, cluster variables plotted at intensity mean light and color. All data have been corrected by $E(B - V) =$ 0.00 for M3, M12 for M15, and 0 02 for M92, but have not been corrected for differential blanketing

are known for horizontal-branch stars. Because the three parameters are subject to observational error, it is important to know the dependence of Y on each separately. Christy (1966) gives a large enough grid of models to permit a preliminary separation, which was done as follows:

1. From Christy's Table 1, T_e (blue edge) was plotted against $\mathfrak{M}/\mathfrak{M}_\odot$. A two-parameter family of curves results which depend on log L and Y. Interpolation among the curves produces a table of T_e and log L for fixed $\mathfrak{M}/\mathfrak{M}_\odot$ and Y values.

2. Log L was converted to M_V by adopting $M_{\text{bol}\odot} = 4.62$ and ΔM_{bol} (RR Lyrae) $=$ 0.00, with $L_{\text{bol}\odot} = 3.86 \times 10^{33}$ ergs sec^{-1}. These values are still uncertain, especially $M_{\text{bol}\odot}$, which Morton and Adams (1968) adopt as 4.84, but the effect on the final answer is small compared with uncertainties in the parameters for horizontal-branch stars themselves (see eqs. [5] and [7]).

3. The temperature T_e was transformed to $(B - V)^o$ by $\theta_e = 0.62 + 0.51\,(B - V)^o$, which is the determination of Oke, Giver, and Searle (1962) for RR Lyrae variables with $\Delta S \simeq 6$. Strom (1969) has recently questioned the result, suggesting that temperatures predicted by this equation are too low by 200°–350° K near $T = 7500°$ K. The effect of such higher temperatures at a given $(B - V)^o$ is discussed later.

4. From steps 2 and 3, the table produced in step 1 now gives Y as a function of M_V, $B - V$, and $\mathfrak{M}/\mathfrak{M}_\odot$. The result is shown in Figure 5, which is the C-M diagram showing lines of constant Y at various $\mathfrak{M}/\mathfrak{M}_\odot$ values. Here, $(B - V)_{\rm BE}{}^{o,c}$ is the reddening-free color of the *blue edge* of the RR Lyrae gap, corrected for line blanketing to a line strength corresponding to $\Delta S \simeq 6$, for which the Oke *et al.* (1962) temperature scale applies.

Figure 5 shows that, as $(B - V)_{\rm BE}{}^{o,c}$ becomes bluer at constant $\mathfrak{M}/\mathfrak{M}_\odot$ and M_V, Y increases. Alternatively, at constant $\mathfrak{M}/\mathfrak{M}_\odot$ and color, Y decreases for fainter M_V. Finally, at constant M_V and color, Y decreases for increasing mass.

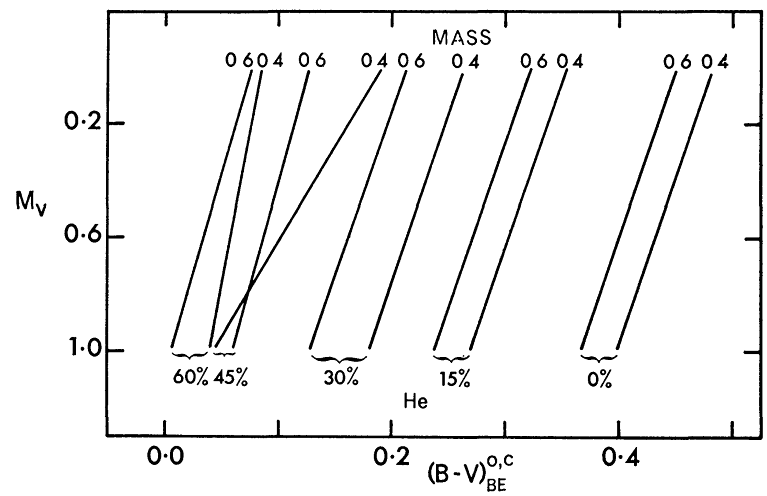

FIG. 5.—Variation of the helium abundance with M_V, mass, and intrinsic $B - V$ color of the blue edge of the instability strip corrected for differential line blanketing to variables of the $\Delta S \simeq 6$ type The normalization of the abscissa depends on the validity of the Oke, Giver, and Searle (1962) color-temperature relation for $\Delta S = 6$ cluster variables.

Some of these changes are difficult to see at a glance from Figure 5, and a different representation proved convenient. Interpolation of Figure 5 produces the three different two-parameter representations shown in Figure 6. In each case, Y is the independent variable, with M_V, $\mathfrak{M}/\mathfrak{M}_\odot$, and color alternately as the ordinate. Lines of constant mass or constant M_V are shown in these plots, keeping either M_V or the color fixed. Two different M_V values are shown in the lower panels of the diagram.

An approximate Y value can now be obtained by entering appropriate parameter values into Figure 6. The mass has been adopted as $\mathfrak{M}/\mathfrak{M}_\odot = 0.55 \pm 0.15$ from work by Newell, Rodgers, and Searle (1969) in NGC 6397. From Figure 4 and Table 7 we take $(B - V)_{\rm BE}{}^{o,c} = 0.175 \pm 0.02$ for M3. The absolute luminosity is taken as $M_V = 0.42 \pm 0.2$ (Sandage 1964*b*) shown as a vertical cross, or $M_V = 0.52$ (Newell *et al.* 1969) as an open circle. The crossed areas in each plot represent the range of Y for the assumed errors in M_V, $\mathfrak{M}/\mathfrak{M}_\odot$, and color.

From Figure 6 it follows that our best value of Y is 0.32, with a total range from 0.24 to 0.42. A similar result, based on the present value of $(B - V)_{\rm BE}{}^{o,c} = 0.175$ has been given by Christy (1966).

It proved useful to represent the correlations of Figures 5 and 6 analytically. The differential dependences of Y on $\mathfrak{M}/\mathfrak{M}_\odot$, $B - V$, and M_V could be determined by linearizing the correlations of Figure 6. By going back through the calibrations of $\theta_e = f(B - V)$ and $M_{\mathrm{bol}\odot}$, these lead to

$$Y = -3.138\theta_e - 0.34\mathfrak{M}/\mathfrak{M}_\odot + 0.40 \log L - 11.344 \,, \tag{5}$$

where θ_e is $5040/T_e$, and L is the bolometric luminosity in ergs sec^{-1}.

A check against Table 1 of Christy (1966) shows that equation (5) gives a very good

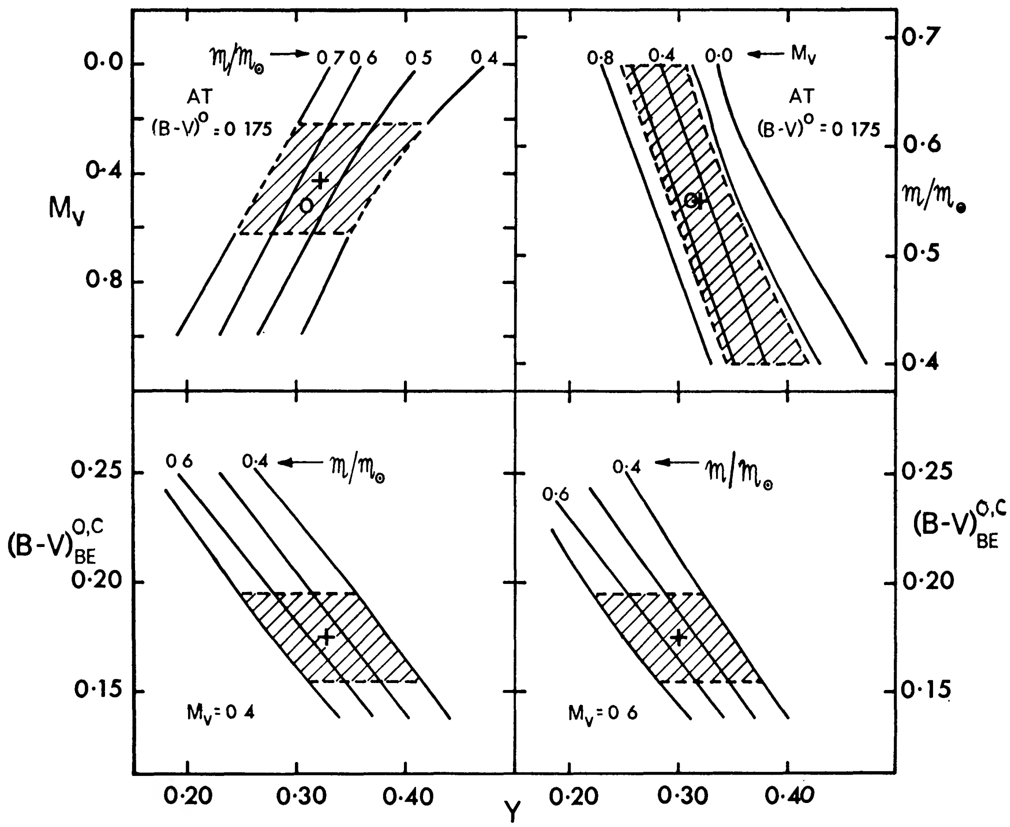

FIG. 6.—Same as Fig. 5, but in three different representations. Cross represents parameter choices of $M_V = +0.42$, $\mathfrak{M}/\mathfrak{M}_\odot = 0.55$; circle represents $M_V = +0\ 52$, $\mathfrak{M}/\mathfrak{M}_\odot = 0\ 55$. Hatched areas give the boundaries for errors of $\Delta M_V = \pm 0.2$ mag, $\Delta\mathfrak{M}/\mathfrak{M}_\odot = \pm 0.15$, and $\Delta(B - V) = \pm 0.02$ mag.

fit to Y within $(Y_{\mathrm{cal}} - Y_{\mathrm{Christy}}) = +0.004 \pm 0.018$ (A.D.), with the largest difference being $\Delta Y = 0.042$. Equation (5) is valid within this accuracy in the range

$$0.7 \geq \mathfrak{M}/\mathfrak{M}_\odot \geq 0.25 \,; \quad 2.8 \times 10^{35} \geq L \geq 1.5 \times 10^{35} \,;$$

$$0.840 \geq \theta_e \geq 0.636 \,; \quad 0.60 \geq Y > 0 \,,$$

which covers all models of interest.

Equation (5) can be transformed back into the observed quantities by adopting $L = 3.86 \times 10^{33}$ ergs sec^{-1}, $M_{\mathrm{bol}\odot} = 4.62$, ΔM_{bol} (RR Lyrae) $= 0.00$, and $\theta_e = 0.51 (B - V)_{\mathrm{BE}}{}^{o,c} + 0.62$, giving

$$Y = -3.138\theta_e - 0.34\mathfrak{M}/\mathfrak{M}_\odot - 0.16\, M_V + 2.830 \,, \tag{6}$$

or

$$Y = -1.60(B - V)_{\mathrm{BE}}{}^{o,c} - 0.34\mathfrak{M}/\mathfrak{M}_\odot - 0.16\, M_V + 0.884 \tag{7}$$

A temperature change of $\sim +250^\circ$ K for RR Lyrae stars near $T \simeq 7500^\circ$ K as advocated by Strom (1969) will change Y by $\Delta Y \simeq 0.06$ toward higher values, which agrees with Strom's discussion.

Errors of ± 0.5 mag in M_V will introduce a variation of $\Delta Y = \pm 0.08$, while changing the mass from 0.5 $\mathfrak{M}_\odot$ to 1.0 $\mathfrak{M}_\odot$ causes an appreciable change of $\Delta Y = 0.17$.

VI. DIFFERENCE OF HELIUM ABUNDANCE BETWEEN M3, M15, AND M92

Figure 4 and Table 7 show that the corrected blue color boundary is constant to within $\Delta(B - V) = 0.025$ mag for M3, M15, and M92. The color difference between M3 and M15, where the RR Lyrae data are most complete, is only $\Delta(B - V)_{\mathrm{BE}}{}^{o,c} = 0.005$ mag. It follows from the discussion in § I that temperature differences cannot be the reason for the Oosterhoff-Sawyer I and II period groupings because the observed color differences are smaller compared with $\Delta(B - V)_{\mathrm{BE}}{}^{o,c} \simeq 0.14$ required for such an explanation. Consequently, either the cluster variables differ in luminosity by $\Delta M_V = 0.24$ between groups I and II, keeping the mass fixed, or formally, a mass ratio of $\mathfrak{M}_{\mathrm{I}}/\mathfrak{M}_{\mathrm{II}} = 1.40$ could also be invoked, keeping M_V fixed. This follows from

$$\frac{P_{\mathrm{II}}}{P_{\mathrm{I}}} \simeq \frac{0.65}{0.55} = \left[\frac{\mathfrak{M}_{\mathrm{I}}}{\mathfrak{M}_{\mathrm{II}}} \frac{R_{\mathrm{II}}{}^3}{R_{\mathrm{I}}{}^3}\right]^{1/2} . \tag{8}$$

A more complete treatment, taking into account the *distribution* of periods across the gap, is given elsewhere (Sandage 1958; Kinman 1959*a*). The same conclusion is reached by Christy (1966, 1968*a*, *b*) following a more satisfactory route. But in addition, Christy demonstrates that a mass difference is not a possible explanation and, therefore, that the luminosity difference is the only remaining possibility.

These precepts and the data of Table 7 permit an estimate of the variation of Y between M3, M15, and M92 using equation (7). Adopting $\Delta M_V = +0.24$ (with M15 being brighter), $\Delta(B - V)_{\mathrm{BE}}{}^{o,c} = 0.005$, and keeping $\mathfrak{M}/\mathfrak{M}_\odot$ constant gives

$$Y(\mathrm{M3}) - Y(\mathrm{M15}) = -0.046 . \tag{9}$$

Similar considerations for M92 with $\Delta(B - V) = -0.020$, $\Delta M_V = 0.24$, and constant mass give

$$Y(\mathrm{M3}) - Y(\mathrm{M92}) = -0.006 . \tag{10}$$

The sense of the changes due to $\Delta(B - V)$ and ΔM_V is most easily seen by inspection of Figure 6, but it also follows from equation (7) with proper adherence to the signs.

The principal conclusion is, then, that the helium abundance of M3, M15, and M92 is the same within the error of the determination, despite the fact that the metal abundances differ by factors of ~ 10 between the clusters. The result is of considerable significance both for the problem of the chemical history of the Galaxy and for a decision on which zero-age main sequence is to be used for a proper photometric fit of color-magnitude diagrams for the four clusters studied here.

It is now apparently justified to keep Y fixed and to vary Z to obtain theoretical differential corrections of the form $\Delta M_V = f(Z/Z_{\mathrm{H}}, Y/Y_{\mathrm{H}})$ to the Hyades main sequence so as to obtain correct distance moduli and absolute luminosities. Only when this is properly done can ages of clusters be reliably determined.

Not enough data are presently available to generalize the result of constant Y to include such clusters as NGC 7006 (Sandage and Wildey 1967) or NGC 362 (Menzies 1967). The question of how constant Y may be (to better than, say, $\Delta Y \simeq \pm 0.05$) over the entire set of globular clusters is still open, in view of these anomalous cases and the work of Hartwick (1968).

VII. SUMMARY

The principal results are:

1. The two-color diagram gives $E(B-V) = 0.00, 0.03, 0.12$, and 0.02 for M3, M13, M15, and M92, respectively.

2. The intrinsic color $(B-V)_{\text{TO}}{}^{o,c}$ for the main sequence turnoff is the same to within 0.01 mag for the four clusters. If there is no dependence of T_e (T.O.) on Z, the spread of ages, Δt, is $\Delta t/t \leq 0.03$, or $\Delta t \lesssim 3 \times 10^8$ years. The conclusion on the value of Δt between the two metal-abundance groups will be modified if the T_e (T.O.) $= f(Z)$ function of Simoda and Iben (1968) is adopted. However, the conclusion would seem to hold between M3 and M13, and between M15 and M92, separately.

3. The corrected blue color boundaries of the RR Lyrae instability strip agree in M3, M15, and M92 to within 0.025 mag.

4. In agreement with Christy (1966), the helium abundance is $Y \simeq 0.32 \pm 0.09$ (total range) if the Oke, Giver, and Searle temperature scale is adopted. The abundance will be higher by about $\Delta Y \simeq 0.06$ if the change in temperature advocated by Strom applies.

5. The helium abundances of M3, M15, and M92 are the same to within the errors, despite the large changes in metal abundance.

6. Enough information is now apparently available to make, in principle, a proper photometric fit of the four globular clusters to the appropriate zero-age main sequence, and thereby to obtain a theoretically correct luminosity calibration and age determination.

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