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A DISCUSSION OF THE SIZES AND EXCITATION OF H II REGIONS

ABSTRACT

The generality of the relation between radio flux density, distance, and the ionizing radiation producing a nebula is demonstrated. Previous estimates for the size of H II regions for some stars of early spectral type must be revised downward when current model atmospheres rather than black-body atmospheres are used to represent the emergent flux.

We shall derive a general method of relating L_c, the total number of Lyman-continuum photons per second exciting a nebula, to D, the distance to the nebula, and S_ν, the total flux density of free-free emission at radio frequency ν. For the case of a completely arbitrary density distribution with no restriction on the geometry of the nebula except that it be completely ionization-bounded, ionization equilibrium requires that the rate of production of Lyman-continuum photons equals the rate of destruction. The main sources of these photons are, of course, the early-type stars in the region. Additional sources are recombinations from the two major constituents of nebulae, H II and He II, to the ground state of the neutral atoms and various bound-bound transitions of He I. Because model atmospheres representative of the exciting stars indicate negligible radiation for $\lambda < 227$ Å, we do not consider the presence of He III. The lack of He III is also borne out by the observations of Palmer *et al.* (1967), who were able to set an upper limit on the abundance due to the non-appearance of a radio recombination line. Lyman-continuum photons are lost in photo-ionization from the ground state, in which the overwhelming majority of H I and He I atoms are found. The equilibrium relation may be written as

$$L_c \geq \int N_e N_i (\beta - \beta_1) dV , \tag{1}$$

where the inequality holds if the nebula is partially density-bounded. In equation (1) the integration is over the entire nebular volume, where N_e and $N_i = N(\text{H II}) + N(\text{He II})$ are the local electron and ion densities, respectively, and β and β_1 are, respectively, the rate coefficients for recombination to all levels and to the ground state, integrated over frequency. The quantity $(\beta - \beta_1)$ is a function of electron temperature, T, and is assumed to be the same for both hydrogen and helium. The value of $(\beta - \beta_1)$, obtained from Seaton (1959), may be approximated very well for electron temperatures generally found in H II regions by

$$\beta - \beta_1 = 4.10 \times 10^{-10} T^{-0.8} \text{ cm}^3 \text{ sec}^{-1} . \tag{2}$$

In equation (1) we have neglected the ionizing photons which are produced by bound-bound He I processes. According to the work of Hummer and Seaton (1964), approximately 65 per cent of the recombinations to excited states of He I will produce Lyman-continuum photons, in which case the right-hand side of equation (1) should be reduced by no more than 6 per cent.

The total flux density S_ν at a frequency which is large enough to insure that self-absorption is negligible is given by

$$S_\nu = \frac{8.61 \times 10^{-76}}{D^2 \nu^{0.1}} \int \frac{N_e N_i}{T^{0.35}} dV \text{ ergs cm}^{-2} \text{ sec}^{-1} \text{ Hz}^{-1} , \tag{3}$$

where D is in parsecs and ν is in GHz. We have used the expression for the free-free emission derived by Oster (1961) in the form given by Mezger and Henderson (1967), except that we have set their quantity $a(\nu,T) = 1$, since it differs from unity by at most a few per cent. Combining equations (1)–(3), we obtain

$$L_c \geq 4.76 \times 10^{65} \nu^{0.1} D^2 S_\nu \frac{\int N_e N_i T^{-0.8} dV}{\int N_e N_i T^{-0.35} dV} \text{ photons sec}^{-1} . \tag{4}$$

As long as the electron temperature does not vary drastically in the plasma, we may usefully re-express equation (4) by expanding the temperature-dependent factors, in a manner similar to that of Peimbert (1967), in a Taylor series about T_0, defined as

$$T_0 = \frac{\int T N_e N_i dV}{\int N_e N_i dV} . \tag{5}$$

When we retain terms in the series to second order, equation (4) becomes

$$L_c \geq 4.76 \times 10^{65} \nu^{0.1} D^2 S_\nu T_0^{-0.45} (1 + 0.484 t^2) , \tag{6}$$

where

$$t^2 = \frac{\int (T - T_0)^2 N_e N_i dV}{T_0^2 \int N_e N_i dV} . \tag{7}$$

For most gaseous nebulae it is a good assumption that $t^2 \ll 1$, and since $T_0^{-0.45}$ will not be significantly different for various nebulae, we have a relation between L_c, S_ν, and D that is to a good approximation independent of the nebular structure. When t^2 is small, spatial variations of density, temperature, and N(He II)/N(H II) affect this relation only to the extent that they define T_0. We may use equation (6) by adopting an appropriate T_0 for all H II regions of perhaps 7000° K (see Rubin 1968*a*) and ignoring t^2. A more refined use of this equation could be made if we knew T_0 and t^2 for individual nebulae. Toward this end, we may employ the expressions of Peimbert which relate T_0 and t^2 to various observationally determined temperatures such as $T_{(\mathrm{Bac/H}\beta)}$, $T_{(\Delta\nu T_l/T_c)}$, and $T_{(4363/5007)}$. Observations of the intensity ratios necessary to determine two or more of these temperatures would provide two or more equations, which could be solved for both T_0 and t^2. For the present use, the intensities must represent measurements for the entire nebula. (A more thorough discussion of the determination of the mean temperature and mean-square temperature variation is presented in Rubin 1968*b*.) Because S_ν is almost unaffected by interstellar absorption, its observation provides a powerful method of obtaining either L_c or D if the other quantity can be ascertained by another method. A similar relation was derived by Moriyama (1960) and by Higgs and Ramana (1968), but it is believed that the present result is the most general.

We now attempt to improve on previous calibrations of the Lyman-continuum radiation that emanates from early-type stars. In two respects the calculation is different from that of Gould, Gold, and Salpeter (1963): (1) We use model stellar atmospheres from Mihalas (1965) and Hickok and Morton (1968) instead of black-body atmospheres. (2) We use recent calibrations of effective temperature, surface gravity, and radii for early-type stars. Table 1 presents values for the effective temperature T_e, surface gravity g, and radius a for early-type stars on the main sequence obtained by Hjellming and Jarecke (1967). These stellar parameters for various spectral types were compiled from a literature search that included both theoretical and observational data. The identification of spectral type with a given set of parameters is the most uncertain step and must be considered with due caution. Using the values of effective temperature and surface gravity, we interpolate in a grid of Mihalas' (1965) model atmospheres to obtain the flux of Lyman-continuum photons emerging from the star,

$$\int_{\nu_1}^{\infty} \mathfrak{F}^s(r = a;\nu)d\nu \, ,$$

which is entered in the fifth column of Table 1. It was necessary to assume that $N(\mathrm{He})/N(\mathrm{H}) = 0.15$ in the stellar atmospheres, for this is the only value for which Mihalas gives an extensive grid. If the exciting stars of H II regions have $N(\mathrm{He})/N(\mathrm{H})$ similar to the nebular material, then the ratio would be closer to 0.10 (Mathis 1962; Palmer *et al.* 1967). A comparison of the emergent Lyman-continuum flux from Mihalas' models with the same properties but with $N(\mathrm{He})/N(\mathrm{H}) = 0.05$ and 0.3 indicates that the major change from decreasing $N(\mathrm{He})/N(\mathrm{H})$ is to decrease the discontinuity at the helium ionization limit (λ = 504 Å), since the opacity due to He II will be less. The effect on the total flux of Lyman-continuum photons does not appear to be more than a few per cent.

TABLE 1

PROPERTIES OF MAIN-SEQUENCE STARS AND THE RESULTING STRÖMGREN RADIUS (SEE TEXT)

Spectral Type	T_e (10^3 ° K)	log g (cm sec^{-2})	$a/a_{\odot}$	$\int_{\nu_1}^{\infty} \mathfrak{F}^s(r=a;\nu)d\nu$ (photons cm^{-2} sec^{-1})*	L_c (photons sec^{-1})*	R_s (pc)
O4	50±5	4 4	8 0	5 4 (24)	2 1 (49)	87
O5	45±4	4 3	6 8	3 1 (24)	8 7 (48)	65
O6	40±4	4 3	5 8	1 5 (24)	3 1 (48)	46
O7	38±3	4 3	5 5	9 5 (23)	1 7 (48)	38
O8.	36±3	4 3	5 2	5 3 (23)	8 7 (47)	30
O9..	35±2	4 3	5 0	3 5 (23)	5 3 (47)	26
O9 5	33±2	4 3	4 7	1 3 (23)	1 7 (47)	18
B0 .	31±2	4 3	4 5	3 5 (22)	4 3 (46)	11
B0 5	29±2	4 3	4 4	8 7 (21)	1 0 (46)	6 8

* Quantities in parentheses are the powers of 10 by which to multiply the corresponding entry.

Hickok and Morton (1968) have computed two model atmospheres, taking into account the detailed blanketing of the strongest lines in the ultraviolet. The Lyman-continuum fluxes for their models with T_e = 37450° and 28640° K are nearly identical with those for Mihalas' model with T_e = 38180° K and an interpolated model that has T_e = 30360° K, respectively (log g = 4 and $N(\mathrm{He})/N(\mathrm{H}) = 0.15$ in all cases). Therefore, it appears that atmospheres that account for line blanketing will have the effect of raising the values of

$$\int_{\nu_1}^{\infty} \mathfrak{F}^s(r = a;\nu)d\nu$$

(for a given T_e) in Table 1. This correction will be larger apparently for the later spectral types than for the earlier ones. As mentioned before, the most uncertain part of Table 1 is the identification of an effective temperature with spectral type. The scale to represent main-sequence stars derived by Morton and Adams (1968), which is based on model atmospheres that include line blanketing, has for the earliest stars much lower effective temperatures than are given in Table 1. For instance, for the earliest star that Morton and Adams list, type O5, T_e = 37500° K. Agreement is much better for the later spectral types, where Morton and Adams have for B0 and B0.5 stars T_e = 30900° and 26200° K, respectively. The corresponding radii obtained from the luminosities presented in their Table 2 are 5.15 and 4.68 $R_\odot$. The lower temperature that Morton and Adams give for B0.5 stars will undoubtedly be in good agreement with the higher value in Table 1 when allowance is made for the difference in the line-blanketed and -unblanketed models. We use the calibration by Hjellming and Jarecke, since they provide values for the stellar surface gravities and radii, which are necessary to obtain L_c from

$$L_c = 4\pi a^2 \int_{\nu_1}^{\infty} \mathfrak{F}^s(r = a;\nu) d\nu \,. \tag{8}$$

Values of L_c for the individual stars in Table 1 are listed in the sixth column. Note that L_c has also been used to represent the total number of Lyman-continuum photons exciting an H II region, in which case a sum over all exciting stars is implied.

When the nebula is assumed to be a Strömgren sphere with radius R_s and the exciting star(s) is at the center, we may write, from equation (1),

$$R_s \leq \left[fL_c \Big/ 4\pi \int_0^{R_s} N_e N_i (\beta - \beta_1)(r/R_s)^2 dr/R_s \right]^{1/3} . \tag{9}$$

This equation allows for a radial variation of density and temperature and an idealized model for density fluctuations. This model (see Rubin 1968*a*) assumes that there are clumps of uniform density randomly distributed in an otherwise empty volume. For any volume in the nebula that is significantly larger than the volume of a clump, the ratio of the latter to the former volume is statistically equal to f, the clumping factor. In equation (1), we have assumed that f is constant throughout the nebula, and N_e and N_i represent local mean values. In the seventh column of Table 1 we have given R_s for constant values of $f = 1$, $N_e = N_i = 1$ cm^{-3}, and $T = 10^4$ ° K in order to compare them with the values obtained by Gould *et al.* (1963). There is good agreement between the O4 star in Table 1 and their model with a black-body surface temperature of 49250° K and $a = 8.25\ R_\odot$. However, the agreement becomes progressively worse for later spectral types mainly because the black body with a surface temperature equal to our effective temperature overestimates the ionizing photon flux increasingly for later spectral types. Thus their model with a surface temperature of 28200° K and $a = 3.48\ R_\odot$ (values which are smaller than the temperature and radius for our B0.5 star) has more than twice the Strömgren radius. Because the stellar parameters are reasonably well calibrated for the later spectral types, it is clear that the amount of ionization caused by these later stars must be revised downward. The values of R_s in Table 1 are for ionization-bounded nebulae. If a nebula is density-bounded or if the matter is clumped ($f < 1$), these values must be considered upper limits as indicated by equation (9).

Frequent use has been made of the so-called excitation parameter, $u \equiv R_s N_e^{2/3}$, to describe the total amount of Lyman-continuum radiation that is creating an H II region. In this definition N_e is actually a complicated average electron density over the nebula. From equation (9), we see that even for constant T, $N_e = N_i =$ constant, and $f = 1$, we have $u = [3L_c/4\pi(\beta - \beta_1)]^{1/3}$; that is, the excitation parameter also depends on the electron temperature of the gas. It would seem preferable not to involve properties of the nebular matter when determining a quantity that is supposed to be a measure of the Lyman-continuum radiation producing the nebula. It is suggested that L_c rather than u be determined directly, because determination of L_c does not require spherical symmetry and does not disguise the pertinent quantity with complicated averages that depend on the detailed density, temperature, and abundance structure of the nebula.

Interpretation of observations of S_ν by Schraml, Mezger, and Altenhoff (now being reduced) at 2 and 6 cm with the NRAO 140-foot telescope will, it is hoped, provide information that will allow us to determine which nebulae are ionization-bounded, which are density-bounded, and which do not yet have the main exciting sources known. Comparisons with observations may also establish whether the current relation of L_c to spectral type is applicable to the exciting stars in H II regions. Estimates of the sizes and emergent fluxes of early-type stars will be of interest in the fields of stellar atmospheres and interiors, where observational data regarding physical parameters for such stars are scarce.

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