THE ASTROPHYSICAL JOURNAL, Vol. 155, March 1969

# ON THE INTERPRETATION OF THE COLOR-MAGNITUDE DIAGRAMS OF M67 AND NGC 188

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*Received July 1, 1968*

## ABSTRACT

A method for determining the helium content $Y$, for a given $Z$, of old galactic clusters is described. This method could be of considerable importance since it is independent of the distance modulus of the cluster and of color corrections such as those due to interstellar reddening and differential line blanketing. It is found that $Y$ is proportional to the width of the gap of the C-M diagram just above the main sequence In order to calibrate this effect, a number of theoretical evolutionary tracks was obtained from the main sequence to beyond the hydrogen-exhaustion phase for different chemical compositions. A statistical analysis of the data of Eggen and Sandage was made which confirms the presence of such a gap in M67 and NGC 188 It shows that the gap width has a well-defined value equal to $0.167 \pm 0.010$ mag in M67. This last result yields $Y = 0\ 38 \pm 0.02$ at $Z = 0\ 03$ and $Y = 0.28 \pm 0.02$ at $Z = 0.06$. Higher $Y$-values would be required if $Z$ is as low as 0 01. The corresponding ages for M67 are about $3.5 \times 10^9$ years for $Y = 0.38$, $Z = 0.03$, and $2\ 6 \times 10^9$ years for $Y = 0\ 28$, $Z = 0.06$. In NGC 188, the gap, although present, does not appear to be fully developed, which leaves some uncertainty as to its chemical composition if this method is used.

## I. INTRODUCTION

Evolutionary tracks for stars with convective cores exhibit a well-known hook just above the main sequence, corresponding to the hydrogen-exhaustion phase of the convective core (Henyey, LeLevier, and Levée 1955; Haselgrove and Hoyle 1956; Hallgren and Demarque 1966; Hallgren 1967; Iben 1967*a*, *b*). This rapid phase of evolution results in a gap in the stellar distribution in the color-magnitude diagram of a star cluster. Eggen and Stoy (1960) noticed such a gap in NGC 2477 and Eggen (1963), in NGC 752. In recent years, photoelectric observations of M67 (Eggen and Sandage 1964) and NGC 188 (Eggen and Sandage 1968) have made possible a detailed comparison of observation with theory. The purpose of this paper is to investigate the dependence of the gap width on chemical composition and to analyze the stellar distribution in the vicinity of the gap observed in M67 and NGC 188 in order to gain some information on the chemical composition of their members.

## II. STELLAR MODELS

Low-mass, Population I stars can convert hydrogen into helium by either the *p-p* chains or the CNO cycle. Stars in the former category have radiative central regions whereas those in the latter have convective cores. The dominant process in any given star is a function of mass and chemical composition, both of which determine the main-sequence position of the star. Stars with radiative central regions on the main sequence may develop a convective core prior to hydrogen exhaustion and the onset of shell burning. If this occurs, the evolution of such a star is qualitatively the same as that of a star with a convective core on the main sequence. As in the case of massive stars ($M \gtrsim 2M_\odot$) hydrogen exhaustion takes place simultaneously over a significant mass fraction of the star, and a small helium core results. As this occurs the discontinuity in mean molecular weight between the convective core and the radiative zone which surrounds it gives rise to an increase in radius with a resulting drop in effective temperature. The effect becomes more noticeable as the core increases in mass. With the exception of a thin outer region, contraction of most of the star sets in and continues until a hydrogen-shell source can provide the necessary luminosity. This contraction is on a Kelvin time scale. Since the radius remains almost constant while the luminosity increases significantly, there is a rapid increase in the effective temperature of the star. This stage of evolution corresponds to a gap just above the main sequence in the C-M diagram of a galactic cluster.

At a given luminosity level, the size of the convective core (if it exists) depends principally on the metal abundance. High $Z$ increases the opacity, leading to steeper radiative temperature gradient, which favors convection. Furthermore, high $Z$ enhances the CNO cycle which also favors convection. Thus, low $Z$ stars may not develop a convective core. Where there is a convective core, one can expect the luminosity difference between the beginning and the end of the hydrogen-exhaustion phase (proportional to the size of the convective core) to increase with increasing $Z$. The region of the hydrogen-exhaustion phase (HEP) in the C-M diagram will be referred to as the "gap."

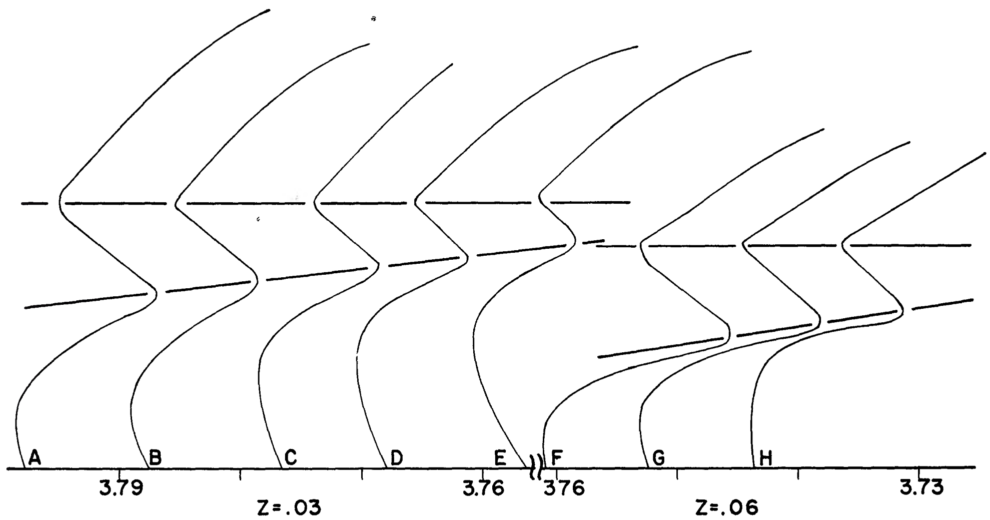


FIG. 1.—Evolutionary tracks for several models exhibiting a gap. The abscissa is log $T_{\rm eff}$ (increasing to the left), and the ordinate is $M_{\rm bol}$. All tracks have been normalized to the same main-sequence luminosity. Horizontal line for tracks $A$–$E$ is 0.55 mag brighter than the main sequence; horizontal line for tracks $F$–$H$ is 0.46 mag brighter. Horizontal and sloping lines stress the steady alteration of gap properties with composition. Initial compositions for tracks $A$–$E$ are all $Z = 0.03$ with $Y$ values, respectively, 0.40, 0.35, 0 30, 0.25, and 0.20. Tracks $F$–$H$ all have $Z = 0.06$ with $Y$ values, respectively, 0.30, 0.25, and 0.20.

The effect of varying $Y$ for fixed values of $Z$ is easily determined. An increase in $Y$ leads to a higher mean molecular weight which in turn raises the internal temperature of the star. This favors the CNO cycle which leads to convection. Models with high $Y$ would show larger gaps than models with the same luminosity and $Z$ but a lower $Y$. At the end of the HEP the luminosity of the stars of varying $Y$ but constant $Z$ is the same if their ZAMS luminosities are the same (Fig. 1). This relationship has been determined by means of detailed evolutionary computations.

One further point remains to be considered. How does the mass of the star affect the extent of the gap for a given composition? It is apparent that a certain range of low-mass stars will not have convective cores because their temperatures never become high enough during core hydrogen burning to provide a significant fraction of the energy through the CNO cycle. As the mass increases, the temperature increases and the CNO cycle becomes more prominent. Since the transition from $p$-$p$ to CNO dominance is very sensitive to the temperature (Schwarzschild 1958, p. 82, Fig. 10.1), the transition occurs over a small range in the mass of the star (of the order of 2 per cent). However, once the mass is high enough to insure the development of a convective core, a further increase in mass over a small range will not modify the mass fraction of the core. The reason for this is the following: over a small mass range the energy generation and opacity laws are very nearly the same, and the resulting models are then very nearly homologous (Schwarzschild, 1958, pp. 103 ff.). There is, in fact, evidence that this effect is nearly constant for a much greater range in mass (up to at least 5 $M_\odot$, Schlesinger 1968). Thus, a calibration of $Y$ to the size of the gap must be made at a luminosity such that a fully developed convective core exists for all relevant values of $Y$.

Stellar models have been evolved from the main sequence through the HEP and into the shell-burning phase for the following composition parameters:

$$Z = 0.06\,, \quad Y = 0.20,\ 0.25,\ 0.30;$$
$$Z = 0.03\,, \quad Y = 0.20,\ 0.25,\ 0.30,\ 0.35,\ 0.40;$$
$$Z = 0.01\,, \quad Y = 0.25\,.$$

For each composition, the mass was adjusted so that the main-sequence model was approximately the same, with $M_{\rm bol}$ in the range 4.50–4.55. Details of the evolutionary

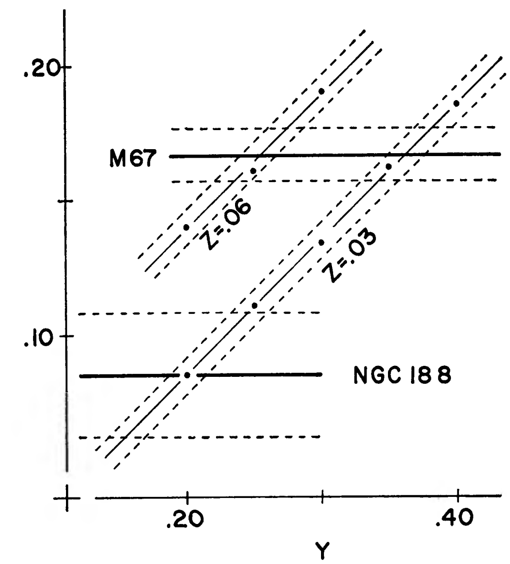


FIG. 2.—Variation of gap width with composition parameters. Ordinate is the width of the gap in magnitudes, and abscissa is the helium content, $Y$. Sloping lines represent gaps from the tracks $A$–$H$ of Fig. 1, and dashed lines bracketing them are our estimates of the accuracy of inferring the gap width from the models. Horizontal lines represent the values of the gap width (and error estimates) determined from observational data on the two clusters.

tracks are given in Table 1. A Henyey program with logarithmic variables was used (Hartwick 1966; Demarque, Hartwick, and Naylor 1968; Schlesinger 1968). The opacities were those of Cox (1965); the energy generation rates those of Reeves (1965), including the three *p-p* chains and the CNO cycle treated as discussed by Demarque and Schlesinger (1969). In the hydrogen convection zone, the mixing-length theory of Böhm-Vitense (1958) and Baker and Kippenhahn (1962) was used, with the mixing length set equal to 1 pressure scale height. Although the treatment of the convection zone affects the radii of the models, it is irrelevant to the physical development of the stellar core.

Figure 1 shows the evolutionary tracks for $Z = 0.03$ and for $Z = 0.06$. The corresponding calibration curves for the gap width as a function of $Y$ for each $Z$ are given in

TABLE 1 EVOLUTIONARY TRACKS

| AGE ($\times 10^9$ yrs) | $M_{bol}$ | LOG $T_{eff}$ | $R/R_\odot$ | LOG $P_c$ | LOG $T_c$ | LOG $\rho_c$ | $X_c$ | Core |
|---|---|---|---|---|---|---|---|---|
| $M = 0.92\ M_\odot$, $X_o = 0.57$, $Y_o = 0.40$ $Z_o = 0.03$ TRACK A | | | | | | | | |
| 0.00 | 4.5391 | 3.7978 | 0.9271 | 17.3575 | 7.2037 | 2.0658 | 0.5700 | CONV |
| 1.00 | 4.4373 | 3.7984 | 0.9691 | 17.3953 | 7.2253 | 2.1288 | 0.4446 | CONV |
| 2.00 | 4.3360 | 3.7966 | 1.0271 | 17.4346 | 7.2503 | 2.2053 | 0.3217 | CONV |
| 2.80 | 4.2591 | 3.7924 | 1.0813 | 17.4714 | 7.2767 | 2.2778 | 0.2127 | CONV |
| 3.64 | 4.1659 | 3.7868 | 1.1580 | 17.5522 | 7.3176 | 2.4009 | 0.0465 | CONV |
| 3.72 | 4.1315 | 3.7885 | 1.1676 | 17.5948 | 7.3288 | 2.4456 | 0.0264 | CONV |
| 3.78 | 4.0530 | 3.7926 | 1.1888 | 17.6891 | 7.3403 | 2.5394 | 0.0088 | CONV |
| 3.80 | 3.9857 | 3.7950 | 1.2115 | 17.8150 | 7.2999 | 2.7035 | 0.0026 | CONV |
| 3.88 | 3.9574 | 3.7944 | 1.2309 | 17.8652 | 7.2743 | 2.7763 | 0.0007 | RAD |
| 4.12 | 3.9023 | 3.7924 | 1.2745 | 17.9399 | 7.2608 | 2.8592 | 0.0000 | RAD |
| 4.52 | 3.8005 | 3.7887 | 1.3584 | 18.0453 | 7.2709 | 2.9494 | 0.0000 | RAD |
| $M = 0.99\ M_\odot$, $X_o = 0.62$, $Y_o = 0.35$, $Z_o = 0.03$ TRACK B | | | | | | | | |
| 0.00 | 4.5249 | 3.7878 | 0.9774 | 17.3301 | 7.1930 | 2.0311 | 0.6200 | CONV |
| 1.00 | 4.4188 | 3.7891 | 1.0202 | 17.3757 | 7.2145 | 2.1026 | 0.4988 | CONV |
| 2.00 | 4.3235 | 3.7880 | 1.0713 | 17.4134 | 7.2382 | 2.1673 | 0.3918 | CONV |
| 3.00 | 4.2355 | 3.7845 | 1.1335 | 17.4539 | 7.2660 | 2.2456 | 0.2535 | CONV |
| 3.60 | 4.1865 | 3.7807 | 1.1802 | 17.4856 | 7.2877 | 2.3067 | 0.1409 | CONV |
| 4.02 | 4.1361 | 3.7785 | 1.2197 | 17.5438 | 7.3130 | 2.3931 | 0.0528 | CONV |
| 4.10 | 4.1086 | 3.7795 | 1.2297 | 17.5796 | 7.3223 | 2.4327 | 0.0341 | CONV |
| 4.18 | 4.0437 | 3.7826 | 1.2491 | 17.6598 | 7.3331 | 2.5147 | 0.0120 | CONV |
| 4.22 | 3.9691 | 3.7852 | 1.2774 | 17.8068 | 7.2760 | 2.7183 | 0.0020 | RAD |
| 4.54 | 3.9048 | 3.7829 | 1.3301 | 17.8935 | 7.2588 | 2.8171 | 0.0001 | RAD |
| 5.14 | 3.7657 | 3.7775 | 1.4538 | 18.0656 | 7.2716 | 2.9677 | 0.0000 | RAD |
| 5.54 | 3.6404 | 3.7697 | 1.5962 | 18.3300 | 7.2796 | 3.2026 | 0.0000 | RAD |

TABLE 1 (continued)

| AGE ($\times 10^9$ yrs) | $M_{bol}$ | LOG $T_{eff}$ | $R/R_\odot$ | LOG $P_c$ | LOG $T_c$ | LOG $\rho_c$ | $X_c$ | Core |
|---|---|---|---|---|---|---|---|---|
| $M = 1.05\ M_\odot$, $X_o = 0.67$, $Y_o = 0.30$, $Z_o = 0.03$ TRACK C | | | | | | | | |
| 0.00 | 4.5675 | 3.7766 | 1.0089 | 17.2978 | 7.1779 | 1.9966 | 0.6700 | RAD |
| 1.00 | 4.4767 | 3.7777 | 1.0467 | 17.3414 | 7.1948 | 2.0654 | 0.5430 | RAD |
| 2.00 | 4.3762 | 3.7785 | 1.0922 | 17.3877 | 7.2182 | 2.1449 | 0.4367 | CONV |
| 3.00 | 4.2908 | 3.7768 | 1.1448 | 17.4269 | 7.2441 | 2.2091 | 0.3408 | CONV |
| 4.00 | 4.2124 | 3.7729 | 1.2084 | 17.4689 | 7.2723 | 2.2783 | 0.1996 | CONV |
| 4.56 | 4.1716 | 3.7691 | 1.2533 | 17.5124 | 7.2960 | 2.3545 | 0.0920 | CONV |
| 4.72 | 4.1499 | 3.7685 | 1.2695 | 17.5421 | 7.3073 | 2.3932 | 0.0590 | CONV |
| 4.84 | 4.1095 | 3.7700 | 1.2844 | 17.5946 | 7.3192 | 2.4531 | 0.0300 | CONV |
| 4.94 | 4.0126 | 3.7738 | 1.3194 | 17.7539 | 7.2818 | 2.6613 | 0.0040 | RAD |
| 5.10 | 3.9816 | 3.7728 | 1.3445 | 17.8029 | 7.2655 | 2.7245 | 0.0009 | RAD |
| 5.72 | 3.8748 | 3.7688 | 1.4389 | 17.9391 | 7.2646 | 2.8552 | 0.0001 | RAD |
| 6.32 | 3.7318 | 3.7615 | 1.5893 | 18.2063 | 7.2739 | 3.0953 | 0.0000 | RAD |
| 7.30 | 3.6469 | 3.7243 | 1.9618 | 19.3485 | 7.3430 | 3.9978 | 0.0000 | RAD |
| $M = 1.13\ M_\odot$, $X_o = 0.72$, $Y_o = 0.25$, $Z_o = 0.03$ TRACK D | | | | | | | | |
| 0.00 | 4.5330 | 3.7678 | 1.0675 | 17.2707 | 7.1693 | 1.9616 | 0.7200 | RAD |
| 1.00 | 4.4433 | 3.7692 | 1.1052 | 17.3121 | 7.1847 | 2.0250 | 0.6048 | RAD |
| 2.00 | 4.3459 | 3.7704 | 1.1499 | 17.3603 | 7.2055 | 2.0990 | 0.4739 | CONV |
| 3.00 | 4.2579 | 3.7699 | 1.2002 | 17.4028 | 7.2314 | 2.1825 | 0.3886 | CONV |
| 4.00 | 4.1825 | 3.7671 | 1.2587 | 17.4432 | 7.2574 | 2.2405 | 0.2770 | CONV |
| 4.80 | 4.1272 | 3.7623 | 1.3201 | 17.4920 | 7.2871 | 2.3236 | 0.1242 | CONV |
| 5.08 | 4.0949 | 3.7611 | 1.3470 | 17.5361 | 7.3038 | 2.3892 | 0.0622 | CONV |
| 5.16 | 4.0765 | 3.7618 | 1.3542 | 17.5622 | 7.3104 | 2.4203 | 0.0428 | CONV |
| 5.28 | 3.9873 | 3.7654 | 1.3879 | 17.7044 | 7.2894 | 2.6010 | 0.0050 | RAD |
| 5.68 | 3.9229 | 3.7634 | 1.4424 | 17.8078 | 7.2566 | 2.7378 | 0.0004 | RAD |
| 6.28 | 3.8246 | 3.7593 | 1.5384 | 17.9415 | 7.2660 | 2.8562 | 0.0001 | RAD |
| 7.65 | 3.6263 | 3.7302 | 1.9269 | 18.8765 | 7.3017 | 3.6527 | 0.0000 | RAD |

TABLE 1 (continued)

| AGE ($X10^9$ yrs) | $M_{bol}$ | LOG $T_{eff}$ | $R/R_\odot$ | LOG $P_c$ | LOG $T_c$ | LOG $\rho_c$ | $X_c$ | Core |
|---|---|---|---|---|---|---|---|---|
| $M = 1.2\ M_\odot$, $X_o = 0.77$, $Y_o = 0.20$, $Z_o = 0.03$ TRACK E | | | | | | | | |
| 0.00 | 4.5529 | 3.7551 | 1.1214 | 17.2403 | 7.1576 | 1.9269 | 0.7700 | RAD |
| 1.00 | 4.4718 | 3.7588 | 1.1443 | 17.2772 | 7.1711 | 1.9826 | 0.6674 | RAD |
| 2.00 | 4.3824 | 3.7601 | 1.1854 | 17.3222 | 7.1870 | 2.0500 | 0.5565 | RAD |
| 3.00 | 4.2906 | 3.7609 | 1.2319 | 17.3703 | 7.2074 | 2.1249 | 0.4205 | RAD |
| 4.00 | 4.2116 | 3.7598 | 1.2844 | 17.4117 | 7.2355 | 2.2152 | 0.3584 | CONV |
| 5.00 | 4.1422 | 3.7566 | 1.3454 | 17.4538 | 7.2628 | 2.2621 | 0.2328 | CONV |
| 5.40 | 4.1181 | 3.7543 | 1.3753 | 17.4769 | 7.2770 | 2.2990 | 0.1571 | CONV |
| 5.88 | 4.0733 | 3.7523 | 1.4171 | 17.5470 | 7.3033 | 2.4052 | 0.0534 | CONV |
| 5.96 | 4.0476 | 3.7530 | 1.4288 | 17.5831 | 7.3087 | 2.4488 | 0.0336 | CONV |
| 6.04 | 3.9910 | 3.7552 | 1.4520 | 17.6815 | 7.2828 | 2.5835 | 0.0094 | RAD |
| 6.12 | 3.9774 | 3.7550 | 1.4622 | 17.7056 | 7.2701 | 2.6243 | 0.0049 | RAD |
| 7.08 | 3.8440 | 3.7494 | 1.5959 | 17.9098 | 7.2639 | 2.8281 | 0.0004 | RAD |
| 8.85 | 3.7019 | 3.7100 | 2.0424 | 19.1615 | 7.3224 | 3.8656 | 0.0000 | RAD |
| $M = 1.11\ M_\odot$, $X_o = 0.64$, $Y_o = 0.30$, $Z_o = 0.06$ TRACK F | | | | | | | | |
| 0.00 | 4.5275 | 3.7610 | 1.1042 | 17.2714 | 7.1906 | 1.9702 | 0.6400 | CONV |
| 1.00 | 4.4431 | 3.7609 | 1.1483 | 17.3095 | 7.2092 | 2.0358 | 0.5229 | CONV |
| 2.00 | 4.3775 | 3.7589 | 1.1950 | 17.3330 | 7.2284 | 2.0853 | 0.4220 | CONV |
| 3.00 | 4.3261 | 3.7547 | 1.2475 | 17.3515 | 7.2478 | 2.1344 | 0.2918 | CONV |
| 4.00 | 4.2867 | 3.7482 | 1.3087 | 17.3842 | 7.2747 | 2.2160 | 0.1506 | CONV |
| 4.40 | 4.2569 | 3.7456 | 1.3425 | 17.4332 | 7.2967 | 2.2960 | 0.0650 | CONV |
| 4.64 | 4.1803 | 3.7485 | 1.3726 | 17.5290 | 7.3199 | 2.3987 | 0.0180 | CONV |
| 4.68 | 4.1124 | 3.7517 | 1.3952 | 17.6135 | 7.3242 | 2.4850 | 0.0042 | CONV |
| 4.72 | 4.0573 | 3.7529 | 1.4236 | 17.7543 | 7.2522 | 2.6936 | 0.0000 | RAD |
| 5.20 | 3.9946 | 3.7492 | 1.4906 | 17.8476 | 7.2473 | 2.7867 | 0.0000 | RAD |
| 5.80 | 3.8969 | 3.7436 | 1.5995 | 18.0125 | 7.2574 | 2.9330 | 0.0000 | RAD |

TABLE 1 (continued)

| AGE ($X10^9$ yrs) | $M_{bol}$ | LOG $T_{eff}$ | $R/R_{\odot}$ | LOG $P_c$ | LOG $T_c$ | LOG $\rho_c$ | $X_c$ | Core |
|---|---|---|---|---|---|---|---|---|
| $M = 1.19\ M_{\odot}$, $X_o = 0.69$, $Y_o = 0.25$, $Z_o = 0.06$ TRACK G | | | | | | | | |
| 0.00 | 4.5230 | 3.7512 | 1.2125 | 17.2436 | 7.1787 | 1.9372 | 0.6900 | RAD |
| 1.00 | 4.4335 | 3.7533 | 1.1948 | 17.2825 | 7.1944 | 1.9999 | 0.5667 | RAD |
| 2.00 | 4.3615 | 3.7520 | 1.2421 | 17.3167 | 7.2156 | 2.0688 | 0.4724 | CONV |
| 3.00 | 4.3072 | 3.7491 | 1.2910 | 17.3389 | 7.2356 | 2.1131 | 0.3775 | CONV |
| 4.00 | 4.2652 | 3.7442 | 1.3464 | 17.3627 | 7.2573 | 2.1687 | 0.2478 | CONV |
| 4.80 | 4.2386 | 3.7389 | 1.3966 | 17.4060 | 7.2840 | 2.2572 | 0.1028 | CONV |
| 5.05 | 4.2129 | 3.7380 | 1.4187 | 17.4455 | 7.2989 | 2.3127 | 0.0525 | CONV |
| 5.15 | 4.1782 | 3.7394 | 1.4326 | 17.4919 | 7.3102 | 2.3640 | 0.0288 | CONV |
| 5.25 | 4.0545 | 3.7444 | 1.4821 | 17.6951 | 7.2711 | 2.6168 | 0.0018 | RAD |
| 5.70 | 4.0043 | 3.7413 | 1.5384 | 17.7831 | 7.2480 | 2.7249 | 0.0001 | RAD |
| 6.40 | 3.9106 | 3.7357 | 1.6480 | 17.9515 | 7.2553 | 2.8778 | 0.0000 | RAD |
| 7.20 | 3.8063 | 3.7207 | 1.8532 | 18.4453 | 7.2683 | 3.3146 | 0.0000 | RAD |
| $M = 1.275\ M_{\odot}$, $X_o = 0.74$, $Y_o = 0.20$, $Z_o = 0.06$ TRACK H | | | | | | | | |
| 0.00 | 4.5065 | 3.7434 | 1.2092 | 17.2157 | 7.1688 | 1.9028 | 0.7400 | RAD |
| 1.00 | 4.4277 | 3.7438 | 1.2516 | 17.2543 | 7.1831 | 1.9624 | 0.6330 | RAD |
| 2.00 | 4.3451 | 3.7444 | 1.2967 | 17.2955 | 7.2019 | 2.0290 | 0.5148 | CONV |
| 3.50 | 4.2584 | 3.7413 | 1.3689 | 17.3334 | 7.2327 | 2.0987 | 0.3940 | CONV |
| 4.50 | 4.2217 | 3.7364 | 1.4234 | 17.3546 | 7.2537 | 2.1513 | 0.2510 | CONV |
| 5.44 | 4.1827 | 3.7310 | 1.4860 | 17.4298 | 7.2920 | 2.2948 | 0.0676 | CONV |
| 5.60 | 4.1370 | 3.7325 | 1.5070 | 17.4938 | 7.3065 | 2.3697 | 0.0299 | CONV |
| 5.64 | 4.1125 | 3.7336 | 1.5167 | 17.5254 | 7.3098 | 2.4036 | 0.0186 | CONV |
| 5.68 | 4.0524 | 3.7360 | 1.5418 | 17.6237 | 7.2922 | 2.5243 | 0.0023 | CONV |
| 6.12 | 4.0004 | 3.7337 | 1.5960 | 17.7383 | 7.2436 | 2.6860 | 0.0061 | RAD |
| 6.72 | 3.9366 | 3.7296 | 1.6749 | 17.8466 | 7.2512 | 2.7824 | 0.0000 | RAD |
| 7.12 | 3.8835 | 3.7259 | 1.7458 | 17.9752 | 7.2553 | 2.8999 | 0.0000 | RAD |

Figure 2 and Table 2. For $Z = 0.01$ and $Y = 0.25$, the gap does not exist at the luminosity level considered, which suggests that the models are below the narrow transition region in which the convective core develops, or that much higher values of $Y$ are needed. Models with masses slightly greater than those in the transition region already show a fully developed gap.

## III. ANALYSIS OF THE EGGEN-SANDAGE DATA

Because the isochrones obtained from evolutionary studies are very narrow, it is tempting to ascribe the scatter in observational color-magnitude diagrams to random observational errors. Presumably systematic effects, over such a small part of the color-magnitude plane as is used to study the gap ($\Delta V \sim 1$ mag, $\Delta(B - V) \sim 0.1$ mag), will merely displace the region, or perhaps warp it somehow. In any event, such systematic effects are probably not important for this purpose. A study was carried out using the M67 and NGC 188 data of Eggen and Sandage (1964, 1968) to see how well the width of the gaps could be determined.

TABLE 2

THE CORRELATION OF GAP SIZE TO CHEMICAL COMPOSITION

| $Y$ | $\Delta M_{\rm bol}$ (GAP) | |
|---|---|---|
| | $Z=0$ 03 | $Z=0.06$ |
| 0.20 | 0 085±0 007 | 0 140±0 07 |
| .25 | .112 | .161 |
| .30 | 134 | 0 190 |
| 35 | 162 | |
| 0 40 | 0 186 | |

### *a*) *M67*

In the absence of precise information concerning random observational measurement errors, we assumed that the errors in $B$- and $V$-measurements were uncorrelated and that the relative sizes of the errors were given by a mean ratio of $V$ to $B$ in the small part of the diagram containing the gap. Accordingly, we took $\langle \delta B^2 \rangle = 1.6 \langle \delta V^2 \rangle$ and $\langle \delta B \delta V \rangle = 0$, leaving $\langle \delta V^2 \rangle$ as a free parameter to be determined by other means.[1] On a color-magnitude diagram, an observational error in $B$ produces a horizontal displacement of the point representing a star, while an error in $V$ produces a diagonal displacement. At the relative scale used by Eggen and Sandage (1964) for their plots ($B - V$ scale five times as large as $V$-scale), a standard deviation departure from "true" position permits the point representing a star to lie in an elliptical region whose major axis is inclined at $2\frac{1}{4}°$ to the horizontal (upward to the right), whose axis ratio is about 8.5, and whose major semiaxis is approximately the measurements dispersion, $\langle \delta V^2 \rangle^{1/2}$. This inclination of the error ellipse (covariance ellipse), approximately along the boundaries of the gap, makes the gap seem real, rather than an illusion of the color-magnitude plot. A fairly careful comparison of the horizontal scatter of points (along the major axis of the ellipse) leads to an estimate of $\langle \delta V^2 \rangle^{1/2} \sim 0.012$–$0.013$ mag. Note that this estimate is based only on the scatter of observed points, with the assumptions stated earlier: (*a*) that the true locus in the color-magnitude diagram is a narrow line, while the ob-

[1] The results that follow from these guesses agree with the conclusions of Sandage (1962, Appendix A). These restrictions are removed in the later discussion.

served width is totally ascribable to observational errors, and (*b*) that the relationships among the various sources of error are as described above.

The points representing the various stars may be slid along the $2\frac{1}{4}°$ (regression) line to the isochrone. For convenience, we chose the vertical line $B - V =$ constant, to represent the isochrone. Stars with two or more observations from Tables 1 and 2 of Eggen and Sandage (1964), for which $12.55 \leq V \leq 13.55$ and $0.55 \leq B - V \leq 0.635$ were included. When the number of stars that project onto this line, fainter than a given $V$-magnitude, is plotted as a function of $V$, a diagram containing two approximately straight-line segments results, with different slopes on the two sides of the gap (Fig. 3). An (integral) distribution of this type results if the probability of finding stars along the line $B - V =$ constant is uniform with one value above the gap, is zero in the gap, and uniform with a second value below the gap.

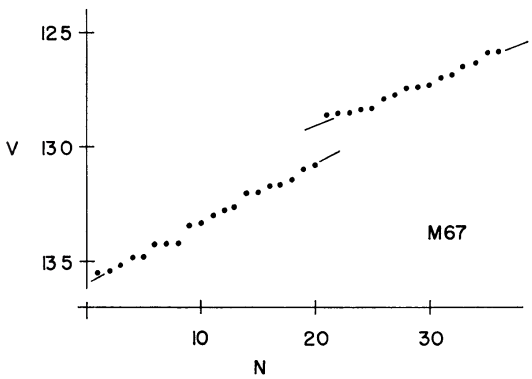


FIG. 3

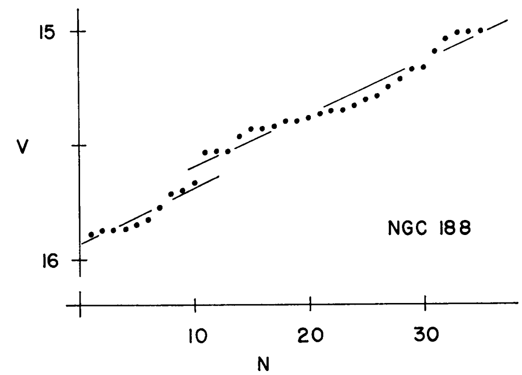


FIG. 4

FIG. 3.—Observational integral distribution of magnitudes of stars for M67. Ordinate is the magnitude, $V$, from Eggen and Sandage (1964), with the measurements projected onto the line $B - V = 0.55$ as indicated in the text. Abscissa is the number of stars from the gap region that are fainter than $V$. Gap occurs between stars 20 and 21.

FIG. 4.—Observational integral distribution of magnitudes of stars for NGC 188. Data are from Eggen and Sandage (1968). Projection is onto the line $B - V = 0.65$, and gap appears between stars 10 and 11.

There is, of course, no a priori reason to assume such a distribution. The hypothesis about the parent distribution consisting of two straight lines can be tested, and any results obtained are then purely observational. The only prejudice injected by comparison with evolutionary calculations is that it is meaningful to project all the representative points onto a curve.

The width of the gap may be determined by making two separate least-squares fits to straight lines—one to the group of stars above the gap and the other to the group below the gap. The hypothesis of uniform distribution in each of the two regions, when tested by the chi-square criterion, is significant at the 10 per cent level (for assumed $\langle \delta V^2 \rangle = 0.012$). The "width" of the gap, as projected onto the line $B - V =$ constant is $0.167 \pm 0.010$ mag. The gap appears to be real when a careful treatment of the observational errors is attempted. The result is independent of the slope of the regression line (the $2\frac{1}{4}°$ line) over rather wide limits. The relative densities of stars above and below the gap is $1.4 \pm 0.2$, with higher density on the bright side of the gap.

This method appears to be the most efficient way to use the observational data for comparison with evolutionary models. The expected distributions of stars along a $B - V =$ constant line, as worked out from models, has the same two-straight-line character as the observed distribution; but the calculated distribution is not zero in the gap (although it falls to about $\frac{1}{10}$ of the value outside the gap). Because of the similarities of the model distributions, a maximum-likelihood fit is not sensitive for choosing among models with a gap narrower than the observed gap. Direct comparison of gap width is preferable.

### *b) NGC 188*

A similar study was carried out for NGC 188 by using data kindly supplied by Eggen and Sandage (1968). This cluster does not work out nearly as well as M67. The gap is not as distinct. Stars with $15.0 \leq V \leq 15.9$ and $0.66 \leq B - V \leq 0.76$ were included. The results obtained are as follows: the width of the gap is $0.085 \pm 0.023$ mag, and the measurements errors work out to $\langle \delta V^2 \rangle^{1/2} \sim 0.023$ mag. This cluster is observationally more difficult—the gap is at $V \sim 15.63$, and there are few well-observed stars. Stars from NGC 188 with only one measurement were included, making it necessary to weight each by its number of measurements. The observational data are shown in Figure 4.

### *c) Discussion*

The ratio of slopes of the lines in the number versus magnitude plot of Figure 3 for M67 is different from that predicted from the models—the model predictions have a ratio of about $\frac{1}{2}$ in the sense quoted above (where the observed ratio is about $1.4 \pm 0.2$). This discrepancy might arise from some kind of observational selection effect, or it might be attributed to modification of the mass spectrum through evaporation of stars from the cluster. To ascribe this difference to evaporation alone would require that evaporation alter the initial mass spectrum in a very strange way. We cannot explain this discrepancy but do not regard this difference as important because it depends sensitively on fine details of the models and of observational effects.

The evolutionary tracks of § II suggest that $Z$ could be determined if the height of the top of the gap over the main sequence could be measured. This situation is complicated, since no (observational) main sequence should exist near the gap. We attempted a purely observational approach to the determination of the magnitude of the main sequence at the same $B - V$ as the gap by linear extrapolation using those stars sufficiently far below the gap to have avoided much evolutionary modification. With M67, very few candidates remain (six); a least-squares linear extrapolation through them leaves the main sequence too far below the gap but also has the main-sequence magnitude at the $B - V$ of the gap uncertain by $\pm 0.15$ mag—nearly twice the size of the effect sought. In principle, this determination might be sharpened by using knowledge of the shape and location of the main sequence; but then the method would no longer rest wholly on observational determinations within this cluster. It would no longer provide a way to sidestep the difficulties and uncertainties of differential line-blanketing, absorption and reddening corrections, and so on.

## IV. COMPARISON WITH THEORY

The observed gap width as determined by the method of the previous section has been plotted in Figure 2. We obtain the following for M67: $Z = 0.03$, $Y = 0.38 \pm 0.02$; $Z = 0.06$, $Y = 0.28 \pm 0.02$. For intermediate values of $Z$, linear interpolation, $Y = (0.491 - 3.6Z) \pm 0.02$, may be used.

For NGC 188 we obtain the following: $Z = 0.03$, $Y = 0.20 \pm 0.04$. For $Z$ between 0.02 and 0.04, the linear relation, $Y = (0.307 - 3.6Z) \pm 0.04$, may be used.

## V. THE AGE OF M67

Estimates of the age of a galactic cluster depend on: (1) the chemical composition of the cluster, which affects the evolutionary tracks of the individual members; and (2) the distance modulus of the cluster, which determines the value of $M_{\mathrm{bol}}$ at the observed turnoff. Since the distance modulus depends on the position of the adopted main sequence, it is also a function of the chemical composition (Demarque and Schlesinger 1969).

Eggen and Sandage (1964) pointed out the difficulty of separating the effects of differential line blanketing from interstellar reddening on three-color photometric data. The slopes of the blanketing and the reddening lines do not differ markedly in the $[(U - B), (B - V)]$-plane. In spite of this they were able to obtain values of $\delta(U - B)$ and $E(B - V)$ consistent with their data. They adopted the values of $\delta(U - B) = +0.035$ and $E(B - V) = +0.06$. We can make use of these data together with evolutionary calculations in order to estimate the age of M67 on the assumption that (*a*) $Z_{67} = 0.03$ or (*b*) $Z_{67} = 0.06$. We will not consider the case $Z_{67} = 0.01$.

*a*) $Z_{67} = 0.03$: This value of $Z$ leads to $Y = 0.38 \pm 0.02$. The composition of M67 may be compared with the chemical composition of the Sun. Present estimates for $Z_\odot$ range from 0.015 to 0.03. Consider both extreme cases: (i) $Z_\odot = 0.03$: If the stars in M67 have a mild ultraviolet excess with respect to the Hyades (Eggen and Sandage 1964), then they must have a solar metal content, since the Sun also has a slight ultraviolet excess with respect to the Hyades. Hence, $Z_{67} = Z_\odot = 0.03$. One can then use the Eggen-Sandage data-fitting with fair accuracy, and the turnoff point is at $M_v = 3.4$; $M_{\text{bol}} = 3.34$. This leads to an age estimate of $3.5 \times 10^9$ years. (ii) $Z_\odot = 0.015$: Since no gap was found with $Z = 0.01$, $Y = 0.25$, we take $Z_{67} = 2Z_\odot = 0.03$, in agreement with Spinrad (1967). We find $\delta(U - B) = -0.03$ and $E(B - V) = 0.14$. Furthermore, since in this case the M67 main sequence is then about 0.25 mag above the Hyades main sequence, the M67 turnoff point is located at $M_v = 2.73$ or $M_{\text{bol}} = 2.67$, and the corresponding age is about $2 \times 10^9$ years.

*b*) $Z_{67} = 0.06$: This value of $Z$ leads to $Y = 0.28 \pm 0.02$. Such a high value of $Z$ implies a high value of $Z_\odot$, say 0.03, and we then have $Z_{67} = 2Z_\odot$. The fitting procedure is the same as in subsection *a*ii, so that the turnoff occurs at $M_{\text{bol}} = 2.67$, and the age of the cluster is $2.6 \times 10^9$ years. However, an examination of the evolutionary tracks for $Z = 0.06$ (Fig. 1) shows that they move more quickly toward low effective temperatures than do the $Z = 0.03$ tracks. This indicates that $Z = 0.06$ is probably too high an estimate for the metal content of M67, since the observed color-magnitude diagram resembles the $Z = 0.03$ evolutionary tracks.

### VI. THE AGE OF NGC 188

A series of consistent values for $\delta(U - B)$ versus $E(B - V)$ were derived by a graphical technique using observations of NGC 188 by Eggen and Sandage (1968). The method is the same as in § V.

*a*) $Z_{188} = 0.03$: Then if $Z_\odot = 0.03$ we have $\delta(U - B) = +0.03$ and $E(B - V) = 0.10$. The turnoff point is at $M_{\text{bol}} = 3.95$, corresponding to an age of $4.6 \times 10^9$ years, for $Y = 0.30$–$0.35$. The reason for this choice will be discussed presently. $Z_\odot = 0.015$ works out to $\delta(U - B) = -0.03$ and $E(B - V) = +0.18$. The turnoff point is then at $M_{\text{bol}} = 3.71$, and the age of the cluster is then $3\ 5 \times 10^9$ years.

*b*) $Z_{188} = 0.06$: The age of the cluster is $5.5 \times 10^9$ years for $Y_{188} = 0.35$ and $Z_\odot = 0.03$.

*c*) *Discussion:* It seems quite probable that $Z = 0.01$ can be excluded due to the low luminosity of the stars near the gap in NGC 188.

It was thought (Iben 1967*a*, *b;* Demarque 1968; Demarque and Schlesinger 1969) that it was not possible to construct a model which had the luminosity of NGC 188 and exhibited a gap with "normal" compositions. We have found that this is not the case; a gap can be produced by using ordinary compositions with the opacity tables given by Cox. Figure 5 illustrates this result. The dotted curve is the result of a computation for a star of $1.0\,M_\odot$ having $X = 0.67$, $Y = 0.30$, and $Z = 0.03$ by using Keller and Meyerott tables (Demarque and Larson 1964); it shows no gap. This same result was found by Hallgren (1967) for the same composition and mass. The solid curve is the evolution of a 0.98 solar-mass star with the same composition as Demarque and Larson, but with Cox opacities. The dashed curve is the approximate location of the observed color-magnitude diagram of NGC 188 (Sandage 1962). The break in the curve is the location of the gap. The computed model has a similar gap (Table 3).

Now consider some possible interpretations of the gap in NGC 188. Are the stars in NGC 188 massive enough to insure a fully developed gap (see § II) or are they just in the region where the gap sets in? On the assumption that the gap is fully developed one can assign the chemical composition given in § V. But then there remains the problem of the difference in composition between NGC 188 and M67, which appear to be very

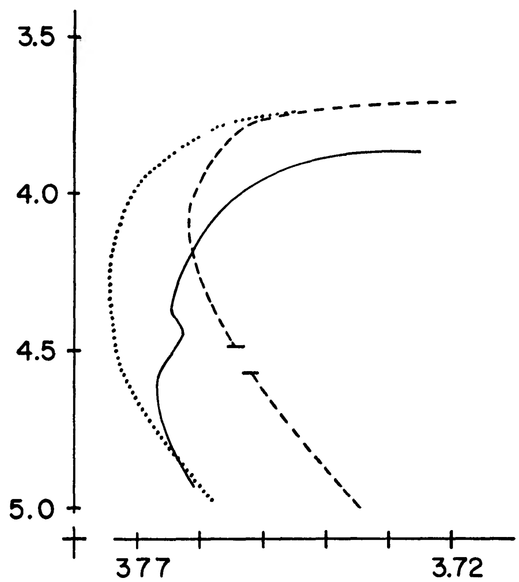


FIG. 5.—C-M diagram for NGC 188. Ordinate is $M_{bol}$ and abscissa is log $T_{eff}$. Solid curve is the new model of the text (Table 3; incipient gap); dotted curve is from Demarque and Larson (1964); and dashed curve is from observation (Sandage 1962), with the gap shown.

TABLE 3

COMPUTED MODEL WITH $M = 0.98\ M_{\odot}$, $=X_0 = 0.67$, $Y_0 = 0.30$, $Z_0 = 0.03$

| Age ($\times 10^9$ yr) | $M_{bol}$ | log $T_{eff}$ | $R/R_{\odot}$ | log $P_c$ | log $T_c$ | log $\rho_c$ | $X_c$ | Core |
|---|---|---|---|---|---|---|---|---|
| 0 00. | 4 9283 | 3 7619 | 0 9145 | 17 2719 | 7 1536 | 1 9945 | 0 6700 | Rad. |
| 1 00 | 4 8543 | 3 7632 | 0 9402 | 17 3084 | 7 1664 | 2 0489 | 5794 | Rad. |
| 2 00. | 4 7690 | 3 7651 | 0 9694 | 17 3498 | 7 1812 | 2 1115 | 4830 | Rad. |
| 3 00. | 4 6796 | 3 7664 | 1 0042 | 17 3978 | 7 1990 | 2 1842 | 3748 | Rad. |
| 4 00. | 4 5981 | 3 7662 | 1 0435 | 17 4421 | 7 2258 | 2 2593 | 2983 | Conv. |
| 5 00. | 4 5224 | 3 7649 | 1 0871 | 17 4884 | 7 2526 | 2 3262 | 2093 | Conv. |
| 5 20. | 4 5074 | 3 7642 | 1 0980 | 17 4941 | 7 2572 | 2 3230 | 1791 | Conv. |
| 5.40 | 4 4929 | 3.7636 | 1 1086 | 17 5086 | 7 2648 | 2 3474 | 1476 | Conv. |
| 5.80 | 4 4573 | 3 7628 | 1 1313 | 17 5462 | 7 2807 | 2 4082 | 0802 | Conv. |
| 6 00 | 4 4197 | 3 7633 | 1 1484 | 17 5956 | 7 2893 | 2 4744 | .0396 | Conv. |
| 6 10 | 4 3813 | 3 7644 | 1 1629 | 17 6536 | 7 2746 | 2 5583 | 0122 | Rad. |
| 6 50 | 4 3237 | 3 7639 | 1 1967 | 17 7231 | 7 2570 | 2 6550 | .0020 | Rad. |
| 7 00 | 4.2494 | 3 7626 | 1 2459 | 17 8225 | 7 2514 | 2 7563 | 0000 | Rad. |
| 8 00. | 4.0655 | 3.7570 | 1 3914 | 18 1050 | 7.2602 | 3 0134 | 0.0000 | Rad. |

similar open clusters. The discrepancy is removed, however, by allowing for the possibility that the gap in NGC 188 is not fully developed. This last possibility seems likely, since below $M_{bol} = 4.6$, we have been unable to obtain a fully developed gap on a stellar model. It is found, as expected, that the occurrence of the gap is very sensitive to the mass of the star. For example, the evolutionary track in Figure 5 should be compared with track $C$ in Figure 1. The only difference between them is the mass of the stars involved. The mass of the star given in Figure 5 is 0.98 $M_{\odot}$ while that of track $C$ is 1.05 $M_{\odot}$. The track in Figure 5 does not have a fully developed gap, but it matches the observations qualitatively. Stars which have evolved into the gap region of NGC 188 may have lower mass than those of M67. Further evidence for this is the fact that NGC 188 appears to be intrinsically fainter than M67. If the gap is not fully developed, the composition of NGC 188 cannot be determined from Figure 2. $Z_{188} = 0.03$, $Y = 0.30$ is preferred because of the shape of the color-magnitude diagram. This agrees closely with the chemical composition of M67. The new estimate of $Z = 0.03$ is at variance with earlier suggestions for the metal content of NGC 188 (Demarque and Schlesinger 1969).

## VII. SUMMARY AND DISCUSSION

The gaps just off the main sequence in the color-magnitude diagrams of M67 and NGC 188 are statistically significant and may be used to make an estimate of the chemical compositions of these clusters. Furthermore, the high-luminosity end of the gap can act as discriminant for $Z$ if it is possible to determine its height above the initial main sequence.

The most consistent picture of the chemical composition of M67 is obtained for the range of $Y = 0.38$–$0.32$ corresponding to a range of $Z = 0.03$–$0.04$. The arguments are as follows: (1) The shape of the isochrones in the C-M diagram is consistent with the observed color-magnitude diagram for $Z = 0.03$. (2) Spectrophotometric observations by Spinrad (1967) indicate that the metal content of red giants in M67 is approximately twice the solar value. (3) The metal content of the Sun is now estimated to be between 0.015 and 0.020 (Lambert 1968; Bahcall, Bahcall, and Shaviv 1968). (4) Lower values of $Z$ would require larger values of $Y$, but this is inconsistent with the belief that low $Z$ values should not correspond with high values of $Y$ for Population I stars.

This result is independent of any corrections due to differential line blanketing or interstellar reddening of the cluster.

The gap in NGC 188 is not fully developed, so it is difficult to make a precise estimate for the cluster composition. Further computations in the region where the gap forms should clarify the picture. A model reproduces the gap with a composition of $Y = 0.30$, $Z = 0.03$. This is probably more representative than the composition given in § IV, where it is assumed that the gap is fully developed.

We would like to thank Dr. A. Sandage for providing us with photoelectric observations for NGC 188 prior to publication, and Mr. B. M. Schlesinger for helpful discussions. This work was partially supported by the National Science Foundation under contract NSF GP-8109.

*Note added in proof.*—The calculations reported in this paper were carried out using the Schwarzschild convection criterion, which ignores effects of gradients in mean molecular weight. This increases the uncertainty of our results but will not affect the arguments and probably will not affect the conclusions appreciably.

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