

HEATING OF H I REGIONS BY ENERGETIC PARTICLES

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ABSTRACT

The heating of interstellar H I regions by energetic particles, originally analyzed by Hayakawa, Nishimura, and Takayanagi, has been reconsidered. A fourfold increase in the relative abundance of carbon and inclusion of C⁺ and O excitation by H atoms increases the loss rate per cubic centimeter by more than an order of magnitude, decreasing the temperatures appreciably. Consideration of the ionization and excitation produced by the secondary electrons which result from ionizing collisions of energetic particles with H atoms indicates that 3.4 eV of kinetic energy are available for heating the gas for each hydrogen atom ionized. Temperature and ionization levels are computed for various values of ζ_f , the rate coefficient for ionization of H by energetic particles, including ionization produced by the secondaries.

The minimum value of ζ_f consistent with the cosmic radiation observed at the Earth is 6.8×10^{-18} sec⁻¹; the corresponding equilibrium temperatures are less than 25° K for n_H greater than 0.1 cm⁻³, much less than the observed temperatures. A much higher value of ζ_f is obtained if the atoms in Type I supernova shells, which have an energy of 2 MeV per nucleon at a velocity of 20000 km sec⁻¹, are assumed to permeate the Galaxy; the range of such protons in the galactic plane is about 700 pc. If at most one-third the shell energy of 10⁵¹ ergs is available in this form, a galactic frequency of one Type I supernova per 100 years gives an upper limit for ζ_f equal to 1.2×10^{-15} sec⁻¹; the computed temperatures range between about 30° and 120° K as n_H decreases from 10 to 0.1 cm⁻³. These theoretical results are somewhat uncertain, but an increase of temperature with decreasing density seems consistent with the 21-cm data. The computed mean electron density in H I regions corresponding to the higher value for ζ_f is about 0.01 cm⁻³. Deviations from equipartition are generally small, since collisions between electrons and positive ions and between positive ions and H atoms couple closely the kinetic temperature of all these components.

I. INTRODUCTION

The possible effect of energetic interstellar particles in heating H I regions was discussed in some detail by Hayakawa, Nishimura, and Takayanagi (1961; subsequently referred to as "HNT"), who determined the flux of low-energy cosmic rays required to account for a kinetic temperature of 125° K in clouds of neutral hydrogen. No origin was suggested for the relatively large flux found for such particles. Since the observational and theoretical bases for this analysis have substantially changed in recent years, the subject is considered anew in the present paper.

From an observational standpoint, accumulating evidence suggests (Spitzer 1968a) that the temperature of at least the denser clouds is substantially less than 125° K, perhaps in the neighborhood of about 60° K. In particular, a comparison of the 21-cm-line profiles seen in emission and absorption (Clark 1965) suggests that a substantial difference in kinetic temperature exists between different clouds, with the strongly absorbing clouds at a temperature perhaps only one-tenth as great as in the clouds which are seen mainly in emission. Thus, the gas producing the bulk of the emission in 21 cm might be at a temperature of at least 1000° K, according to Clark, with the densest clouds at a temperature somewhat less than 100° K. It is of interest to investigate what distribution of energetic suprathermal particles, with kinetic energies much larger than those of the gas, is required to explain the observations. Possible sources of such particles are also of great interest.

In a steady state, the temperature of the interstellar gas is determined by the condition that the losses of kinetic energy by the gas are just equal to the gains. Hence we may write

$$\Lambda \equiv \sum_{\zeta, n} \Lambda_{\zeta n} = \sum_{\zeta, n} \Gamma_{\zeta n} \equiv \Gamma, \quad (1)$$

where $\Lambda_{\zeta\eta}$ is the loss of kinetic energy per cubic centimeter resulting from interactions between components of types ζ and η in the interstellar gas, while $\Gamma_{\zeta\eta}$ represents the corresponding gain. Subscripts H, e , i , and R will denote H atoms, electrons, positive ions, and cosmic rays in the following discussion. Since the heating and cooling rates may depend not only on the kinetic temperatures but also on the electron density, the ionization level must also be considered. If a steady state is again assumed, the equilibrium between atoms of an element X in the stages of ionization r and $r + 1$ is given by

$$n(X_r)(\beta_{r1f} + \zeta_{r1f}) = n_e n(X_{r+1})a_r, \quad (2)$$

where β_{r1f} is the rate coefficient for ionization of an atom in the stage of ionization r and in the ground level by a photon, and ζ_{r1f} is the corresponding rate for ionization by energetic particles, while a_r is the radiative recombination coefficient. Equation (2) is based on the assumption, generally valid for interstellar space, that ionization transitions from excited levels up to the free state may be ignored. Doubly ionized atoms are not considered in the present discussion, and the quantities β_{r1f} , ζ_{r1f} , and a_r will be denoted here more simply by β_f , ζ_f , and a . For neutral and ionized hydrogen we shall denote $n(X_r)$ by $n(\text{H I})$ and $n(\text{H II})$, respectively, with their sum denoted by n_{H} , with all these particle densities defined per cubic centimeter.

In the first section below, the quantities occurring in equations (1) and (2) will be evaluated for reactions involving energetic particles, and the possible origin of such particles in supernova shells will be discussed. Reactions between the thermal particles in the interstellar gas, primarily H atoms, electrons, positive ions, and neutral heavy atoms, are discussed in a subsequent section, and, in particular, deviations from equipartition of kinetic energy between different components of the gas are analyzed. Computed temperatures and densities together with general conclusions are given in a final section.

II. IONIZATION AND HEATING BY ENERGETIC PARTICLES

a) Ionization Rate

For a hydrogen atom in the ground state, the cross-section σ_i for ionization by an energetic charged particle whose velocity is β times c (the velocity of light) is given by the familiar formula (Bethe 1933, eq. [55.13])

$$\sigma_i = \frac{1.23 \times 10^{-20} Z^2}{\beta^2} \left(6.20 + \log \frac{\beta^2}{1 - \beta^2} - 0.43\beta^2 \right), \quad (3)$$

where Z is the atomic number of the energetic ion, and the logarithm is to base 10. Equation (3) is valid for positive ions with an energy in excess of about 0.3 MeV per nucleon, corresponding to a value of about 2.6×10^{-2} for β .

This ionization process produces free electrons, which we shall call "first-generation secondary electrons," with a wide distribution of energy. The more energetic of these electrons can produce additional ion pairs in the interstellar gas, which must be taken into account in the computation of the ionization rate ζ_f for H atoms; we shall use the term "secondaries" to refer to all electrons produced, including higher generations as well as the first. According to Dalgarno and Griffing (1958), the total number of secondary electrons produced in hydrogen is $\frac{5}{3}$ times the number of first-generation secondaries produced. Hence we have

$$\zeta_f = \frac{5}{3} \int 4\pi j(E) \sigma_i dE, \quad (4)$$

where $j(E)dE$ is the differential spectrum, defined here as the intensity of energetic particles per square centimeter per second per unit of solid angle with a kinetic energy within the range dE ; σ_i is given in equation (3). The factor $\frac{5}{3}$ in equation (4) was not included in the ionization-rate computations by HNT.

The intensity of cosmic rays is relatively well known at energies exceeding a few GeV but becomes increasingly uncertain at lower energies; the solar wind, with its trapped magnetic field, tends to sweep the lower-energy particles back out into interstellar space, and the correction that must be applied to $j_E(E)$ —the intensity measured at Earth—to yield the true $j(E)$ outside the planetary system becomes large and uncertain at low E . However, a lower limit for ζ_f may be found if $\log j(E)/j_E(E)$ is assumed to equal twice $\log j_M(E)/j_E(E)$, where $j_M(E)$ is the intensity measured near the planet Mars. This lower limit on ζ_f , based primarily on cosmic-ray fluxes measured at Earth, we denote by ζ_E .

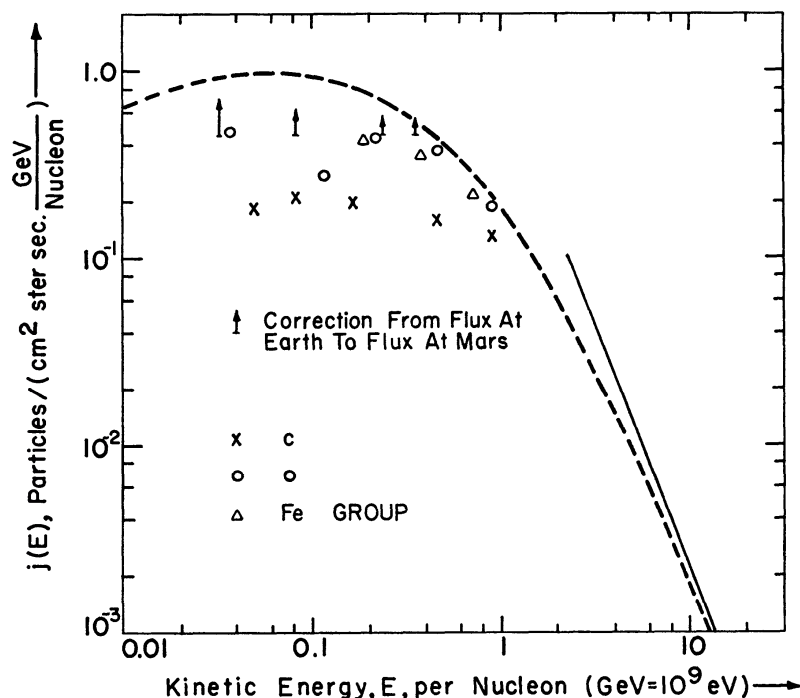


FIG. 1—Cosmic-ray intensities. Solid line shows the differential proton spectrum above 10 GeV (10^{10} eV), while plotted points represent measured values for heavy particles above the atmosphere, converted to equivalent proton fluxes. The length of each arrow represents the ratio of fluxes at Mars and at the Earth. Dashed curve gives the adopted minimum flux in interstellar space.

Measures of $j_E(E)$ for nuclei of various charges down to energies as low as 20 MeV per nucleon have been made by Comstock, Fan, and Simpson (1966), while values of $\log j_M(E)/j_E(E)$ have been reported by O'Gallagher and Simpson (1967), based on direct measures at energies of about 200 and 360 MeV and found by an indirect method at lower energies, based on temporal variations observed at Earth. The measured values of j_E , together with the corrections, are shown in Figure 1, converted to proton densities using the relative abundances given by Webber (1967), together with a hydrogen/helium ratio of 10 at each energy. The straight line shows the proton spectrum obtained above 10^{10} eV (Webber 1967), while the dashed curve is the lower limit assumed here, given by the analytical form

$$j(E) = \frac{0.90}{(0.85 + E_G)^{2.6}} \frac{1}{(1 + 0.01/E_G)} \text{ particles cm}^{-2} \text{ sec}^{-1} \text{ sterad}^{-1}, \quad (5)$$

where E_G is the kinetic energy in units of 10^9 eV.

If equations (3) and (5) are substituted into equation (4) and the integral is multiplied by an additional factor 1.89 to allow for ionization by heavy ions, based on the abundances given by Webber (1967), we obtain for ζ_E

$$\zeta_E = 6.8 \times 10^{-18} \text{ sec}^{-1}. \quad (6)$$

If the spectrum had been assumed flat below about 100 MeV per nucleon, decreasing only at much lower energies, the order of magnitude of ζ_f would not be affected. However, if the straight-line spectrum observed at relativistic energies were extrapolated down to 10 MeV, ζ_f would be increased by some two orders of magnitude; in this way HNT obtain a value of $1.0 \times 10^{-15} \text{ sec}^{-1}$ for ζ_f . Thanks to measures of $j_E(E)$ at low energy with satellites and space probes, the lower limit on ζ_f in equation (6) is more than two orders of magnitude greater than the value used in interstellar-heating computations some 20 years ago (Spitzer 1948a).

There is also a small contribution to ζ_f resulting from ionization by the secondary products of nuclear reactions produced in nuclear collisions of the primary cosmic rays. This process is discussed below and shown to be negligible.

b) Heating Rate

If each secondary electron contributes an energy E_h to heating the gas—i.e., to increasing its kinetic energy—the heating rate Γ_{HR} resulting from collisions between cosmic rays and H atoms is given by

$$\Gamma_{HR} = n_H \zeta_f \langle E_h \rangle, \quad (7)$$

where the brackets denote an average value over all free electrons produced. The kinetic energy gained by atoms and heavy ions interacting directly with energetic particles is some three orders of magnitude less than the energy gained by electrons and may be neglected. The mean kinetic energy of the first-generation electrons liberated by primary collisions of heavy ions with H atoms varies between about 35 and 50 eV (Bethe 1933) as the velocity of the primary particles varies from 0.05 to 0.95 c . However, only a small fraction of this energy is available for increasing the kinetic energy of the gas, since most of the energy goes into ionization and excitation. Only by elastic collisions with electrons or with atoms can kinetic energy of the secondary electrons be made available for the gas generally. The energy lost to hydrogen atoms in elastic collisions is generally several orders of magnitude less than the energy lost in inelastic collisions, for free electrons with an energy slightly exceeding 10.2 eV—the first excitation energy of hydrogen. The energy transferred in an elastic collision is reduced by the very low mass ratio of the electron to the atom, a reduction only partly offset by the increased cross-section for elastic collisions as compared with the inelastic ones.

Thermal electrons in the gas can provide an important energy sink for the secondaries. However, from the ionization and excitation cross-sections referred to below and the rate of energy loss to charged particles given by Spitzer (1962), it may be shown that the energy lost in elastic collisions with electrons will be less than the corresponding loss in inelastic collisions with H atoms provided that n_e/n_H , the ratio of particle densities, is less than about 10^{-2} ; this result refers to secondary electrons moving with kinetic energies between about 15 and 30 eV, the range of major interest. If ionization of the interstellar hydrogen by cosmic rays is not very important, n_e/n_H is 5×10^{-4} , and energy loss from the secondaries to the thermal electrons may be neglected. For this reason the computations in this paper have all been based on the assumption that the secondary electrons lose energy only by inelastic encounters until their kinetic energy falls below $3E_1/4$, or 10.2 eV, the excitation energy of the first excited level in hydrogen; E_1 denotes the binding energy of the H atom. As we shall see below, under some conditions the ionization

by energetic particles can increase n_e/n_H substantially above 10^{-2} , in which case the energy available for heating the gas will be greater than the values assumed here.

At kinetic energies below $3E_1/4$ the secondaries will, in general, lose their kinetic energy by elastic collisions with the thermal electrons; the energy lost in elastic collisions with H atoms is less by at least one order of magnitude, provided that n_e/n_H is 5×10^{-4} or greater. Some inelastic processes will also occur but these are generally unimportant; for example, for 5 eV electrons the energy-loss rate by the excitation of the O I level at 1.97 eV is about $\frac{1}{40}$ the elastic-energy-transfer rate to H atoms and is an even smaller fraction of the energy-transfer rate to the thermal electrons.

On the basis of the assumptions described above, the heating energy, E_h , is the energy of the secondary electron when it first enters the energy region below $3E_1/4$ and can no longer suffer an inelastic collision with hydrogen atoms. If the distribution of electron energies when the secondaries first arrived in the range below $3E_1/4$ (as a result of some previous inelastic collision) were uniform with energy, then $\langle E_h \rangle$ in equation (7) would equal $3E_1/8$, or 5.1 eV. In fact $\langle E_h \rangle$ will be somewhat smaller, since the distribution of secondary electrons when first produced tends to be peaked at the lower energies. If we consider only those first-generation secondaries with initial energies between 0 and $3E_1/4$, the mean kinetic energy of this group—which includes 67 per cent of all the first-generation secondary electrons—is $0.25E_1$, or 3.4 eV.

We must also consider the more energetic first-generation secondaries, which are responsible for all the second and higher generation electrons produced. As an electron of initial kinetic energy, E , slows down by inelastic collision, the number of additional electrons which it will produce, on the average, may be denoted by ϕ_e . The total mean energy available for heating the gas from this initial electron will be denoted by $(1 + \phi_e)E_h$; E_h is again the mean heating energy available per electron, including those of higher generations produced by the initial electron being considered. If the number of first-generation secondary electrons with an initial energy in the range dE equals $n_s(E)dE$, then $\langle E_h \rangle$ is given by

$$\langle E_h \rangle = \frac{\int_0^\infty (1 + \phi_e) E_h n_s(E) dE}{\int_0^\infty (1 + \phi_e) n_s(E) dE}. \quad (8)$$

If E does not much exceed E_1 , we can evaluate E_h and ϕ_e , which are both functions of E . We give the functional form of these quantities for different ranges of E ; throughout each range the same set of physical processes is possible.

1. $E < \frac{3}{4}E_1$.—In this range, as we have already seen, the full energy is available for heating; ϕ_e is zero, and E_h equals E .

2. $\frac{3}{4}E_1 < E < E_1$.—The only inelastic process is excitation; ϕ_e again vanishes and

$$E_h = E - \frac{3}{4}E_1. \quad (9)$$

3. $E_1 < E < \frac{3}{2}E_1$.—Either one ionization or one excitation is possible, with a relative probability proportional to the ionization and excitation cross-sections, σ_i and σ_e . Hence we have, on the average,

$$E_h(1 + \phi_e) = \frac{\sigma_e}{\sigma_e + \sigma_i} (E - \frac{3}{4}E_1) + \frac{\sigma_i}{\sigma_e + \sigma_i} (E - E_1) \quad (10)$$

and

$$\phi_e(E) = \frac{\sigma_i}{\sigma_e + \sigma_i}. \quad (11)$$

Both σ_i and σ_e are functions of E within this range.

4. $\frac{3}{2}E_1 < E < \frac{7}{4}E_1$.—In this range, only one ionization is possible, but, if one excitation occurs instead, then another excitation must also occur. As a result, equation (10) is again applicable if $3E_1/4$ in the first term on the right-hand side is changed to $3E_1/2$. Equation (11) is again valid.

These results for E_h and ϕ_e are shown in Figure 2; the values of σ_i used were taken from Kieffer and Dunn (1966), while those for σ_e were taken by adding to the values for the $1s-2p$ transition measured by Fite, Stebbings, and Brackman (1959) an average of the values for the $1s-2s$ transition measured by Stebbings *et al.* (1960) and by Lichten and Schultz (1959). For values of E above $7E_1/4$, the energy distribution of the next generation of secondary electrons must be known to indicate what inelastic collisions are possible after ionization has occurred. In the range between $1.75E_1$ and $2E_1$, for example, an electron can first produce a secondary and will still have enough energy left over for

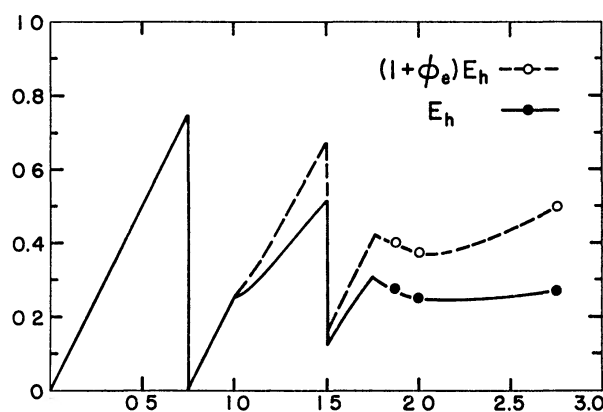


FIG. 2.—Heating energy available per electron. Dashed line gives values of $(1 + \phi_e)E_h$ (vertical scale), the total heating energy available from a secondary electron of initial energy E (horizontal scale); number of additional electrons produced by the secondary electron is ϕ_e . Solid line gives E_h , the mean heating energy available per free electron. For energies above $7E_1/4$, values of E_h were computed only at the few points indicated, and a smoothed curve has been interpolated.

excitation if the secondary has very little kinetic energy. The distribution of kinetic energies for secondaries produced by 25 V electrons is not well known, but by extrapolating to lower energies the data given by Mott and Massey (1965; see Fig. 72, p. 492) for somewhat higher energies, E_h and ϕ_e have been estimated at $15E_1/8$ and also at several higher energies, and these points are plotted in Figure 2.

If $n_e(E)$ is assumed constant in equation (8), evaluation of the integrals from zero to $7E_1/4$ gives $\langle E_h \rangle$ equal to $0.32E_1$, or 4.3 eV; if the curves are extrapolated up to $11E_1/4$, as shown in Figure 2, $\langle E_h \rangle$ becomes 4.0 eV. If a weighted average is taken, according to the initial distribution of energies of the first-generation electrons produced by energetic protons, the average up to $11E_1/4$ is 3.4 eV. While the weighting does not have a very large effect, the weighted values are probably closer to the truth, and we shall adopt, accordingly,

$$\langle E_h \rangle = 0.25E_1 = 3.4 \text{ eV} . \quad (12)$$

The efficiency, ϵ_R , of heating by energetic particles equals the ratio of $\langle E_h \rangle$ to the known total energy loss of 36 eV per free electron produced (Dalgarno and Griffing 1958); hence we have

$$\epsilon_R = 0.094 . \quad (13)$$

As noted earlier, when n_e/n_H exceeds 10^{-2} , the secondaries will lose a significant fraction of their energy above $3E_1/4$, and ϵ_R will rise appreciably above the value in equation (13); however, ϵ_R cannot exceed 0.6, about the fraction of the energy lost by the primary ion which appears in the form of kinetic energy in the first-generation secondaries produced.

Some additional heating will be produced by the mesons which result from nuclear collisions and which decay promptly into electrons and positrons with relativistic energies. The production rate for this process has been computed by Jones (1963). To find an upper limit for the heating rate, we may neglect possible escape of these fast electrons from the galactic plane and compute the rate at which these electrons produce next-generation secondaries. In addition to excitation and ionization losses, bremsstrahlung losses are also important, becoming dominant for electron energies greater than about 400 MeV. When all such losses are taken into account, the fraction of the electron and positron kinetic energy which is available for heating the gas varies from 0.08 to 0.03 as the initial energy varies from 10 to 1000 MeV, respectively. In this computation the total number of secondaries produced by the relativistic electrons is assumed to be $\frac{5}{3}$ times the number of first-generation secondary electrons, exactly as before, and the mean kinetic energy available for heating has been assumed given by equation (12). When allowance is made for the loss of energy to neutrinos and photons in the nuclear collisions which produce mesons, and in the decay of these mesons into electrons, the over-all heating efficiency of this process is an order of magnitude or more below that given in equation (13).

The over-all heating rate computed in this way is about 5 per cent of the minimum heating rate computed from equations (6) and (12). We shall neglect this effect in the subsequent computations. Ionizations produced in this manner are also negligible.

c) Heating by Particles Ejected from Supernovae

The possible importance of supernovae in producing cosmic rays of relativistic energies has been discussed by many authors. From the standpoint of gas heating, it seems likely that the atoms in the main supernova shell may be much more effective than the particles of relativistic energy. The velocity of expansion of a Type I supernova shell is now believed to be about $20000 \text{ km sec}^{-1}$, according to a recent survey by Minkowski (1968). A proton moving at this velocity will have a kinetic energy of 2 MeV, and, as we shall see below, its range in the galactic plane will be about 700 pc. More energy is probably available in this form than in relativistic particles, and also the efficiency of conversion into gas thermal energy is more than an order of magnitude greater.

It is, of course, not certain that all the initial energy of the expanding envelope will be available in the form of 2 MeV protons traversing the interstellar gas. In fact, for motions transverse to a magnetic field, a hydromagnetic shock will certainly develop, and the material will cool as it expands; a theory of such expansion, assuming a spherical shock, has been developed by Sedov (1959). In a direction parallel to the interstellar magnetic field, it is less obvious what will happen. If the magnetic field is ignored entirely, the electrons, with much less energy than the positive ions, and a correspondingly shorter mean free path may be expected to form a shock front. The energetic protons and positive ions would be quite unaffected by such a front, and could penetrate the cold gas, moving as free particles. If the protons all had the same energy, this situation would be unstable against longitudinal plasma oscillations. However, a small dispersion of velocities would stabilize the situation, according to Noerdlinger (1961). Other instabilities may still appear to slow down the positive ions (Kahn 1962, 1964), but the subject is still unclear.

If an initial interstellar magnetic field is taken into account, the problem becomes much more complex. According to the analysis by Kulsrud *et al.* (1965), a supernova produces a field-free cavity, surrounded by a compressed layer of high field. On this pic-

ture, which replaces the supernova envelope by an expanding conducting shell, 2 MeV protons would be unable to escape from the cavity, except near the two stagnation points at which the magnetic field vanishes. If the spread of particle energies in the supernova explosion were considered, one might expect that strongly relativistic particles would escape in abundance through these two openings and that the pressure of these particles might push the interstellar gas aside, opening up two large holes through which the particles of lower energy could escape freely. It is not obvious exactly how all these particles would finally be spiraling around the lines of force of the interstellar magnetic field, but some fine-scale mixing would presumably occur.

Evidently the problem is very complicated, and no very definite statements can be made at present. To give an upper limit on ζ_f , which we denote by ζ_s , we shall assume that one-third the kinetic energy of the supernova envelope is available in the form of energetic protons and other positive ions. We shall also assume in computing ζ_s that such particles are uniformly distributed throughout the galactic disk, confined by the galactic magnetic field.

We compute the energy that could be available in such ions. The initial energy of a Type I supernova shell has been estimated as 10^{51} ergs (Minkowski 1968) corresponding to a mass of $M_\odot/4$ moving at a velocity of $20000 \text{ km sec}^{-1}$. The average frequency of supernovae in galaxies has been estimated as one per 50 years (Katgert and Oort 1967), with the value for our own Galaxy about twice this great. We adopt here for the Galaxy the conservative figure of one every 50 years and assume that half of these are Type I. If the Galaxy is taken to be a cylinder with a radius of 10000 pc and a thickness of 200 pc, we find that the total power in Type I supernova shells is $1.7 \times 10^{-25} \text{ erg cm}^{-3} \text{ sec}^{-1}$. We ignore Type II supernova shells; the velocity of these shells is about 6000 km sec^{-1} , and the range of a proton of this initial velocity is only about 14 pc if the hydrogen density is 0.85 per cm^3 ; the energies liberated in these Type II explosions are likely to be deposited in the neighborhood of the supernovae. If we divide the computed power density from Type I supernovae by three, on the assumption that the kinetic energy is available in only one direction (parallel to the magnetic field), and divide by the energy loss of 36 eV per free electron produced, the ionization rate per atom becomes

$$\zeta_s = \frac{1.0 \times 10^{-15}}{\bar{n}_H} \text{ sec}^{-1} = 1.17 \times 10^{-15} \text{ sec}^{-1}, \quad (14)$$

where $\bar{n}_H$, the effective particle density of hydrogen (which we take equal to $n(\text{H I})$, the density of neutral hydrogen), has been set equal to 0.85 cm^{-3} . In accordance with equation (7), the heating rate Γ_{HR} corresponding to ζ_s is given by

$$\Gamma_{HR} = 6.3 \times 10^{-27} \bar{n}_H \text{ erg sec}^{-1} \text{ cm}^{-3}. \quad (15)$$

The lifetime of these energetic protons is determined by the total rate of energy loss. According to the data summarized by Whaling (1958), a 2 MeV proton loses energy at the rate of $3.8 \times 10^8 \text{ eV per gm cm}^{-2}$ traveled, corresponding to an energy loss of about 60 eV per primary ion produced. For a mean hydrogen density of 0.85 per cm^3 , we find

$$\tau \equiv \frac{E}{dE/dt} = 1.9 \times 10^{12} \text{ sec}. \quad (16)$$

When account is taken of the increase in dE/dt as the proton slows down, the actual lifetime is about $2\tau/3$. The energy density U_R is obtained approximately on multiplying by τ the power dissipated per cubic centimeter. In this way we find

$$U_R = \frac{1}{3} \times 1.7 \times 10^{-25} \times 1.9 \times 10^{12} = 1.1 \times 10^{-13} \text{ erg cm}^{-3}, \quad (17)$$

about one-tenth the energy density observed in the relativistic cosmic radiation reaching Earth. A more detailed computation, including retardation, gives a value of U_R about half that obtained from equation (17). The range of a 2 MeV proton in hydrogen is

$3.0 \times 10^{-3} \text{ gm cm}^{-2}$, according to Whaling (1958), corresponding to 700 pc for the density of 0.85 cm^{-3} assumed above.

Uncertainties in the frequency of supernovae and in the mass of the shell are sufficiently great so that equations (14) through (17) could well be in error by half an order of magnitude. Moreover, the fraction of supernova energy that is available in this form may well be substantially less than one-third. It seems unlikely that ζ_r can much exceed the value of ζ_s in equation (14) unless some additional source of energy, much greater than is available from supernovae, can be invoked.

III. COLLISIONS BETWEEN THERMAL PARTICLES

a) Cooling Rates for Inelastic Collisions

The temperature computations made by HNT were based on the consideration of only one cooling rate, Λ_{ei} , resulting from the excitation of ions by electrons. The ions considered were C^+ , Si^+ , and Fe^+ ; following Takayanagi and Nishimura (1960), the rate coefficients for this process were taken from Seaton (1955, 1958). The same rate coefficients have been adopted here.

More important cooling rates in the dense H I clouds are provided by Λ_{Hi} and Λ_{Ha} , the rates of energy loss per cubic centimeter resulting from collisions between H atoms, on the one hand, and ions and atoms, respectively, on the other. As shown by Dalgarno and Rudge (1964), Λ_{Hi} can be an order of magnitude greater than Λ_{ei} , if n_e/n_H is as low as 5×10^{-4} . Since the de-excitation cross-section σ_{kj} varies less rapidly with particle energy than does the excitation cross-section, σ_{jk} , we write Λ_{Hi} in the approximate form

$$\Lambda_{Hi} = \frac{n_H n_i E_{jk} g_k}{g_j} \langle u \sigma_{kj} \rangle e^{-E_{jk}/kT}, \quad (18)$$

where u is the relative thermal velocity of hydrogen atoms and ions. According to the cross-section given by Dalgarno and Rudge, $u \sigma_{kj}$ is independent of u , and we have

$$\frac{g_k u \sigma_{kj}}{g_j} = \frac{4\pi}{3} \left(\frac{\alpha E_1}{\mu} \right)^{1/2} a_0^2, \quad (19)$$

where μ is the reduced mass for an H atom interacting with an ion, and E_1 is the binding energy of an H atom; α is the polarizability in atomic units (equal to 4.5 for atomic H), and a_0 is the radius of the first Bohr orbit in hydrogen. Numerically, equations (18) and (19) may be combined to yield

$$\Lambda_{Hi} = 2.16 \times 10^{-21} n_H n_i E_{jk} (\text{eV}) 10^{-5040 E_{jk} (\text{eV}) / T} \text{ erg cm}^{-3} \text{ sec}^{-1}, \quad (20)$$

where $E_{jk} (\text{eV})$ denotes the excitation energy in electron volts, and μ has been set equal to m_H .

The corresponding cooling rate for collisions with neutral atoms is given by an equation identical with (18) but with different cross-sections. Cross-sections for the excitation of the $J = 1$ and $J = 0$ fine-structure levels in O I, with energies of 0.0197 and 0.0281 eV, respectively, have been computed by Smith (1966). Oxygen is much the most abundant neutral element in H I regions, other than H and He, and neither nitrogen nor neon, the next most abundant, have excited levels within 2 eV of the ground level. Hence these two transitions in O I give the dominant contribution to Λ_{Ha} for temperatures below 1000° K .

The relative abundances of H, C, Si, and Fe we take from Aller (1961), who gives

$$n_H : n_O : n_C : n_{Si} : n_{Fe} = 10^6 : 890 : 400 : 32 : 3.7. \quad (21)$$

The C, Si, and Fe atoms will mostly be singly ionized, and if allowance is made also for the electrons from Mg and S atoms, the total particle density, n_i , of the positive ions other than hydrogen is given by

$$\frac{n_i}{n_H} = 5 \times 10^{-4}. \quad (22)$$

If hydrogen is not ionized appreciably by cosmic radiation, n_e/n_H will also be given by equation (22). Since the electron density and ion density are each increased by about a factor 3 over the values assumed by HNT, the cooling rate for a particular hydrogen density is increased by more than an order of magnitude. The increased abundance of C, based primarily on the detailed analysis of solar abundances by Goldberg, Müller, and Aller (1960), is not entirely certain, especially since under interstellar conditions an appreciable fraction of the carbon atoms might be locked up in grains.

Cooling by molecular hydrogen can be very effective when the interstellar temperature much exceeds 100° K. However, recent theoretical work by Stecher and Williams (1967) indicates that H_2 molecules will be dissociated by ultraviolet light in the absorption lines between 1100 and 912 Å and that the ratio $n(H_2)/n(H)$ may be less than 10^{-6} . At this level, the cooling rate produced by H_2 will be negligible at temperatures below 1000° K, as shown by the detailed reaction rates computed by Takayanagi and Nishimura (1960). The observational upper limit of 0.1 for $n(H_2)/n(H)$ obtained by Carruthers (1967) is consistent with the assumption that $n(H_2)/n(H)$ may be neglected.

b) Radiative Recombination

The recombination coefficient, α , for electron-proton collisions has been evaluated by many authors. However, for conditions in interstellar space, recombination directly into the ground state should be ignored, since the emitted photon in the Lyman continuum is promptly absorbed by a neighboring neutral atom (see discussion by Gould *et al.* 1963). Accordingly, we here take for α

$$\alpha = \alpha^{(2)} \equiv \sum_{j=2}^{\infty} \alpha_j. \quad (23)$$

The function $\alpha^{(2)}$ may be expressed in the form (see Spitzer 1948b)

$$\alpha^{(2)} = \frac{2.07 \times 10^{-11}}{T^{1/2}} \phi_2, \quad (24)$$

where ϕ_2 varies slowly with T . The computations by Seaton (1951), based on a series expansion for the Gaunt factors, show that ϕ_2 increases from 2.27 to 3.89 as T drops from 1000° to 31.3° K (see Spitzer 1968b).

c) Equipartition between Electrons and H Atoms

Since Λ_{H_i} , one of the major cooling rates, acts on the H atoms, while energy in interstellar space is gained primarily by electrons, H atoms will tend to be cooler than the electrons. In the non-steady state, where a gas cools by ion-electron collisions, Takayanagi and Nishimura (1960) have pointed out that elastic encounters between H atoms and electrons are relatively ineffective and cannot reduce $T_e - T_H$ below about $T_e/4$, where T_H and T_e are kinetic temperatures for H atoms and ions, respectively. A similar conclusion might be expected to apply if the heating were produced by Γ_{HR} , with cooling by Λ_{H_i} .

However, another mechanism is available which reduces the deviations from equipartition in H I regions. The electron and positive ion temperatures, T_e and T_i , will be closely coupled together by electrostatic interactions, which have a very large cross-section at low temperatures. Collisions between H atoms and positive ions, including C^+ and protons, will then couple T_i and T_H together more closely than T_e and T_H are coupled by electron-atom collisions.

To compute $T_e - T_H$ in general, we let Λ_H be the total energy lost by H atoms in inelastic collisions of all sorts and assume that this loss is equal to the energy gained in elastic collisions with ions. This flow of energy away from the ions is balanced by energy gained in elastic collisions of the ions with the electrons. The heat flows from both the thermal electrons and from the energetic secondaries directly to the H atoms will be

ignored, since these flows will usually not exceed about 10 per cent of the main heat flow considered here and will in any case only reduce the deviations from equipartition. The heat flow from the electrons to the ions may be computed from the equipartition time, t_{eq} , defined by Spitzer (1962), and we obtain

$$\Lambda_{\text{H}} = \sum_i \frac{3n_i k (T_e - T_i)}{2t_{\text{eq}}(i,e)}, \quad (25)$$

where $t_{\text{eq}}(i,e)$ denotes the equipartition time for positive ions, with electrons as field particles. In this discussion of equipartition, protons are included in the sum over positive ions. If we insert numerical results for t_{eq} (see Spitzer 1962, eq. [5-31]), with $\ln \Lambda$ set equal to 20, equation (25) yields

$$T_e - T_i = 6.1 \times 10^{16} \frac{T_e^{3/2} \Lambda_{\text{H}}}{n_e \sum n_i m_{\text{H}} / m_i}. \quad (26)$$

Equation (25) may also be applied to encounters between H atoms and positive ions, where now $t_{\text{eq}}(i,\text{H})$ is the equipartition time for positive ions colliding with H atoms. Since the energy flow is now away from the ions into the H atoms, the sign of the right-hand side of equation (25) must be changed; i.e., $T_i - T_{\text{H}}$ replaces $T_e - T_i$. For the equipartition time, we write approximately

$$\frac{1}{t_{\text{eq}}(i,\text{H})} = \frac{2n_{\text{H}} m_{\text{H}} \langle \sigma u \rangle}{m_i}, \quad (27)$$

where u is the relative velocity, which we equate to the velocity of an H atom. The factor 2 in equation (27) results from the exchange of energy rather than of momentum in the definition of t_{eq} . For the cross-section σ , we use the theoretical value by Osterbrock (1961), who gives σu equal to $2.3 \times 10^{-9} \text{ cm}^3 \text{ sec}^{-1}$, independent both of temperature and of ion mass. At temperatures in the neighborhood of 100° K , approximately this same value of $\langle \sigma u \rangle$ applies for the momentum exchange between protons and H atoms (Dalgarno 1957), although the physical mechanism involves charge exchange and is thus quite different in detail from the momentum exchange between heavier positive ions and H atoms. With this value of $\langle \sigma u \rangle$ inserted into equation (27), equation (25) gives for $T_i - T_{\text{H}}$

$$T_i - T_{\text{H}} = 1.05 \times 10^{24} \frac{\Lambda_{\text{H}}}{n_{\text{H}} \sum n_i m_{\text{H}} / m_i}. \quad (28)$$

If equation (26) is divided by equation (28), we obtain

$$\frac{T_e - T_i}{T_i - T_{\text{H}}} = 5.8 \times 10^{-8} T_e^{3/2} \frac{n_{\text{H}}}{n_e}. \quad (29)$$

If n_{H}/n_e is 2000, the right-hand side of equation (27) equals $(T_e/420)^{3/2}$ and is small for the low equilibrium temperatures found in H I clouds (see Table 1 below). For the less dense regions, where high temperatures are shown under some conditions in Table 1, the corresponding ionization levels are higher, and $T_e - T_i$ is still small compared to $T_i - T_{\text{H}}$.

To compute $T_i - T_{\text{H}}$ approximately, we may set Λ_{H} equal to $\Lambda_{\text{H}i}$, which is evaluated from equation (20). If the only two ions considered are C^+ (with E_{jk} of 0.0079 eV) and protons, equation (28) yields

$$T_i - T_{\text{H}} = \frac{18^\circ \times 10^{-40^\circ/T}}{(n_p/n_i) + (1/12)}, \quad (30)$$

where n_p denotes the particle density of protons, equal to $n_e - n_i$. As we shall see below, either T is low in H I regions or n_p/n_i is large, with the result that $T_i - T_{\text{H}}$ may generally be ignored. Consideration of $\Lambda_{\text{H}a}$ does not change this result. We conclude that,

under the conditions considered here, all deviations from equipartition among the thermal components of the interstellar gas are negligible.

IV. COMPUTED TEMPERATURES AND ELECTRON DENSITIES

The equations given in the previous sections have been combined to compute electron densities and temperatures in H I regions. The resultant values depend only on the value of ζ_f/n_H ; while Γ_{HR} is proportional to $\zeta_f n_H$, all the other rates are proportional to n_H^2 , and the equilibrium will remain unaltered if ζ_f and n_H are changed proportionally. For each value of ζ_f/n_H , equations (1) and (2) must be solved simultaneously. We assume that n_i , which we here take to be the particle density of positive ions heavier than hydrogen, is unaffected by the cosmic-ray flux, and equation (2) has the solution

$$2n_e = n_i + (n_i^2 + 4\zeta_f n_H/a)^{1/2}. \quad (31)$$

In the computations, Λ_{ei} and Γ_{ei} were taken from Takayanagi and Nishimura (1960), revised in accordance with the abundances in equation (21). Values of Λ_{Hi} were obtained from equation (20), while Λ_{Ha} was computed from the cross-sections given by Smith (1966), evaluating $u\sigma_{kj}$ at the rms velocity for each temperature; since $u\sigma_{kj}$ changes relatively slowly with kinetic energy, no detailed average over a Maxwellian distribution seems required. The loss rates Λ_{ea} and Λ_{Hm} were both ignored; a was taken from equation (24).

The resultant values of T_E , the equilibrium temperature, and of n_e/n_H are shown in Figures 3 and 4, respectively.

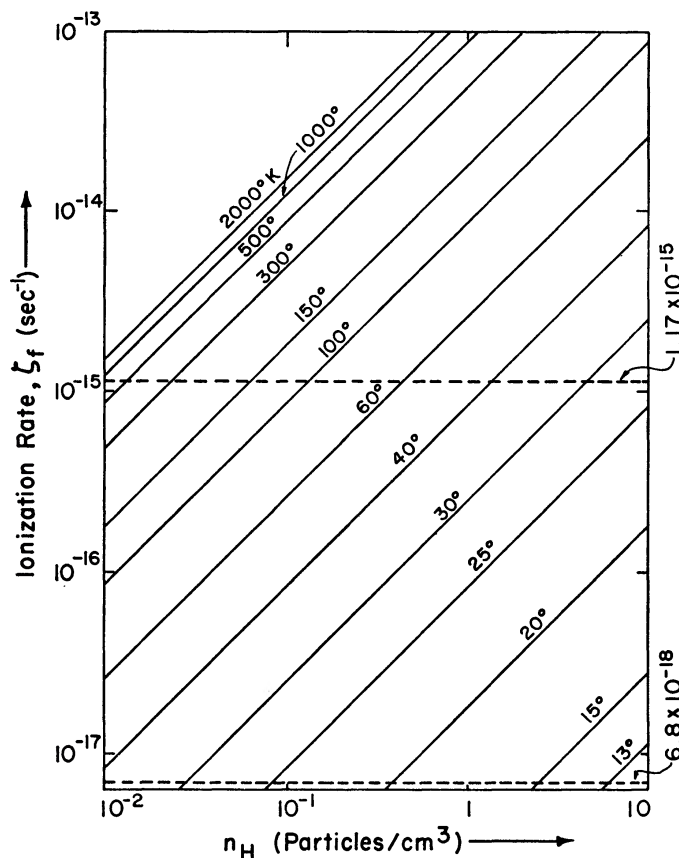


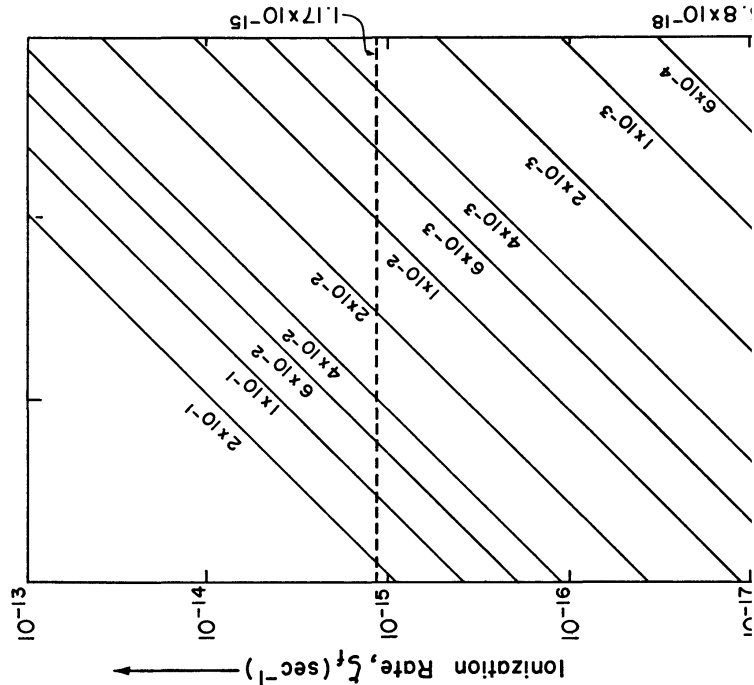
FIG. 3.—Computed temperature. Diagonal lines give temperatures of H I regions for different ratios of the ionization rate, ζ_f , to n_H , the particle density of hydrogen. Two dashed lines give rough upper and lower limits for ζ_f .



TABLE 1

TEMPERATURE AND RELATIVE IONIZATION

IONIZATION RATE, ξ_f		HYDROGEN DENSITY n_H (cm^{-3})							
		0.01	0.03	0.1	0.3	1.0	3	10	
$\xi_F = 6.8 \times 10^{-18} \text{ sec}^{-1}$	$\left\{ \begin{array}{l} 10^3 \times (n_e/n_H) \\ T(^{\circ}\text{K}) \end{array} \right\}$	7.5 38	4.2 30	2.2 24	1.4 20	0.8 17	0.6 14	0.5 12	
	$\left\{ \begin{array}{l} 10^3 \times (n_e/n_H) \\ T(^{\circ}\text{K}) \end{array} \right\}$	270 860	100 250	42 120	21 70	10 45	5.5 33	3.0 27	



(15), obtained on the assumption that the protons in Type I supernova shells permeate the Galaxy as free particles.

The temperatures computed for the higher value of ζ_f seem to be of the same order as indicated by the observations, with values of T below 100°K for the dense clouds and rising somewhat above 100°K for the less dense regions. Since there is little observational information on the variation of T with n_H , a detailed comparison between theory and observation is not possible. Moreover, the temperatures given in Table 1 for ζ_f equal to ζ_s may be somewhat low since, as noted earlier, when n_e/n_H is as great as 10^{-2} , the thermal electrons will absorb an appreciable fraction of the kinetic energy of the secondary electrons before these are slowed down to an energy less than $3E_1/4$. The maximum possible increase of ϵ_R above the value given in equation (13) is a factor of about 6; inspection of Figures 3 and 4 indicates that for n_H equal to 0.1 cm^{-3} an increase of the heating rate by this factor would increase T up to about 400°K . In view of the large uncertainties in ζ_f a more detailed analysis has not been attempted.

In a general way, the observations seem not inconsistent with the assumption that the power available for heating the gas is an appreciable fraction of the energy available in supernova shells. Both observations and theory are uncertain, but the change in T with density suggested by the observations seems to be qualitatively explained by the theory. The electron density of about 0.01 cm^{-3} for the average H I region in this case is an order of magnitude higher than the values generally assumed.

In principle, the observations of interstellar absorption lines could discriminate between the upper and lower values of ζ_f , since the ionization equilibrium for interstellar atoms depends on n_e . Data for the Ca I lines are too fragmentary to be useful. If the particle densities of Na I in average clouds, as obtained by Spitzer (1948*b*), are interpreted with the theory given by Seaton (1951), one finds that the density of Na atoms in the gas must be an order of magnitude less than would be expected from the adopted value of n_H and the usual cosmical composition. This result would seem to favor the lower value of ζ_f . However, it is possible that most of the Na atoms within the clouds may be locked up in grains, a conclusion indicated some time ago for Ca atoms. If O and C atoms are also locked up in the grains to any substantial extent, the equilibrium temperature for each value of ζ_f will be increased above the values computed here. Measurements of lines from neutral and ionized atoms of the same element, such as Ca in the visible, or Mg, C, Si, or S in the ultraviolet, could determine n_e directly.

The temperature to be expected in an intercloud medium with a density of $0.1\text{ H atom cm}^{-3}$ or less should be studied further, since the present theory begins to break down for this condition. A more detailed analysis, taking into account the direct heating of thermal electrons by energetic secondaries, would be of interest. Another effect which should be considered is the propagation of diffuse ultraviolet flux from early-type stars into this relatively rarefied gas. At sufficiently low density the hydrogen might well be kept ionized by ultraviolet radiation escaping from early-type stars in regions of relatively low n_H .

The higher value of ζ_f discussed here would have other consequences, which will be discussed elsewhere (Spitzer 1968*b*). With the increased value of n_e , H I regions would produce appreciable absorption of the low-frequency non-thermal radiation reaching the Galaxy. In addition a large flux of 2 MeV protons would disrupt very small grains, making interstellar evolution of grains from small nuclei very difficult.

The calculations carried out here provide a ready determination of the cooling time t_T at constant density, defined here by the relation

$$\frac{1}{t_T} \equiv \frac{\Lambda - \Gamma}{3nk(T - T_E)/2} \quad (32)$$

Values of t_T for various values of T and for n_H equal to 1 and 10 per cm^{-3} are given in Table 2. The computations are for the lower value of ζ_f ; however, for the higher values of

T the cooling rate much exceeds the heating rate even for ζ_f equal to its upper limit, and t_T is the same for both cases. These cooling times are even shorter than those obtained by Takayanagi and Nishimura (1960) and suggest even more strongly that collisions between clouds cannot increase very significantly the average temperature in H I regions.

Note added in proof (April 22, 1968).—Since this material was submitted several related discussions have appeared. The heating of the gas in H I regions by suprathermal particles has been discussed by Balasubrahmanyam *et al.* (1967) and by Pikel'ner (1967). Both of these papers conclude that temperatures of about 100° K can be expected in interstellar clouds if the ionization rate per H atom resulting from the energetic particles is about $10^{-15} \text{ sec}^{-1}$, about equal to the upper limit ζ_f considered here.

The paper by Balasubrahmanyam and his colleagues makes a number of modifications in the analysis by HNT. The recombination coefficient at 100° K is taken to be about $10^{-12} \text{ cm}^3 \text{ sec}^{-1}$, about one-tenth that adopted by HNT and about one-fifth the more exact value of $\alpha^{(2)}$ used here. The heating efficiency is set equal to about one-half, substantially greater than the value of ϵ_R used here. These assumptions increase n_e for a given ζ_f and also give too high a T_E , even though the cooling by collisions between hydrogen and oxygen atoms is taken roughly into account.

TABLE 2
COOLING TIMES

$n_H (\text{cm}^{-3})$	t_T (years) FOR GIVEN T (° K)			
	30°	100°	300°	1000°
1	1.5×10^5	1.3×10^5	1.4×10^5	2.3×10^5
10	2.4×10^4	1.6×10^4	1.6×10^4	2.5×10^4

The analysis by Pikel'ner neglects cooling by oxygen atoms and follows HNT in assuming 91 C atoms for 10^6 H atoms, 1/4.4 of the carbon abundance assumed in equation (21). In addition his assumed value of 10 eV for $\langle E_k \rangle$, again following HNT, is too great by a factor 3; heating of the gas by plasma waves excited by the primary particles, a process which has been ignored in the present paper but which Pikel'ner takes into account, is relatively unimportant as long as n_e/n_H is only a few per cent. As a result of these changes, the temperatures obtained by Pikel'ner are substantially higher than those obtained here. In addition the relative ionization which he obtains is increased by his low value of α , which corresponds to ϕ_2 equal to 1.2, about a third of the value adopted here.

As a result of all these differences, the analysis in this paper gives the same temperatures as found by Pikel'ner only if the hydrogen density is taken to be about one-thirtieth of his values. In view of the uncertain composition of the interstellar gas, it is impossible to decide a priori what the correct relationship is between n_H and temperature, but it would seem that to justify the relation given by Pikel'ner would require either a lower relative abundance of C^+ than we have assumed here or a more energetic source of suprathermal particles.

The arguments in favor of a high n_e in H I regions have been discussed by Pottasch (1968) as well as by Pikel'ner. Both authors point out that appreciable ionization in H I regions would help to explain both the Faraday rotation produced in the galactic plane and the absorption of low-frequency extragalactic radio waves in the plane. Because of uncertainties in the mean magnetic field and in the detailed density distribution of hydrogen in H I and H II regions, these arguments are not quantitatively very firm at

the present time, though they are certainly consistent with an appreciable value of n_e/n_H in H I regions.

Habing and Pottasch (1967) have pointed out the effects which an increased n_e in H I regions will have on the ionization equilibrium of Na and C atoms (see above in the discussion of Table 1); a more quantitative treatment of this problem will appear elsewhere (Spitzer 1968b). A generally low abundance of the heavy elements relative to H in the interstellar gas, a possibility already mentioned above, would appear to offer a consistent reconciliation of all these theoretical arguments. Unfortunately preliminary interstellar line data from rockets suggest precisely the opposite conclusion, with an anomalously high abundance of C⁺ and O (Stone and Morton 1967) and a low abundance of H (Jenkins 1968). Evidently further observations are required.

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