THE ASTROPHYSICAL JOURNAL, Vol. 154, October 1968

ENERGY DISTRIBUTIONS, K CORRECTIONS, AND THE STEBBINS-WHITFORD EFFECT FOR GIANT ELLIPTICAL GALAXIES

J. B. OKE AND ALLAN SANDAGE
Mount Wilson and Palomar Observatories, Carnegie Institution of Washington, California Institute of Technology

Received January 25, 1968

ABSTRACT

The energy distribution $F(\lambda)$ in the range 3350 Å $\leq \lambda \leq$ 5850 Å has been measured with a spectral resolution of 50 Å for a number of nearby giant E and S0 galaxies and for M31 and M32. The distribution in the near-infrared to λ = 10800 Å has been obtained for NGC 3379 and NGC 4472. The various distributions, apart from that for M32, are nearly identical and have been averaged to obtain a standard $F(\lambda)$ from which new K_B, K_V, and K_R corrections for the effects of redshifts are calculated, as listed in Table 4.

New observations of $B - V$ colors for the several brightest galaxies in thirty-one clusters, covering the redshift range $0 < z \leq 0.20$, are listed in Table 6. Figure 2 shows the observed color change with redshift. Agreement of the observations with the predicted color change, $K_B - K_V$, shows, following an earlier result by Whitford, that the Stebbins-Whitford effect is absent to the 3 per cent level in the broad-band $B - V$ system and that no variation of the shape of $F(\lambda)$ with time is present larger than that implied by $\Delta(B - V) \leq 0$ 03 mag for redshifts as large as $z = 0$ 20 The shape change of $F(\lambda)$ because of evolution of the stellar content in the lookback time must, therefore, be below the limit of present detectability.

I. INTRODUCTION

The most pressing current problem in observational cosmology is the determination of the Hubble constant H_0 and the deceleration parameter q_0. The best means of finding q_0 still appears to be the non-linearity in the Hubble diagram, $m_{\text{bol}} = f(\Delta\lambda/\lambda_0)$, due to deceleration of the Universe. But this approach has always been complicated by several important corrections to the observational data, which are (*a*) the K dimming, (*b*) the aperture effect, and (*c*) the evolutionary change in absolute luminosity of distant galaxies during the light-travel time.

Each correction is a function of redshift, which means its value must be known with high accuracy if a systematic error in q_0 is to be avoided; viz., errors in the corrections that are functions of z ($\equiv \Delta\lambda/\lambda_0$) will give spurious contributions to the observed departure from linearity of the Hubble diagram. An estimate of the required precision can be made from the well-known series expansion (cf. Heckmann 1942; Robertson 1955; McVittie 1956) of the exact equation (Mattig 1958):

$$m_{\text{bol}} = 5 \log z + 1.086(1 - q_0)z + O(z^2) + \text{const} . \tag{1}$$

At a given redshift, the error in q_0 for a given error Δm is

$$|\Delta q_0| = 0.921\Delta m/z . \tag{1'}$$

The largest redshift now available for a galaxy is $z = 0.461$ for 3C 295 (Minkowski 1960). The total error in m_{bol} must, therefore, be kept below ± 0.1 mag at this redshift limit if q_0 is to be determined systematically to within ± 0.2.

Progress in understanding the three corrections has proceeded in recent years to about this level, and it appears that the observational problem is now tractable. We discuss here a new determination of the K correction, preparatory to a revised solution for q_0. The other two corrections will be discussed in later papers.

II. THEORY OF THE K CORRECTION

The K correction is a purely technical effect that occurs when a continuous energy distribution $F(\lambda)$ is redshifted through *fixed* spectral-response bands of a detector such as a photographic plate or a photoelectric photometer with specified filters. It is composed of two terms. (a) If $F(\lambda)$ is not flat, i.e., $F(\lambda_0) \neq F[\lambda_0/(1+z)]$, then the flux at the effective wavelength λ_z(eff) in the rest frame of a galaxy of redshift z, transformed from the effective wavelength λ_0(eff) of the detector by $\lambda_0(\text{eff})/(1+z)$, will differ from the flux of a galaxy at rest. (b) The effective rest-frame band width of the detector will differ between the redshifted galaxy and the galaxy at rest by the factor $1+z$; the band width, expressed in *proper* Ångstrom units, is smaller in the redshifted galaxy, giving a correction 2.5 log $(1+z)$ mag. Effect a is selective, i.e., a function of λ. It would be zero if $F(\lambda)$ were flat. Effect b is non-selective and is independent of the form of $F(\lambda)$. If fluxes are expressed as energy per unit wavelength interval, the defining equation for the K term at effective wavelength λ_i is

$$K_i = 2.5 \log (1+z) + 2.5 \log \left\{ \int_0^\infty F(\lambda_0) S_i(\lambda) d\lambda \Big/ \int_0^\infty F[\lambda_0/(1+z)] S_i(\lambda) d\lambda \right\} , \quad (2)$$

where K_i is expressed in magnitudes.

The problem can be equivalently formulated if the flux is expressed per unit frequency interval, $F(\nu)$. Under redshift z, all rest-frame frequencies ν_0 are shifted to frequencies ν_{new} by

$$\nu_{\text{new}} \equiv \nu(z) = \nu_0/(1+z) . \quad (3)$$

The effective band width, $\Delta\nu$, in the rest frame of each galaxy is now *increased* by the factor $1+z$ because *more* proper cycles per second are observed by a detector with a fixed band pass in the redshifted spectrum than in the unshifted spectrum. The complete K term in this case is

$$K_i = -2.5 \log (1+z) + 2.5 \log \left\{ \int_0^\infty F(\nu_0) S_i(\nu) d\nu \Big/ \int_0^\infty F[\nu_0(1+z)] S_i(\nu) d\nu \right\} . \quad (4)$$

Equations (2) and (4) are equivalent, since $F(\nu) \propto \lambda^2 F(\lambda)$.

The sign of the K term is defined so that the transformation from observed heterochromatic magnitudes in spectral band i to a bolometric scale is

$$m_{\text{bol}} = m_i - K_i + \Delta m_{\text{bol}}(i) , \quad (5)$$

where the bolometric correction is Δm_{bol} (i).

The present definition is the same as given elsewhere (Humason, Mayall, and Sandage 1956, Appendix B), from a slightly different viewpoint. We should emphasize again that the correction is a purely instrumental effect. It would disappear if intensity measurements of redshifted galaxies were made with a detector whose spectral acceptance band was shifted by $1+z$ at all wavelengths. The detector band width would then automatically increase by the factor $1+z$, and the redshifted spectrum of all galaxies would be measured at the same rest-frame effective wavelength. Such observations could be made using a spectrum scanner with variable channel widths and positions. Baum's (1962) method of intermediate band-width photometry is similar in some respects to this ideal, in that he read his constructed $F(\lambda)$ functions at the *proper* wavelength (corrected for redshift). Only the band-width difference then remains as a correction. Except for Baum's results, all other measurements of faint galaxian intensities to date have been made with fixed-band detectors, and the full K correction is required.

We should also mention that the preceding discussion differs from that of Hubble

(1936, p. 535), where he added two other terms to equation (2), each equal to 2.5 log $(1 + z)$, to obtain what he considered to be the "total correction to observed magnitudes due to redshift." He heuristically called these two factors the "energy" and "number" effects. We have not adopted this procedure but rather correct the observed heterochromatic magnitude by our K term to what would be observed, to within a constant bolometric correction, by a bolometer put above the Earth's atmosphere. The absolute bolometric luminosity, L_{bol}, in the rest frame of any galaxy of redshift z is related to this hypothetically observed bolometric luminosity at the Earth by Robertson's equation (1938)

$$l_{\text{bol}} = \frac{L_{\text{bol}}}{4\pi R_0^2 r^2 (1 + z)^2}, \tag{6}$$

where r is the co-moving radial coordinate of the Robertson-Walker line element. Hubble's two factors are included as $(1 + z)^2$ in equation (6), which, when combined with the null geodesic of the metric, gives the physically correct relation between observed quantities $m_i - K_i = f(z,q_0)$ (cf. Sandage 1961), with K_i defined as in equations (2) and (4) above.

III. PREVIOUS CALCULATIONS OF K—THE STEBBINS-WHITFORD EFFECT

In his first evaluation of the K term, Hubble (1936) assumed $F(\lambda)$ to be the Planck function with $T = 6000°$ K, justified from the solar-type spectra of E galaxies in the blue region. Greenstein (1938) later showed that a temperature near $T \simeq 4200°$ K was more nearly correct and, even more importantly, that the true $F(\lambda)$ could not be approximated adequately by the Planck function because of the composite nature of the stellar content.

The first measurement of $F(\lambda)$ was made by Stebbins and Whitford (1948) for M32 using a broad-band six-color system, reduced to absolute intensity by reference to the Sun. When the M32 distribution, assumed to be typical of giant E systems, was redshifted through the measuring bands, a discrepancy was found between the calculated color change $K_b(z) - K_v(z)$, and the observed relation $C_p = f(z)$. If substantiated, this difference would have had the most serious consequences because it would suggest that $F(\lambda)$ is itself a function of redshift or, equivalently, a function of time. The rest-frame value of $F(\lambda)$ for galaxies at different redshifts would then be dissimilar, and the assumption of constant L_{bol}, necessary to derive $m_{\text{bol}} = f(z,q_0)$, would be false.

The Stebbins-Whitford effect was further discussed by Whitford (1954) using new observations of colors to improve the empirical $C_p = f(z)$ relation, but still adopting the six-color $F(\lambda)$ function for M32. The discrepancy remained.

The K corrections calculated from $F(\lambda)$ for M32 were tabulated by Humason *et al.* (1956). These were used in the first determination of q_0, in which a correction was attempted for the Stebbins-Whitford effect. It was clear, however, that an improved value of q_0 must await the solution of the color anomaly.

De Vaucouleurs (1948) suggested that part of the discrepancy arose because of inadequate spectral resolution of the six-color system, especially in the region of the precipitous drop in $F(\lambda)$ for solar-type spectra near $\lambda_0 = 3900$ Å. He noted that the broad bands might excessively smooth the true $F(\lambda)$, falsifying the calculation of the color change $K_b - K_v$. It is now known that this smoothing, combined with the abnormally blue $F(\lambda)$ for M32, provides the solution. Whitford (1957) showed that the true color anomaly, if it existed at all, was much smaller than initially calculated in 1948. A further discussion was given by Whitford (1962).

Code (1959) and Oke (1962) undertook moderately high-resolution spectrum scans of nearby giant ellipticals to strengthen Whitford's conclusion. Although their separate $F(\lambda)$ functions for NGC 4374 were in general agreement, a small systematic difference was present, which amounted to about 0.15 mag at $\lambda = 6000$ Å when the two functions were normalized at $\lambda = 4000$ Å (Sandage 1966*a*, Fig. 1). A substantial part of this difference may have been due to different absolute calibrations of α Lyr used by each author and not to the observations themselves.

New interim K corrections were computed (Sandage 1966*a*) for each function, and these were used in a second approximation for the observed Hubble diagram (Sandage 1966*a*, *b*, *c*). The color effect, $K_B - K_V$, calculated separately for each distribution, differed by about 15 per cent at $z = 0.15$. In order to clarify the various kinds of discrepancies, we have made new spectrophotometric observations, presented in the next section.

IV. NEW SPECTROPHOTOMETRIC OBSERVATIONS

A program to obtain $F(\lambda)$ for giant E and S0 galaxies was carried out between 1960 and 1964. The objects studied included many of the brightest galaxies in the Virgo Cluster, the two brightest galaxies in the Coma Cluster, and a few other bright, nearby systems, including M31 and M32. Observations were made largely with the Cassegrain scanner attached to the 100- or 60-inch telescope at Mount Wilson; a few observations were obtained with the prime-focus scanner on the 200-inch telescope. An entrance aperture of 10″–14″ was usually used, and no attempt was made to study the color changes with distance from the center of the galaxy. An exit slit of 50 Å was used, and measurements were usually made every 50 Å through the spectrum. (Since the Cassegrain scanner has a slightly non-linear drive, steps with a nominal size of 50 Å are not always spaced exactly 50 Å apart.) The measurements were made by alternately observing galaxy plus sky and sky alone. Charge-integration techniques were employed for most observations. The observations were reduced to absolute flux measurements by comparison with standard stars using techniques described elsewhere (Oke 1965). The fluxes are based on the calibration of α Lyr (Oke 1964), except that ultraviolet points are made fainter by 0.06 mag to conform with a new calibration. The fluxes are expressed as $AB_\lambda = -2.5 \log F(\lambda) +$ const., where $F(\lambda)$ is the flux in ergs sec^{-1} cm^{-2} Å^{-1}.

Most galaxies were observed more than once. The observations on different nights vary greatly in accuracy, depending mostly on the sky brightness. A comparison of results on different nights indicates that the mean fluxes, expressed in magnitudes, at a given wavelength have standard deviations of approximately ± 0.07 mag for $\lambda > 4000$ Å and ± 0.13 mag in the ultraviolet for most of the observed galaxies. The fluxes for M31 and M32, based mainly on measurements at Palomar, have standard deviations of less than 0.02 mag at all wavelengths.

When the giant elliptical and S0 galaxies with the best-determined fluxes were compared, it was found that within the accuracy of the measurements they were identical from λ3375 to λ5840. To improve the accuracy, the fluxes of eight program galaxies have been averaged; the resulting values of AB are listed in Table 1. The galaxies represented in the table are NGC 3115 (E7), 4278 (E1), 4374 (E1), 4472 (E2), 4486 (E0), 4594 (Sa), 4649 (E2), and 5866 (S0). It should be noted that, although one Sa spiral is included, the data refer to the central region, which has the same color distribution as E and S0 systems. For the averaged fluxes the errors are ± 0.03 mag except in the ultraviolet, where they are 0.05.

The energy distributions from λ3375 to λ5841 for M31 and M32, also in terms of AB_λ, are given in Table 2. The magnitudes have been normalized so that M31, M32, and the average elliptical galaxy from Table 1 agree in the red. Comparison of the three energy distributions shows that the average elliptical galaxy and M31 are almost identical. On the other hand, M32 is much bluer than M31, being brighter in the ultraviolet by 0.3 mag.

Spectrophotometric data in the infrared are very limited. Complete data are available only for NGC 3379 and NGC 4472, those for the former being of better quality. On the average the two agree very well, and the results for NGC 3379 are listed in Table 3. The values of AB have again been normalized so that they match the values of AB in Tables 1 and 2. The data from Tables 1–3 are shown in Figure 1.

For the calculation of K from equation (2), it is necessary to adopt an energy distribution which is suitable for the average giant elliptical galaxy. It was decided to use the results in Table 2 for M31, together with the infrared data in Table 3. This should be an adequate representation, since the data in Tables 1 and 2 agree closely.

These results, together with those of Lasker (1966), and supplemented by broad-band color data by many people, show that giant E and S0 galaxies have very similar values of $F(\lambda)$ over the present range of zero observed wavelengths. The conclusion agrees with similarity of integrated spectra for many E galaxies (Humason, in Humason *et al.* 1956; Morgan 1962), and identical resolution results for all elliptical systems of the Local Group (Baade 1944*a*, *b*). Giant E and S0 galaxies form a remarkably homogeneous class

TABLE 1

MEAN FLUX VALUES FOR EIGHT GIANT GALAXIES NORMALIZED TO M31 IN THE RANGE FROM λ_0 = 3975 TO λ_0 = 5600 Å

λ(Å)	AB_λ	λ(Å)	AB_λ	λ(Å)	AB_λ	λ(Å)	AB_λ	λ(Å)	AB_λ
3376	2 26	3888 .	2 12	4396	1 11	4900	0 76	5398	0 57
3427	2 35	3939	2 02	4448	1 02	4950	70	5448	59
3479	2 19	3990	1 74	4498	0 91	5000	73	5497	.61
3530	2 19	4041	1 39	4549	0 88	5050	.77	5547	59
3581	2 38	4092	1 37	4599	0 78	5100	75	5596	58
3633	2 13	4143	1 45	4649	0 82	5150	.82	5645	58
3684	1 89	4194	1 42	4699	0 88	5200	88	5694	56
3735.	1 97	4245	1 35	4750	0 85	5250	72	5743	.60
3786	2 03	4296	1 36	4800	0.76	5300	.66	5793	58
3837.	2 31	4347..	1 23	4850	0 74	5349	0 58	5842	0 53

TABLE 2

MEAN FLUX IN MAGNITUDES PER UNIT WAVELENGTH INTERVAL FOR M31 AND M32

λ(Å)	AB_λ		λ(Å)	AB_λ		λ(Å)	AB_λ	
	M31	M32		M31	M32		M31	M32
3325	2 49	. . .	4225	1 34	1 16	5050	0 79	0 67
3375	2 58	2 11	4275	1 35	1 18	5100	.80	71
3425	2 25	1 96	4325	1 20	1 14	5150	89	79
3475	2 28	1 97	4375	1 12	1 02	5200	84	74
3525	2 34	1 94	4425	1 04	0 94	5250	72	73
3575	2 40	2 03	4475	0 93	0 84	5300	64	61
3625	2 14	1 86	4525	0 89	0 79	5350	62	61
3675	1 97	1 74	4575	0 82	0 75	5398	58	63
3725	2 20	1 89	4600	0 85	0 72	5448	61	59
3775	2 09	1 74	4649	0 90	0 77	5497	66	62
3825	2 40	1 89	4699	0 81	0 68	5547	63	.59
3875	2 18	1 77	4750	0 82	0 70	5596	59	.63
3925	2 04	1 75	4800	0 75	0 65	5645	58	59
3975	1 76	1 55	4850	0 79	0 74	5694	60	.60
4025	1 36	1 23	4900	0 71	0 64	5743	46	.53
4075	1 40	1 20	4950	0 69	0 62	5792	53	.62
4125	1 44	1 21	5000	0 80	0 70	5841 .	0 51	0.63
4175	1 47	1 23						

with regard to stellar content. A difference does exist in the ultraviolet between M31 and the dwarf elliptical M32—a difference partly responsible for the apparent Stebbins-Whitford effect in the earlier work.

The existence of a well-defined $F(\lambda)$ shows that the K correction obtained therefrom will have almost negligible cosmic scatter locally. Of course, if $F(\lambda)$ is a function of time because of evolution of the stellar content, K will also be a function of time. Such a

TABLE 3

MEAN FLUX FOR NGC 3379 IN THE NEAR-INFRARED NORMALIZED TO TABLES 1 AND 2

λ(Å)	AB_λ	λ(Å)	AB_λ	λ(Å)	AB_λ	λ(Å)	AB_λ	λ(Å)	AB_λ
5800	0 54	6600	0 48	7450	0 50	8400	0 59	10050	0 64
5850	48	6650	51	7500	49	8450	60	10100	63
5900	58	6700	53	7550	47	8500	65	10150	52
5950	51	6750	.54	7700	58	8550	72	10200	53
6000	48	6800	53	7750	58	8600	62	10250	.64
6050	47	6850	54	7800	55	8650	60	10300	60
6100	48	6950	54	7850	50	8700	67	10400	66
6150	51	7000	52	7900	55	8750	64	10450	65
6200	50	7050	.51	7950	57	8800	58	10500	75
6250	53	7100	52	8000	59	9700	71	10550	.69
6300	62	7150	61	8050	.50	9800	.72	10600	.71
6350	.57	7200	67	8100	54	9850	67	10650	.66
6400	57	7250	62	8250	56	9900	63	10700	65
6450	50	7300	53	8300	58	9950	60	10750	66
6500	53	7350	52	8350	0 53	10000	0 61	10800	0.65
6550	0 52	7400	0 49						

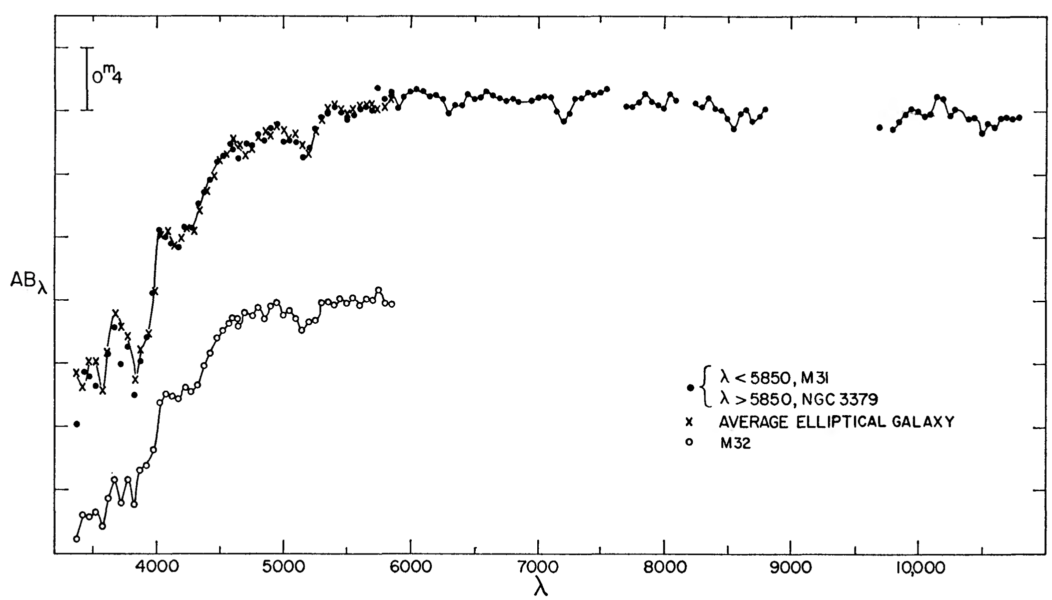

FIG. 1.—Spectral energy distributions of galaxies expressed in magnitudes, $AB_\lambda = -2.5 \log F(\lambda) +$ const. Measurements have not been made in regions of heavy atmospheric absorption near λλ6870, 7600, 8200, and from λ8800 to λ9700.

difference will appear as a real Stebbins-Whitford effect at some level of accuracy. This is the effect we have looked for but have failed to find, as discussed in § VI.

V. CALCULATION OF K

Equation (2) was used to calculate K_B, K_V, and K_R, adopting $S(\lambda)$ for the natural b and v color systems as given by Matthews and Sandage (1963). The sensitivity functions for zero air mass were used, which give a color-transformation coefficient of 1.00 relating $b - v$ to $B - V$. Hence, $K_b - K_v = K_B - K_V$. A similar development for the R_s photometric system of Sandage and Smith (1963) is given in the Appendix.

Table 4 gives the results for the total K correction, i.e., the sum of the band-width term and the selective term of equation (2), calculated with $F(\lambda)$ defined by M31 from Table 2, and extended to the red by Table 3. Similar calculations for M32, again using

TABLE 4

K TERMS IN B, V, AND R FOR THE STANDARD GIANT ELLIPTICAL

z	2 5 log $(1+z)$	K_B	K_V	K_{R_s}	K_B-K_V	$K_V-K_{R_s}$	$(B-V)_c$	$(V-R_s)_c$
0 00	0 00	0 00	0 00	0 00	0 00	0 00	1 04	0 82
.02	02	0.11	04	02	07	02	1 10	0 84
04	04	0 22	.09	04	13	.05	1 16	0.88
06	.06	0 33	14	06	19	08	1 23	0 91
08	.08	0.44	18	08	24	10	1 29	0 93
10	10	0 55	22	10	33	12	1 36	0 95
.12	12	0 67	27	13	.40	14	1 43	0 96
14	14	0 78	32	16	46	16	1 50	0 98
.16	16	0 89	37	19	.52	.18	1 56	1 00
18	18	1.00	.44	23	56	21	1 60	1 04
20	20	1 10	51	27	59	24	1 64	1 07
22	22	1 20	.58	30	62	.28	1 66	1 10
24	23	1 30	65	33	65	32	1 69	1 15
26	25	(1.40)	73	35	67	38	1 71	1 20
0 28	0 27	(1 51)	0 83	0 39	0 68	0 44	1 73	1 27

Table 3 for $\lambda > 5800$ Å, are given in Table 5. Also listed in each table are the calculated $(B - V)_c$ and $(V - R_s)_c$ colors obtained from the transformation equations given by Matthews and Sandage (1963) and in the Appendix below.

We should emphasize that the K corrections and the calculated colors refer to the nuclei of giant galaxies and not to the integrated light. This is because $F(\lambda)$ of Tables 1–3 were obtained from observations of the central 10″–14″ of the relevant galaxies. This presents a slight problem when comparison is attempted between observed integrated colors and calculated colors, because a color gradient is known to exist in elliptical systems (de Vaucouleurs 1960; Tifft 1963). However, since the color difference between the nucleus and the integrated light is nearly constant with a value of about 0.1 mag, the calculated and the observed colors should differ by only a zero-point constant, again of about 0.1 mag. The calculated K terms may, however, contain a systematic error due to this effect, because the energy distribution of the nucleus will differ somewhat in shape from the $F(\lambda)$ distribution for the integrated light. We cannot for the moment assess the effect accurately, but spectrophotometric observations of the integrated light for Virgo Cluster ellipticals, using a 4-inch telescope of short focal length, are now in progress by Oke and Schild, and a second approximation to these results will soon be available. We believe that the systematic effect in K_V and K_R will be less than 0.04 mag for $z \leq 0.20$. It has been neglected in the remaining discussion.

VI. COMPARISON OF $K_B - K_V$ WITH $B - V$ MEASUREMENTS FOR CLUSTER GALAXIES

To search for a possible residual Stebbins-Whitford effect, we have measured $B - V$ colors of the several brightest galaxies in thirty-one clusters with redshifts in the range $0 < z \leq 0.20$. The observations were obtained with the 100- and 200-inch reflectors. Most clusters are from Humason's list of redshifts (Humason *et al.* 1956, Table III), but a few are clusters in which a radio source occurs as the brightest member. The detailed photometric measurements will be given elsewhere (Sandage 1968), but a summary is listed in Table 6. Nearly all the clusters are in high galactic latitude, $|\beta| > 40°$. Corrections for galactic reddening were computed with a cosecant law but were found to be negligibly small in most cases and have been ignored. The mean observed colors are

TABLE 5

K TERMS IN B, V, AND R FOR THE DWARF ELLIPTICAL M32 + NGC 3379 (INFRARED)

z	2 5 log $(1+z)$	K_B	K_V	K_{R_s}	K_B-K_V	$K_V-K_{R_s}$	$(B-V)_c$	$(V-R_s)_c$
0 00 .	0 00	0 00	0 00	0 00	0 00	0 00	0 93	0 80
02 .	02	0 09	04	02	05	02	0 98	0 82
04. .	.04	0 18	08	04	10	04	1 03	0 84
06 .	.06	0 28	12	06	16	06	1 09	0 86
08	08	0 37	15	08	22	07	1 15	0 88
10	10	0 47	19	10	.28	09	1 21	0 89
12	12	0 57	22	13	35	09	1 27	0 89
14.	14	0 66	26	16	40	10	1 33	0 90
16	16	0 76	31	19	45	12	1 38	0 92
18	18	0 85	37	.22	48	15	1 42	0 95
20.	20	0 94	42	25	52	17	1 45	0 98
22	.22	1 04	49	28	.55	.19	1 48	1 01
.24	23	1 13	55	31	.58	24	1 51	1.05
26	.25	1 23	62	34	61	28	1 54	1 09
0 28 ..	0 27	1 34	0 69	0 36	0 65	0 33	1 57	1 15

tabulated, averaged from measurements of several of the brightest galaxies in each cluster, each galaxy measured through several apertures. The average deviations (A.D.) of the measurements from the mean $B - V$ are also listed. The identification of the measured galaxies is given. For non-NGC galaxies, the numbering system is that of Humason's marked charts (Humason *et al.* 1956, Pls. I and II), supplemented where necessary with additional numbers listed in the final column of Table 6.

The run of measured $B - V$ with redshift is shown in Figure 2, where the average deviations are given as error bars. The line is the calculated $K_B - K_V$ term of Table 4, but normalized to the observed points by a zero-point shift of 0.97 mag. We could equally well have plotted the line using $(B - V)_c$ from Table 4 with a zero-point shift of 0.07 mag, justified by the discussion of § V.

The absence of a systematic difference between the calculated curve and the observations is taken as proof that the Stebbins-Whitford effect does not exist at the ± 0.03 mag level and that any time variation of $F(\lambda)$ due to stellar evolution must be smaller than 3 per cent over the $B - V$ band pass in the lookback time corresponding to $z \leq 0.20$. This result justifies the view that our K terms must be nearly correct, at least to the ± 0.05-mag level, and that no substantial error will result from the use of Table 4 for the calculation of q_0 by means of the $m_i - K_i = f(z,q_0)$ Hubble diagram. Our present limits

TABLE 6

OBSERVED $B - V$ COLORS FOR THE SEVERAL BRIGHTEST GALAXIES IN THIRTY-ONE CLUSTERS

Cluster	z	$B-V$	A D	Identification
Virgo.. .	0 003	0 97	±0 02	4472+4486
Peg I.....	0128	1 04	02	7619
0122+3305	0170	1 05	01	507
Coma.....	0222	1 06	01	4889
2308+0720	0428	1 08	04	7503
2322+1425	0439	1 09	.	7649
1145+5559 .	0516	1 11	02	UMa 1; N24, N25
0106−1536	0526	1 06	03	N2, N4, N10
1024+1039 .	0649	1 18	02	Leo; N1, N2
1239+1852 .	0718	1 19	01	Virgo 2; N2, N4
1520+2754	0722	1 19	02	Cor Bor; N2, N3, N6
0705+3506	0779	1 24	03	Gemini; N1, N2
1513+0433	0944	1 25	03	N1, N2, N3
1431+3146	1312	1 38	07	Bootis; N1, N4
1055+5702 .	1345	1 45	06	UMa 2; N2, N5, N10
1153+2341	1426	1 43	03	N1+N1a, N2, N4
1534+3749	.1532	1 42	.02	N1, N2, N4, N6
0025+2223 .	.1594	1 50	03	N1, N2, N4
1228+1050	.1651	1 52	04	Virgo 3; N1, N2, N4
0138+1832	1730	1 50	03	N1
1309−0105	1745	1 48:	.	N1+N2
1304+3110	.1831	1 54	06	N1+N1A, N2, N3
0925+2044	1917	1 64	04	N1, N2, N10
1253+4422	1979	1 52	07	N3, N5, N8, N9
0855+0321	2018	1 57	03	N1+N2+N2a, N4, N5
3C 40	0180	1 03	01	N1, N2, N3
3C 66.. .	.0215	1 08	..	Only 3C 66
3C 465..	0301	1 12	03	N1+N2, a, b, c, d
3C 338..	0303	1 04	02	N4, N6
3C 317 .	0351	1 05	02	N2, N3
3C 219..	0 1745	1 53	±0 03	Only 3C 219

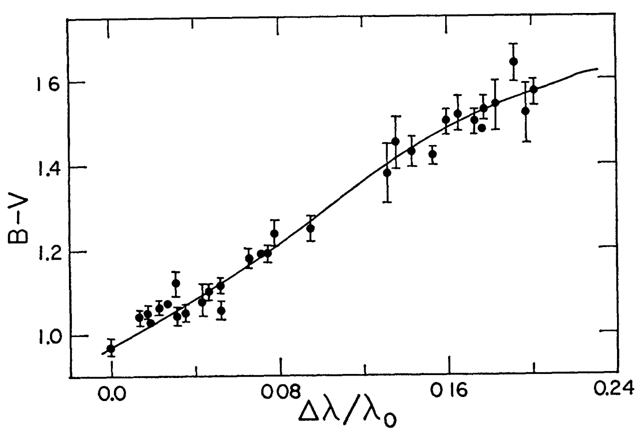

FIG. 2.—Comparison of measured $B - V$ colors for the several brightest galaxies in thirty-one clusters with calculated color differences, $K_B - K_V$, using eq. (2) and the function $F(\lambda)$ for the nucleus of M31, extended to the red from Table 3. The calculated curve has been normalized to the observations by a small zero-point shift. The systematic agreement between the observations and the theory shows the absence of a Stebbins-Whitford effect to the 3 per cent level.

on the time variation of $F(\lambda,t)$ are important for the problem of dL/dt because of evolution of the stellar content of elliptical galaxies. The results of the present paper are a prelude to discussion of this further problem. In particular, Figure 2 shows that the conclusion of Tinsley (1968) on the size of dL/dt is too pessimistic.

APPENDIX

CHARACTERISTICS OF THE R PHOTOMETRIC SYSTEM

To compute the magnitudes and colors of any given distribution $F(\lambda)$, it is necessary to integrate this function over appropriate filter, photocell, and telescope sensitivity functions, which define a "natural" photometric system. To convert to a standard system defined by specific stars, e.g., the Johnson-Morgan UBV system, it is convenient to follow the procedure used by observers in relating any two color systems. An example is the natural calculational $(b - v)_0$ system of Matthews and Sandage (1963), which is related to $B - V$ by

TABLE A1

ADOPTED RESPONSE FUNCTION FOR THE R_s MAGNITUDE

λ(Å)	$S(\lambda)_0$	λ(Å)	$S(\lambda)_0$	λ(Å)	$S(\lambda)_0$
5800...	0 000	6300	1 32	7800 .	0 38
5900	0 030	6400	1 29	8000 .	27
5950 .	0 190	6600 .	1 16	8200	.17
6000. .	0 490	6800	1 04	8400	.08
6050.	0 790	7000	0 90	8600	.01
6100...	1 09	7200	0 76	8800 .	0 00
6150... .	1.24	7400	0 64		
6200... . .	1.31	7600 .	0 50		

$$B - V = 1.00(b - v)_0 + 0.91 ,$$

where the subscript refers to observations at zero air mass.

The $V - R_s$ system used here has been described by Sandage and Smith (1963), who also give standard-star magnitudes and colors for this system. Since the cell sensitivity is not known accurately, a "natural" r magnitude is defined here by the product of an RG1 filter of thickness 2 mm and an S20 cathode with good red sensitivity. The adopted relative response is listed in Table A1. To convert $(v - r)_0$ to $V - R_s$, the same procedures are used that were employed to convert $(b - v)_0$ to $B - V$.

A number of stars are listed in Table A2 for which $V - R_s$ has been measured with a photometer (Sandage and Smith 1963; Mannery and Wallerstein 1967) and for which blue and red scans exist, so that $F(\lambda)$ is known. In the second column, $(v - r)_0$ has been computed using the v response given by Matthews and Sandage (1963), the response given above in Table A1, and the $F(\lambda)$ from the scans. The last column is the observed value of $V - R_s$. A comparison yields

$$V - R_s = 1.065(v - r)_0 + 1.28 . \tag{A1}$$

The third column gives $V - R_s$ calculated from equation (A1) using the values of $(v - r)_0$.

The coefficient 1.065 in equation (A1) implies that the effective wavelength of the response function in Table A1 is too small, provided that the v response function is correct. Therefore, the

K corrections, computed from the $S(\lambda)$ function of Table A1, are slightly incorrect compared with the R_s system defined from actual stars as in Sandage and Smith (1963), because an incorrect effective wavelength was used. To test the size of the error, a red response function which yielded a color coefficient in equation (A1) of 1.18 was used to calculate another set of values,

TABLE A2

CALCULATED AND OBSERVED $V - R$ COLORS FOR THE CALIBRATING STARS

Stars	$(v-r)_0$ (Scans)	$V-R_s$ (Eq (A1))	$V-R_s$ (Obs)
10 Lac....	−1 33	−0 14	−0 11
α Lyr....	−1.16	+0 04	0 00
HD 19445	−0.78	+0.45	+0 45
HD 22879	−0 77	+0 46	+0 47
99 Her....	−0.77	+0.46	+0 47
BD−17°4708.	−0.80	+0 43	+0 47
16 Cyg..	−0 73	+0 50	+0 50
HD 175305....	−0 60	+0 64	+0 64
HD 122563..	−0.48	+0 77	+0 76
HD 221170..	−0 43	+0 82	+0 81
HD 165195...	−0 25	+1 01	+1 04

K'_R. At $z = 0.24$ the difference between the K corrections was 0.025 mag. The values of K_R in Tables 4 and 5 have therefore been corrected so that they correspond to a color coefficient in equation (A1) of 1.00.

REFERENCES

Baade, W. 1944*a*, *Ap. J.*, **100**, 137.

———. 1944*b*, *ibid.*, p. 147.

Baum, W. A. 1962, in *Problems of Extragalactic Research*, ed. G. C. McVittie (New York: Macmillan Co.), p. 390.

Code, A. 1959, *Pub. A.S.P.*, **71**, 118.

Greenstein, J. L. 1938, *Ap. J.*, **88**, 605.

Heckmann, O. 1942, *Theorien der Kosmologie* (Berlin: Springer), p. 63.

Hubble, E. P. 1936, *Ap. J.*, **84**, 517.

Humason, M. L., Mayall, N. U., and Sandage, A. R. 1956, *A.J.*, **61**, 97.

Lasker, B. M. 1966, *A.J.*, **71**, 862.

McVittie, G. C. 1956, *General Relativity and Cosmology* (London: Chapman & Hall).

Mannery, E. J., and Wallerstein, G. 1967, *A.J.*, **72**, 1357.

Matthews, T. A., and Sandage, A. R. 1963, *Ap. J.*, **138**, 30.

Mattig, W. 1958, *Astr. Nach* , **284**, 109.

Minkowski, R. 1960, *Ap. J.*, **132**, 908.

Morgan, W. W. 1962, *Ap. J.*, **135**, 1.

Oke, J. B. 1962, in *Problems of Extragalactic Research*, ed. G. C. McVittie (New York: Macmillan Co), p. 34.

———. 1964, *Ap. J.*, **140**, 689.

———. 1965, *Ann. Rev. Astr. and Ap.*, **3**, 23.

Robertson, H. P. 1938, *Zs. f. Ap.*, **15**, 69.

———. 1955, *Pub. A.S.P.*, **67**, 82.

Sandage, A. 1961, *Ap. J.*, **133**, 355.

———. 1966*a*, "Radio Sources and the Expansion of the Universe," in *Proceedings of the Meeting on Cosmology for the Galileo Celebration*, Vol. 2, Tomo 3, ed. L. Rosino (Florence, Italy), p. 104.

———. 1966*b*, in *High Energy Astrophysics*, ed. L. Gratton (Internat. School of Phys. "Enrico Fermi," Course 35 [New York: Academic Press]), p. 12.

———. 1966*c*, "Annual Report of the Director of Mount Wilson and Palomar Observatories," *Carnegie Yrb.*, **66**, 162.

———. 1968, *Ap. J.* (in press).

Sandage, A , and Smith, L. L. 1963, *Ap. J.*, **137**, 1057.
Stebbins, J., and Whitford, A. E. 1948, *Ap. J.*, **108**, 413.
Tifft, W. G. 1963, *A.J.*, **68**, 302.
Tinsley, B. M. 1968, *Ap. J.*, **151**, 547.
Vaucouleurs, G. de. 1948, *C.R.*, **227**, 466.
———. 1960, *Ap. J. Suppl.*, **5**, 233.
Whitford, A. E. 1954, *Ap. J* , **120**, 599.
———. 1957, *Sky and Tel.*, **16**, 222.
———. 1962, in *Problems of Extragalactic Research*, ed. G. C. McVittie (New York: Macmillan Co.), pp. 27 and 40.