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# EFFECTIVE TEMPERATURES OF THE CENTRAL STARS OF PLANETARY NEBULAE

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ABSTRACT

The effective temperatures of the central stars of thirty-five planetary nebulae are determined on the assumption that the central stars are non-gray and that there is complete absorption in the $He^+$ Lyman continuum in each case The model atmospheres of Böhm and Deinzer that correspond to the Harman and Seaton sequences of stars with masses between 0.5 and 1 solar mass are assumed to be representative of the atmospheres of the central stars of planetary nebulae.

Upper and lower limits are set to the characteristic optical thickness at the Lyman limit in the $H^0$ continuum for each object under consideration.

## I. INTRODUCTION

On the assumption that there is complete absorption in the continuum of $He^+$ for each of thirty-five planetary nebulae, Harmon and Seaton (1966) were able to determine in each case the temperature of the central star and that fraction of the radiation emitted by the central star in the $H^0$ Lyman continuum that is converted to $H^0$ line radiation in the nebula. The central stars were assumed to radiate like black bodies.

Capriotti (1967; hereinafter referred to as "Paper I") used the results of Harmon and Seaton in order to estimate an upper and lower limit to the optical thickness at the Lyman limit in the $H^0$ continuum for each of thirty-five planetary nebulae.

In the following sections, new estimates of the temperatures of the central stars of the thirty-five planetary nebulae are made using non-gray stellar atmospheres constructed by Böhm and Deinzer (1966). New estimates of the optical thicknesses in the $H^0$ continua are made using these new temperatures.

## II. DETERMINATION OF THE CENTRAL STAR TEMPERATURES

The flux in the $He^+$ Paschen-$\alpha$ line can be related to the photographic magnitude, $m_{pg}$, and the central star temperature and gravity by the equation

$$W(T_s, g) = 8.2 \frac{F(4686)}{h\nu(4686)} 10^{(0\ 4m_{pg}+4\ 70)}, \tag{1}$$

where $F(4686)$ is the energy flux at Earth in the $He^+$ (4686) line. $W(T_s,g)$ is a function of the stellar effective temperature and is given by

$$W(T_s, g) = \int_{4\nu_0}^{\infty} F_\nu(T_s, g) \frac{d\nu}{h\nu} \Big/ \int_0^{\infty} S_{pg} F_\nu(T_s, g)\, d\nu, \tag{2}$$

where $S_{pg}$ is the filter function for the photographic region. $F_\nu(T_s,g)$ is the emergent flux distribution for a star of effective temperature $T_s$ and gravity $g$. By multiplying the numerator and denominator of equation (2) by

$$\int_{\nu_0}^{\infty} B_\nu(T_s) \frac{d\nu}{h\nu},$$

we can write

$$W(T_s,g)=\left[\int_{\nu_0}^{\infty}B_\nu(T_s)\frac{d\nu}{h\nu}\Big/\int_0^{\infty}S_{pg}F_\nu(T_s,g)\,d\nu\right]\times\left[\int_{4\nu_0}^{\infty}F_\nu(T_s,g)\frac{d\nu}{h\nu}\Big/\int_{\nu_0}^{\infty}B_\nu(T_s)\frac{d\nu}{h\nu}\right], \tag{3}$$

where $B_\nu(T_s)$ is the Planck function.

Here we use the emergent flux distributions that were determined by Böhm and Deinzer (1966) for the Harmon-Seaton (1966) sequences of stars with masses between 0.5 and 1 solar mass. Therefore, the gravity $g$ depends on the temperature $T_s$ in a known way. The first ratio in equation (3) is nearly equal to

$$\int_{\nu_0}^{\infty}B_\nu(T_s)\frac{d\nu}{h\nu}\Big/\int_0^{\infty}S_{pg}B_\nu(T_s)\,d\nu$$

when the Böhm and Deinzer flux distributions are used. The latter ratio is denoted by $F(T_s)$ and tabulated by O'Dell (1963). Equations (1) and (3) were used to determine the effective temperatures of the central stars of the thirty-five objects under consideration.

In Table 1 the black-body temperatures listed in Paper I are given in the second column and the effective temperatures for the non-gray models are given in the third column. One can see that the temperature of the central star found from the non-gray model is larger in each case than the assigned black-body temperature. This is a result of the fact that a Böhm and Deinzer model of a particular effective temperature shows a depletion in the continuum beyond the $He^+$ Lyman limit (due primarily to the absorption by Ne, He, N, C, and O ions in the stellar atmosphere) when compared to a black body with the same temperature.

## III. CALCULATIONS OF THE OPTICAL THICKNESSES IN THE $H^0$ LYMAN CONTINUA

The probability of absorption of direct radiation from the central star beyond the Lyman limit is defined to be

$$\bar{G}_1=\int_{\nu_0}^{\infty}\left\{1-\exp\left[-\tau\left(\frac{\nu_0}{\nu}\right)^3\right]\right\}F_\nu(T_s,g)\frac{d\nu}{\nu}\Big/\int_{\nu_0}^{\infty}F_\nu(T_s,g)\frac{d\nu}{\nu}. \tag{4}$$

This can be expressed in terms of $G_1$, which is defined in Paper I as the probability of absorption of the central star radiation beyond the Lyman limit, assuming the central star radiates as a black body, by

$$\bar{G}_1=\left[\!\left[\int_{\nu_0}^{\infty}\left\{1-\exp\left[-\tau\left(\frac{\nu_0}{\nu}\right)^3\right]\right\}F_\nu(T_s,g)\frac{d\nu}{\nu}\Big/\int_{\nu_0}^{\infty}\left\{1-\exp\left[-\tau\left(\frac{\nu_0}{\nu}\right)^3\right]\right\}B_\nu(T_s)\frac{d\nu}{\nu}\right]\!\right]\div\left[\int_{\nu_0}^{\infty}B_\nu(T_s)\frac{d\nu}{\nu}\Big/\int_{\nu_0}^{\infty}F_\nu(T_s,g)\frac{d\nu}{\nu}G_1\right]. \tag{5}$$

The determination of $\bar{G}_1$ then reduces to the evaluation of the above integrals since $G_1$ is already known for various combinations of $T_s$ and $\tau$. The integrals were evaluated numerically by fitting polynomials between successive absorption edges and then integrating the resulting polynomial. The integrals involving $B_\nu(T_s)$ were done in the same manner so as to avoid systematic errors. The value of the second ratio in equation (5), done by this method, differed by less than 1 per cent from the value found by considering the area under a plot of $B_\nu(T_s)$ versus $\nu$. The values of $\bar{G}_1$ are given in Table 2 as a function of $\tau$ and $T_s$. The values of $T_s$ were those used by Böhm and Deinzer. Since the largest value of $T_s$ is 150000° K, we must extrapolate when a star's effective temperature appears greater than 150000° K.

TABLE 1

EFFECTIVE TEMPERATURES OF CENTRAL STARS

| | $T_s$ (Paper I) | $T_s$ | $\bar{y}(T_s)$ | $\bar{G}_1$(min) | $\bar{G}_1$(max) | $\tau$(min) | $\tau$(max) |
|---|---|---|---|---|---|---|---|
| NGC 650-1 | 182000 | 195000? | 0 251 | 0 49 | 0 61 | 17 | 30 |
| IC 351 . | 91000 | 108000 | 028 | 06 | 14 | 0 34 | 0 92 |
| NGC 1501 | 72000 | 88000 | 0063 | 021 | 035 | 0 08 | 0 19 |
| NGC 1535 | 74000 | 90000 | 0071 | 039 | 063 | 0 18 | 0 32 |
| NGC 2022 . | 91000 | 108000 | 028 | 034 | 055 | 0 17 | 0 30 |
| NGC 2371-2 | 100000 | 118000 | 047 | 053 | 085 | 0 32 | 0 50 |
| NGC 2392 | 68000 | 84000 | 004 | 008 | 013 | < 0 05 | 0 065 |
| NGC 3242 | 93000 | 109000 | 028 | .065 | .10 | 0 40 | 0 62 |
| NGC 3587 | 105000 | 123000 | 056 | .24 | .34 | 2 0 | 3 4 |
| NGC 6058 | 72000 | 88000 | 0063 | 016 | .026 | 0 07 | 0.10 |
| NGC 6309 | 95000 | 112000 | 036 | 054 | 087 | 0 25 | 0.50 |
| NGC 6439 | 72000 | 88000 | 0063 | 045 | .074 | 0 22 | 0 34 |
| NGC 6445 | 182000 | 206000? | .282 | .67 | 77 | 40 | 100 |
| NGC 6543 | 66000 | 82000 | 0032 | 25 | 36 | 1 4 | 2 4 |
| NGC 6572 | 61000 | 77000 | 0015 | 17 | 25 | 0 73 | 1 4 |
| NGC 6751 | 76000 | 92000 | 0089 | 031 | 056 | 0 12 | 0 34 |
| NGC 6772 | 112000 | 130000 | 074 | 26 | 37 | 2 5 | 4 2 |
| NGC 6778 | 85000 | 100000 | 018 | 12 | 19 | 0 65 | 1 2 |
| NGC 6781 | 91000 | 110000 | 032 | 13 | 21 | 0 73 | 1 3 |
| NGC 6804 | 72000 | 88000 | 0071 | 014 | 023 | 0 07 | 0 09 |
| BD 30°3639 | 46000 | 60000 | 00008 | 001 | 002 | < 0 05 | < 0 05 |
| NGC 6818 | 195000 | 222000? | 355 | 49 | 61 | 25 | 40 |
| NGC 6826 | 69000 | 85000 | 0045 | 094 | .14 | 0 44 | 0 69 |
| NGC 6853 | 132000 | 148000 | 120 | 45 | 58 | 7 9 | 15 5 |
| NGC 6881 | 69000 | 85000 | 0045 | 015 | 025 | 0 07 | 0 10 |
| NGC 6891 | 56000 | 71500 | 0006 | 038 | 062 | 0 18 | 0 27 |
| NGC 6894 | 98000 | 116000 | 043 | 20 | 30 | 1 5 | 2 5 |
| NGC 6905 | 102000 | 120000 | 050 | 11 | 17 | 0 72 | 1 2 |
| NGC 7008 | 98000 | 116000 | 045 | 057 | 092 | 0 33 | 0 55 |
| NGC 7009 | 81000 | 98000 | 0063 | 056 | .09 | 0 28 | 0 43 |
| NGC 7026 | 98000 | 116000 | 043 | 20 | 29 | 1 5 | 2 5 |
| NGC 7139 | 98000 | 116000 | 043 | 27 | 38 | 2 1 | 3 8 |
| IC 5217 | 74000 | 90000 | 0074 | 07 | 11 | 0 36 | 0 50 |
| NGC 7354 | 102000 | 120000 | 050 | 11 | 18 | 0 73 | 1 3 |
| NGC 7662. | 100000 | 118000 | 0 047 | 0 10 | 0 15 | 0 58 | 1 0 |

Following the arguments in Paper I, upper and lower limits can be placed on $\bar{G}_1$ and correspondingly on $\tau$ by the relations

$$\bar{G}_1(\min) = 1.22\bar{y}(T_s)\frac{F(\mathrm{H}\beta)}{F(4686)} \tag{6}$$

and

$$\bar{G}_1(\max) = \frac{\bar{G}_1(\min)}{0.4+0.6G_1(\min)}, \tag{7}$$

where $\bar{y}(T_s)$ is here defined to be

$$\bar{y}(T_s) = \int_{4\nu_0}^{\infty} F_\nu(T_s,g)\frac{d\nu}{\nu} \Big/ \int_{\nu_0}^{\infty} F_\nu(T_s,g)\frac{d\nu}{\nu}, \tag{8}$$

A theoretical curve was constructed of $\bar{y}(T_s)$ versus $T_s$. Thus for the temperatures determined in § I and the $F(H\beta)/F(4686)$ ratio given in Paper I, $\bar{G}_1$(min) and $\bar{G}_1$(max) can be determined. By interpolation in Table 2 or by interpolation in a $\bar{G}_1$ versus $T_s$ plot, with $\tau$ being the free parameter, $\tau$(min) and $\tau$(max) can be found. Table 1 lists the values of $\bar{G}_1$(max), $\bar{G}_1$(min), $\tau$(max), and $\tau$(min) for thirty-five planetary nebulae assuming, as in Paper I, that $T_e \simeq 1 \times 10^4$ ° K. It can be seen from Table 1 that only ten of the thirty-five nebulae have minimum optical depths in the $H^0$ Lyman continua that are greater than unity.

As pointed out above, the models of central stars made by Böhm and Deinzer (1966) indicate ultraviolet deficits for frequencies beyond the $He^+$ Lyman limit. The following analysis is carried out in order to provide insight into the effects of the ultraviolet deficits of a non-gray model when it is used to determine $G_1$ and $\tau$.

TABLE 2

$\bar{G}_1$, PROBABILITY OF ABSORPTION OF DIRECT RADIATION OF CENTRAL STAR BEYOND THE LYMAN LIMIT FOR A NON-GRAY SOURCE

| $\tau$ | $T_s$(°K) | | | | |
|---|---|---|---|---|---|
| | 46250 | 63100 | 91200 | 10600 | 15000 |
| 0 05 . | 0 017 | 0 012 | 0 011 | 0 011 | 0 011 |
| 0 10 | 035 | 029 | 025 | 022 | 022 |
| 0 50 | 155 | 125 | .102 | 084 | .064 |
| 1 00 | 276 | 221 | 185 | 151 | .116 |
| 2 50 | 499 | 419 | 350 | 296 | 231 |
| 5 00 | 682 | .601 | 506 | .437 | 352 |
| 7 50 | 766 | 689 | 595 | 526 | 432 |
| 10 00 | 817 | .750 | 659 | 590 | 490 |
| 25 00 | 934 | 902 | 833 | 786 | 677 |
| 50 00 | 977 | 965 | 927 | 902 | .803 |
| 75 00 | 991 | 984 | 960 | .948 | 897 |
| 100 00 | 0 996 | 0 992 | 978 | 971 | 966 |
| 250 00 . | . | . | 997 | 997 | 966 |
| 500.00 . | | .. | 0 998 | 0 999 | 988 |
| 750 00 . | . | | | | .994 |
| 1000 00 . | .. | | | ... | 0.996 |

From equations (3), (6), and (8) we obtain

$$\bar{G}_1(\min) = 1.22W(T_s,g)\frac{F(H\beta)}{F(4686)}\left[\int_0^\infty S_{pg}B_\nu(T_s)\,d\nu \Big/ \int_0^\infty B_\nu(T_s)\frac{d\nu}{h\nu}\right]\ldots$$
$$\left[\int_{\nu_0}^\infty B_\nu(T_s)\frac{d\nu}{\nu}\Big/\int_{\nu_0}^\infty F_\nu(T_s,g)\frac{d\nu}{\nu}\right]. \tag{9}$$

The product $1.22W(T_s,g)\ F(H\beta)/F(4686)$ is determined observationally for each object and is therefore independent of the choice of central star model. It turns out that

$$\int_{\nu_0}^\infty F_\nu(T_s,g)\frac{d\nu}{\nu}\Big/\int_{\nu_0}^\infty B_\nu(T_s)\frac{d\nu}{\nu}\simeq 1$$

when the emergent flux distributions are those of Böhm and Deinzer. The ratio

$$\int_0^\infty S_{pg}B_\nu(T_s)\,d\nu\Big/\int_{\nu_0}^\infty B_\nu(T_s)\frac{d\nu}{h\nu}$$

decreases as $T_s$ increases. The value of the effective temperature determined from the non-gray model is greater than the value of the effective temperature determined from the black-body model in each case. Therefore, $\bar{G}_1$(min) < $G_1$(min), where $G_1$(min) is the minimum value of the probability of absorption by the nebula of direct radiation of the central star beyond the Lyman limit for a black-body source.

The characteristic optical thickness increases with an increase in $G_1$(min) when the effective temperature is held constant, but it also increases with an increase in effective temperature when $G_1$(min) is held constant. Although $G_1$(min) < $G_1$(min), $T_s$(non-gray) > $T_s$(black body). Therefore, it is difficult to predict whether the value of a characteristic optical thickness at the Lyman limit obtained from any non-gray model should be larger or smaller than the value obtained using the black-body model. It is fact, however, that nearly all the values of the characteristic optical thicknesses obtained from the Böhm and Deinzer models are smaller than the respective values obtained from the black-body models.

It should be noted that we did not strictly follow the Harmon and Seaton sequences. We don't allow for the fact that two central stars with two different surface gravities may have the same temperatures. Only the hottest stars considered here are affected and they don't fall on a Harmon and Seaton sequence anyway.

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