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RECOMBINATION OF THE PRIMEVAL PLASMA*

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ABSTRACT

A theory is presented for the plasma recombination that would have taken place when the Universe had expanded and cooled to a Primeval Fireball temperature of about 4000° K. The computed residual ionization of the hydrogen following this recombination is in the range of 2×10^{-5} to 2×10^{-4}, depending on the assumed cosmological model. In the closed cosmological model the matter temperature would have effectively decoupled from the radiation at a temperature of 1200° K, while in the lowest density model the matter temperature would not have fallen much below the radiation temperature before the galaxies formed. Also computed is the effect of the recombination radiation on the spectrum of the Primeval Fireball.

I. INTRODUCTION

It is becoming clear that the details of the decoupling of matter from the Primeval Fireball (if it exists) should play an important role in the origin of the galaxies. The recombination of the plasma fixes the epoch at which non-relativistic bound systems like galaxies can start to form (Peebles 1965), and one would like to know with some assurance the matter-temperature variation at this time, for the temperature fixes the critical Jeans length for formation of bound systems (Gamow 1948; Peebles 1965; Dicke and Peebles 1967). It has been pointed out to me that one also would like to know the residual ionization in the matter, because the electrons and ions mediate the formation of molecular hydrogen and so help determine the rate of energy radiation by clouds that formed at the Jeans limit. Finally, the rapidity of the recombination determines the extent to which density irregularities in the plasma would have been dissipated during the recombination (Peebles 1967).

Progress at Princeton toward a reasonable picture for the origin of the galaxies has been slow at best, but we have obtained a reasonably simple theory of the initial recombination of the plasma, and it is the purpose of this paper to present this theory. The question to be considered is the degree to which the recombination is inhibited by the presence of recombination radiation. This problem has been mentioned previously by Shklovskii (1967) and by Novikov and Zeldovič (1967). The residual ionization obtained below is similar to the value stated by Novikov and Zeldovič. In the following treatment of the problem, the very complicated recombination process is reduced to simpler terms by means of a number of approximations valid in the conditions of interest, so that the physical situation can be described by a few variables, and the recombination equation can be integrated by hand, without having to abandon any of the essential elements of the problem.

In the following computation it is assumed that the Universe is expanding in a strictly isotropic and homogeneous way. There is some reason to believe that this is the case through the epoch of decoupling (e.g., Peebles 1967), but of course the assumption fails once the first-bound systems (galaxies, or perhaps proto-globular clusters, cf. Dicke and Peebles 1968) have formed. In the assumed conditions the matter could not have been hotter than the Primeval Fireball, so that the situation envisaged by Weymann (1966), where a hot plasma is augmenting the intensity of the Primeval Fireball radiation, could

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not yet have obtained. The only appreciable perturbation to the spectrum of the Primeval Fireball due to the initial recombination would be provided by the plasma recombination radiation, which would appear on the high-energy tail of the spectrum. Finally, for simplicity, it is assumed that at the epoch of recombination the matter is pure hydrogen.

The adopted gravity theory is General Relativity without the Cosmological Constant. Throughout, all quantities are expressed in proper units, as measured with conventional rods, clocks, and scales. The cosmological equation is (Tolman 1934)

$$\left(\frac{1}{a}\frac{da}{dt}\right)^2 = \tfrac{8}{3}\pi G\rho + \frac{K}{a^2}\,, \tag{1}$$

where $a(t)$ is the expansion parameter, $\rho(t)$ is the total mass density, and K is a constant. It is known that in the present epoch the left side of this equation does not differ from the first term on the right by more than a factor of about 40 (Oort 1958), so that at the epoch of recombination, which is at a redshift $a_0/a \sim 1000$, the second term on the right side of the equation can always be neglected. In this approximation, equation (1) can be integrated to obtain the equation

$$t = \left(\frac{3}{2\pi G\rho_m}\right)^{1/2}\left[\tfrac{1}{3}\left(1+\frac{\rho_r}{\rho_m}\right)^{3/2} - \frac{\rho_r}{\rho_m}\left(1+\frac{\rho_r}{\rho_m}\right)^{1/2} + \tfrac{2}{3}\left(\frac{\rho_r}{\rho_m}\right)^{3/2}\right]. \tag{2}$$

Here ρ_r and ρ_m are the mass densities in radiation and (non-relativistic) matter at epoch t. They are fixed by the present density of matter, which is thought to be in the range of 1.8×10^{-29} gm cm^{-3} to 4.5×10^{-31} gm cm^{-3}, by the present value of the Fireball Radiation temperature T, which is taken to be 2.7° K (Stokes, Partridge, and Wilkinson 1967), and by the conditions that ρ_r varies as T^4, and ρ_m varies as T^3. The above-mentioned mass densities range from the value needed to just close the Universe if the present value of Hubble's constant is $H_0 = (1 \times 10^{10}\, y)^{-1}$ to the estimated mass density due to galaxies (Oort 1958). In the following discussion, orders of magnitude are established using the cosmologically flat model. Here the radiation mass density is relatively small during the epoch of recombination, and we have the approximate time

$$t = \frac{0.93 \times 10^{12}}{T_4^{3/2}}\ \text{sec}\,, \qquad n = 5.5 \times 10^5 T_4^3\ \text{cm}^{-3}\,, \tag{3}$$

where T_4 is the radiation temperature in units of 10^4 ° K, and n is the number density of hydrogen atoms plus free protons.

II. APPROXIMATE THEORY OF THE RECOMBINATION

a) *Thermal Contact*

Near the end of the epoch of recombination, the most important means for transferring energy between radiation and matter would be Thomson scattering of the free electrons (Weymann 1966). Since the heat capacity of the radiation is greater than that of the matter by the factor $aT^3/nk \sim 10^8$, it is an excellent approximation to suppose that the spectrum of the radiation over the peak is unaffected by this heat transfer.

The characteristic time for the protons and atomic hydrogen to relax to a thermal, Maxwell-Boltzmann velocity distribution is on the order of the mean free time between atomic collisions. Near the epoch of recombination, this relaxation time is on the order of 10^6 sec, which is 10^6 times smaller than the characteristic expansion time of the Universe (eq. [3]). Therefore, it is a good approximation to suppose that the nucleons are in kinetic thermal equilibrium, although of course the nucleon temperature need not be equal to the radiation temperature.

When the electrons are near thermal equilibrium with the radiation, the rate of transfer of energy per unit volume between radiation and free electrons is (Weymann 1965; Hensel 1967)

$$\frac{d\mathcal{E}_{e\gamma}}{dt} = \frac{4\sigma a T^4 n x_e k}{m_e c}(T - T_e) , \tag{4}$$

where T_e is the electron temperature, n is the nucleon number density, and nx_e is the free electron number density. Assuming for the moment that the electrons preserve thermal contact with the nucleons, thermal contact between matter and radiation is broken when the ratio of the heat content of the matter to the heat-transfer rate (eq. [4]) exceeds the characteristic expansion time scale of the Universe (eq. [3]), or when the quantity

$$\frac{T}{T - T_e}\frac{t}{\mathcal{E}_m}\frac{d\mathcal{E}_{e\gamma}}{dt} = 4.6 \times 10^6 T_4^{5/2}\frac{x_e}{1 + x_e} \tag{5}$$

approaches unity. According to the results presented below, this characteristic number is unity when $T_4 = 0.12$, $x_e = 5 \times 10^{-5}$.

The dominant means of transferring energy from electrons to the protons and atoms is the long-range coulomb interaction with the ions. On taking the rate of energy transfer for this process from Spitzer (1962), we find that the ratio of the rate of energy transfer between electrons and radiation (eq. [4]) to the rate of energy transfer between electrons and atoms and protons via coulomb scattering is

$$\frac{\text{radiation–electrons}}{\text{electrons–nucleons}} = \frac{2.3 \times 10^{-3} T_4^{5/2}}{x_e \log \Lambda} , \tag{6}$$

where $\log \Lambda \cong 16$ for conditions of interest. According to the results of the recombination theory presented below, the maximum value of this ratio is 0.02, at $T_4 = 0.16$. In the lower density models, the numerical coefficient in equation (6) is larger, but this is almost balanced out by the larger residual ionization x_e. We conclude therefore that the matter, electrons, protons, and ions, would always be close to kinetic thermal equilibrium and that the matter temperature would follow the radiation temperature until the fractional ionization is quite low. Accordingly, we assume that matter and radiation have the same temperature through the greater part of the plasma recombination, where the recombination radiation plays an important role. Once the ionization is fixed, one can obtain a better measure of the temperature variation of the matter (§ III).

b) Ionizing Radiation

The time variation of the spectrum of the electromagnetic radiation is given by the equation

$$\frac{\partial}{\partial t}\left(\frac{\nu n_\nu}{n}\right) = \frac{\nu}{a}\frac{da}{dt}\frac{\partial}{\partial \nu}\left(\frac{\nu n_\nu}{n}\right) + \frac{\nu J_\nu}{n} . \tag{7}$$

Here n_ν is the number of photons per unit volume and frequency interval, n is the total nucleon number density, hydrogen atoms plus free protons, and J_ν is the net rate of production of photons per unit volume and frequency interval. The product $\nu n_\nu / n$ is introduced because this conveniently eliminates the time variation of n_ν due to decreasing photon density and band width.

Using detailed balance, and neglecting the exceedingly small correction for stimulated emission, we find that the net photon production rate is given by the formula

$$J_\nu = \sigma_\nu c\left[\frac{8\pi\nu^2}{c^3}\frac{h^3}{(2\pi m_e k T)^{3/2}} e^{-(h\nu - B_1)/kT} n_e^2 - n_\nu n_{1s}\right] . \tag{8}$$

Here σ_ν is the cross-section for photo-ionization of hydrogen by radiation at frequency ν, B_1 is the binding energy of hydrogen in the ground state, and n_e and n_{1s} are the number densities of free electrons and of hydrogen atoms in the ground state. The computation of the density of ionizing photons properly would involve double integration of equations (7) and (8) over time and frequency. However, we avoid this by observing that, once the recombination is at all appreciable, the mean free time for an ionizing photon is on the order of 10^3 sec, some nine orders of magnitude smaller than the characteristic expansion time scale for the Universe (eq. [3]). This means that the ionizing part of the radiation in effect is in close thermal contact with the matter, so that the two terms on the right-hand side of equation (8) must very nearly balance each other at each frequency above the absorption edge. That is, the spectrum of the ionizing radiation must be given to good accuracy by the formula

$$n_\nu = \frac{n_I h}{kT} e^{-(h\nu - B_1)/kT}\,, \qquad h\nu \geq B_1\,. \tag{9}$$

Here n_I is the total number density of the ionizing photons. In writing this equation, we have ignored the relatively slow variation of phase space with frequency because the exponential factor varies so rapidly with frequency. On substituting equation (9) in equation (7), dividing through by ν, and integrating the result over frequencies $\nu \geq B_1/h$, we find the desired equation for the number density of ionizing photons,

$$\frac{d}{dt}\left(\frac{n_I}{n}\right) = -\frac{n_I}{n}\frac{B_1}{kT}\frac{1}{a}\frac{da}{dt} + \frac{J}{n}\,. \tag{10}$$

Here J is the net rate of production of ionizing photons per unit volume,

$$J = \alpha_{1s} n_e^2 - \sigma_I n_I c n_{1s}\,, \tag{11}$$

where α_{1s} is the recombination coefficient for transitions direct to the ground state. The cross-section for photo-ionization of hydrogen should be folded against the photon distribution (eq. [9]), but, in the epoch of recombination, the radiation spectrum decreases very rapidly with increasing frequency, so it will be adequate to take σ_I to be the cross-section at the absorption edge.

Equations (10) and (11) combined determine the net rate of recombination direct to the ground state. This rate is controlled by the first term on the right-hand side of equation (10), which gives the rate at which ionizing radiation is being redshifted below the absorption edge. We obtain a simple estimate of the effect of this term by noting that the coefficient of n_I/n in this term is a factor $B_1/kT \cong 40$ times larger than the characteristic expansion rate of the Universe at the epoch of recombination. The characteristic time rate of change of the left-hand side of equation (10) is more nearly comparable to the characteristic expansion rate. Therefore, in a rough approximation, we can ignore the left side of equation (10) and solve equations (10) and (11) algebraically, to obtain the formula

$$n_I \cong \alpha_{1s} n_e^2 \bigg/ \left(\sigma_I n_{1s} c + \frac{B_1}{kT}\frac{1}{a}\frac{da}{dt}\right). \tag{12}$$

On substituting this result in equation (11), we find that the net rate of recombinations direct to the ground state is given by the approximate equation

$$J \cong \alpha_{1s} n_e^2 \frac{B_1}{kT}\frac{1}{a}\frac{da}{dt} \bigg/ \left(\sigma_I n_{1s} c + \frac{B_1}{kT}\frac{1}{a}\frac{da}{dt}\right). \tag{13}$$

The rate of recombination direct to the ground state therefore is inhibited by the factor $\sigma_I nctkT/B_1 \sim 10^7$ (eq. [3]) and, to all intents and purposes, the process can be ignored.

c) *Resonance-Line Radiation*

Once the degree of recombination of the plasma becomes at all appreciable, the mean free time of the resonance recombination line photons becomes very short, on the order of 30 sec when $T = 4000°$ K, as compared to a time scale of 10^{12} sec for the expansion of the Universe. The radiation in the line therefore is in intimate contact with the matter, and the radiation spectrum must be sensibly flat (thermal) across the rather narrow line. The cosmological redshift is pulling the line radiation toward longer wavelengths, so the spectrum varies only smoothly just longward of the line. On the other hand, if the recombination is feeding photons into the line, there will be a sharp rise in intensity just shortward of the line. To obtain the magnitude of this rise, we integrate equation (7) over frequency, from a frequency ν^- just below the line to a frequency ν^+ just above the line. This yields the formula

$$\frac{d}{dt}\left(\frac{n_a}{n}\right) = \frac{\nu_a}{n}\frac{1}{a}\frac{da}{dt}\left[n_{(\nu+)} - n_{(\nu-)}\right] + \frac{R}{n}, \tag{14}$$

where

$$n_a = \int_{\nu-}^{\nu+} n_\nu d\nu \tag{15}$$

is the number of photons per unit volume in the line, and R is the net rate of production of the resonance line photons per unit volume. To solve equation (14), we note that n_a is on the order of $n_{(\nu-)}\,\Delta\nu$, where $\Delta\nu$ is the line width, so that equation (14) amounts to a first-order linear differential equation in $n_{\nu-}$, where in the linear term the coefficient of $n_{\nu-}$ is a factor $\nu_a/\Delta\nu \sim 10^5$ times larger than the characteristic rate of time variation of $n_{\nu-}$. Thus, to excellent accuracy, we can ignore the left-hand side of equation (14) to obtain the formula

$$n_{(\nu-)} = n_{(\nu+)} + Ra/(\nu_a da/dt)\ . \tag{16}$$

It is convenient to introduce the dimensionless variable

$$\mathfrak{N} = c^3 n_\nu / 8\pi\nu^2\ . \tag{17}$$

If a portion of the Universe were imagined placed in a box, this variable would be the number of photons per radiation mode in the box. By equation (16), the number of photons per mode in the line is

$$\mathfrak{N}_a = \mathfrak{N}_+ + R\lambda_a{}^3 a/(8\pi da/dt)\ . \tag{18}$$

d) *Distribution of Excited States of Hydrogen*

Recombination direct to the ground state being strongly inhibited, the recombination rate depends on how quickly resonance photons can be redshifted out of the lines, thus permitting allowed radiative transitions from the excited states to the ground state, and on how quickly atoms can populate the ground state via two-quantum emission from the 2s state. The former question is greatly complicated by the existence of numerous resonance lines and by the existence of numerous excited states to which electrons can recombine and from which hydrogen can be photodissociated.

We deal with the numerous levels in the following way. We note first that the excited states of hydrogen are spread over an energy range of 3.4 eV and that through the epoch of recombination there are still substantial numbers of primeval (thermal) photons in this range of energy. When the temperature is 4000° K and the recombination is proceeding most rapidly, there are for each nucleon some 10^5 thermal photons more energetic than 3.4 eV. When the temperature is 2000° K and recombination is effectively finished (cf. Table 1 below), the ratio still amounts to 20. We take it then that radiative transitions between excited states of hydrogen, and between excited states and the continuum, involve energy exchange with radiation not appreciably perturbed from a thermal spectrum. Since the relaxation time for the excited levels is 10^{-7} sec, some 20 orders of magnitude faster than the expansion time, we conclude that the relative populations of the excited states of hydrogen are fixed by thermal equilibrium with the radiation. That is, the number density of atoms in the n,l level satisfies the equation

$$n_{nl} = n_{2s}(2l + 1)e^{-(B_2-B_n)/kT} , \tag{19}$$

where B_n is the binding energy of hydrogen in the nth principal quantum number. In this picture, only the population of the ground state of the atom is out of equilibrium with all the other bound states.

The resonance-line radiation is in intimate contact with the matter, because the mean free time of a resonance photon is so very short, so by detailed balance (and ignoring the very small correction for stimulated emission) the number of photons per mode (eq. [17]) in the nth resonance line satisfies the equation

$$\mathfrak{N}_n = \mathfrak{N}_\alpha e^{-(B_2-B_n)/kT} , \tag{20}$$

where $\mathfrak{N}_\alpha$ is the number of photons per mode in the Lyman-α resonance line,

$$\mathfrak{N}_\alpha = n_{2s}/n_{1s} . \tag{21}$$

Because the exponential factor in equation (20) is small, in the range of 6×10^{-3} to 5×10^{-5} when $T = 4000°$ K, we conclude that most higher order resonance-line photons eventually are absorbed and split up into Lyman-α photons and lower energy photons, the lower energy radiation being lost in the sea of primordial radiation. We take it then that the radiation spectrum is not appreciably affected by redshifted resonance photons shortward of the Lyman-α resonance, so that in equation (18), for the number of photons per mode in the Lyman-α resonance line, we can replace $\mathfrak{N}_+$ with the thermal value, to obtain the formula

$$\mathfrak{N}_\alpha = e^{-(B_1-B_2)/kT} + R\lambda_\alpha^3 a/(8\pi da/dt) . \tag{22}$$

e) Recombination Rate

Neglecting recombination direct to the ground state, we find that the net rate of recombination is given by the expression

$$-\frac{d}{dt}\left(\frac{n_e}{n}\right) = \sum_{n>1}\left(\frac{\alpha_{nl}n_e^2}{n} - \frac{\beta_{nl}n_{nl}}{n}\right) , \tag{23}$$

where α_{nl} is the coefficient for recombination to the (n,l) level, and β_{nl} is the rate of photo-ionization of an atom in this level. Since the lower energy part of the spectrum of the radiation is little perturbed by the recombination, the ratio of β_{nl} to α_{nl} is given by the Saha formula (when $n \geq 2$).

On making use of equations (19) and (21), we can reduce equation (23) to the form

$$-\frac{d}{dt}\left(\frac{n_e}{n}\right) = \frac{\alpha_c n_e^2}{n} - \beta_c \mathfrak{N}_\alpha \frac{n_{1s}}{n} , \tag{24}$$

where

$$\alpha_c = \sum_{n>1} \alpha_{nl} \tag{25}$$

and

$$\beta_c = \sum_{n>1}(2l + 1)\beta_{nl}e^{-(B_2-B_n)/kT} = \alpha_c e^{-B_2/kT}(2\pi m_e kT)^{3/2}/h^3 . \tag{26}$$

In writing down the second line of equation (26), we have taken α_{nl} and β_{nl} to be related through the Saha formula.

According to the above discussion, it is a good approximation to suppose that each net recombination (that is, excess of recombination over photo-ionization) results in the production of a Lyman-α photon or else in a two-quantum decay from the (2s) level to the ground state. The net recombination rate (eq. [24]) therefore is related to the rate R of production of Lyman-α photons (eq. [14]) by the equation

$$\alpha_c n_e^2 - \beta_c \mathfrak{N}_\alpha n_{1s} = R + \Lambda_{2s,1s}[n_{2s} - n_{1s}e^{-(B_1-B_2)/kT}] \,. \tag{27}$$

The second term on the right side of this equation represents the net rate of two-quantum decays, the decay rate from the 2s state being $\Lambda_{2s,1s} = 8.227 \text{ sec}^{-1}$ (Spitzer and Greenstein 1951). Since the atomic collision rate is on the order of 10^{-7} sec^{-1}, it is evident that collisional de-excitation could not be important. In writing down the net rate of two-quantum emission, we have assumed that the rate of the inverse two-quantum absorption process is given by the thermal rate. That is, we have supposed that the perturbation to the radiation spectrum longward of the Lyman-α line does not cause an important perturbation to the rate of the two-quantum excitation of hydrogen. The validity of this assumption is considered below.

The desired equation for $\mathfrak{N}_\alpha$ is obtained by substituting equation (21) into equation (27) and then eliminating R from equations (22) and (27). This yields the equation

$$\mathfrak{N}_\alpha = e^{-(B_1-B_2)/kT} \frac{[1 + K(\alpha_c n_e^2 e^{(B_1-B_2)/kT} + \Lambda_{2s,1s} n_{1s})]}{[1 + K(\beta_c + \Lambda_{2s,1s}) n_{1s}]} \,, \tag{28}$$

where

$$K = \lambda_\alpha^3 a/(8\pi da/dt) \,. \tag{29}$$

Finally, on substituting equation (28) into equation (24), we find that the net rate of recombination of the plasma is given by the formula

$$-\frac{d}{dt}\left(\frac{n_e}{n}\right) = \left[\frac{\alpha_c n_e^2}{n} - \beta_c \frac{n_{1s}}{n} e^{-(B_1-B_2)/kT}\right] C \,, \tag{30}$$

where

$$C = \frac{[1 + K\Lambda_{2s,1s} n_{1s}]}{[1 + K(\Lambda_{2s,1s} + \beta_c) n_{1s}]} \,. \tag{31}$$

It is of interest to compare the rate at which hydrogen is allowed to recombine because photons are being redshifted out of the Lyman-α line with the rate of recombinations via the two-quantum emission process. By equations (22), (27), and (29) this ratio is

$$\begin{aligned} \frac{\text{Lyman-}\alpha \text{ rate}}{\text{Two-quantum rate}} &= R/[\Lambda_{2s,1s}(n_{2s} - n_{1s}e^{-(B_1-B_2)/kT})] = 1/[K\Lambda_{2s,1s} n_{1s}] \\ &= 0.0022 \frac{n_{1s}}{n} T_4^{-3/2} \,. \end{aligned} \tag{32}$$

The last line is evaluated using equation (3). Since the coefficient is small, it is concluded that the bulk of the recombination would have to be accomplished via two-quantum emission, while the last remnants of the plasma (when T_4 is small enough) would recombine via the allowed $2p-1s$ transition. It will be noted that when the ratio (32) is small, allowed recombinations are unimportant, and in this limit the factor C (eq. [31]) multiplying the ordinary recombination rate in equation (30) reduces to $C \cong \Lambda_{2s,1s}/(\Lambda_{2s,1s} + \beta_c)$, which is just the probability that an excited atom can decay via two-quantum emission before it is photodissociated.

III. RESULTS

In the physical situation of interest, the rate of recombination of the plasma is given by the simple formula (30). The rate coefficient for recombination to excited states of hydrogen has been tabulated by Boardman (1964). In the temperature range of interest, the coefficient is given to adequate accuracy by the formula

$$\alpha_c = \frac{2.84 \times 10^{-13}}{T_4^{1/2}} \text{ cm}^3 \text{ sec}^{-1} . \tag{33}$$

Results of numerical integration of equation (30), using equations (2) and (33), are shown in Tables 1 and 2. The third column in Table 1 is the factor C (eq. [31]) by which the ordinary net rate of recombination is inhibited by the presence of the resonance-line radiation.

TABLE 1

RECOMBINATION OF THE PLASMA*

Temperature (° K)	Fractional Ionization x_e	C†
5000	0.996	0 00017
4500	.92	0.00018
4000	.40	0.00059
3500	.072	0 0027
3000	.0098	0 020
2500	00092	0.25
2000	.000123	0.96
1500	0.000053	1.00

* Flat cosmological model, $\rho_0 = 1.8 \times 10^{-29}$ gm cm^{-3} is the present mean mass density.

† Equation (31).

Next, it is straightforward to compute the effect of the recombination radiation on the shape of the high-energy tail of the Primeval Fireball spectrum. It should be emphasized that this perturbation is at best of theoretical interest because the perturbation becomes appreciable only shortward of 200-μ wavelength, where the brightness is some eight orders of magnitude below the peak of the thermal curve and some five orders of magnitude below the estimated brightness of the radiation from interstellar dust (Partridge and Peebles 1967). On using equation (12) for the ionizing radiation, and ignoring the very small second term in the denominator, we find that the ratio of the radiation intensity at the Lyman limit to the value which the intensity would have if the radiation spectrum were truly thermal is given by the expression

$$\frac{n_\nu(912)}{n_\nu(912)_{th}} = \frac{n_e^2}{n_{1s}} \left(\frac{n_{1s}}{n_e^2}\right)_{th} , \tag{34}$$

where the variables with subscript "*th*" refer to the values which the quantities would have at thermal equilibrium. This equation says that the intensity of the ionizing radiation is determined by the degree of ionization of the matter, and since there will be some residual ionization much greater than the thermal equilibrium value, there will be an anomalous excess on the high energy tail of the radiation spectrum. The effect this radiation would have (if it survived) on the presently observed radiation spectrum is shown as curve *2* in Figure 1.

The radiation intensity just longward of the Lyman-α resonance line is larger than the value for a strictly thermal spectrum by the factor

$$\frac{n_\nu(1215)}{n_\nu(1215)_{th}} = \mathfrak{N}_a e^{(B_1-B_2)/kT} . \tag{35}$$

This ratio is given by equation (28). The effect this would have on the presently observed radiation spectrum is shown as line *3* in Figure 1.

According to equation (32) the recombination is generally accomplished by two-quantum decay from the $2s$ state rather than by production of a Lyman-α photon. The

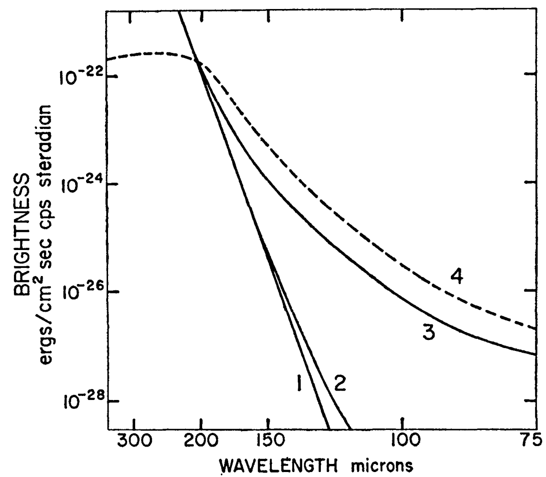

FIG. 1.—Spectrum of the Primeval Fireball radiation as it would be observed now is shown for a purely thermal spectrum (curve *1*), taking account of the redshifted ionizing recombination radiation (curve *2*), and taking account of the redshifted resonance line radiation (curve *3*). The dashed curve (*4*) shows the separate contribution due to two-quantum decay. The curves are computed for the flat cosmological model.

probability that one of the photons issuing from the two-quantum decay is in the frequency range ν to $\nu + d\nu$ is

$$dP = \phi\left(\frac{\nu}{\nu_a}\right)\frac{d\nu}{\nu_a} , \tag{36}$$

where the function ϕ has been tabulated (Spitzer and Greenstein 1951). According to equations (7) and (32), the contribution to the presently observed spectrum due to two-quantum decay is given by the formula

$$\left(\frac{\nu n_\nu}{n}\right)_0 = \int \frac{dn_{1s}}{n} \frac{K\Lambda_{2s,1s}n_{1s}}{(1 + K\Lambda_{2s,1s}n_{1s})} \frac{\lambda_a}{\lambda_{(t)}} \phi\left(\frac{\lambda_a}{\lambda_{(t)}}\right) . \tag{37}$$

In the integral, the wavelength $\lambda(t)$ varies in direct proportion to the expansion parameter. The spectrum (37) as it would be observed in the present Universe is shown as a dashed line in Figure 1.

Now it is necessary to check that the two-quantum decay process is not inhibited by the inverse process, absorption of the two-quantum recombination radiation to excite atoms from the ground state back to the $2s$ level. In this self-absorption process, when one of the photons is near the limiting wavelength λ_a, the other is supplied by the sea of soft primeval photons, and we can take it that this latter supply is little perturbed by the recombination radiation. Then by detailed balance we find that the probability per unit time that the photon with wavelength λ will be absorbed is

$$p = \frac{\Lambda_{2s,1s}}{8\pi} n_{1s}\lambda^2\lambda_a \phi\left(\frac{\lambda_a}{\lambda}\right) e^{-(B_1-B_2-hc/\lambda)/kT} . \tag{38}$$

The total depth for absorption of a photon by the inverse two-quantum process is the time integral of p, where the wavelength varies directly as the expansion parameter. As shown in Figure 1, the expected distribution of the radiation from two-quantum decay is roughly level longward of the peak at 250-μ wavelength (because ϕ is nearly flat over the range of its argument, 0–1). For the flat cosmological model the total depth (eq. [38]) for absorption of a photon whose wavelength now would be 250 μ is 0.05. The total depth decreases with increasing wavelength. The maximum total depth is 1.6 when the present wavelength of observation is 180 μ, and the depth decreases again at still shorter wavelengths. Since the absorption peak is fairly narrow and appears on the shoulder of the two-quantum emission spectrum, it is concluded that the plasma recombination is

TABLE 2

TEMPERATURE VARIATION OF THE MATTER

Radiation Temperature (° K)	Cosmological Model					
	ρ_0† $=1.8\times10^{-29}$ gm cm^{-3}		$\rho_0=2\ 7\times10^{-30}$ gm cm^{-3}		$\rho_0=4.5\times10^{-31}$ gm cm^{-3}	
	T_m	$10^5\ x_e$*	T_m	$10^5\ x_e$*	T_m	$10^5\ x_e$*
2000	1920	12.3	1980	33	2000	100
1500	1280	5 3	1440	14 3	1490	43
1000	680	3 2	870	9.0	970	27
500	197	2 2	310	6 4	430	19
200	33	1.7	61	5.2	110	16

* x_e is the fractional ionization of the hydrogen.

† ρ_0 is the present mean mass density in the Universe.

not seriously inhibited by this self-absorption effect. In the open cosmological models, the absorption probability is even less important because the density is lower.

Finally, we can obtain now a more detailed computation of the expected temperature variation of the matter. By equation (4) the energy equation for the matter can be written in the form

$$\frac{dT_m}{dt} = \frac{8\sigma a T^4}{3m_e c} x_e(T - T_m) - \frac{2T_m}{a}\frac{da}{dt} . \tag{39}$$

The second term on the right side of this equation describes the adiabatic cooling due to the assumed uniform expansion of the matter. The first term takes account of the heat transfer from the radiation to the electrons. The temperature variation is computed in the following way. During the epoch where the factor C (eq. [31]) differs appreciably from unity, the matter temperature is in any case very nearly equal to the radiation temperature, so we can greatly simplify the recombination theory by setting the two temperatures equal. Having determined the time variation of the ionization x_e, we can next integrate equation (39) to find a better approximation to the (very small) temperature difference between matter and radiation. Once the factor C has reached unity, it is straightforward to integrate equations (30) and (39), where now we can take the recombination rate to be fixed by the matter temperature. The results of this integration are shown in Table 2. In the flat model the temperature variation does approach the adiabatic law ($T_m \propto T^2$) in the temperature range shown, as if the matter temperature cooled adiabatically from being equal to the radiation temperature at 1200° K. In the lower density models, the matter temperature remains more nearly equal to the radiation temperature because the residual fractional ionization is higher.

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