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INTRINSIC POLARIZATION OF RAPIDLY ROTATING EARLY-TYPE STARS

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ABSTRACT

Chandrasekhar pointed out that radiation emerging from a plane-parallel semi-infinite pure-scattering atmosphere will be partially polarized. This fact combined with the gravity darkening, the rotational distortion, and the dominance of electron scattering in early-type stars should lead to some intrinsic polarization. In this paper we calculate the polarization present in such stars as a function of their rotational velocity and angle of inclination. It is demonstrated that one may have a degree polarization as high as 1.70 per cent (0.037 mag) for stars rotating at the limit of stability. Several suggestions are presented for detecting this effect in real stars.

I. INTRODUCTION

Scattering by free electrons becomes a major source of opacity for very hot stars. It was pointed out by Chandrasekhar (1946) that the light emitted by a medium whose opacity is due to electron scattering will be partially polarized. Attempts to observe this effect at the limb of an eclipsing binary led to the discovery of interstellar polarization.

This polarization effect will of course cancel when integrated over a spherical surface. Not all stars are spherical, however, and if electron scattering is the major source of continuous opacity, the continuum can exhibit an intrinsic polarization. Two obvious causes of non-sphericity are axial rotation in early-type stars and tidal distortion in close binary systems. In this paper we will calculate the amount of intrinsic polarization expected from a rapidly rotating star whose continuous opacity is due entirely to scattering by free electrons.

In the next section we shall develop the theory appropriate for estimating the polarization to be expected from a rotationally distorted star where the atmosphere may be assumed to be a gray atmosphere. The assumptions involved will be similar to those used in the rotation studies of Collins (1965, 1966) and Collins and Harrington (1966). Thus we shall include effects on the radiation field due to gravity darkening, limb darkening, aspect, and shape distortion.

In the final section we shall discuss the possible implications of the results and indicate some methods by which the effect can be observed.

II. THEORY

Let $\boldsymbol{n}$ be the unit vector normal to the surface of the star. The radiation emerging in the direction of the observer will make an angle with $\boldsymbol{n}$ whose cosine will be denoted by μ. The integrated intensity of this radiation can be broken into two components, I_l and I_r, where I_l is the component polarized with the electric vector in the plane defined by $\boldsymbol{n}$ and the line of sight, and I_r is the component polarized perpendicular to this plane.

We can write I_l and I_r as follows:

$$I_l = Ff_l(\mu) , \tag{1}$$

$$I_r = Ff_r(\mu) , \tag{2}$$

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where F, the emergent flux, is proportional to the local surface gravity to the extent that von Zeipel's theorem is applicable. Chandrasekhar's third approximation was used for $f_l(\mu)$ and $f_r(\mu)$ (see Chandrasekhar 1946, eqs. [122] and [123]).

The geometry involved is illustrated in Figure 1. The X, Y, Z coordinate system is defined so that the star's axis of rotation is along Z and the observer is in the XZ plane. A point on the stellar surface is specified by the spherical coordinate angles θ and ϕ. The X', Y', Z' coordinate system is obtained by a rotation about the Y axis of $(\pi/2-i)$ so that the X' axis points to the observer, who sees the star with an angle of inclination i.

Let the intensity polarized parallel to the Y' axis which is contributed by a particular surface element be called I_{eq}, and that polarized parallel to the Z' axis, I_{pol}. The observer

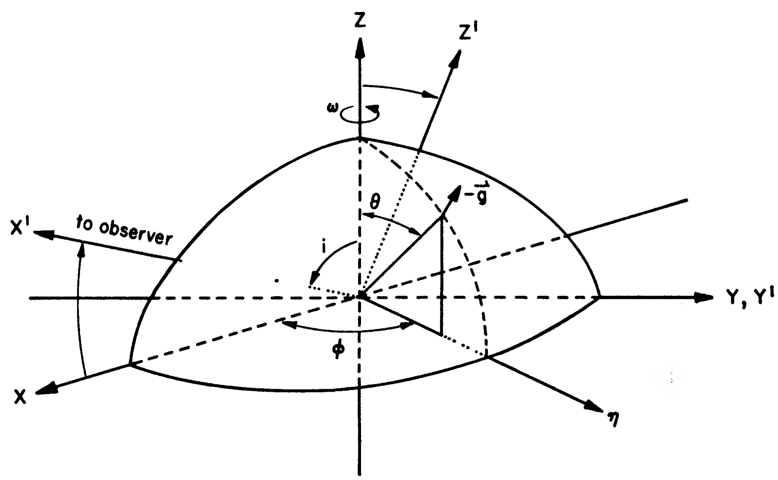

FIG. 1.—Coordinate systems defining angles used in this study

will see the unit normal $\boldsymbol{n}$ projected on the $Y'Z'$ plane. If we let ξ denote the angle between this projection and the Z' axis, then the contributions of I_l and I_r to I_{eq} and I_{pol} are[1]

$$I_{\text{eq}} = I_r \cos^2 \xi + I_l \sin^2 \xi \tag{3}$$

and

$$I_{\text{pol}} = I_r \sin^2 \xi + I_l \cos^2 \xi \,. \tag{4}$$

The observed flux is found by integrating I_{eq} and I_{pol} over the star's surface:

$$F_{\text{eq}} = \int I_{\text{eq}} \mu dA \,, \tag{5}$$

$$F_{\text{pol}} = \int I_{\text{pol}} \mu dA \,. \tag{6}$$

The development of the equations for the shape and local gravity of a rotating star can be found in Collins (1963, 1965) and in Collins and Harrington (1966), where the assumption of constant polar radius is relaxed. As in the previous papers, we assume here that the mass distribution of the star may be approximated by a Roche model, and that the star is rotating as a rigid body so that von Zeipel's theorem applies.

The radius of a point on the surface, whose co-latitude is θ, is $x(\theta,\mathfrak{w})R_p(\omega)$. $R_p(\omega)$ is the polar radius and $\mathfrak{w}$ (defined by eq. [5] of Collins and Harrington 1966) is a measure

[1] For the transformation of the Stokes parameters under rotation of axes, see Chandrasekhar (1960).

of the angular velocity. For these calculations it was convenient to use the trigonometric form of the solution of the cubic equation defining $x(\theta,\mathfrak{w})$:

$$x(\theta,\mathfrak{w}) = \frac{3}{\mathfrak{w}\sin\theta}\cos\{\tfrac{1}{3}[\pi + \cos^{-1}(\mathfrak{w}\sin\theta)]\}. \tag{7}$$

The vector $\boldsymbol{n}$ will make an angle with the Z axis which we will call δ (see Fig. 2). From the $\boldsymbol{r}$ and $\boldsymbol{\theta}$ components of the surface gravity it follows that

$$\delta = \theta - \tan^{-1}\left[\frac{8x\mathfrak{w}^2\sin\theta\cos\theta}{(27/x^2) - 8x\mathfrak{w}^2\sin^2\theta}\right]. \tag{8}$$

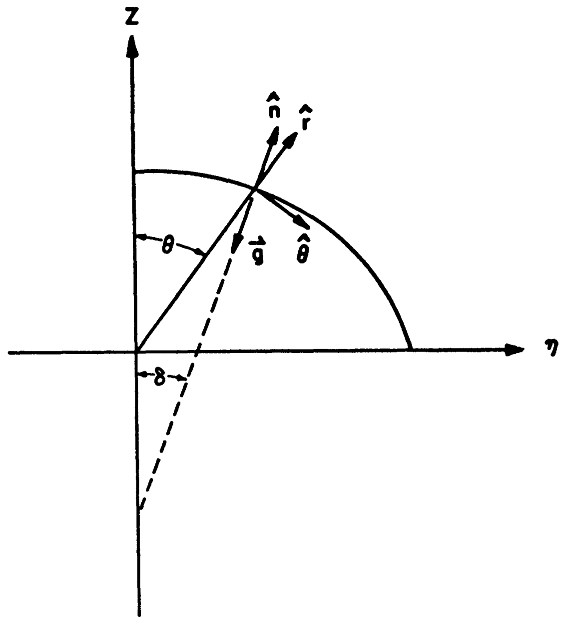

FIG. 2 —Cross-section of star in $Z\eta$ plane, defining angle

If we express $\boldsymbol{n}$ in the $X'Y'Z'$ system, then the X' component is just μ, and the ratio of the Y' to Z' components is the tangent of the angle ξ:

$$\xi = \tan^{-1}\left(\frac{\sin\delta\sin\phi}{\sin i\cos\delta - \cos i\sin\delta\cos\phi}\right). \tag{9}$$

By use of von Zeipel's theorem, the local flux, F, which appears in equations (1) and (2), is proportional to the local gravity, $|g|$. To evaluate equations (5) and (6), we note that, if we take the absolute value of μ, the symmetry allows us to perform the integration over the quarter section of the surface bounded by the XZ and XY planes, regardless of i. Since we are only interested in ratios of F_{eq} and F_{pol}, we can collect all the constant terms into some constant C'. Thus we have for F_{eq} and F_{pol}:

$$F_{\text{eq}} = C'\int_0^{\pi/2}\int_{-1}^{1}[f_r(\mu)\cos^2\xi + f_l(\mu)\sin^2\xi]\frac{|\boldsymbol{g}||\mu|x^2}{\boldsymbol{r}\cdot\boldsymbol{n}}d(\cos\theta)d\phi, \tag{10}$$

$$F_{\text{pol}} = C'\int_0^{\pi/2}\int_{-1}^{1}[f_r(\mu)\sin^2\xi + f_l(\mu)\cos^2\xi]\frac{|\boldsymbol{g}||\mu|x^2}{\boldsymbol{r}\cdot\boldsymbol{n}}d(\cos\theta)d\phi. \tag{11}$$

The quadrature used to evaluate these integrals was the same as in the earlier papers, e.g., Collins (1963). A total of 190 quadrature points was employed.

The amount of polarization is defined by

$$P = (F_{\mathrm{eq}} - F_{\mathrm{pol}})/(F_{\mathrm{eq}} + F_{\mathrm{pol}}) . \tag{12}$$

Calculations were carried out for various values of $\mathfrak{w}$ and i; the results are presented in Figure 3, where the polarization P is plotted against $\mathcal{W}$, the fraction of the breakup angular velocity; $\mathcal{W}$ was related to $\mathfrak{w}$ by equation (47) of Collins and Harrington (1966).

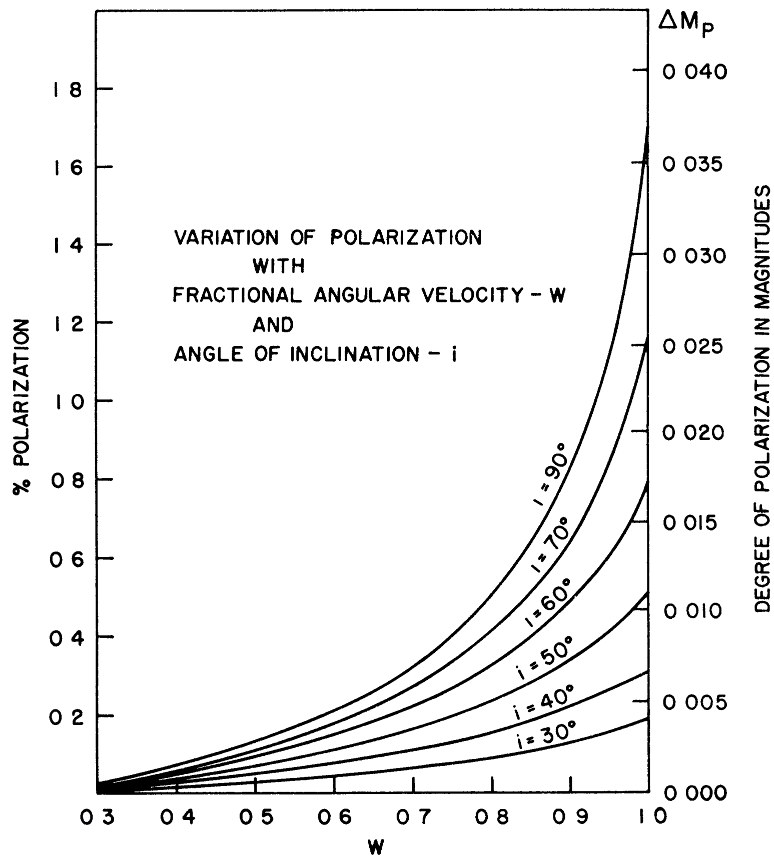

FIG. 3.—The percentage of intrinsic linear polarization as a function of $\mathcal{W}$; the fraction of breakup angular velocity, plotted for several values of the angle of inclination, i.

Figure 4 shows the same information but plotted in a different manner. This figure is useful in displaying the variation of the degree of polarization with changing angle of inclination.

III. DISCUSSION OF THE RESULTS

In the previous section we saw how the effects of rotational distortion, gravity darkening, and electron scattering can all be combined to yield an intrinsic polarization for early-type stars. One property of interest which cannot be obtained from this type of analysis is the wavelength variation of the degree of polarization. This results from the gray-atmosphere approximation which assumes that the effects of scattering will be the same at all wavelengths. In reality this will not be so and we should expect the polarization to be the most prominent in the part of the spectra of early-type stars where the ratio σ/κ_ν is a maximum.

One may indeed ask to what extent the gray-atmosphere assumption may affect the magnitude of the result. The initial reaction to this question is that these calculations should represent an upper limit on the degree of polarization as the gray atmosphere is the appropriate one for pure scattering. However, this need not be true in this case. It is true that any addition of a pure absorption coefficient will tend to dilute the polarizing effect of the electron scattering, but this may have a beneficial effect on the net polarization. This results from the fact that the contribution to the net polarization from the equatorial regions is a negative one in the sense that it brings about a cancellation of the polarization generated in the polar regions. Thus if the polarization generated in

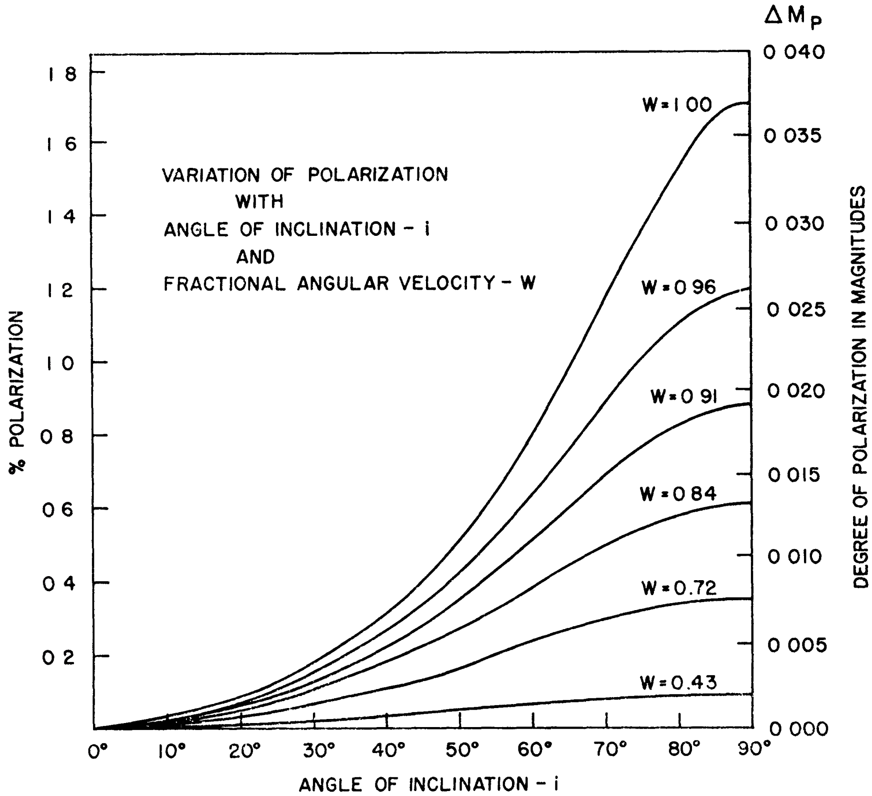

FIG. 4 —The percentage of polarization as a function of the angle of inclination, i, plotted for several values of W, the fraction of breakup angular velocity.

the equatorial regions is diluted by a decrease of the ratio σ/κ_ν, then the loss of the cancelling radiation contributed by these latitudes may more than offset the addition of a pure absorption component in the extinction coefficient appropriate to the poles. This would then lead to a net increase in the degree of polarization. Since the temperature decreases from pole to equator, one would expect the ratio of σ to κ_ν to decrease rapidly as one goes to the lower latitudes. This effect will be offset by the decrease in the surface gravity with lower latitudes which would tend to increase the state ionization. Thus it is not possible to estimate the nature of the effect that introducing non-gray atmospheres into the problem will have without carrying out the detailed calculations required. A study by one of the authors (Collins) is currently under way to ascertain the quantitative as well as qualitative nature of the effect of non-gray atmospheres on the problem. Due to the increased numerical difficulties the results of this study will have to wait until a later discussion.

One result of the presence of a pure absorption component in the opacity is that there should be a spectral type for which the intrinsic polarization is a maxima. This spectral class would probably be in the early B stars (i.e., B0–B3). Thus any attempt at detection of this effect should be concentrated on the early Be stars which show large values of $v \sin i$. One must be careful to compensate for the effects of interstellar polarization. This can be accomplished in several ways only two of which will be suggested here.

First, one could observe stars contained in binary systems one member of which would be a broad-line early-type Be star while the other component of the system should be a sharp-line star. It is clear from Figures 3 and 4 that a low value of $v \sin i$ is a sufficient condition for the intrinsic polarization to be small. Thus any polarization of the sharp-line component could be attributed to an interstellar origin and thus subtracted from the Be star in question. This possibility is being investigated in about ten multiple systems at the present time by one of the authors (Collins).

A second method which might be used to detect this effect would be to compare the polarization of the continuum with that of the emission lines of the star. Inasmuch as the emission lines of a Be star probably arise in an optically thin layer, the radiation would be unpolarized. It is acknowledged that such measurements would in practice be very difficult to perform, but it should be noted that the rewards are great. Not only would quantitative determination of this effect provide valuable information concerning the atmospheres of rapidly rotating stars, but the determination of the position angles of the axes of rotation of the Be stars in clusters could provide insight into some of the problems of cluster dynamics.

Finally, a related problem might be mentioned here, although the calculations in this paper do not directly apply. It has been suggested by Limber (1964) that the Wolf-Rayet phenomena occur as the result of rotational instability. If this were the case, we would expect the star to be surrounded by a flattened gaseous envelope; such a configuration should certainly produce polarized radiation. The problem would be complicated by the optical depth of the envelope, which would scatter the light from the star. However, we still would expect the emission lines to exhibit a different degree of polarization from the continuum.

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