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POLARIZATION OF THE CONTINUUM BACKGROUND IN PLANETARY NEBULAE

ELIA M. LEIBOWITZ
Harvard College Observatory, Cambridge, Massachusetts
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ABSTRACT

This work derives an expression for the expected polarization of the continuum background of planetary nebulae in a given "window" in the visible spectrum. The polarization discussed here results from electron-scattering of the radiation of the nucleus star by the electron gas in the nebula. It is found that in one nebula the effect should be measurable.

I. INTRODUCTION

The continuum background of the radiation of planetary nebulae in the visible region is produced by the following seven major processes: (i) hydrogen recombination and free-free transitions; (ii) hydrogen two-photon emission; (iii) He I recombination and free-free transitions; (iv) He I two-photon emission; (v) He II recombination and free-free transitions; (vi) He II two-photon emission; (vii) electron-scattering.

Little attention has been paid to process vii as the emissivity due to this mechanism is considered negligible in magnitude (Seaton 1955). This neglect is justified in general, in view of the very small cross-section for this process ($\sigma = 0.66 \times 10^{-24}$ cm^2).

The situation is slightly different, however, if we consider the continuum in restricted regions of the spectrum, where some of the other processes make a minimal contribution to the continuum. It will be shown that in such "windows" in the spectrum, under certain physical and geometrical conditions, electron-scattering as a source of the continuum is not negligible.

An immediate consequence to this should be readily observed, namely, the continuum in the proper windows of nebulae fulfilling the required conditions should be polarized. The electron-scattering component of the continuum is highly polarized since in the geometry of planetary nebulae the nucleus star may be regarded as a point source. The polarization is reduced, however, by the other components generated by the other six processes, which are unpolarized.

We shall use the following simplifying assumptions: (1) The nebula is spherically symmetric (shell), or cylinderically symmetric (ring). (2) The electron density N_e and the electron temperature T_e are constant throughout the observed region. (3) The densities of H^+ and He^+—N_{H^+} and N_{He^+}—are constant in the observed region. (4) The nucleus star radiates in the visible region of the spectrum as a black body.

II. THE EMISSION FUNCTIONS

The dependence on wavelength of the seven processes in the visible region is rather weak, except for i, iii, and v. The emission functions of the continuum produced by these three processes rise to maxima near the series limits of the corresponding elements. For example, ϵ_1, the emissivity by process i, has a pronounced peak near the Balmer limit. By observing the continuum far away from series limits we reduce the contribution from recombination considerably. In particular, the present calculations are made for the filter $4100 < \lambda < 4340$ Å. This region is relatively free of H or He II absorption lines and the filter eliminates the peak of the hydrogen recombination function near the Balmer limit at $\lambda = 3646$ Å.

Seaton (1955, 1960) calculated the emission functions of the processes i, ii, and v. To a first approximation we consider the emissivity by process iii as equal to that by process i. Using Seaton's tables we calculated the emission by these four processes in our window. Drake, Victor, and Dalgarno (1969) have recently calculated the probability and the frequency distribution of the two-photon decay in He I. The transition 2^1S–1^1S is the one responsible for process iv, since the probability for the two-photon decay 2^3S–1^1S is 4.02×10^{-9} sec^{-1}, comparable to 51.3 sec^{-1} in the 2^1S–1^1S transition. The number of transitions per unit time per unit volume is proportional to $\alpha_B(\text{He I})X(\text{He I})N_eN_{\text{He}^+}$, where α_B is the rate coefficient of recombinations on all levels of the ion except the ground level; $X(\text{He I})$ is the fraction of the recombinations which populate the 2^1S level (Hummer and Seaton 1964). We used Hummer and Seaton's results that for $T_e = 10^4$ ° K, $\alpha_B(\text{He I})/\alpha_B(\text{H I}) = 1.05$ and $\alpha_B(\text{H I}) = 2.58 \times 10^{-13}$ cm^{-1}. We took $X(\text{He I}) = 1$ in view of the very long lifetime of the 2^3S level against two-photon decay and because a photon emitted in the 2^1P–1^1S transition will be reabsorbed in the nebula. The emission due to process vi is calculated by use of the frequency distribution and the probability for the two-photon process in the Ly-α transition of hydrogenic systems, given by Spitzer and Greenstein (1951). From Hummer and Seaton (1964) we took for $T_e = 10^4$; $\alpha_B(\text{He II}) = 1.528 \times 10^{-12}$ cm^{-3} sec^{-1} and $X(\text{He}^+) = 0.285$, where $X(\text{He}^+)$ is the fraction of the recombinations to the $n = 2$ level which populates the $2S$ state. The emissivities due to the two-photon processes in H and He I were then multiplied by the correction factor $(1 + 0.6 \times 10^{-4} N_e)^{-1}$, allowing for collisional de-excitation of the upper level in the corresponding transitions (Seaton 1960). With $N_e = 10^4$ cm^{-3}, $T_e = 10^4$ ° K, the emissivity within our window, due to hydrogen atoms in units of ergs cm^{-3} sec^{-1} is

$$4\pi\epsilon(\text{H I}) = 17.4 \times 10^{-27} N_e N_{\text{H}^+} . \tag{1}$$

Similarly the emissions due to He I and He II are, respectively,

$$4\pi\epsilon(\text{He I}) = 27.4 \times 10^{-27} N_e N_{\text{He}^+} , \tag{2}$$

$$4\pi\epsilon(\text{He II}) = 75.5 \times 10^{-27} N_e N_{\text{He}^{++}} . \tag{3}$$

We take $N(\text{He})/N(\text{H}) = 0.2$ and consider two extreme cases of (*a*) a nebula in which all the helium is doubly ionized and (*b*) a nebula in which helium is only singly ionized. In the limit of case *a* the contribution to the continuum in our window from helium is given by equation (3). With $N_{\text{H}^+} = 10^4$ cm^{-3}, the emission due to all processes other than electron-scattering in case *a* is (in cgs units)

$$4\pi\epsilon_a/N_e = 32.5 \times 10^{-23} . \tag{4}$$

In case *b*, the contribution from helium to the continuum is given by equation (2). The total emission other than by electron-scattering in case *b* is therefore

$$4\pi\epsilon_b/N_e = 22.9 \times 10^{-23} . \tag{5}$$

III. THE EMISSION FUNCTION OF THE SCATTERED LIGHT

The emission per cubic centimeter per second due to electron scattering is given by

$$4\pi\epsilon_{sc} = \int_{\nu_1}^{\nu_2} J_\nu \sigma N_e d\nu , \tag{6}$$

where J_ν is the intensity of the incident light at the point in question, integrated over the entire solid angle (van de Hulst 1953), and ν_1 and ν_2 are the limits of the window. Let

$$I = \int_{\nu_1}^{\nu_2} I_\nu d\nu$$

be the intensity of the light in the window at the surface of the central star. The physical attenuation of the intensity in the nebula is due to absorption by electron free-free transitions and by photo-ionization, mainly from the $n = 3$ level of hydrogen. The intensity at a distance r from the central star is

$$I(r) = Ie^{-k\rho(r-r_0)} , \tag{7}$$

where r_0 is the inner radius of the nebula, k is the mean absorption coefficient per gram, and ρ is the density of the gas. From a figure given by Aller (1963) one finds that for hydrogen in thermodynamic equilibrium, k in our interval in the spectrum is about 1.6×10^2 and with $N_{H^+} = 10^4$, $\rho k = 2.64 \times 10^{-18}$. This numerical value of k is certainly an upper limit to the mean absorption coefficient in a planetary nebula, since in the thermodynamic state of a planetary nebula, the excited levels of hydrogen are less populated than in thermodynamic equilibrium with the same electron density and temperature (e.g., b_3—the factor determining the departure of the population of the third level of hydrogen in planetary nebulae from the equilibrium population of that level—under typical conditions in planetary nebulae is 0.12 [Seaton 1959]). Therefore it is safe to conclude that along a distance in the nebula of less than about 5×10^{17} cm, absorption of the stellar continuum in the window by the nebula can be neglected. In most of the known nebulae, the thickness of the observed region is smaller than this value.

Let R be the radius of the nucleus. At a distance r from the center

$$J_\nu(r) = \pi I_\nu (R/r)^2 , \tag{8}$$

where I_ν depends on the surface temperature of the star T_S. Equations (6) and (8) give

$$4\pi\epsilon_{sc}/N_e = \pi I \sigma (R/r)^2 . \tag{9}$$

IV. THE POLARIZATION FUNCTION[1]

Figure 1 shows the geometry of the problem. The nucleus star is at the origin; the x-axis is in the direction of the line of sight. Consider a point P along a line of sight that intersects the plane of the sky at a point Q at a distance ρ from O. P is at a distance r from the center and the radius vector $\boldsymbol{OP}$ forms an angle θ with $\boldsymbol{OQ}$.

Let ϵ_t be the emission function at P for light with the electric vector vibrating tangentially; ϵ_r is the emission function for light vibrating in the radial direction:

$$\epsilon_t = \tfrac{3}{4}\epsilon_{sc} + \tfrac{1}{2}\epsilon , \tag{10}$$

$$\epsilon_r = \tfrac{3}{4}\epsilon_{sc} \sin^2\theta + \tfrac{1}{2}\epsilon , \tag{11}$$

where ϵ is either ϵ_a or ϵ_b. The coefficients $\frac{3}{4}$ and $\frac{1}{2}$ are implied by the condition

$$\int (\epsilon_t + \epsilon_r) d\Omega = 4\pi(\epsilon_{sc} + \epsilon) ,$$

where the integration on the left-hand side is over the entire solid angle around the point P.

[1] In this section we follow the approach of van de Hulst (1950) to a similar problem in the solar corona.

Consider a model of a shell nebula with an inner radius r_0 and an outer radius S. The intensity of light vibrating tangentially, emitted per second per unit solid angle by a column with a cross-section of 1 cm^2, is

$$K_t = 2\int_{X_{\min}}^{X_{\max}} \epsilon_t dX \ , \tag{12}$$

where $X_{\max}^2 = S^2 - \rho^2$, and $X_{\min} = 0$ if $\rho \geq r_0$ and $X_{\min}^2 = r_0^2 - \rho^2$ if $r_0 > \rho$. Similarly, for light vibrating radially,

$$K_r = 2\int_{X_{\min}}^{X_{\max}} \epsilon_r dX \ . \tag{13}$$

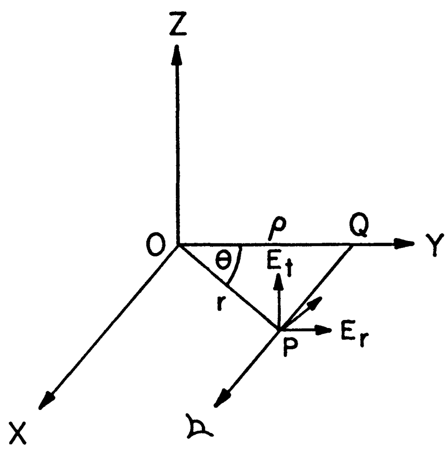

FIG. 1.—Geometry in the scattering process. Nucleus star is at O. A scattering electron is at P. E_t and E_r are the components of the electric vector of the scattered radiation in the tangent and the radial directions, respectively.

We define

$$\alpha = 10^{13}\pi I\sigma = 10^{13}(r/R)^2\, 4\pi\epsilon_{sc}/N_e \ ,$$

$$\beta = 10^{23}\sum_{\mathrm{i}}\int_{\nu_1}^{\nu_2}\gamma_{\mathrm{i}}\, dh\nu = 10^{23}\, 4\pi\epsilon/N_e \ ,$$

where γ_{i} are the rate coefficients for the mechanisms i–vi, and the sum is over the processes producing the continuum in the nebula in question. The integration is over the "window" in the spectrum; ϵ is either ϵ_a or ϵ_b.

In practical observation, the observed intensities are averages over the area of the nebula exposed to the slit of the polarimeter. Consider a narrow rectangular slit, mounted in a radial direction, as shown in Figure 2, exposing an area of the nebular image extending from an inner radius ρ_1 to the outer edge S. The intensity observed through this slit is to a first approximation

$$\langle K\rangle = A\int_{\rho_1}^{S} K d\rho \ , \tag{14}$$

where A is a constant of no consequence to our final result. It is useful to define the dimensionless parameters $\zeta = \rho_1/S$ and $\lambda = \max(r_0/S, \rho_1/S)$ and the geometric functions

$$B(\zeta,\lambda) = \tfrac{1}{2}\pi \log(1/\lambda) - \zeta/\lambda + \zeta ,$$

$$F(\zeta,\lambda) = \sin^{-1}(\zeta/\lambda) - \sin^{-1}\zeta + (\zeta/\lambda)\sqrt{(1-\zeta^2/\lambda^2)} - \zeta\sqrt{(1-\zeta^2)} ,$$

$$G(\zeta,\lambda) = \pi/2 - \sin^{-1}\zeta + \lambda^2[\sin^{-1}(\zeta/\lambda) - \pi/2] - \zeta\sqrt{(1-\zeta^2)} + \zeta\sqrt{(\lambda^2-\zeta^2)} .$$

Carrying out the integrations (12) and (13) and then (14) we obtain

$$\langle K_t\rangle/A = 3a \times 10^{10}R^2B(\zeta,\lambda) + \beta S^2G(\zeta,\lambda) , \tag{15}$$

$$\langle K_r\rangle/A = \tfrac{3}{2}a \times 10^{10}R^2[B(\zeta,\lambda) - \tfrac{1}{2}F(\zeta,\lambda)] + \beta S^2G(\zeta,\lambda) . \tag{16}$$

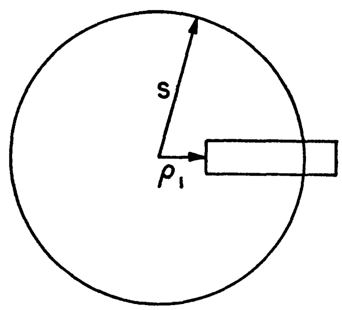

FIG. 2.—Slit of the polarimeter on the background of the image of the nebula

The observed polarization is

$$P = (\langle K_t\rangle - \langle K_r\rangle)/(\langle K_t\rangle + \langle K_r\rangle) = \frac{2B(\zeta,\lambda) + F(\zeta,\lambda)}{6B(\zeta,\lambda) - F(\zeta,\lambda) + \frac{8}{3}G(\zeta,\lambda)q} , \tag{17}$$

where

$$q = 10^{10}(\beta/a)(S/R)^2 .$$

With $\zeta = 0$

$$P = \frac{\log(1/\lambda)}{3\log(1/\lambda) + \frac{4}{3}(1-\lambda^2)q} . \tag{18}$$

For a nebula with $\lambda = \frac{1}{2}$, i.e., the inner radius is half the outer radius, with $\zeta = 0$ we obtain

$$P = \frac{0.693}{2.08 + q} . \tag{19}$$

In a similar way we obtain an expression for the polarization of a ring nebula observed with a similar slit. Let $2d$ be the dimension of the observed volume in the direction of the line of sight and let $\eta = d/S$. For $\eta \leq \lambda$ and $\zeta \leq \lambda$,

$$P = \frac{\eta/\lambda - \eta + \tan^{-1}(1/\eta) - \tan^{-1}(\lambda/\eta)}{3(\eta/\lambda - \eta) - \tan^{-1}(1/\eta) + \tan^{-1}(\lambda/\eta) + \frac{8}{3}\eta(1-\lambda)q} , \tag{20}$$

with $\eta = \lambda = \frac{1}{2}$,

$$P = \frac{1.23}{1.77 + q} .$$

V. RESULTS AND DISCUSSION

The polarization of the continuum background in planetary nebulae is determined by the dimensionless parameter $q = 10^{10} (\beta/\alpha) (S^2/R^2)$, representing the inverse of the ratio of scattered light to atomic continuum. In order to have a measurable polarization which is of the order of 1 per cent or larger, q must be of the order of 100 or smaller. The value of q depends strongly on S, the radius of the nebula, and on R, the radius of the nucleus star. It depends also on T_S, the surface temperature of the central star through the quantity α. The value of q depends through β also on N_{H^+}, the density of hydrogen ions in the nebula, on $Y = N_{\mathrm{He}}/N_{\mathrm{H}}$, on N_e, the electron density in the nebula and on T_e, the electron temperature in the nebula. Thus in a search for polarization of the continuum in planetary nebulae we have to look for an object having the following properties: large central star, small nebular radius, high surface temperature of the central star, high electron density and high electron temperature in the nebula.

In looking for candidates for polarization we used O'Dell's (1963) and Seaton's (1966) data concerning the features of planetary nebulae and their nuclei. The radii of the central stars can be derived from Seaton's list by the expression (Landolt-Börnstein 1965) $\log (R/R_\odot) = \frac{1}{2} \log (L/L_\odot) - 2 \log T_S + 7.51$, where L is the luminosity of the central star and T_S is its effective surface temperature. In computing the q-values of different nebulae we used the values $\beta = 22.9$ for case b, for all the nebulae. In so doing we introduce only a small error since q depends weakly on β, and β itself is an insensitive function of the physical parameters of planetary nebulae. The value of α is obtained from Figure 3, in which we plot α as a function of T_S, using the black-body intensity (Allen 1963). Applying case b to the nebula BD+30°3639, we find the results shown in the accompanying tabulation.

	Seaton	O'Dell
T_S (° K)	46000	20000
α..	4	1
S (cm).	3 5×10^{17}	1.54×10^{17}
R (cm)	9 0×10^{11}	29 7×10^{11}
q	89 2	6 15

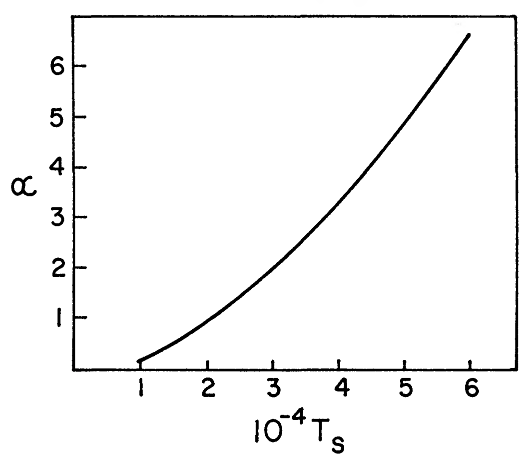

FIG. 3.—$\alpha = 10^{13}\, \pi\sigma I$ in the region $4100 < \lambda < 4340$ Å versus black-body temperature

From equation (19) we obtain the corresponding polarizations:

$$P_{\text{Seaton}} = 0.76 \text{ per cent} , \quad P_{\text{O'Dell}} = 8.4 \text{ per cent} .$$

The choice of $\lambda = \frac{1}{2}$ in equation (19) is admittedly arbitrary, but expressions (17) and (18) are quite insensitive to the value of λ for $\frac{1}{3} < \lambda < \frac{3}{4}$. With the data of both sources, O'Dell as well as Seaton, a measurable polarization is predicted in the nebula BD+30°3639. The differences between the two sets of data result from two main reasons: (*a*) O'Dell's temperature T_S is obtained by assuming that the nebula is optically thick in the H I Lyman continuum. Harman and Seaton (1966) showed that the temperature deduced from the surface brightness of the nebula in hydrogen lines is inconsistent with the temperature derived from the surface brightness in He II lines. They interpreted the smaller value, obtained from hydrogen lines, as a result of an incomplete absorption of the stellar H I Lyman continuum by the nebula. (*b*) Both sources derive the radius of the nebula by the use of the surface brightness in the Hβ line, assuming constant mass M of all planetaries. They differ however in the adopted value of M. O'Dell (1962) obtained $M = 0.14\ M_{\odot}$ by the method of statistical parallaxes and from distances deduced by independent methods. Seaton obtained $M = 0.38\ M_{\odot}$ from distances to nebulae, in which N_e—the electron density—could be estimated from relative intensities of forbidden lines. As our two results for P in the nebula BD+30°3639 differ from each other by an order of magnitude, polarization measurement of the continuum background in this nebula may serve as a test between the two sets of parameters and as an indirect independent measure to the optical thickness of this nebula.

An attempt to carry out the proposed measurements was recently made by Dr. W. Liller. So far no observational results were obtained since the recording of the nebulosity of BD+30°3639 in the continuum was unsuccessful.

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