# THE CHEMICAL COMPOSITION OF THE LAMBDA BOOTIS STARS 

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ABSTRACT
A curve-of-growth abundance analysis has been carried out of $\lambda$ Boo, 29 Cyg, $\pi^{1}$ Ori, $\theta$ Hya, and $\gamma$ Aqr and of the comparison stars $\beta$ Ari, $\rho$ Peg, a Lyr, and HD 161817. It is found that only $\lambda$ Boo, 29 Cyg, and $\pi^{1}$ Ori form a distinct group from the composition point of view. These stars are young main-sequence A stars of low turbulent velocities with metal deficiencies of a factor of $\sim 3$. Oxygen abundances are normal. In $\lambda$ Boo itself, Ca and Mg are deficient with respect to iron. It is suggested that these stars constitute a type of Ap star and that their composition does not reflect that of the material from which they formed.

## I. INTRODUCTION

The $\lambda$ Bootis stars are a group of A stars whose spectra exhibit anomalously weak lines for their colors. In terms of their rotational and kinematic properties they are indistinguishable from normal A stars of the solar neighborhood. In recent papers (which list the stars that have been classified as $\lambda$ Bootis stars and also provide a bibliography of earlier literature), Eggen (1967), Oke (1967), and Sargent (1967) discuss the nature of the $\lambda$ Bootis stars. Oke has shown that in the absolute magnitude-effective temperature array these stars fall on or near the Hyades main sequence.

The question arises whether the $\lambda$ Bootis stars are really metal-poor stars and, if so, whether their composition is related to that of the Population II stars or to that of the peculiar stars of Population I. We should also like to know whether the stars that have been called $\lambda$ Bootis stars really form a homogeneous group. In this paper we report the results of an analysis of five $\lambda$ Bootis stars ( $\lambda$ Boo, 29 Cyg, $\pi^{1}$ Ori, $\theta$ Hya, and $\gamma$ Aqr) that was undertaken in an attempt to answer these questions.

The spectral analysis of $\lambda$ Bootis stars is unusually difficult. The stars have rotationally broadened shallow lines whose measurement is subject to large uncertainties. Few lines are unblended, even in A stars, when $v \sin i \sim 100 \mathrm{~km} \mathrm{~s}^{-1}$. Measurable lines mostly fall on the flat portion of the curve of growth, so that the microturbulent velocity must be determined accurately if abundances are to be derived. Our abundance analysis is mainly based on curve-of-growth techniques because the small number of available lines does not justify more detailed methods.

Because of these difficulties and in order to test our methods of analysis, we have studied two normal stars, $\rho$ Peg and $\beta$ Ari, that have rotational velocities comparable with those of the $\lambda$ Bootis stars. As a further test of the curve-of-growth techniques, we analyzed two sharp-lined stars, a Lyr and HD 161817, for which Hunger (1955) and Kodaira (1964), respectively, have published detailed model atmosphere analyses. We can evaluate the significance of the abundance results obtained for the $\lambda \underline{\text { Bootis stars by }}$ comparing them with the results obtained for these comparison stars.

Our results are in qualitative agreement with those of previous workers. In particular, we agree with Burbidge and Burbidge (1956), who analyzed $\lambda$ Boo and 29 Cyg, that these stars are metal deficient, and we find, as we had previously suggested (Bowen 1963) and as has recently been shown by Kodaira (1967b), that the oxygen abundance is normal.

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## II. THE OBSERVATIONS

Most of the spectrograms used in this work were obtained in the period 1962-1963 at Mount Wilson, using the 16 -inch and 32 -inch cameras of the coudé spectrograph of the 100 -inch telescope. A few additional spectrograms were obtained with the 32 -inch camera of the coudé spectrograph of the 74-inch telescope at Mount Stromlo. Several of these latter were obtained by using an image-tube camera on loan from the Department of Terrestrial Magnetism of the Carnegie Institution of Washington. Table 1 summarizes the material we have used. The plate calibrations are based on step slits (Mount Wilson) or linear wedges (Mount Stromlo) impressed on the plate carrying the stellar spectrogram, and in all cases the non-uniformity of illumination of slits or wedges (and, in the case of image-tube spectra, the effects of field distortion) was directly measured by means of a "square window"' test and the appropriate corrections applied. The spectrograms were traced with a linearizing microphotometer, keeping the tracing scale the same for one spectral region in all program stars. All tracings of each spectral region were carefully intercompared, and pains were taken to insure that consistent local continua were drawn.

TABLE 1
Number and Type of Spectrograms Used

| Spectral Range | Emulsion | Dispersion <br> ( $\AA_{\mathrm{Mm}} \mathrm{mm}^{-1}$ ) | $\lambda$ Boo | 29 Cyg | $\pi^{1}$ Ori | $\theta$ Hya | $\gamma$ Aqr | p Peg | $\beta$ Ari |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Blue. | IIaO baked | 4 5, 10 | 5 | 5 | 5 | 4 | 1 | 2 | 1 |
| Red... .. | fIIaD, IIaF, 103aF | 7,15 | 3 | 6 | 2 | 3 | 1 | 1 | 1 |
|  | \{image-tube | 10 | . . . |  | 3 |  | 3 |  |  |
| Infrared. . | $\left\{\mathrm{IN}+\mathrm{NH}_{3}\right.$ | 20, 40 | 5 | 3 | 4 | 2 | 1 | 1 | 1 |
|  | qimage-tube |  |  |  | 1 |  | 1 |  |  |

The results for the Balmer-line profiles are given in Table 2, which contains the percentage depression in the line as a function of distance $\Delta \lambda$ from the line center in angstroms. In the column labeled $n$ we state the number of spectrograms contributing to the tabulated line profile. We identified all other lines which were present in the spectra of at least two program stars and, after eliminating serious blends, were left with the line list of Table 3. Because of the large rotational broadening, contributions of blending lines within $1 \AA$ of the line center have been considered. We believe that we have insured that the contribution of blends to the equivalent widths in Table 3 is in all cases smaller than 25 per cent. For identifications and for information on blending we have relied heavily on the studies of $a \mathrm{Lyr}$ (Hunger 1955), $a \mathrm{CMa}$ (Kohl 1964), and $\gamma$ Ser (Kegel 1962). Our program stars cover a range of effective temperature, and a few of the selected lines, while essentially unblended in most of them, are probably quite seriously blended in others. All such details are given in the remarks to Table 3. Finally, we have measured two serious blends, those of the infrared Ca ir lines with neighboring Paschen lines. Our procedures here are illustrated (and defined) in Figure 1.

In Table 3 we give our final mean equivalent widths (or corresponding upper limits) in milliangstroms. They have been rounded to the nearest $10 \mathrm{~m} \AA$. Grain fluctuations alone will introduce errors of perhaps $\pm 50 \mathrm{~m} \AA$ in these measurements, but systematic errors (mainly resulting from slight uncertainties in the location of the continuum in these very broad-lined stars) are far larger than this. We have compared the equivalent widths measured by Burbidge and Burbidge (1956) for 29 Cyg and $\lambda$ Boo with those that we have measured. Our equivalent widths for 29 Cyg are systematically 20 per cent
smaller than theirs; for $\lambda$ Boo, ours are 30 per cent larger. In the worst cases, equivalent widths of individual lines differ by a factor of 2 in the two series of measurements. We have found similar differences between our own measurements from individual spectrograms of the same weak-lined star.

In Table 3 the letters $a, b$, and $c$ following each tabulated equivalent width (or upper limit) indicate our degree of confidence in the determination. For measurements of class $a$ at least two plates giving accordant results were used, the line was free from blending, and the profile could be drawn unambiguously. Measurements were called class $b$ if they

TABLE 2
Balmer-Line Profiles

| Line | $n$ | $\Delta \lambda(\AA)$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 2 | 5 | 10 | 15 | 20 | 30 | 40 |
| $\lambda$ Boo: |  |  |  |  |  |  |  |  |  |
|  | 5 | 56 | 48 | 34 | 24 | 17 | 13 | 6 | 4 |
| H $\beta$ | 5 | 67 | 57 | 45 | 32 | 23 | 18 | 10 | 6 |
| H $\gamma \ldots$ | 3 | 74 | 64 | 50 | 34 | 24 | 17 | 8 | 4 |
| 29 Cyg : |  |  |  |  |  |  |  |  |  |
| Ha $\mathrm{H} \beta .$. | 2 | 65 72 | 49 55 | 35 40 | 24 27 | 17 19 | 12 14 | 7 | 3 |
|  |  |  |  |  |  |  | 13 | 7 | 3 |
|  |  |  |  |  |  |  | 10 | 4 | 2 |
| H $\beta$. ${ }^{\text {. }}$ | 3 | 68 | 56 | 43 | 30 | 21 | 14 | 4 | 1 |
| $\mathrm{H}_{\gamma}$.. | 5 | 74 | 66 | 51 | 37 | 26 | 18 | 9 | 4 |
| $\theta$ Hya: ${ }^{\text {a }}$ |  |  |  |  |  |  |  |  |  |
| H $\beta$.. | 5 | 67 | 54 | 40 | 24 | 14 | ${ }_{9}$ | 3 | 1 |
| $\mathrm{H}_{\gamma} \ldots$. | 2 | 75 | 66 | 51 | 29 | 17 | 10 | 4 | 1 |
| $\gamma$ Aqr: |  |  |  |  |  |  |  |  |  |
| H $\beta$.. | 2 | 62 | 54 | 41 | 24 | 14 | 7 | 2 |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| $\mathrm{Ha} \ldots$, $\mathrm{H} \beta \ldots$ | ${ }_{3}^{2}$ | 52 60 | 43 5 | 33 40 | 22 <br> 24 | 16 15 | 11 8 | 5 3 | $\stackrel{2}{1}$ |
| $\mathrm{H}_{7} \ldots$. | 2 | 75 | 63 | 47 | 28 | 15 | 8 | 4 | 1 |
| $\beta$ Ari: ${ }^{\text {c/. }}$ |  |  |  |  |  |  |  |  | 3 |
| H $\beta$... | 2 | 72 | 55 | 41 | 27 | 19 | 13 | 6 | 2 |
| H $\gamma \ldots .$. | 1 | 73 | 58 | 41 | 28 | 18 | 12 | 4 | 1 |

depended on only one good-quality plate but otherwise satisfied the criteria for class $a$ lines, or if the drawing of the line profile involved some judgment. Finally, measurements of class $c$ were those where blending effects were more serious and an appreciable portion of the line profile had to be reconstructed.

In addition to the data of Tables 2 and 3, our investigation requires knowledge of the spectral energy distribution of the program stars corrected for the blanketing due to lines. These data were obtained for us by Dr. J. B. Oke and have been discussed separately by him (Oke 1967). The spectral energy distribution of $\beta$ Ari was taken from Baschek and Oke (1965). The spectral energy distributions are put on an absolute scale by adopting an energy distribution for a Lyr, the primary standard. In this work we assumed (following Bessell 1967) that the spectral energy distribution of $\alpha$ Lyr is identi-
equivalent widths (mA)

| $\lambda$ | ion (mult.no.) | $\lambda$ Boo | 29 Cyg | $\pi^{1}$ Ori | $\theta$ Hya | Y Aqr | $\rho \mathrm{Peg}$ | B Ari |  | remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3933.7 | Ca II (1) | 460 a | 1370 a | 720 a | 290 | 430 b | 1210 a | 2940 | b | 1 |
| 4045.8 | Fe I (43) | 130 | 160 a | 110 | 60 | 70 | 110 | 370 | b | 2 |
| 4063.6 | Fe I (43) | 100 | 130 a | 110 a | 50 | 80 | 70 b | 360 | b |  |
| 4071.7 | Fe I (43) | 100 b | 110 b | 90 b | 60 | 70 | 70 c | 240 | c | 3 |
| 4077.7 | Sr II (1) | 100 | 190 c | 90 c | 50 | 80 | 130 | 480 | c | 2, 3 |
| 4202.0 | Fe I (42) | 80 b | 70 | 70 | $\leq 40$ | $\leq 40 \mathrm{~b}$ | 60 | 280 | b |  |
| 4215.5 | Sr II (1) | 90 b | 130 b | 50 c | <40 | $\leq 40 \mathrm{~b}$ | 60 | 250 | b |  |
| 4226.7 | Ca I (2) | 100 | 200 a | 130 | $\leq 40$ | $\leq 40 \mathrm{~b}$ | 80 | 460 | b |  |
| 4235.9 | Fe I (152) | $\leq 80$ c | 60 b | $\leq 50$ b | S40 | $\leq 40 \mathrm{~b}$ | $\leq 40$ b | 150 | c |  |
| 4246.8 | Sc II (7) | 80 c | 100 b | 550 b | S40 | $\leq 40 \mathrm{~b}$ | 110 | 210 | b | 2 |
| 4260.5 | Fe I (152) | $\leq 80$ a | 80 a | 70 b | $\leq 40$ | $\leq 40 \mathrm{~b}$ | 100 | 190 | b | 2 |
| 4290.2 | Ti II (41) | $\leq 80$ b | 100 b | 80 b | $\leq 40$ | $\leq 50 \mathrm{~b}$ | <40 | 360 | b | 3 |
| 4300.1 | Ti II (41) | 580 a | 140 a | 100 a | 40 | $\leq 50 \mathrm{~b}$ | $\leq 120$ b | 380 | b | 3 |
| 4383.6 | Fe I (41) | 110 b | 170 b | 150 b | 70 | 100 | 140 | 440 | c | 3 |
| 4395.0 | Ti II (19) | 5100 a | 130 a | 110 | 70 | 70 | 90 | 270 | b |  |
| 4404.8 | Fe I (41) | 100 | 120 a | 100 | $\leq 40$ | 50 | $\leq 40$ | 180 | b | 2 |
| 4415.1 | Fe I (41) | $\leq 80$ b | 100 b | 90 c | $\leq 40$ | $\leq 40 \mathrm{~b}$ | 60 | 240 | c |  |
| 4443.8 | Ti II (19) | 110 | 110 a | 70 b | 50 | 80 | 140 | 280 | b |  |
| 4454.8 | Ca I (4) | 80 | 80 b | 550 c | $\leq 40$ | 50 | 50 | 210 | c | 2 |
| 4468.5 | Ti II (31) | 580 b | 110 b | 90 b | 70 | 100 b | 120 b | 310 | c |  |
| 4471.6 | He I (14) |  |  |  | 50 | 100 | 110 |  |  |  |
| 4481.2 | Mg II (4) | 150 a | 190 a | 260 a | 300 | 460 b | 500 a | 670 | b | 4 |
| 4491.4 | Fe II (37) | $\leq 80$ b | 50 b | $\leq 50$ b | 70 | 80 | 80 b | 140 | c |  |
| 4501.3 | Ti II (31) | $\leq 80$ a | 90 a | 70 | 50 | 70 b | 50 | 210 | b |  |
| 4508.3 | Fe II (38) | 100 a | 80 a | 550 a | 50 | 70 b | 80 a | 200 | b |  |
| 45153 | Fe II (37) | $\leq 80$ a | 80 a | $\leq 50$ | 60 | 90 b | 80 b | 180 | b |  |
| 4522.6 | Fe II (38) | $\leq 80$ b | 100 b | 70 c | 70 | 60 c | 80 c | 200 | c |  |
| 4572.0 | Ti II (82) | 580 a | 120 | 90 a | 60 |  | 80 a | 200 | b |  |
| 4583.8 | Fe II (38) | 80 c | 100 a | 140 a | 90 | 110 b | 200 | 360 | b | 5 |
| 4629.3 | Fe II (37) | 100 b | 50 a | $\leq 50$ a | 90 | 80 b | 100 | 180 | b |  |
| 4703.0 | Mg I (11) | $\leq 80$ a | 50 b | $\leq 50$ | S40 | $\leq 40 \mathrm{~b}$ | 100 | 150 | b |  |
| 4923.9 | Fe II (42) | $\leq 50$ a | 120 b | 150 b | 120 | 130 c | 90 | 350 | b |  |
| 5018.4 | Fe II (42) | 70 a | 200 b | 150 b | 90 |  | 180 b | 550 | c |  |
| 5172.7 | Mg I (2) | 50 b | 120 b | 130 a | $\leq 70$ | $\leq 90$ c | 110 b | 370 | b |  |
| 5183.6 | Mg I (2) | 80 b | 160 | 140 a | 90 | $\leq 90$ c | 110 b | 370 | b |  |
| 5890.0 | Na I (1) | 140 c | 250 b |  | 150 |  | 160 | 450 | b | 6 |
| 5895.9 | Na I (1) | 110 | 210 b |  | 120 |  | 110 | 330 | b | 6 |
| 6347.1 | Si II (2) | $\leq 50$ a | 580 b | 60 c | 150 | 140 b | 220 b | 270 | b |  |
| 6371.4 | Si II (2) | 550 a | 580 b | 550 b | 100 | 100 b | 110 b | 240 | b |  |
| 7774 | 0 I (1) | 690 a | 710 a | 670 | 260 | 470 b | 740 b | 720 | b | 7 |
| 8498.0 | Ca II (2) | 250 | 390 c | 250 c |  |  |  | 600 | c | 8 |
| 8542.1 | Ca II (2) | 280 c | 640 c | 440 c |  |  |  | 930 | c | 8 |
| 8598.4 | H I (9) | 2680 b | 2820 | 2780 b | 3840 | 3140 b | 3300 b | 2390 | b | 1 |

## Remarks:

| $a, b, c$ | An indication of the quality of the equivalent width (or limit), a being the most reliable. |
| :---: | :---: |
| 1. | The $K$ line and Paschen 14 are measured with respect to a local continum drawn through the highest points of the overlapping hydrogen line wings. |
| 2. | In the hot stars $\theta$ Hya, $\gamma$ Aqr and $\rho$ Peg these lines could be appreciably blended with lines of V II, Cr II, Mn II, and Zr II. |
| 3. | These lines occur in the wings of $\mathrm{H} \gamma$ or $\mathrm{H} \delta$. In no case does the absorption in the Balmer line wing exceed 15 per cent. |
| 4. | Unresolved doublet $\lambda 4481.13$ and $4481.33 \AA$. |
| 5. | Equivalent widths include contribution from Fe II (37) $\lambda$ ¢ 482.8. |
| 6. | Equivalent widths uncertain because of blending with telluric water vapor. <br> In 29 Cyg and $\rho$ Peg sharp interstellar D lines are superimposed on the broad stellar components. The equivalent widths of the interstellar components $D_{1}$ and $D_{2}$ are in 29 Cyg 25 and $45 \mathrm{~m} \AA$, and are in $\rho$ Peg 40 and 50 mA . |
| 7. | Total equivalent width of the triplet $\lambda 7772.0,7774.2$ and $7775.4 \AA$. |
| 8. | Equivalent widths defined in Figure 1. |

cal with that predicted from the Mihalas (1966) Balmer-line blanketed model with parameters $\theta_{e}=0.525$ and $\log g=4.0$. All available empirical evidence is in accord with this assumption, which deviates only slightly from the assumptions made by Oke.

One further observational datum is required in this investigation: the rotational velocities of the program stars. Hyland (1967) has computed relations between the halfwidth of Mg II $\lambda 4481$ and $v \sin i$ and between the central depth of the same line (normalized to a standard equivalent width) and $v \sin i$. We determined the profile of Mg II


Fig. 1 -Observed profiles of two infrared Ca II lines. Both are blended with neighboring Paschen lines. The equivalent widths of the Ca ir line in Table 3 refer to the area below the interpolated Paschen line profile, which is shown as a dashed line in the figure. This dashed line was adopted as the local continuum (a) $\lambda$ Boo (observations based on 3 spectrograms); (b) 29 Cyg (3); (c) $\pi^{11}$ Ori (4); (d) $\theta \mathrm{Hya}$ (2); (e) $\gamma \mathrm{Aqr}$ (1); (f) $\rho \mathrm{Peg}$ (1); (g) $\beta$ Ari (1).
$\lambda 4481$ for all our program stars and used Hyland's correlations to determine the projected equatorial rotational velocities. They are given in Table 4. Our results confirm the rotational velocities earlier determined by Slettebak (1954) from low-dispersion material and show that, as far as rotation is concerned, our comparison stars $\beta$ Ari and $\rho$ Peg are closely similar to our other program stars.

For comparison with the analysis of our program stars we have analyzed the published material on two well-studied stars, a Lyr and HD 161817. For HD 161817, equivalent widths, Balmer-line profiles, and the spectral energy distribution were taken from Kodaira (1964, 1967a). For a Lyr, equivalent widths come from Hunger (1955) and Matsushima and Groth (1960), Balmer-line profiles come from Baschek and Oke (1965), and the energy distribution is (by assumption) that adopted by Bessell (1967).

## III. THE ATMOSPHERIC PARAMETERS

The basic atmospheric parameters for the $\lambda$ Bootis stars, the effective temperature $\theta_{e}=5040 / T_{e}$ and the surface gravity $g$, have been determined by Oke (1967) from his photoelectric spectrophotometric observations and from our $\mathrm{H} \gamma$ line profiles. We repeated the fitting of observations and models, using an independent procedure suggested to us by Dr. A. W. Rodgers. We gave equal weight to the information contained in hydrogen-line profiles and spectral energy distributions.

As parameters to characterize the line profiles we used the total widths of $\mathrm{H} \beta$ and $\mathrm{H} \gamma$ at 10,20 , and 30 per cent depression from the continuum. The continuous energy distribution can be described by four flux ratios $\lambda 3636 / \lambda 3390, \lambda 5000 / \lambda 4032, \lambda 4019 / \lambda 3646$, and $\lambda 6790 / \lambda 4475$. The observed value of each of these parameters defines a curve in the

TABLE 4
ATMOSPHERIC PARAMETERS

| Parameter | $\lambda$ Boo | 29 Cyg | $\pi^{1}$ Ori | $\theta$ Hya | $\boldsymbol{\gamma}$ Aqr | $a \mathrm{Lyr}$ | $\rho \mathrm{Peg}$ | $\beta$ Ari | HD 161817 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\theta e$ | 060 | 063 | 059 | 045 | 048 | 053 | 053 | 063 | 065 |
| $\log g$. | 4.0 | 39 | 4.0 | 42 | 4.0 | 3.8 | 35 | 3.8 | 3.1 |
| $\theta$ | 068 | 072 | 066 | 051 | 0.54 | 058 | 058 | 0.72 | 074 |
| $\log P_{P}(1)$ | 19 | 1.7 | 20 | 25 | 24 | 22 | 21 | 16 | 14 |
| $\log P_{e}(2)$ | 22 | 15 | 1.9 | 28 | 25 | 21 | 19 | 14 | 11 |
| $\log P_{6} \ldots \ldots \ldots$. | 2.1 | 16 | 2.0 | 2.7 | 2.4 | 2.2 | 20 | 1.5 | 1.2 |
| $\log \Gamma_{4}$ | 9.5 | 93 | 9.6 | 10.0 | 9.9 | 98 | 9.6 | 93 | 9.0 |
| $\log \Gamma_{6}$ | 125 | 126 | 12.4 | 11.1 | 112 | 11.6 | 11.4 | 125 | 121 |
|  |  |  |  |  |  |  |  | $\beta$ Ari I $\beta$ Ari II |  |
| $F(\Omega=0):$ |  |  |  |  |  |  |  |  |  |
| Fe I. | 10 | 13 | 13 | $\geq 1.1$ | $\geq 1.2$ | 12 | 14 | $\leq 1.1 \quad 19$ | 1.3 |
| Fe II...... |  | 13 | 14 | 10 | 1.1 | 12 | 1.1 |  | 1.3 |
| Mean $F(\Omega=0)$. | 10 | 13 | 1.3 | 11 | 1.2 | 1.2 | 1.3 | $\begin{array}{llll}1 & 1 & 1 & 7\end{array}$ | 13 |
| Errors in $F(\Omega=0)$ | $\pm 02$ | $\pm 01$ | $\pm 0.2$ | $\pm 01$ | $\pm 02$ | $\pm 0.1$ | $\pm 02$ | $\pm 0.2 \pm 02$ | $\pm 01$ |
| $\boldsymbol{\xi}\left(\mathrm{km} \mathrm{s}^{-1}\right) \ldots .$. | 1.6 | 40 | 40 | 21 | 30 | 31 | 40 | $\begin{array}{ll}23 & 306\end{array}$ | 40 |
| $v \sin i\left(\mathrm{~km} \mathrm{~s}^{-1}\right) \ldots$ | 135 | 90 | 110 | 100 | 75 |  | 115 | 80 |  |

$\left(\theta_{e}, \log g\right)$-diagram. We obtained these curves by interpolating in the model atmosphere grid of Mihalas (1965). These models are flux constant, assume radiative equilibrium and LTE, and ignore the effects of Balmer-line blanketing. The assumed helium abundance is $N(\mathrm{He}) / N(\mathrm{H})=0.15$ by number. The intersection of the curves so defined gives $\theta_{e}{ }^{\prime}$ and $\log g^{\prime}$, which differ from the true parameters $\theta_{e}$ and $\log g$ by small corrections:

$$
\begin{gather*}
\theta_{e}=\theta_{e}^{\prime}+0.02,  \tag{1}\\
\log g=\left\{\begin{array}{ll}
\log g^{\prime}+0.2 & \text { if } \theta_{e}<0.55 \\
\log g^{\prime} & \text { if } \theta_{e}>0.55
\end{array} .\right. \tag{2}
\end{gather*}
$$

Equation (1) takes into account the influence on the fluxes of the opacity in the Balmer lines and results from a comparison of the models used with the blanketed models of Mihalas (1966). Equation (2) is a correction based on the empirical work of Strom and Peterson (1968) and Hyland (1967). Mihalas calculated profiles using the theory by Griem (1962), which leads to systematically low surface gravities when used to interpret
the observations of hot stars. No correction to $\log g$ is needed for the cool stars because in these the wings of the hydrogen lines are independent of gravity.

The resulting atmospheric parameters are given in Table 4. They are in essential agreement with the results obtained by Oke. The curves in the $\left(\theta_{e}, \log g\right)$-plane defined by the various observational parameters led to a unique well-defined intersection in all cases except that of $\rho$ Peg. Here the fitting was much improved if we assumed a small reddening of $E(B-V)=0.01$, and this seemed quite reasonable in view of the existence of interstellar Na D-lines in $\rho$ Peg.

According to Oke (1967) the energy distributions of $\theta$ Hya and $\gamma$ Aqr are affected by cool companions with $\theta_{e} \sim 0.7$. Their contribution to the total flux is about 6 per cent at $\lambda 4000$ and 12 per cent at $\lambda 5500 .{ }^{1}$ The influence of a cool companion on the $\mathrm{H} \gamma$ profile is negligible. Accordingly, for these stars we adopted Oke's values for the effective temperatures and determined $\log g$ from the $\mathrm{H} \gamma$ profile.

In what follows, representative temperatures $\theta$ and electron pressures $\log P_{e}$ are required that describe conditions in the layers of the atmospheres where the measured absorption lines are formed. Assuming that the optical depth (in the continuum at $5000 \AA$ ) $\tau^{*}=0.2$ is representative of line formation (cf. Hunger 1955, 1960; Kodaira 1964), we take $\theta=\theta\left(\tau^{*}\right)$ and $P_{e}(1)=P_{e}\left(\tau^{*}\right)$ from the blanketed model atmospheres of Mihalas (1966). Our finally adopted $\log P_{e}$ is a mean of $\log P_{e}(1)$ and $\log P_{e}(2)$ which is derived from the ionization equilibrium of iron (see §V). The quantities $\theta, \log P_{e}(1)$, $\log P_{e}(2)$, and $\log P_{e}$ are given in Table 4.

## IV. CURVES OF GROWTH

Reading the measured equivalent widths into a curve of growth, we read out $\eta_{0}$, the ratio of line to continuous absorption coefficients in the line center. These values of $\eta_{0}$ can then be used to yield information about the composition. In this section we outline the assumptions and approximations that were adopted in this work to obtain the $\eta_{0}$ values. We ignored the depth dependence of the damping $\gamma$, of the thermal velocity $\xi_{0}$, of the microturbulent velocity $\xi$, and, most importantly, of $\eta_{0}$ itself. We assumed line formation in LTE, and we also assumed that the monochromatic Planck function is a linear function of monochromatic optical depth. As for notation,

$$
\begin{gather*}
v^{2}=\xi_{0}{ }^{2}+\xi^{2}  \tag{3}\\
R_{c}=1-B_{\lambda}(0) / F_{\lambda},  \tag{4}\\
\Omega=\log \frac{W}{2 R_{c} \Delta \lambda_{D}} ; \quad \frac{\Delta \lambda_{D}}{\lambda}=\frac{\Delta \omega_{D}}{\omega}=\frac{v}{c}, \tag{5}
\end{gather*}
$$

where a line with equivalent width $W$ is located at wavelength $\lambda$, the emergent flux in the continuum from the star at this wavelength is $F_{\lambda}$, and $B_{\lambda}(0)$ is the Planck function at the same wavelength corresponding to the surface temperature. With the stated assumptions, a unique curve relates $\Omega$ and $\eta_{0}$ for each value of the damping parameter $a=\gamma / 2 \Delta \omega_{D}$. Such curves have been computed by Hunger (1956) and Wrubel (1949).

We interpolated in Mihalas' Balmer-line blanketed model grid to determine $B_{\lambda}(0)$ and $F_{\lambda}$ and hence $R_{c}$ for each equivalent width given in Table 3. The triplet $\mathrm{O}_{\mathrm{I}} \lambda 7774$ is unresolved on our spectrograms. For this line a special curve of growth was constructed, assuming (as is justified) that the absorption coefficient of the components do not overlap. We made the same assumption (which is here a fair approximation) for the doublet Mg II $\lambda 4481$.

[^1]The construction of the $\Omega$-values and the determination of the $\eta_{0}$-values require a determination of the damping constants and turbulent velocities.
a) Damping

The total damping $\gamma$ consists of the radiation damping $\gamma_{\mathrm{rad}}$ and damping by collisions with electrons $\gamma_{4}$ and with neutral hydrogen atoms $\gamma_{6}$.

The collisional broadening can be written as

$$
\begin{equation*}
\boldsymbol{\gamma}_{k}\left(10^{8} \sec ^{-1}\right)=\Gamma_{k} C_{k}^{2 /(k-1)} \tag{6}
\end{equation*}
$$

where the $\Gamma_{k}$ depend on the atmospheric parameters only (Unsöld 1955). The interaction constants $C_{k}$ are defined by the frequency shift in circular frequency units caused by a perturber at a relative distance $r(\mathrm{~cm})$ from the radiating atom

$$
\begin{equation*}
\Delta \omega=C_{k} / r^{k} . \tag{7}
\end{equation*}
$$

In Table 4 we give the values of $\Gamma_{4}$ and $\Gamma_{6}$ evaluated for the representative temperatures and pressures $\theta$ and $\log P_{e}(1)$.

The radiation damping and the interaction constants are listed in Table 5. The $\gamma_{\mathrm{rad}^{-}}$
TABLE 5
data for Damping Calculations

| Ion | Multiplet | $\begin{gathered} \gamma_{\text {rad }} \\ \left(10^{8} \mathrm{~s}^{-1}\right) \end{gathered}$ | $-\log C_{6}$ | $-\log C_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| OI. | 1 | 11 | 30.9 | 139 |
| NaI. | 1 | 06 | 30.9 | 145 |
|  | $\{2$ | 1.0 | 30.7 | 137 |
| Mg I.. | \{11 | 4.5 | 294 | 120 |
| Mg II. | 4 | 72 | 30.4 | 132 |
| Si II. . | 2 | 122 | 310 | 134 |
|  | \{ 2 | 18 | 311 | 148 |
| CaI. . | $\{4$ | (1.8) | 301 | 132 |
|  | \{ 1 | 19 | 316 | 15.6 |
| Ca II. | $\{2$ | 17 | 316 | 146 |
| ScII. |  | 10 | 310 | (13 4) |
| Ti II. |  | (16) | 315 | (13 4) |
| FeI... |  | 16 | 31.0 | (13.4) |
| Fe II.. |  | 16 | 315 | (13 4) |
| Sr II | 1 | 20 | 31.3 | (148) |

and $C_{6}$-values were taken from Hunger (1955, 1960), Kegel (1962) and Kodaira (1964), the $C_{4}$-values from Griem (1964). Numbers in parentheses are our estimates.

## b) Microturbulence

Since most of the lines are on the flat part of the curve of growth, knowledge of the microturbulent velocity $\xi$ is crucial. Only for Fe I and Fe II are enough lines available to determine $\xi$.

For each star we plotted two empirical curves of growth, $F=\log 10^{6} \mathrm{~W} / \lambda$ versus $\log$ $g f \lambda-\theta \cdot \epsilon$, from the $\mathrm{Fe}_{\mathrm{I}}$ and Fe II lines, respectively. The quantity $\epsilon$ is the excitation potential in electron volts of the lower level. The oscillator strengths $g f$ are taken from Corliss and Warner (1964) for Fe I, and from Warner (1967) for Fe ir (Multiplet No. 42 of Fe II from Roder 1962, reduced to the absolute scale of Warner). By comparing the
empirical curve of growth with the theoretical curve, $\Omega$ versus $\eta_{0}$, for the appropriate damping parameter $a$, we obtained a vertical shift of the two ordinates, expressed as the value of $F$ for $\Omega=0$, and hence $\xi$.

Since the knowledge of $\xi$ is required for the calculation of $a$, we carried out one further iteration step in the fitting procedure.

In Table 4 we give the results $F(\Omega=0)$ for $\mathrm{Fe}_{\mathrm{I}}$ and Fe II separately, the adopted mean value and its estimated uncertainty, and finally, the microturbulent velocity $\xi$. We wish to emphasize that the data will not allow curve-of-growth fittings with $F(\Omega=0)$ values outside the quoted uncertainties. For $\lambda$ Boo only the lines of Fe I permit a reasonable fit of the curve of growth. In the case of $\beta$ Ari we obtained two solutions (called $\beta$ Ari I and II in the following) from the lines selected by us. The further analysis (see $\S \mathrm{VI} c$ ) will show that the large value of $\xi$ is the appropriate one.

After having determined the damping constants and microturbulent velocities, we finally computed $\Omega$ corresponding to each $W$ and read from the appropriate curve of growth the associated value of $\eta_{0}$.

By the use of a procedure identical with that described above for the program stars, the atmospheric parameters and $\eta_{0}$-values were derived for $a \mathrm{Lyr}$ and HD 161817. The sources of the data used for these stars are listed in § II. Kodaira's (1964) model atmosphere analysis of HD 161817 gives $\theta=0.73, \log P_{e}=1.34$ at mean $\tau=0.2$. Hunger's (1955) model IV for a Lyr gives $\theta=0.59, \log P_{e}=2.31$ at mean $\tau=0.2$. The agreement with the atmospheric parameters derived by our procedures and given in Table 4 is excellent. The agreement between the values of the turbulent velocities found by these authors and those found in our independent redetermination is also excellent.

## V. ABUNDANCES RELATIVE TO ALPHA LYRAE

The program stars (and also HD 161817) were analyzed differentially with respect to $a$ Lyr. First we improved the representative parameters by taking account of the relative strengths of Fe I and Fe Ir lines. We adopted the $\theta$-values of Table 4 and obtained new values, $\log P_{e}(2)$, for the representative electron pressures from the $\eta_{0}$-values of lines of Fe I and Fe II.

We denote by $N_{r s}$ the number of atoms, per gram of stellar material, in ionization state $r$ and in excitation state $s$ of the species responsible for a particular line at wavelength $\lambda$, whose oscillator strength is $f$. The continuous opacity, per gram of stellar material, is $\kappa$. Then

$$
\begin{equation*}
\eta_{0}=C \frac{N_{r s} f \lambda}{\kappa v}, \tag{8}
\end{equation*}
$$

where $C$ is an atomic constant independent of the line and $v$ is the previously defined Doppler velocity.

We adopted the $f$-values discussed in § IV for the Fe lines and take $\kappa$ from Bode's (1965) tables at $\theta$ and $\log P_{e}(1)$. (In considering the degree of ionization of Fe , we form ratios of $\eta_{0}$ so that $v$ cancels and only the wavelength dependence of $\kappa$ really enters, not its absolute value.) We thus computed $C N_{r s}$ for each line and, applying Boltzmann's equation, $C N_{r 0}$. Forming a mean of $C N_{r 0}$ by giving weight 4 to class $a$ lines (see Table 3 ), weight 2 to $b$, and 1 to $c$, we applied Saha's equation, with $\theta$-values from Table 4, to find $\log P_{e}(2)$, the representative electron pressure demanded by the ionization equilibrium of Fe . The values of $\log P_{e}(2)$ are given in Table 4.

The agreement between $\log P_{e}(1)$ (obtained from spectral energy distributions and Balmer-line profiles alone) and $\log P_{e}(2)$ (obtained solely from atomic line strengths) shows that the choice of $\tau=0.2$ as the representative layer of line formation is satisfactory. The finally adopted representative electron pressure is $\log P_{e}$, the mean of $\log$ $P_{e}(1)$ and $\log P_{e}(2)$. We iterated our entire procedure with these new values of $\log P_{e}$
but found that the effects on the damping constants and through these on the derived $\eta_{0}$-values could be ignored.

In a customary notation if $X(*)$ and $X(a)$ are the values of any quantity $X$ in a program star and in a Lyr, respectively, we write

$$
\begin{equation*}
[X]=\log X(*)-\log X(a) \tag{9}
\end{equation*}
$$

Then for any line

$$
\begin{equation*}
\left[\eta_{0}\right]=\left[N_{r s}\right]-[\kappa]-[v] . \tag{10}
\end{equation*}
$$

For any atomic species [v] may be computed from the $\xi$ - and $\theta$-values in Table 4. We obtained [ $\kappa$ ] from Bode's (1965) tables. Hence [ $N_{r s}$ ] was computed and reduced to [ $N$ ], where $N$ is the total number of atoms per gram of the line-producing element, by obvious applications of the Saha and Boltzmann equations.

We derived mean values of $[N]$ from the lines of each ion, weighting the individual determinations by line quality as previously explained. The results are given in Table 6.

TABLE 6
Logarithmic Abundances [ $N$ ] Relative to a Lyrae

| Ion | $n$ | $\lambda$ Boo | 29 Cyg | $\pi^{1}$ Ori | $\theta$ Hya | $\gamma$ AqR | $\rho$ Peg | $\beta$ Ari |  |  | $\underset{161817}{\mathrm{HD}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  | I | II | III |  |
| OI. | 1 | +0.6 | +02 | -0.1 | -1.6 | -07 | +02 | +05 | -11 | -09 | $-01$ |
| NaI. | 2 | -15 | -1.1 |  | 00 |  | -09 | +03 | -18 | -16 | -2 8 |
| MgI.. | 3 | -19 | -13 | -14 | $\leq 00$ | $\leq-0.1$ | +0 1 | +07 | -07 | -0 5 | -13 |
| $\mathbf{M g}$ II. . | 1 | -0.4 | -0.6 | -0.4 | 0.0 | +05 | +0.5 | +1.6 | +0.2 | +04 | -0.7 |
| Si II.... | 2 | $\leq-1.2$ | $\leq-08$ | $\leq-12$ | -0 4 | -07 | -0 1 | +08 | -06 | -05 | -1.5 |
| CaI.. | 2 | -03 | -0.4 | -07 | $\leq 07$ | $\leq 01$ | +02 | +1.5 | -0 1 | +0.2 | -1.2 |
| Ca II. | 3, 1 | -1.4 | -0.9 | -10 | +0 2 | 00 | +05 | -0 2 | -02 | -02 | -08 |
| ScII. | 1 | +0.2 | -05 | -08 | $\leq 07$ | $\leq 0.2$ | $\leq 06$ | +18 | -0.4 | -0 4 | -10 |
| Ti II. | 7 | +02 | -0.5 | -0 5 | +05 | +0 3 | +00 | +18 | -0 2 | 00 | -07 |
| FeiI. | 8 | +0.1 | -0 5 | -0 5 | +0.4 | +03 | +0 1 | +15 | -0.2 | 00 | -11 |
| SriI. | 2 | +0.5 | -0.2 | -0.9 | $\leq 09$ | $\leq 05$ | +03 | +18 | -0 1 | +0.2 | -08 |

Here $n$ is the number of lines used in forming the mean. We have given the iron abundance derived from the Fe ir lines only. The abundance derived from the Fe r lines necessarily agrees with the tabulated value (to an accuracy of $\pm 0.1$ in [ $N$ ]) because of our derivation of the representative electron pressure. Columns $\overline{\mathrm{I}}$ and II of $\beta$ Ari refer to the two possible turbulent velocities allowed by the curve-of-growth fitting procedure, and the significance of column III will be explained in § VIc.

Of the sources of error associated with the numbers in Table 6, we believe the most important to be the systematic errors in the determination of the Doppler velocity $v$. As a guide to the magnitude of these errors, we obtained from Baschek and Traving (1964) the change in the [ $N$ ]-values in each star that would result from a change $\Delta \log v=$ 0.1. The results are given in Table 7. The error will be of the same sign for all elements, of course. In Table 4 we list the errors in $F(\Omega=0)$, or, what is the same thing, in $\log v$, that result from the uncertainties of the curve-of-growth fitting. Comparing these with the numbers of Table 7, we conclude, first, that the abundance [ $N$ ] of the iron-group elements has an uncertainty of about $\pm 0.5$ in all our broad-lined program stars; second, that abundances relative to iron are very much better determined than absolute abundances; third, that the Ca abundances indicated by Ca ir lines (all on the damping portion of the curve of growth) are free from errors due to this source.

Additional errors arise from the equivalent-width determinations themselves. In this
analysis of rotating stars these errors are larger than would be acceptable in most abundance studies. From a comparison of our equivalent widths measured on different spectrograms, and from a comparison of these measurements with those of Burbidge and Burbidge (1956), we conclude that errors as large as $\pm 0.5$ in the value of $[N]$ indicated by a single line must frequently occur.

The abundance results for different elements in Table 6 are not equally reliable. The results for Fe and Ti are the best. Those for Sc and Sr should be treated with caution, as they are based on few and uncertain equivalent widths. We caution the reader that the results of Table 6 certainly show analysis errors whose origins we have not been able fully to understand. We shall discuss these analysis effects in connection with an examination of the derived abundances in the comparison stars.

## VI. THE COMPARISON STARS

The analyses of the comparison stars are test analyses. The standard stars $\rho$ Peg and $\beta$ Ari show whether it is possible to derive significant abundance results from rotating stars at all. The star HD 161817, previously studied by model atmosphere techniques, reveals the limitations of curve-of-growth methods.

TABLE 7
Change in [ $N$ ] for a Change in Velocity Parameter $\Delta$ log $V=0.1$

| Ion | $\lambda$ Boo | 29 Cyg | $\boldsymbol{\pi}^{1}$ Ori | $\theta$ Hya | $\boldsymbol{\gamma}$ Aqr | $\rho \mathrm{Peg}$ | $\beta$ Ari II | a Lyr |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| OI. | 03 | 04 | 0.4 | 02 | 0.4 | 0.4 | 0.2 | 0.4 |
| NaI. | 0.4 | 0.5 |  | 0.4 |  | 0.3 | 03 | 05 |
| Mg I. | 0.2 | 02 | 02 | 02 | 02 | 0.1 | 02 | 0.2 |
| Mg II.. | 02 | 0.2 | 02 | 03 | 02 | 0.3 | 0.2 | 0.3 |
| Si II. | 01 | 0.2 | 01 | 02 | 0.2 | 03 | 0.1 | 0.2 |
| CaI . | 04 | 02 | 02 | 00 | 01 | 0.1 | 0.4 | 0.1 |
| Ca II. | 00 | 0.0 | 0.0 | 0.1 | 00 | 00 | 00 | 0.0 |
| Sc II. . | 03 | 0.2 | 0.1 | 01 | 01 | 0.2 | 0.1 | 0.1 |
| Ti II. . | 03 | 02 | 02 | 01 | 0.1 | 0.2 | 0.3 | 01 |
| Fe II.. | 03 | 02 | 02 | 0.2 | 02 | 0.2 | 03 | 0.2 |
| Sr II. . | 0.4 | 04 | 0.1 | 0.2 | 02 | 0.2 | 04 | 0.1 |

a) $\rho$ Pegasi

The atmospheric parameters for $\rho$ Peg and $a \mathrm{Lyr}$ are very similar. However, $a \mathrm{Lyr}$ has sharp lines, while $\rho$ Peg has rotationally broadened lines. Table 6 shows that our analysis yields the same abundances for the two stars within a factor of 3 . The only exception to this statement is the sodium abundance. In his analysis of $a$ Lyr, Hunger (1955) derived sodium abundances from coarse and fine analyses differing by a factor of 10. It appears from this result and from our abundance in $\rho$ Peg that the Na D-lines are very modelsensitive and that curve-of-growth analysis is inadequate to deal with them.

We conclude that, with the exception of sodium, and despite the high rotation of $\rho$ Peg, a reasonably accurate analysis can be made. The errors in the abundance determination are larger than those obtained in an analysis of sharp-lined stars for two reasons. One is that errors in continuum location have larger effects on equivalent widths in highly rotating stars, and the other is that the number of lines available for study is severely curtailed by the more severe blending in such stars.

$$
\text { b) } H D 161817
$$

Kodaira (1964, 1967a) gives the abundances in HD 161817 relative to the solar composition. He finds oxygen to be normal and all other elements to be deficient by a factor of about 10. In Table 6 we give abundances in HD 161817 relative to a Lyr. Comparing
our results with Kodaira's, on the assumption that $a$ Lyr and the Sun are identical in compositions, we find that, with the exception of sodium, the abundances agree within a factor of 2 . As in $\rho \mathrm{Peg}$, this star shows, according to our analysis, a large (and spurious) sodium underabundance.

$$
\text { c) } \beta \text { Arietis }
$$

The small number of selected usable Fe I and Fe ir lines does not allow a unique determination of the microturbulent velocity $\xi$ for this star. The two possible solutions of the fitting procedure, however, lead to very different abundances ( $\beta$ Ari I and $\beta$ Ari II in Table 6). Since $\beta$ Ari is an MKK standard A5 V star, we exclude the low $\xi$-value which would give large overabundances for most elements. Another strong reason for the choice of a high $\xi$-value ( $\beta$ Ari II) is that the abundances of Ca derived from lines of Ca I should agree with those derived from Ca II . Since the Ca I lines are sensitive to turbulence while the Ca ir lines, being on the damping portion of the curve of growth, are unaffected by turbulence, this abundance criterion is a good turbulence discriminator.

Since the determination of the iron abundance is the most reliable, we adjusted the velocity parameter to give the same iron abundance in $\beta$ Ari as in $a$ Lyr. The resulting abundances are shown in Table 6 under the heading $\beta$ Ari III. The adjustment in the Doppler velocity required to go from the actual fitting to $\beta$ Ari III is only a small decrease of $\log v$ by 0.07 , which is well within the error limits of the curve-of-growth fitting. The microturbulent velocity for $\beta$ Ari III is $9.1 \mathrm{~km} \mathrm{~s}^{-1}$, a relatively large value as judged by the thermal velocity of hydrogen of $10.8 \mathrm{~km} \mathrm{~s}^{-1}$. Recently, Conti and Strom (1968) found comparable turbulent velocities in some normal A stars in the Pleiades.

We now discuss the abundance results for $\beta$ Ari III. With the exception of the sodium and oxygen abundances, all elements are normal within a factor of 3 . The sodium discrepancy is similar to that found in $\rho \mathrm{Peg}$ and HD 161817. This is a clear example of an analysis effect. Another example is to be found in the magnesium abundances derived from lines of Mg I and Mg II, respectively. Comparison of our results with those of Kodaira for HD 161817 shows that the correct magnesium abundance is to be obtained by averaging the results of the two determinations. But this leaves the fact that the two determinations themselves differ by nearly a factor of 10 . It is important to notice that these analysis effects are quantitatively very similar in the two comparison stars $\beta$ Ari and HD 161817, which have closely similar atmospheric parameters. They will therefore cancel in a differential comparison of similar stars. For this reason, in what follows, we shall compare abundances in $\lambda$ Boo, 29 Cyg , and $\pi^{1}$ Ori with those found in $\beta$ Ari.

Our result for oxygen in $\beta$ Ari is quite unexpected. The underabundance of oxygen with respect to iron is not dependent on turbulence. Our methods of analysis gave an oxygen abundance for HD 161817 that is in excellent agreement with that obtained by Kodaira. There seems no reason to doubt our result except a reluctance to believe that MKK standards have large abundance anomalies. ${ }^{2}$

The discussion of the three comparison stars shows that it is possible to obtain abundance results accurate within a factor of 3 for stars with rotational velocities of about $100 \mathrm{~km} \mathrm{~s}^{-1}$. It shows, too, that intercomparison of stars of similar atmospheric parameters will keep to a minimum the residual analysis errors.

## VII. THE COMPOSITION OF LAMBDA BOOTIS STARS

The attempt to classify the program stars by considering their chemical compositions leads to the conclusion that only three of them ( $\lambda$ Boo itself, 29 Cyg, and $\pi^{1}$ Ori) form a more or less homogeneous group from the abundance point of view. This is already apparent in Table 6, where those stars show large deficiencies of Ca , as indicated by the

[^2]turbulence independent Ca II lines. These stars also show abnormally large $\mathrm{O} / \mathrm{Fe}$ ratios.
In order to eliminate the uncertainties caused by the different atmospheric parameters of the three stars and $a$ Lyr, we have determined the abundances of $\lambda$ Boo, 29 Cyg, and $\pi^{1}$ Ori relative to the standard star $\beta$ Ari. By analogy with the notation [ $N$ ] which has been used for abundances relative to a Lyr, we have introduced the notation $\{N\}$ for logarithmic abundance differences with respect to $\beta$ Ari. Values of $\{N\}$ are given in Table 8. We found oxygen to be deficient in $\beta$ Ari by a factor of 8 . In the first line for O I in Table 8 we give the oxygen abundance relative to $\beta$ Ari; in the second line (in parentheses), the oxygen abundance relative to a fictitious star with normal oxygen but with all other parameters the same as $\beta$ Ari.

TABLE 8
Logarithmic Abundances $\{N\}$ Relative to $\beta$ Arietis

| Ion | $\lambda$ Boo |  | 29 Cyg | $\boldsymbol{\pi}^{1} \mathrm{ORI}^{\text {r }}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | I | II |  |  |
| OI. | +1.5 | +10 | $+1.2$ | +10 |
|  | (0 6)* | $\left(\begin{array}{ll}0 & 1\end{array}\right)$ | (0 3) | $\left(\begin{array}{ll}0 & 1\end{array}\right)$ |
| Mg I . . | -14 | -18 | $-0.8$ | -09 |
| Mg II. | $-0.8$ | -12 | -10 | -09 |
| Si II. . | $\leq-07$ | $\leq-09$ | $\leq-03$ | $\leq-0.7$ |
| CaI... | - 05 | -13 | $-05$ | $-09$ |
| Ca II. . . | $-1.1$ | -11 | $-0.6$ | -08 |
| Sc II | +0 6 | -0 1 | -02 | -05 |
| Ti II. | +02 | -04 | -05 | -04 |
| Fe II.... . | +01 | -05 | -05 | -0 5 |
| Sr II ..... | +03 | $-05$ | -04 | -1.1 |

* The significance of the numbers in parentheses is explained in the text.


## a) $\lambda$ Bootis

Column I of Table 8 for $\lambda$ Boo results directly from the $[N]$-values in Table 6. The turbulent velocity in $\lambda$ Boo is less well determined than that for 29 Cyg and $\pi^{1}$ Ori. In $\lambda$ Boo the Fe ir lines scattered too widely to provide a curve-of-growth fit, and only the Fe i lines were used. Therefore, the analysis has been repeated (column II), assuming that the Doppler velocity for iron was $\Delta \log v=0.2$ larger than the value given in Table 4. The differences between the relative abundances in $\lambda$ Boo in columns I and II are those resulting from our "best" and limiting fit of the curves of growth. They therefore give the errors inherent in our analysis. The ionization equilibrium of Ca provides a further criterion that can be used to fix the microturbulent velocity. From it we conclude that column II represents the parameters for $\lambda$ Boo best. It does not violate the empirical evidence from the weakly determined curve-of-growth fitting. Our finally adopted turbulence in $\lambda$ Boo is very similar to that of 29 Cyg and $\pi^{1}$ Ori.

We conclude from Table 8 that $\lambda$ Boo, as well as being the type-star, is conveniently the most extreme example of its class from the composition viewpoint. The significant characteristics of its composition are that (1) iron and titanium are deficient by about a factor of 3; (2) oxygen has nearly normal abundance; (3) magnesium and calcium are deficient with respect to the iron group. The amount by which the iron-group elements are deficient depends on the turbulence. Without violating the Ca -ionization equilibrium and the curve-of-growth fitting, we estimate a possible range of $-0.7 \leq\{N(\mathrm{Fe})\} \leq$ -0.2 . The qualitative result of a high $\mathrm{O} / \mathrm{Fe}$ ratio and low $\mathrm{Mg} / \mathrm{Fe}$ and $\mathrm{Ca} / \mathrm{Fe}$ ratios in $\lambda$ Boo is independent of the choice of turbulence.

Our abundance results for O and Mg are in excellent agreement with Kodaira's (1967b) analysis of the infrared oxygen and magnesium lines. The abundances derived by Burbidge and Burbidge (1956) for $\mathrm{Mg}, \mathrm{Ca}, \mathrm{Fe}$, and Sr relative to the normal star 95 Leo are about a factor of 3 smaller than ours, also showing a stronger deficiency for Mg and Ca than for Fe . The factor of 3 arises because our equivalent widths are systematically larger by 30 per cent and because our velocity parameter is smaller by $\Delta$ log $v=0.1$. Burbidge and Burbidge simply assumed the microturbulence to be the same as in 95 Leo ( $\xi=4 \mathrm{~km} \mathrm{~s}^{-1}$ ).
b) 29 Cygni and $\pi^{1}$ Orionis

The compositions of these two stars are very similar to one another. The curve-ofgrowth fitting gives a better determined turbulence than in the case of $\lambda$ Boo. Since in both stars the abundances from neutral and ionized calcium lines agree, we can assume that the values of $\xi$ in Table 4 are correct.

From Table 8 we conclude that iron and titanium are deficient relative to $\beta$ Ari by a factor of 3 , as in $\lambda$ Boo. Oxygen is normal. The deficiency of Mg and Ca relative to the iron-group elements is less pronounced than in $\lambda$ Boo.

The star 29 Cyg has been analyzed by Burbidge and Burbidge (1956). We agree roughly with their Fe and Sr abundances, while our abundances for Mg and Ca are smaller than theirs by a factor of 2.5 . If we compare our results with the analysis of the infrared $\mathrm{O}_{\mathrm{I}}$ and Mg ir lines by Kodaira (1967b), we find excellent agreement for $\pi^{1}$ Ori. For 29 Cyg we agree only with Kodaira's oxygen abundance, whereas his magnesium abundance is larger by a factor of 10 . In view of the very good agreement of the $\mathbf{M g}$ abundance in the other stars, we are inclined to assume that Kodaira's measurement of the equivalent width of Mg II, $\lambda 7896$, in 29 Cyg is too large.

Our abundance analysis shows that $\lambda$ Boo, 29 Cyg , and $\pi^{1}$ Ori not only have similar atmospheric parameters (including microturbulent velocity) but also form a group of related stars from the standpoint of their chemical composition. The composition characteristics of the $\lambda$ Boo group are (1) deficiency of about a factor of 3 in the iron-group elements; (2) nearly normal oxygen abundance; (3) deficiencies of Mg and Ca that are at least as large as those of the iron-group elements and sometimes are larger.

In the following discussion we shall use these composition characteristics to define the $\lambda$ Bootis stars.

## viII. ARE $\gamma$ AQR, $\boldsymbol{\theta}$ HYA, AND ADS $3910 \mathrm{~B} \boldsymbol{\lambda}$ bootis stars?

We now discuss the abundances in Table 6 for the hot stars $\gamma$ Aqr and $\theta$ Hya which have been related to the $\lambda$ Bootis group. We will show that these stars, and ADS 3910B, do not share the abundance characteristics of $\lambda$ Boo, 29 Cyg , and $\pi^{1}$ Ori.

## a) $\gamma$ Aquarii

The abundances of $\mathrm{Mg}, \mathrm{Ca}, \mathrm{Ti}$, and Fe in $\boldsymbol{\gamma}$ Aqr do not differ from those in $a \mathrm{Lyr}$ by more than a factor of 3 . Oxygen and silicon possibly are slightly deficient; however, we note that from Table 7 the oxygen line is very sensitive to changes in the microturbulence. Kodaira (1967b) obtained normal $O$ and Mg abundances from the infrared lines. In any case, $\gamma$ Aqr shows neither a deficiency of the iron group nor a large $\mathrm{O} / \mathrm{Fe}$ ratio and hence does not belong to the $\lambda$ Bootis group.

The star $\gamma$ Aqr is probably a normal one. It was originally proposed as a $\lambda$ Bootis star because it was believed to be situated below the main sequence. A rediscussion of its position by Eggen (1967) and Oke (1967), however, puts it on or near the main sequence.
b) $\theta$ Hydrae

Morgan, Keenan, and Kellman (1943) describe $\theta$ Hya as a weak-lined A0 star similar to $\lambda$ Boo with less pronounced spectral peculiarities. Energy distribution and hydrogen
lines show that $\theta$ Hya is a late $B$ star. As in the case of many Ap stars, its original classification as an A star was based on the incorrect assumption of a normal helium abundance. The He I line $\lambda 4471$ is weaker than in the slightly cooler star $\gamma$ Aqr, corresponding to a helium deficiency of a factor of 8 relative to $\gamma$ Aqr.

Oxygen clearly is deficient (cf. also Kodaira 1967b); the other elements are normal. Thus $\theta$ Hya does not share the abundance characteristics of the $\lambda$ Bootis stars. It is a peculiar B star, possibly more related to sharp-lined stars like 3 Cen A or a Scl. Also, from the kinematical properties, $\theta$ Hya as a member of the old Wolf 630 group is distinguished from the $\lambda$ Bootis stars (Eggen 1967).
c) $A D S 3910 B$

Slettebak (1963) suggested that the B star ADS 3910 B (HR 1754), whose spectrum has no helium lines, could be a $\lambda$ Bootis star. Sargent (1966) supported this view in showing that this star, having kinematic properties of Population I, shows large deficiencies of $\mathrm{He}, \mathrm{C}, \mathrm{Mg}, \mathrm{Si}, \mathrm{Ca}$, and Fe by different amounts, He being most deficient, Si and Fe least deficient.

If the membership of ADS 3910 B in the $\lambda$ Bootis group is to be decided, it is essential to determine its oxygen abundance. We obtained coudé image-tube spectra of the triplet O i $\lambda 7774$ in ADS 3910 B and A with 20 and $40 \AA \mathrm{~mm}^{-1}$, respectively, with the 32 -inch camera of the 74 -inch telescope at Mount Stromlo. In the spectrum of component B the triplet could not be detected, whereas component A shows a roughly normal strength of $W(\mathrm{~A})=460 \mathrm{~m} \AA$. Giving a conservative estimate of $W(\mathrm{~B})<100 \mathrm{~m} \AA$, we derive from the atmospheric parameters given by Sargent (1966) an oxygen deficiency of at least $\log N_{\mathrm{B}} / N_{\mathrm{A}}<-1.6$ in ADS 3910 B relative to component A. Therefore, ADS 3910 B does not show the same abundance characteristics as $\lambda$ Boo.

## IX. DISCUSSION

Stars have been called $\lambda$ Bootis stars if they appeared to be weak-lined A stars. Such stars were found to be subluminous in the Hertzsprung-Russell diagram, and this characteristic has sometimes been used in defining the class. It is now clear that the stars considered to be $\lambda$ Bootis stars in the recent literature (e.g., Sargent 1965) do not form a homogeneous group.

The fact that they appear subluminous in the Hertzsprung-Russell diagram merely reflects errors in spectral classification. They are not subluminous for their effective temperatures. We can, however, use composition as a classification criterion. If we define $\lambda$ Bootis stars as stars whose composition resembles that of $\lambda$ Boo itself, we find that 29 Cyg and $\pi^{1}$ Ori are $\lambda$ Bootis stars. The stars $\theta$ Hya and ADS 3910 B are peculiar B stars of a different type, and $\gamma$ Aqr seems to be normal. Probably $\xi$ Aur, which is not included in this study, is also a $\lambda$ Bootis star, since Kodaira (1967b) found that it has normal oxygen and deficient magnesium. In what follows we shall use the term $\lambda$ Bootis stars in this restricted sense.

One important cause of the weakness of the metal lines in the $\lambda$ Bootis stars is that their microturbulent velocities $\xi \leq 4 \mathrm{~km} \mathrm{~s}^{-1}$ are on the lower edge of the large range of turbulence that is characteristic of middle A-type main-sequence stars. However, in addition, a deficiency of the metal-to-hydrogen ratio by a factor of about 3 relative to the normal composition is required to explain the weak lines.

In the luminosity-effective temperature array $\lambda$ Boo, 29 Cyg, and $\pi^{1}$ Ori fall along the Hyades evolved main sequence (Oke 1967). There is no indication that they are aligned along a horizontal branch. Their gravities are larger than for evolved low-mass stars on the horizontal branch and confirm their main-sequence character. Thus the $\lambda$ Bootis stars are young stars, distinguished from other young stars of the same effective temperature only by their low turbulence and their abnormal composition.

Do stars with the composition characteristics of the $\lambda$ Bootis group, but with high
microturbulence, exist? We would surely expect them to do so if this composition were merely the anomalous composition of the interstellar material out of which these stars formed. Such stars would have a weak Ca II K -line (the strength of which is independent of $\xi$ ) for their color and hydrogen-line type. Their metal lines would not be particularly weak. They would have high oxygen-to-magnesium ratios. A study of the infrared spectra of A stars that have weak K-lines might reveal stars of this type if they do exist. At the present time it appears that no examples are known.

Kodaira (1967a, b) has pointed out the similarity between the composition of the Population II star HD 161817 and that of the $\lambda$ Bootis stars. This similarity is a very striking feature of our own results. However, we emphasize that the deficiency of Ca and Mg with respect to Fe , which we believe to be firmly established in the case of $\lambda$ Boo itself, is not a characteristic of the abundances in the metal-poor stars of Population II (Baschek 1959; Wallerstein 1962; Kodaira 1964). This effect (which is, admittedly, clearly established only in $\lambda$ Boo itself) and the absence of known metal-poor A stars of Population I that have high turbulence velocities lead us to the tentative conclusion that the surface composition of $\lambda$ Bootis stars does not represent the "low abundance end" of a spread in composition of the matter from which young stars are formed. We are led to suggest instead that the $\lambda$ Bootis stars are related to the young peculiar A stars, that they form a separate type of Ap stars, and that their surface composition does not reflect that of the material from which they formed. The general idea that these stars are related to the Ap stars and that their abundance peculiarities are to be attributed to surface processes was first suggested by Sargent (1965). We think that our results, although hardly conclusive, are most readily understood in terms of this idea, although we emphasize, in agreement with Oke (1967) and Kodaira (1967b), that the proposal that $\lambda$ Bootis stars are in a post-red-giant phase, no longer seems tenable.

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    $\dagger$ Now at Mount Wilson and Palomar Observatories.

[^1]:    ${ }^{1}$ At $\lambda 8500$ the cool companion contributes 18 per cent to the continuum. To the combined spectrum it contributes infrared Ca II lines with equivalent widths $\leq 300 \mathrm{~m} \AA$. These are about 10 times weaker than the neighboring Paschen lines, and it is not surprising that we do not see them. Our spectrograms thus neither confirm nor disprove Oke's hypothesis.

[^2]:    ${ }^{2}$ We wish to emphasize, however, that the oxygen deficiency in $\beta$ Ari cannot be considered established beyond doubt. A careful study of the effects of depth-dependent turbulence on the line strengths oxygen is required first.

