

SPECTROSCOPIC BINARIES AND COLLAPSED STARS*

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ABSTRACT

Lists are given of single-line, spectroscopic binaries with large mass functions. The absence of a secondary spectrum in these systems could, in principle, result from the secondary star's being either a collapsed star or a massive neutron star. For all these systems, however, other explanations are possible in the light of present observations. Statistical considerations suggest that few, if any, of the systems in these lists contain collapsed or neutron-star secondaries. None of these binary systems coincide with any published X-ray source position.

I. NON-ECLIPSING, SINGLE-LINE BINARIES WITH LARGE MASS FUNCTIONS

Theoretical considerations suggest that some massive stars should terminate their evolution by gravitational collapse (see, e.g., Wheeler 1966; Thorne 1967). The end products of such collapse—"neutron stars" of $M \lesssim 1.5 M_{\odot}$, and "collapsed stars" of $M \gtrsim 1.5 M_{\odot}$ —have never been observed and, indeed, should be virtually impossible to observe directly because of their small size.

Zel'dovich and Guseynov (1965) have proposed that collapsed stars and neutron stars might be found among the unseen companion stars of single-line spectroscopic binaries. Motivated by this suggestion, we have searched the *Sixth Catalogue of the Orbital Elements of Spectroscopic Binary Systems* (Batten 1968) for systems which could harbor collapsed or neutron-star secondaries. The results of this search are contained in Tables 1 and 2, which extend a table of seven systems given by Zel'dovich and Guseynov (1965).

Our search of the binary catalogue followed the procedure of Zel'dovich and Guseynov. For each system, the mass, M_1 , of the primary star was estimated from its spectral type; and an approximate lower limit, $M_{2 \min}$, to the mass of the unseen companion was then calculated from the observed mass function, $f(M) = M_2^3 \sin^3 i / (M_1 + M_2)^2$, using $\sin i = 1$ (orbit seen edge-on).

The systems divide into three classes. *Class I systems*, in which the minimum mass of the unseen star is less than the Chandrasekhar limit ($1.4 M_{\odot}$), are of little interest to the search for collapsed and neutron stars since the unseen companion may well be a white dwarf.

The *class II systems*, in which the unseen secondary star is larger than $1.4 M_{\odot}$ but is less massive than the primary star, are listed in Table 1. For most of the class II systems the difference between the primary and the secondary mass is rather large, so that the secondary is probably too dim for its lines to have been detected on the available spectrograms. Of the four systems in which the two stars have similar masses (nos. 2, 6, 18, and

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TABLE 1*
SINGLE-LINE BINARIES OF CLASS II
($M_1 > M_{2 \text{ min}} \geq 1.4 M_\odot$)

No.	Star	α_{1900}	δ_{1900}	Magnitude	Spectral Type	Period (days)	e	K_1 (km sec ⁻¹)	$f(M)$	$\frac{M_1}{M_\odot}$	$\frac{M_{2 \text{ min}}}{M_\odot}$
1. B47 . . .	ω Cas	01 ^b 48 ^m 2	+68°12'	4.99	B8	69.92	0.30	29.64	0.164	4.5	1.9
2. B74 . . .	μ Cet	02 39.5	+9 42	4.28	F0 IV	1202.2	.460	14.4	0.261	1.8	1.4
3. B87 . . .	HR 976	03 09.8	+34 19	6.24	A2m	5.54348	.038	62.0	0.137	3.0	1.4
4. B100 . . .	17 Tau	03 39.0	+23 48	3.69	B6 III	100.46	.522	26.0	0.114	6.0	1.9
5. B117 . . .	μ Per	04 07.6	+48 09	4.14	G0 Ib	283.272	.057	20.67	0.25	10.	3.6
6. B130 . . .	ρ Tau	04 28.2	+14 38	4.65	F0 V	488.5	.094	18.5	0.317	1.8	1.5
7. B153 . . .	103 Tau	05 02.0	+24 08	5.41	B2 V	58.31	.189	36.73	0.28	12.	4.2
8. B171† . . .	η Ori A abc	05 19.4	-2 29	3.32	B0.5 V	9.2 yr	.1	17.5	1.84	30.	16.
9. B175 . . .	χ Aur	05 26.2	+32 07	4.77	B5 Iab	655.16	.171	20.53	0.56	25	8.6
10. B179 . . .	37 φ^1 Ori	05 29.3	+09 25	4.42	B0 IV	8.4 yr	.22	13.3	0.696	17	7.5
11. B184 . . .	θ^2 Ori	05 30.5	-05 29	5.07	O9.5 Vp	21.0315	.131	105.8	2.50	20.	14.
12. B195 . . .	HD 40005	05 51.1	+16 21	6.91	B3 V	3.306	0	77	0.157	10.	3.0
13. B234 . . .	30 τ CMa	07 14.5	-24 47	4.48	O9 III	154.90	.30	49.5	1.68	20.	12.
14. B241 . . .	HD 59543	07 25.3	-13 46	6.94	B5	17.911	.52	45.6	0.10	7.0	2.0
15. B242 . . .	σ Pup	07 26.1	-43 06	3.23	K5 III	257.8	.17	18.55	0.164	5.0	2.0
16. B256 . . .	α Pup	07 48.8	-40 19	3.73	G5 III	2660.	.4	11.8	0.35	3.2	2.2
17. B297 . . .	κ Vel	09 19.0	-54 35	2.49	B2 IV	116.65	.19	46.5	1.16	11.	7.3
18. B394 . . .	α Dra	14 01.7	+64 51	3.66	A0 III	51.42	.38	46.9	0.436	3.5	2.5
19. B442 . . .	HD 145849	16 08.1	+36 41	5.51	K5, gK4	2150.	.6	16.0	0.469	4.5	3.0
20. B444 . . .	σ Sco	16 15.1	-25 21	2.89	B1 III	34.2	.36	34.0	0.112	15.0	3.4
21. B476 . . .	α Her B	17 10.1	+14 30	5.39	G0 II-III	51.578	.0220	36.122	0.2583	4.1	2.2
22. B528 . . .	β Sct	18 41.9	-4 51	4.21	G5 II	834.	.35	16.65	0.33	5.0	2.7
23. B540 . . .	δ^1 Lyr	18 50.2	+36 51	5.51	B3	88.352	.37	39.7	0.523	10.	4.9
24. B544 . . .	HD 176318	18 54.6	+38 07	5.70	B6 V	2.91225	.169	79.3	0.150	5.0	1.9
25. B567 . . .	201 G Sgr	19 27.3	-40 15	5.90	A3 III	4.625	.09	72.5	0.18	3.0	1.6
26. B568 . . .	HD 184398/9	19 29.2	+55 31	6.37	K2 II-III	108.5707	.054	22.1	0.1222	4.0	1.6
27. B580† . . .	10 S Sge	19 51.5	+16 22	5.87-7.02	F6-G5 Ib	676.2	.246	15.03	0.217	10.	3.4
28. B596 . . .	HD 191473	20 05.0	+36 57	8.6	B0.5 V	4.2876	.0447	106.48	0.535	15	6.2
29. B601 . . .	22 Vul	20 11.2	+23 12	5.17	G2 Ib	251.0	.050	26.8	0.50	10.	4.8

* The entries in this table include all single-line, non-eclipsing binaries listed in Batten's (1968) catalogue, for which $M_1 > M_{2 \text{ min}} \geq 1.4 M_\odot$, and for which our own survey of the literature yielded no convincing evidence (e.g., doubling of lines) for the presence of a normal secondary star. As a result of our survey of the literature, we have adopted different spectral types for some of the primary stars from those given by Batten. Since references to the literature on each star are readily available in Batten (1968) and in Martinov (1961), we do not give them here. In the first column are given our identifying number for each system and also Batten's

number. The column $M_1/M_\odot$ is our estimate of the mass of the primary star, based on its spectral type. The column $M_{2 \text{ min}}/M_\odot$ is a lower limit on the mass of the secondary star, which is obtained from the mass function, $f(M)$, using our estimated primary mass, M_1 , and using $\sin i = 1$ (orbit seen edge-on). The remaining notation and entries are those of Batten.

† Primary is double with mass ratio 11.2/10.6.

‡ Primary is a Cepheid variable.

TABLE 1—Continued

No.	Star	α_{1900}	δ_{1900}	Magnitude	Spectral Type	Period (days)	e	K_1 (km sec ⁻¹)	$f(M)$	$\frac{M_1}{M_\odot}$	$\frac{M_{2\min}}{M_\odot}$
30. B605	35 Cyg	20 ^h 14 ^m 8	+34°40'	5 16	F5 Ib	2440.0	.506	9.57	0 142	10	2.9
31. B616	HD 194184	20 19 1	-41 07	6.15	K3 III	117 776	.240	22.62	0 130	4.3	1.7
32. B617	HD 194495	20 20.8	+21 10	7 09	B7	4.9052	133	82.2	0.275	6.0	2.8
33. B618	θ Cep	20 27.9	+62 39	4.21	A5m, F5 IV	840.6	.03	13.85	0.232	2.0	1.4
34. B622	26 Vul	20 31.8	+25 32	6.22	A4 III	11.088	.2843	58.7	0.205	2.3	1.4
35. B634	HD 198784	20 47 6	+37 36	6.97	B3	3.30353	.018	63.81	0.082	10	2.3
36. B639	HD 199579	20 53 1	+44 33	5.96	O5-6e	48.608	.0988	42.22	0.374	25	7.3
37. B644§	HD 203025	21 14.6	+58 10	6.42	B2 IIIe	225 44	.226	21.9	0.3956	14.	5.3
38. B650	HD 204188	21 21.8	+18 58	6.03	A3m	21.724	0	41.45	0 1607	1.4	1.4
39. B683	HD 209961	22 02.0	+47 45	6.27	B2 V	2.1721	0	127 7	0.47	12.	5.2
40. B714	π Cep A	23 04.7	+74 51	4.42	G2 III	556.2	0.281	23 02	0.624	3 0	2.7

§ Primary is double with mass function 0.0353.

TABLE 2
SINGLE-LINE BINARIES OF CLASS III
($M_{2\min} > M_1$ and $M_{2\min} \geq 1.4M_\odot$)

No.	Star	α_{1900}	δ_{1900}	Magnitude	Spectral Type	Period (days)	e	K_1 (km sec ⁻¹)	$f(M)$	$\frac{M_1}{M_\odot}$	$\frac{M_{2\min}}{M_\odot}$
1. B127	71 Tau	04 ^h 20 ^m 6	+15°23'	4.51	F0 V	5200.	0.241	15 1	1.70	1.8	3 7
2. B141	KS Per	04 41.8	+43 06	7 60-7 70	A0 II, Ape	360.47	.28	51.4	4.41	12., 5 0, or 1.0	15., 10., or 6.0
3. B145	ω Eri	04 48.0	- 5 37	4.39	A9 IV	3057	46	18.1	1.318	1.9	3.3
4. B155	HD 33232	05 03 7	+40 53	8 1	B3n, e	3710.	.31	31.5	10.4	10.	22.
5. B189	49 Ori	05 34.0	- 7 16	4.81	A4 IV	445 74	.549	28.6	0 635	2.2	2.4
6. B232	δ Gem	07 14.2	+22 10	3.52	F0 IV	2238 6	.353	27 1	3.80	1.8	6.3
7. B508	ξ Pav	18 14.0	-61 33	4.36	K2 III	2214.	.264	17.92	1.188	4.0	4.4
8. B576	HD 187399	19 44.7	+29 10	7 7	A0, Bge, B9e β	27.97	.35	104.5	2.72	4.0	6.8
9. B615	HD 193928	20 17.8	+36 36	9 43	WN6+	21.64	0	130	4.94	10	14.
10. B655	ξ Aqr	21 32.4	- 8 18	4.69	A7 V	8016.	0.544	11.31	0.711	2.0	2.4

NOTES TO TABLE 2

General—The entries in this table include all single-line, non-eclipsing binaries listed in Batten's (1968) catalogue, for which (i) $M_{2 \min} > M_1$, (ii) $M_{2 \min} \geq 1.4 M_\odot$; and (iii) our own survey of the literature yielded no convincing evidence for the presence of a normal secondary star. See note (*) to Table 1 for further discussion.

1. 71 Tau = HD 28052.—The broad, poor lines of the primary (only H and Ca II are measureable) may well conceal a second spectrum. The second set of lines reported by Frost, Barrett, and Struve (1929) would seem to imply a period near 100^d , while Abt (1965) derived a period of 5200^d from the primary spectrum and saw no lines at all that could be attributed to the second star. Abt suggests that in this and other long-period, well-separated binaries, the secondary is itself double or multiple, thus producing a much smaller luminosity than would be produced by a single star of the same mass.

2. K5 Per = HD 30353.—The hydrogen deficiency of the primary suggests to Wallerstein, Greene, and Tomley (1967) that it is a highly evolved, overluminous star with present mass about $1 M_\odot$ and absolute magnitude about -3 . If this is the case, the secondary will have a mass of about $6 M_\odot$, which corresponds to a main-sequence spectral type of B5 with absolute magnitude near -1 . Such a secondary might well have remained undetected in the observations that have thus far been carried out.

3. ω Eri = HD 31109.—This system has both a rather large separation and an overluminous (evolved) primary. The more massive secondary might therefore be either itself multiple, or single but fainter than the primary.

4. HD 33232.—The observed spectrum is a complicated one, with lines of various elements yielding different values of the orbital elements, especially K_1 (Merrill 1934). The values tabulated are those derived from the hydrogen lines. The lines of Fe II, Si II, Si III, and Mg II show larger amplitudes, while the amplitudes obtained from Ca II (probably affected by interstellar lines) and He are smaller. The mass function might therefore be seriously in error; and the secondary star might be little, if at all, more massive than the primary. As in other long-period systems, a double or multiple secondary is also a possibility.

5. 49 Ori = HD 37507.—Lines of a second star, consistent with the period of $445^d 74$ and implying a mass ratio of about 2, were reported on five plates by Frost *et al.* (1929). These lines were not detected by Abt (1965).

6. δ Gem = HD 56986.—This system, like ω Eri, may include either a multiple secondary or an overluminous primary. The large mass ratio (at least 3) makes it seem unlikely that the primary can be sufficiently luminous to mask a secondary of normal spectrum.

7. ξ Pav = HD 168339.—This system also has a rather large separation and a primary which may be overluminous.

8. HD 187399.—This close system is completely inclosed in an expanding cloud giving hydrogen absorption lines at -95 km sec^{-1} . (The system velocity is -19 km sec^{-1} .) There is also hydrogen emission which gives velocities near that of the system. No good spectral type is available for the primary, and its mass is, therefore, poorly determined. If it is an overluminous, evolved star, the complex spectrum could well conceal any lines due to the secondary.

9. HD 193928.—The primary is a Wolf-Rayet star. At least six systems are known consisting of a Wolf-Rayet and an O or early B star. In each case the early-type star is the more massive, by a factor of as much as 3–5, without being necessarily the more luminous. The mass ratio of less than 2 for this system is therefore consistent with the secondary star's being sufficiently less luminous than the primary that its spectrum is completely lost among the complicated WN lines.

10. ξ Aqr = HD 205767.—This long-period system could contain a double or multiple secondary.

40 of Table 1), three contain a primary which has evolved off the main sequence and is probably somewhat overluminous for its mass, thereby concealing the secondary spectrum. The remaining system (no. 6, ρ Tau) has been reported to show two sets of spectral lines on some plates (Lee 1910), although the velocities found from the second set are quite unreliable. It is unlikely that further optical studies can yield convincing evidence for collapsed or neutron stars among these systems. If any of them should prove to be a strong source of X-rays or gamma rays, however, the case for a collapsed or neutron-star secondary would be greatly strengthened (see § II below).

The systems of *class III*, those in which the unseen secondary is more massive than $1.4 M_\odot$ and more massive than the primary, are by far the most interesting, since, other things being equal, the more massive star should be the brighter and should contribute the stronger lines to the combined light of the system. Table 2 lists the class III systems and suggests reasons for the absence of the secondary spectra. Three factors seem to be important. First, in long-period systems of large separation (nos. 1, 3, 4, 6, 7, and 10), the secondary star may itself be double or multiple, rendering it much fainter than ex-

pected, since two main-sequence stars of mass $\mathcal{M}/2$ give less light than one star of mass $\mathcal{M}$. Second, in many of the systems (nos. 2, 3, 5, 6, 7, and 9), the observed primary star has evolved well off the main sequence and could easily be more luminous than a main-sequence star of larger mass. Third, several of the systems (nos. 2, 4, 8, and 9) have complicated spectra, including emission components, which may help to mask the lines of the secondary star.

For all of these class III systems and for numbers 2, 6, 18, and 40 of class II an effort should be made to detect the secondary, both in high-resolution spectrograms and in color measurements over the entire accessible spectrum. Presence of a normal secondary could be definitely established by detection of its spectral lines or could be strongly suggested by color anomalies. (If the observed light of a system in fact comes from two stars, its intensity distribution might not be normal for the observed spectral type as deduced from the lines.) Failure to detect the secondary star cannot, on the other hand, rule out its normalcy.

TABLE 3*
SINGLE-LINE ECLIPSING BINARIES OF CLASS II
($\mathcal{M}_1 > \mathcal{M}_{2\text{ min}} \geq 1.4 \mathcal{M}_\odot$)

No.	Star	No.	Star	No	Star	No	Star
B8 . .	TV Cas	B191 .	32 TU Cam	B380 .	UX UMa	B556	FL Lyr
B15 . .	ZZ Cas	B194	Z Ori	B453	W UMi	B578	V505 Sgr
B22 .	YZ Cas	B199	SV Gem	B504	μ Sgr	B664	EK Cep
B176 . .	EY Ori	B237	AR Mon	B534	β Lyr	B721	AR Cas
B183 . .	θ^1 Ori	B263 . .	UU Cnc	B535 .	HS Her	B727	UU Cas

* For detailed spectroscopic data on these stars see Batten (1968) The numbers given here for each star are those of Batten.

II. X-RAYS AND GAMMA RAYS FROM MASS ACCRETION

As pointed out by Zel'dovich and Guseynov (1965), the discovery of X-rays or gamma rays from a single-line binary system, such as those in Tables 1 and 2, would constitute positive evidence for the presence of a collapsed star or neutron star: Matter flowing out from the primary star should accrete onto the collapsed or neutron-star secondary. During this accretion, infalling particles of matter acquire velocities approaching that of light. Collisions of the infalling particles with each other and with the surface of a neutron star should produce gamma rays and X-rays (see Salpeter 1964; Shklovsky 1967; Cameron and Mock 1967).

None of the binary systems of Tables 1 and 2 fall within the error limit of any published X-ray position (tables and references in Friedman, Byram, and Chubb 1967 and in Gorenstein, Giacconi, and Gursky 1967). The positions of the systems in Tables 1 and 2 should be compared with additional X-ray and gamma-ray positions as these become available.

III. ECLIPSING BINARIES OF CLASSES II AND III

For the purpose of comparison with the systems of Tables 1 and 2, we have also compiled from Batten's catalogue lists of single-line *eclipsing* systems in which the mass of the unobserved star is greater than $1.4 \mathcal{M}_\odot$. These are given in Tables 3 and 4. In these systems, the secondary is certainly not either a collapsed star or a neutron star, since the small size of such stars (radius $\lesssim 20$ km) would prevent them from eclipsing any normal star. Their intrinsic faintness, of course, precludes their being the stars eclipsed, even at secondary minima.

The gravitational lens effect is also unimportant for a binary system containing one

SINGLE-LINE ECLIPSING BINARIES OF CLASS III
($M_{2\min} \geq M_1$ and $M_{2\min} \geq 1.4M_\odot$)

No.	Star	α_{1900}	δ_{1900}	Magnitude	Spectral Type	Period (days)	e	K_1 (km sec ⁻¹)	$f(M)$	$\frac{M_1}{M_\odot}$	$\frac{M_{2\min}}{M_\odot}$
1. B149	ϵ Aur	04 ^h 54 ^m 8	+43°40'	3 0	A8 Ia	9890.	0.172	14.71	3 12	13	13.
2. B501	W Ser	18 04.1	-15 34	9 0-9.9	F5 III†	14.15667	.37	66.	0.35	1.4	1.4
3. B534.	β Lyr†	18 46.4	+33 15	3.3-4.2	B8pe	12.908	.017	185.0	8.5	50.	41.
4. B631	V367 Cyg	20 44.2	+38 55	7 4-8.0	A2	18.5972		93.2	1.56	3 0	4.4
5. B696.	CQ Cep.	22 32.9	+56 23	9 0-9.5	WN6+	1.6410	0 0	295	4.38	10.	13.

* The entries in this table include all spectroscopic, eclipsing binaries listed in Batten's (1968) catalogue, for which (i) only one set of spectral lines is seen, (ii) $M_{2\min} \geq M_1$, and (iii) $M_{2\min} \geq 1.4 M_\odot$. For notation see footnote (*) to Table 1.

† The lines are largely shell spectrum, and the spectral type is very poorly known.

‡ This system is actually class II by our definitions; but it is shown here because of its remarkable masses.

TABLE 4*

collapsed star or neutron star; the "lens" is too small and too near the normal star to produce a noticeable effect.¹

The twenty eclipsing binaries for which $\mathcal{M}_{2\min} < \mathcal{M}_1$ resemble the class II systems discussed above in that, in most cases, the disparity of masses is sufficiently large that one would not really expect to see the second set of lines. Of the four eclipsing systems for which $\mathcal{M}_{2\min} \geq \mathcal{M}_1$, one has a large separation and may harbor a multiple secondary, three have non-main-sequence primaries, and all four have complex spectra, including emission lines or a shell spectrum. These eclipsing stars are, therefore, very much like the non-eclipsing class III systems, a fact which certainly does not support the conclusion that the latter include many collapsed or neutron stars.

A second consideration indicates that collapsed stars and neutron stars are, at best, infrequent among the systems of Tables 1 and 2. Single-line binaries with large mass functions are more common among the eclipsing systems in Batten's catalogue than among the non-eclipsing systems. Ten class III systems and forty class II systems were found among about 400 single-line, non-eclipsing binaries, while about seventy eclipsing systems yielded four of class III and twenty of class II.

In spite of possible differences in the selection effects entering into the eclipsing and non-eclipsing samples, it seems highly unlikely that a statistically significant fraction of the class III and III systems harbors collapsed or neutron stars, although a small number of them may do so.

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¹ In quantitative terms, the fractional change in the flux of the normal star due to the gravitational lens effect, when the two stars and the observer are lined up perfectly, is $\delta\mathfrak{F}/\mathfrak{F} = (1 + 8r_g a/R_1^2)^{1/2} - 1 \ll 1$, where $r_g = 2G\mathcal{M}_2/c^2$ is the gravitational radius of the collapsed or neutron-star secondary, R_1 is the radius of the normal primary, and a is the separation of the two stars.

