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A NEW DETERMINATION OF THE HUBBLE CONSTANT FROM GLOBULAR CLUSTERS IN M87

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ABSTRACT

An apparent blue distance modulus of $(m - M)_{AB} = 31.1$ is derived for the Virgo Cluster from Racine's measurement of $B(\text{first}) = 21.3$ for the brightest globular cluster in M87, using a calibration of $M_B(\text{first}) = -9.83$ for cluster B282 in M31. This assumes that the brightest globular clusters in the two galaxies have the same absolute magnitude. The assumption provides an *upper limit* to H in the present context. For this distance modulus, the absolute luminosity of NGC 4472—the first-ranked E galaxy in the Virgo Cluster—is $M_B = -21.68$.

A new redshift–apparent magnitude relation (the Hubble diagram) for first-ranked cluster members (Fig. 1) shows that $M_B(\text{first})$ has a remarkably small dispersion of ± 0.3 mag in a sample of forty-one clusters. Using M_B from NGC 4472 and reading the Hubble diagram at $cz = 10^4$ km sec^{-1}, which is beyond the local anisotropy in the velocity field, we find that $H = 75.3_{-15}^{+19}$ km sec^{-1} Mpc^{-1}, using the precepts in the text. Systematic errors are discussed. It seems possible that H could be as small as 50 km sec^{-1} Mpc^{-1} ($H^{-1} = 19 \times 10^9$ years) with the present data.

The local anisotropy in the velocity field at the Virgo Cluster is estimated to be $H_\infty / H_{\text{VC}} = 1.17 \pm 0.09$.

I. INTRODUCTION

The value of the Hubble constant is not well known at present. Work by Holmberg (1958), van den Bergh (1960), Sersic (1960), Sandage (1958, 1962), and others has shown that H probably lies in the range $125 > H > 50$ km sec^{-1} Mpc^{-1}, but a more precise value is not available.

The determination of H is difficult. Two requirements exist: (1) Accurate distances to galaxies of known redshifts are needed. (2) The redshifts must be large compared to the random virial velocities. These boundary conditions restrict the range of moduli within which H can be measured to $32 \gtrsim m - M \gtrsim 30$. At $m - M \simeq 30$, the cosmological redshift is about 750 km sec^{-1}, which is only a few times the mean random motion. Beyond $m - M \simeq 32$, precision distance indicators fade below plate limit.

A further complication arises from the local anisotropy of the velocity field (de Vaucouleurs 1958, 1966; Kristian 1967) for redshifts less than $\sim$4000 km sec^{-1}. Measured radial velocities differ from those for a pure Hubble flow because of this local perturbation, called the "shear field" by Kristian and Sachs (1966). One method of procedure is to map the velocity field for nearby galaxies in order to separate the cosmological and shear components. Kristian and Sandage are attempting to do this using angular sizes of H II regions for galaxies with redshifts less than 2000 km sec^{-1}, but the data are not yet sufficient for a solution.

II. A NEW METHOD

A second procedure has recently become available from Racine's (1968) measurement of the luminosity function of 2000 globular clusters in the E galaxy M87 in the Virgo Cluster. On the assumption that the brightest globular clusters have the same absolute luminosity in galaxies which have sufficiently large cluster population, the distance to M87, and hence to the E cloud of the Virgo Cluster, can be found. This distance can then be used to calibrate the absolute luminosity of the first-ranked E galaxy in this cluster.

It has previously been shown (Hubble 1936; Humason 1936; Humason, Mayall, and Sandage 1956, hereinafter referred to as "HMS"; Sandage 1968) that the brightest E galaxy in clusters has a remarkably small dispersion in absolute luminosity. The calibration of M_B(brightest) then permits calculation of H by entering the observed redshift–apparent magnitude relation for the first-ranked cluster galaxy at redshifts larger than $\sim$4000 km sec^{-1}, which is the outer boundary of the local anisotropy. It is important to note that the velocity of the Virgo Cluster is not required, and the effects of the local shear field are thereby circumvented.

III. M_B FOR THE BRIGHTEST VIRGO CLUSTER E GALAXY

a) *Distance to M87*

Racine's photometry of the globular clusters in M87 shows that the luminosity function at the bright end is steep and begins at $P = 21.2$ mag. Converting this to the B system of Johnson and Morgan by $B \simeq P + 0.1$, we find that $B = 21.3$ for the brightest globular cluster in M87.

The absolute luminosity of the brightest globular cluster, M_B(first), can be estimated from the data from M31 and, less reliably, from the galactic system. Photoelectric photometry of the clusters in M31 by Kron and Mayall (1960), Hiltner (1960), and Kinman (1963) have been summarized by Kinman (1963). There is a well-defined upper luminosity limit at $V = 14.1$ if the cluster M II is excluded. This cluster is difficult to measure because of the close proximity of two moderately bright stars (see Pl. II of Mayall and Eggen 1953). Until the question of contamination can be re-examined, M II is excluded from further discussion here, and we adopt cluster B282 in M31 as the brightest, at $V = 14.13$ and $B = 15.01$. Clusters H12 and H42 are near this upper limit, at $V = 14.23$, $B = 15.04$, and $V = 14.22$, $B = 15.25$, respectively. We choose to consider here only those clusters in M31 where absorption within this galaxy may be negligible. We have not considered clusters where M_B appears to be brighter than that for B282 after uncertain absorption corrections are applied (Kinman 1963, group C).

The apparent blue modulus of M31 is taken to be $(m - M)_{AB} = 24.84$, obtained from the photometry of Cepheids by Baade and Swope (1963) in their outlying field IV and using the calibration of the (P,L)-relation given elsewhere (Sandage and Tammann 1968). This calibration is based on Cepheids in open clusters in the galactic system, the absolute luminosities of which rest ultimately on photometric parallaxes via the distance to the Hyades (Wayman, Symms, and Blackwell 1965). The effect of a systematic error in this distance, following Hodge and Wallerstein (1966), is discussed later. The adopted data then give $M_B = -9.83$ for B282 in M31.

How constant M_B(first) may be from galaxy to galaxy is not known, either empirically or theoretically. On statistical grounds alone the assumption is likely to be invalid for galaxies with few globular clusters. But clusters in galaxies such as M31 or the galactic system may form an adequate sample. Mindful that future work may clarify the problem, we can proceed quantitatively by assuming that M_B(first) is a stable statistic independent of the total population N as long as N is large enough. It should be noted, as discussed later, that this assumption provides an *upper limit* to *H*. *If* some form of the Scott effect (1957) applies to globular clusters, it can only decrease the value of H in the present context, since M_B(first) in M87 with 2000 clusters would be brighter than M_B(first) in M31 with $\sim$250 clusters.

Partial justification for the assumption of constant M_B may be found in the nearly vertical asymptote of Racine's luminosity function for clusters in M87. Another check comes from comparing M_B(first) in the galactic system with B282 in M31. Gascoigne and Burr's (1956) photometry of ω Cen gives the total apparent luminosity as $P_t = 4.25$, or $B_t = 4.35$. The horizontal branch occurs at $V = 14.5$ (*Roy. Obs. Ann.*, No. 2, 1966). The middle of the RR Lyrae domain is at $B = 14.85$ (Saunders 1963), which gives an apparent blue modulus $(m - M)_{AB} = 14.05$ if M_B(RR) $= +0.8$ (i.e., $M_V = +0.5$, $\langle B - V \rangle = +0.3$). Hence $M_B = -9.7$ for ω Cen, which is the brightest known globular cluster in the galactic system.[1] M_B for ω Cen compares well with $M_B = -9.83$ for B282 in M31, but since the value is based on the somewhat less reliable route via M_B for the RR Lyrae stars rather than through the Cepheid (P,L)-relation, we adopt $M_B = -9.83$ in the following discussion.

Combining $M_B = -9.83$ with Racine's value $B = 21.3$ gives $(m - M)_{AB} = 31.1$ for M87. The possible range of uncertainty is discussed later. The new modulus may be compared with an older value of $(m - M)_{AB} \simeq 30.8$, which follows from Baum's (1955) photoelectric data that gave a 6.0-mag difference between the first-ranked globular clusters in M31 and M87. Sandage (1958) obtained $(m - M)_{AB} \simeq 30.8$ from other considerations.

b) *The First-ranked Cluster Galaxy*

The brightest E galaxy in the Virgo Cluster is NGC 4472 (Holmberg 1958; de Vaucouleurs 1961*b*). These authors give a mean integrated visual magnitude of $V_t = 8.45$. The color index $B - V = +0.97$ (de Vaucouleurs 1961*a*) gives $B_t(4472) = 9.42$. If $(m - M)_{AB} = 31.1$, then $M_B(4472) = -21.68$, which is independent of the absorption between us and the Virgo Cluster.

IV. THE HUBBLE CONSTANT

Figure 1 shows the most recent data for the redshift–apparent magnitude relation for first-ranked cluster galaxies in forty-one clusters. All clusters in Humason's Table III of the Humason-Mayall catalogue (HMS 1956) are plotted. The redshift of the most distant cluster was taken from Minkowski (1960); the next two most distant, from Baum (1962). Also included are new clusters observed in a current program on radio sources which will be discussed elsewhere (Sandage 1969).

Dots represent galaxies that are radio-quiet to 9 flux units at 178 MHz, crosses are radio sources in the 3CR (Bennett 1962) that are brightest cluster members, and open triangles are Baum's data (1962) transformed to the zero point of the present magnitude system. The abscissa is the photoelectrically measured V magnitude corrected for (1) aperture effect to an isophote of about 25 mag per square second by a method which will be given elsewhere, (2) K-dimming taken from tables given by Oke and Sandage (1968), and (3) galactic absorption using $A_V = 0.18$ cosec $(b - 1)$.

The Virgo Cluster is represented by four points (two dots and two triangles) corresponding to Baum and Holmberg's photometry and using $cz = 1136$ km sec^{-1} for the mean of all galaxies in the neighborhood of the cluster (HMS 1956, Table II), or $cz = 950$ km sec^{-1} for the E cloud alone (de Vaucouleurs 1961*b*). The Fornax Cluster is represented by two points at $cz = 1526$ km sec^{-1} and $cz = 1437$ km sec^{-1}, depending on whether NGC 1375 is included or excluded from the mean (HMS 1956, Table I; Mayall and de Vaucouleurs 1962). The box in the lower left-hand corner is the range over which Hubble (1929) first formulated the velocity-distance relation.

A listing of the data and a discussion of Figure 1 are in preparation. For the present purpose it is sufficient to note that the line has been drawn with the theoretical slope $dV_c/d \log z = 5$ for $z \rightarrow 0$ and has been fitted to the data only in zero point. The equation of the line is $V_c = 5 \log cz - 6.78$.

The most striking feature of Figure 1 is the small scatter of the first-ranked cluster galaxies about this line. The formal dispersion, read as a magnitude difference, is $\sigma \simeq \pm 0.3$ mag. The size of the dispersion shows that the absolute luminosity of the brightest galaxy in clusters with $N \gtrsim 30$ members is a remarkably stable statistic.

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[1] According to Gascoigne and Burr's photometry, combined with a modulus of $(m - M)_{AB} = 13.50$ from Tifft's photometry (1963), 47 Tuc has $M_V = -8.70$, which does not rival ω Cen.

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We can now read V_c from Figure 1 at a redshift of, say, 10^4 km $\sec^{-1}$, which is well beyond the local anisotropy. The equation of the line gives $V_c = 13.22$ at $\log cz = 4.0$. Because $\langle B - V \rangle = +0.97$ for E galaxies corrected for K-effect (Oke and Sandage 1968), then $B_c = 14.19$. If we assume that $\langle M_B \rangle = -21.68$ from our calibration, then the apparent blue modulus for a cluster at $\log cz = 4.0$ at the galactic pole is $(m - M)_{AB} = 35.87$.

The true modulus must now be obtained. This requires correction for the galactic half-thickness layer.[2] Assuming $A_B(b = 90°) = 0.25$ mag gives $(m - M)_T = 35.62$ for a distance of 1.33×10^2 Mpc. No correction for different cosmological models is necessary at this redshift.

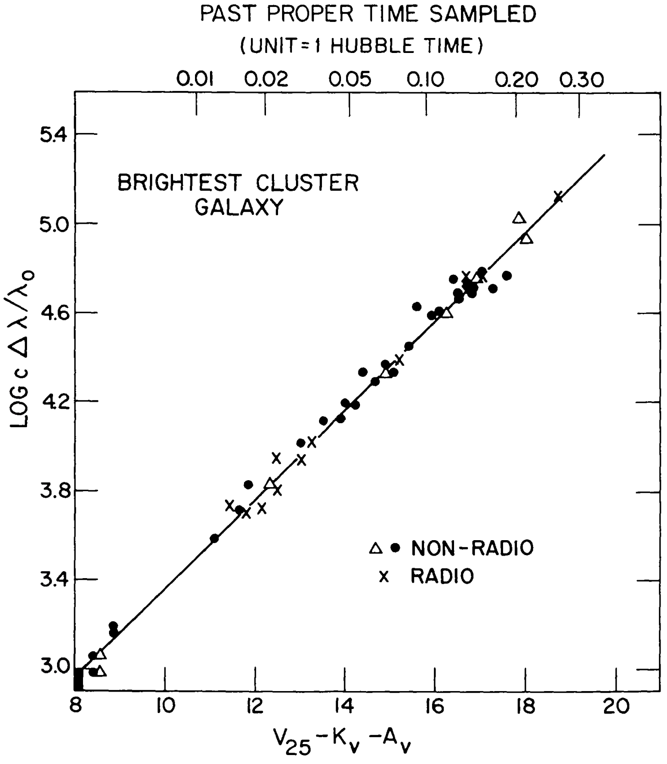

FIG. 1.—Hubble diagram for first-ranked cluster galacies in forty-one clusters. Abscissa is the photoelectric V magnitude corrected for aperture effect, K-dimming, and galactic absorption. Dots represent galaxies that are radio-quiet to 9 flux units at 178 MHz; crosses, radio sources from the 3CR catalogue that are first-ranked cluster members; triangles, Baum's data transformed to the V-magnitude system of the dots and crosses. Ordinate is redshift, corrected for galactic rotation.

The Hubble constant is then $H = cz/D = 75.3$ km $\sec^{-1}$ Mpc^{-1}, or $H^{-1} = 12.9 \times 10^9$ years.

V. ESTIMATE OF ERRORS

The value is uncertain because of three random and four systematic effects. The random errors with their estimated uncertainties are (1) an uncertainty of perhaps ± 0.3 mag in Racine's value of $B = 21.3$ for the brightest globular cluster in M87, due to a combination of transfer errors to SA 57 and the difficulty of photometry on the M87 background; (2) a possible error of ± 0.2 mag in V_t for NGC 4472; (3) $\sigma = \pm 0.3$ mag

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[2] The absorption correction A_V in Fig. 1 was only to the pole and not to outside the galactic system, since A_V was adopted to be proportional to cosec $b - 1$.

for the brightest cluster galaxy about the mean line of Figure 1. If these combine in the usual way, the resulting uncertainty in the distance modulus would have a formal value of about ± 0.5 mag. Neglecting the systematic errors for the moment, these random effects give $H = 75_{-15}^{+19}$ km sec^{-1} Mpc^{-1}, or $H^{-1} = 13_{-2.7}^{+3.7} \times 10^9$ years.

The systematic errors concern (1) the adopted value of M_B for the brightest globular cluster, (2) the value of the galactic half-thickness absorption, and (3) a possible difference in the distance to M87 and NGC 4472 because of the finite angular diameter of the E cloud of the Virgo Cluster. NGC 4472 lies near the edge of the projected distribution on the plane of the sky (de Vaucouleurs 1961*b*, Fig. 2) and therefore is at the distance of the core. The uncertainty arises because M87, seen projected on the core, could be located anywhere along the line of sight through the core. The maximum difference in distance, if M87 is at the periphery of the three-dimensional distribution, whose angular radius is about 5°, is $\delta r/r = \pm 0.09$, which introduces an uncertainty of 9 per cent in the value of H. However, due to the strong density gradient of core members, M87 may be near the cluster center in space, and the back-to-front effect has been neglected.

The systematic effect in M_B is composed of three parts.

A. M_B(first) may be a function of the total number of globular clusters in a given galaxy, becoming brighter for a larger sample. This would make M_B(first) brighter by ΔM for the first-ranked cluster in M87 (2000 clusters), compared with M31 ($\sim$250 clusters), and will reduce H by a factor of $1 + 0.46\Delta M$ for small ΔM.

B. If highly absorbed clusters in M31 have M_B brighter than it is for B282, H will again be decreased.

C. If the modulus of the Hyades must be increased by ΔM_H (Hodge and Wallerstein 1966), the distance to M31 via the Cepheids increases, and M_B for cluster B282 becomes brighter by ΔM_H. This again reduces H by the factor given above.

If the galactic half-thickness absorption is, say, 0.50 mag rather than 0.25 mag in B, all distances obtained by the use of apparent-modulus methods are decreased, and H becomes larger by a factor of 1.12.

The value of H would be 55_{-11}^{+14} ($H^{-1} = 17.7 \times 10^9$ years) if M_B(brightest globular cluster) were, say, $M_B = -10.5$, and we still adopt $A_B(\frac{1}{2}) = 0.25$ mag. H would be 12 per cent higher if $A_B = 0.50$ mag.

VI. THE LOCAL ANISOTROPY AT THE VIRGO CLUSTER

Adopting a consistent set of calibrations, one can obtain the deviation of the Virgo Cluster from the pure cosmological expansion field by finding H_{VC} from the cluster data alone and comparing it with H_∞ derived above. If $(m - M)_{AB} = 31.1$ and $(m - M)_T = 30.85$, then $D = 14.8$ Mpc for the Virgo Cluster. De Vaucouleurs (1958) gives $cz = 950 \pm 70$ km sec^{-1} for the redshift of the Virgo E cloud. Hence, $H_{\mathrm{VC}} = 64 \pm 5$ km sec^{-1} Mpc^{-1}, where the quoted error is due to the uncertainty in cz alone. Therefore, the ratio of the asymptotic value of H to the local value is

$$H_\infty / H_{\mathrm{VC}} = 75/64 = 1.17 \pm 0.09 \, .$$

The actual error is, of course, larger because of a possible difference of M_B(NGC 4472) from $\langle M_B(\text{first}) \rangle$.

The present determination can be compared with de Vaucouleurs's (1958) value of 1.58 based on an explicit model for the local anisotropy. The smallness of our value can be seen graphically from the smallness of the deviation of the Virgo Cluster from the line in Figure 1. It should be emphasized again that part of the observed deviation could be due to a difference of absolute magnitude of NGC 4472 from the mean of the other first-ranked cluster galaxies, rather than to a difference, Δz, due to the local gravitational perturbation over a scale of 10 Mpc. With this in mind, our present data provide no clear evidence that a significant anisotropy occurs at the Virgo Cluster, but this does not change the importance of the de Vaucouleurs discovery (1958, 1966) for other directions in the sky.

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