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A COMPOSITE PERIOD-LUMINOSITY RELATION FOR CEPHEIDS AT MEAN AND MAXIMUM LIGHT

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ABSTRACT

Modern two-color data for Cepheids in the Large and Small Magellanic Clouds, M31, and NGC 6822 are combined into composite period-luminosity relations in M_B and M_V at mean and maximum light. The data are consistent with the view that the *P-L* relations are the same from galaxy to galaxy in both slope and zero point.

The *P-L* relations show dispersions of ± 0.60 mag in M_B and ± 0.50 mag in M_V about the center line. The magnitude residuals, R_B and R_V, from the center line for individual stars, correlate well with the color residuals, $\delta(B - V)$, from the ridge line of the period-color relation. Such a correlation is expected from the theory of the Cepheid instability strip in the color-magnitude diagram. Theory and observation agree well, and this is taken as a proof that the observed dispersion in the *P-L* relation is primarily intrinsic.

The absolute calibrations of the *P-L* relations are made using nine Cepheids in the galactic system whose absolute luminosities are known by photometric parallax methods Tables of the adopted period-luminosity and period-color relations are given in the Appendix.

I. INTRODUCTION

Distances to nearby galaxies beyond the Local Group can be found from various resolved components of the stellar content. These include (1) Cepheids, (2) brightest blue stars, (3) brightest red stars, (4) angular sizes of H II regions, (5) globular clusters, and (6) normal novae. For galaxies with $(m - M) > 32$, the redshift itself must be used, but only when the Hubble constant has been well established.

There are several observational programs now in progress for a redetermination of this parameter. Due to the recent semi-empirical understanding of the Cepheid period-luminosity relation, including its intrinsic dispersion (Sandage 1958; Kraft 1961), these variables still seem to constitute the most accurate distance indicator against which all other calibrators can be compared. But Cepheids can usually be detected and measured only near maximum light in galaxies where $28 > (m - M) > 25$, and it is necessary to have a calibrated *P-L* relation in M_B(max) and M_V(max) if distances are to be determined in this way. It is also necessary to establish, or to make some assumptions about, the uniqueness of the *P-L* relation so as to assess possible systematic differences between galaxies.

A composite *P-L* relation is obtained here from modern two-color data for Cepheids in the Large Magellanic Cloud (LMC), the Small Magellanic Cloud (SMC), M31, and NGC 6822. The five available Cepheids in open clusters of the galactic system, together with four possible members of the extended h and χ Per association are used for the absolute calibration. The uniqueness of the *P-L* relation cannot be precisely established, although the evidence is consistent with the statement that no difference of slope of the ridge line of the *P-L* relation exists between the four galaxies over the period range $0.4 < \log P < 2.2$, or between these galaxies and the nine calibrating Cepheids in the range $0.49 < \log P < 1.13$. Although this does not prove uniqueness of the zero point, it is shown in a following paper on NGC 2403 that, if the distances obtained here are

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adopted, then the secondary distance indicators agree within the accuracy of the method (± 0.2 mag), in the LMC, the SMC, NGC 6822, and NGC 2403.

II. COMPOSITE *P-L* RELATION AT MEAN LIGHT

Modern two-color data exist for more than one hundred Cepheids in galaxies in the Local Group. Photoelectric measurements by Gascoigne and Kron (1965) are available for thirteen Cepheids in the LMC and fourteen in the SMC in the period range $4^{d}.5 < P < 134^{d}$. Photographic data, interpolated from photoelectric sequences, are also available for sixty-four stars in the SMC (Arp 1960), fifty-three stars in the LMC (Woolley, Sandage, Eggen, Alexander, Mather, Epps, and Jones 1962; Dickens 1966), twenty stars in M31 (Baade and Swope 1963), and thirteen stars in NGC 6822 (Kayser 1967). Separate discussions of some of these data had previously suggested (Sandage 1962; Woolley *et al.* 1962) that the slopes of the *P-L* relations differ between the several galaxies, and therefore that the construction of a composite curve is unjustified. The present paper suggests that this view is too pessimistic, and that the apparent differences in slope may not in fact be real, but result from small-sample statistics in the presence of the intrinsic dispersion of the *P-L* relation.

Figure 1 shows the composite *P-L* relation at mean light in both B and V wavelengths, obtained by combining the available data, using true distance moduli quoted in the lower right of panel *b*. Intensity means, $\langle B\rangle_{\rm int}$ and $\langle V\rangle_{\rm int}$, have been used as tabulated in the original references, but corrected for absorption by $A_B = 4E(B - V)$ and $A_V = 3E(B - V)$ with assumed galactic reddening of $E(B - V) = 0.05$ for the LMC and SMC, 0.16 for M31, and 0.27 for NGC 6822. Justification for these values comes from the authors cited.

The adopted true moduli were obtained by combining the individual *P-L* relations, using vertical shifts such that the composite relation had the smallest dispersion, read at constant period. The calibration was then achieved using the standard Cepheids discussed in § III. The moduli adopted here from the Cepheids alone agree remarkably well with values previously obtained by using all distance indicators as discussed elsewhere (Sandage 1962; van den Bergh 1967).

Figure 1 indicates that the *P-L* relation has intrinsic scatter amounting to ± 0.6 mag about the ridge line in B, and ± 0.5 mag in V—a total scatter which is considerably larger than can be attributed to errors of observation. That the scatter is real also follows from theory, if the Cepheid instability strip in the M_V, $B - V$ plane has a finite width in color. This is because the upper and lower envelope lines of the *P-L* relation are the traces of the blue and red color boundaries of the instability strip in the color-magnitude diagram (Sandage 1958).

The theory of the mapping follows from the assumptions that $P\sqrt{\rho} = Q$. The most recent linearized version of the theory is discussed by Sandage and Gratton (1963), where the equation

$$\log P + 0.239\, M\langle_V\rangle - 0.602\, (\langle B\rangle - \langle V\rangle) = 0.838 + \log Q \qquad (1)$$

is derived using Kraft's (1961) version of Oke's temperature scale for supergiants given by

$$\log T_e = 3.886 - 0.175\, (\langle B\rangle - \langle V\rangle) \qquad (1.0 > B - V > 0.4)\,, \qquad (2)$$

together with a linearized bolometric correction of

$$M_{\rm bol} = M_V + 0.145 - 0.322(B - V) \qquad (1.0 > B - V > 0.4)\,. \qquad (3)$$

Equation (3) follows from Oke's work on δ Cep quoted by Kraft (1961, Table 5). The equation in M_B corresponding to equation (1) is

$$\log P + 0.239\, M\langle_B\rangle - 0.841\, (\langle B\rangle - \langle V\rangle) = 0.838 + \log Q\,. \qquad (4)$$

An important property of equations (1) and (4) is their explanation of the smaller observed intrinsic dispersion of the *P-L* relation in M_V than in M_B, read at constant period. For a given total width, $\Delta(B - V)$, of the instability strip, the expected variations of M_V and M_B are

$$\Delta M_V = 2.52\Delta(B - V)\,, \qquad \Delta M_B = 3.52\Delta(B - V)\,, \tag{5}$$

which follow from equations (1) and (4) if Q does not vary with $B - V$ across the strip, but is at most a function of P.

The most stringent test of the theory is its detailed application to each variable in

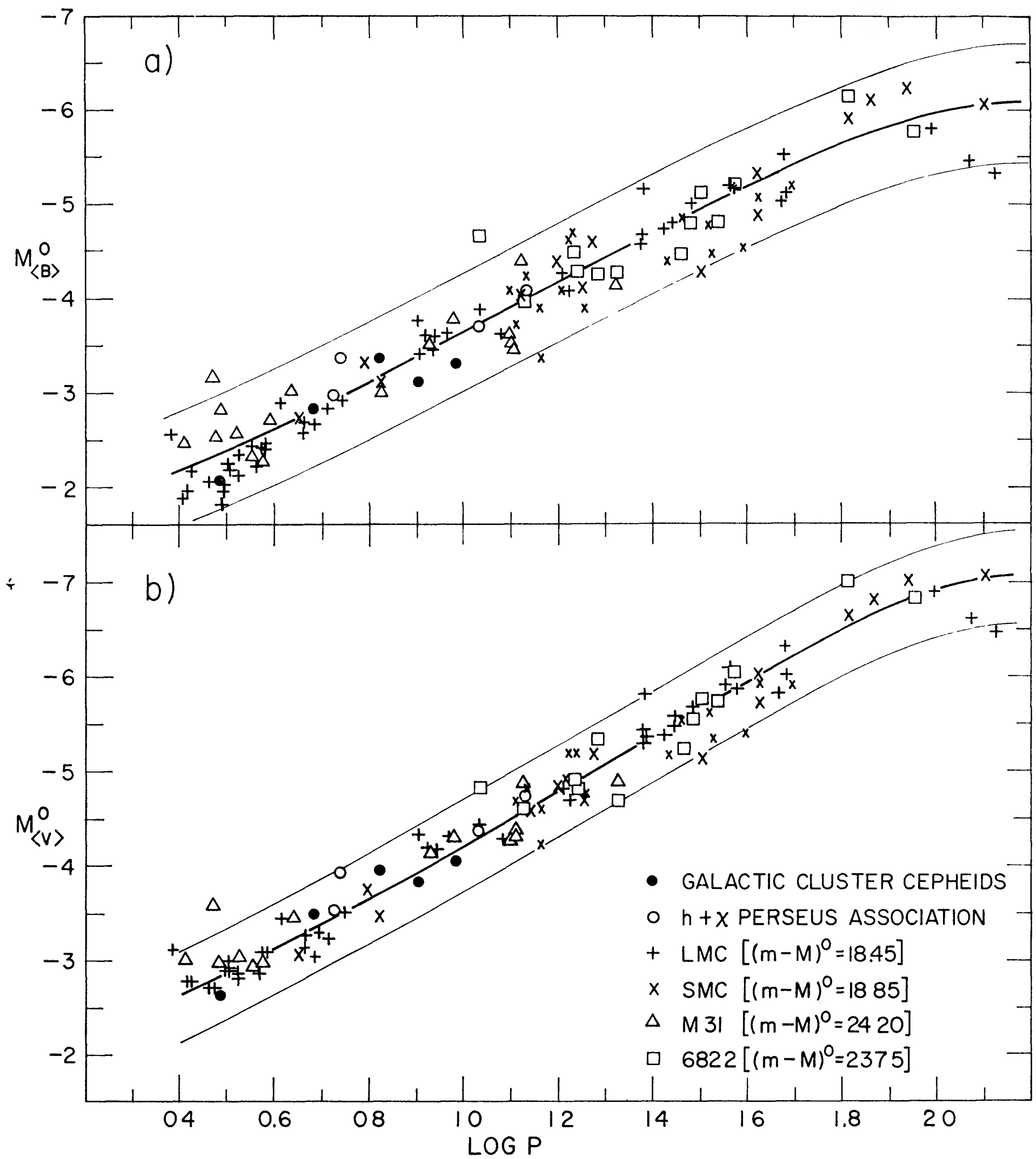

FIG. 1.—The composite period-luminosity relation at mean intensity in B and V wavelengths derived from the sources indicated at the lower right. The absolute calibration was made by using the nine Cepheids of the galactic system shown as open and filled circles. The photographic data from the SMC are plotted with smaller crosses than the Gascoigne and Kron photoelectric data.

Figure 1. In the absence of knowledge of a possible $Q = f(P)$ variation, the most direct test is through equations (5) which are the predicted correlations between the residuals of each star in magnitude and color from the ridge lines of the *P-L* and *P*-color relations. These ridge lines are the traces of the center of the instability strip of the color-magnitude diagram onto the *P-L* and *P*-color planes. Equations (5) define the slope of the lines of constant period in the color-magnitude diagram. The slope will be independent of period as long as Q is not a function of both P and $B - V$. Excluding this possibility, the independence permits us to combine stars of different period into the same deviation diagram.

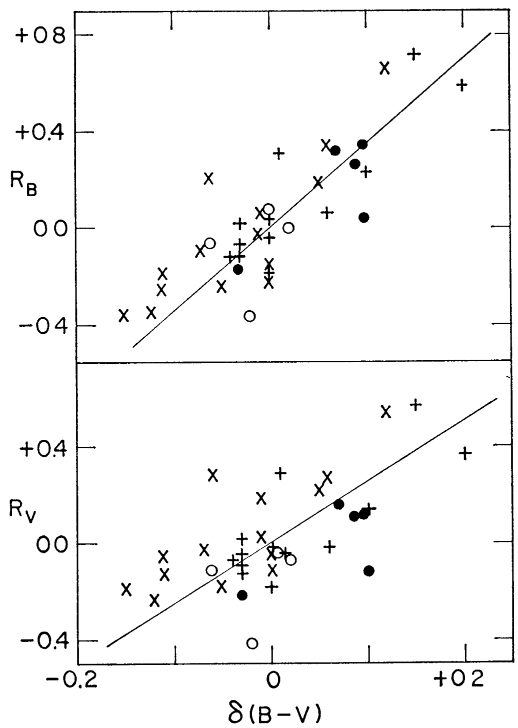

FIG. 2.—Correlation of the magnitude and color residuals at mean light for the photoelectric stars of Gascoigne and Kron and for the nine calibrating Cepheids The coding is the same as in Fig. 1. A positive sign indicates the star is fainter and redder than the ridge lines of Figure 1 and equations (6) and (7). Lines with the theoretical slopes of equations (5) are drawn.

Figure 2 shows the observed correlations for the Cepheids studied photoelectrically by Gascoigne and Kron (1965). The fourteen variables of the SMC are shown by crosses, and the thirteen stars in the LMC by plus signs. The nine calibrating Cepheids discussed later are shown as open and closed circles. The magnitude residuals, denoted by R_B and R_V, are taken relative to the central lines of Figure 1, *a* and *b*, and the positive sign is used when the star is fainter than this ridge line. The color residuals, denoted by $\delta(B - V)$, are relative to the mean period-color relation defined by the twenty-seven Magellanic Cloud stars themselves and given by

$$\langle B\rangle - \langle V\rangle = 0.438 \log P + 0.06\,, \tag{6}$$

where intensity means are used for B and V. A positive sign to $\delta(B - V)$ indicates that the star is observed to be redder than given by equation (6).

Although there is considerable scatter, Figure 2 shows that the residuals are correlated in the required sense that the fainter stars are redder. Furthermore, the lines are drawn in the diagram with the *theoretical* slopes of equations (5), and the agreement seems quite satisfactory. However, the apparent agreement can be taken as confirmation of equations (1) and (4) *only* if there is no differential reddening across the face of the clouds. Such reddening, with its accompanying absorption, will produce an almost identical correlation if the normal ratios of absorption-to-reddening of $\Delta M_B = 4E(B - V)$ and $\Delta M_V = 3E(B - V)$ apply. However, we do not believe that such patchy absorption exists for the present material for reasons discussed later in this section, and do, therefore, accept Figure 2 as indicating that the observed dispersion of Figure 1 is in substantial part intrinsic.

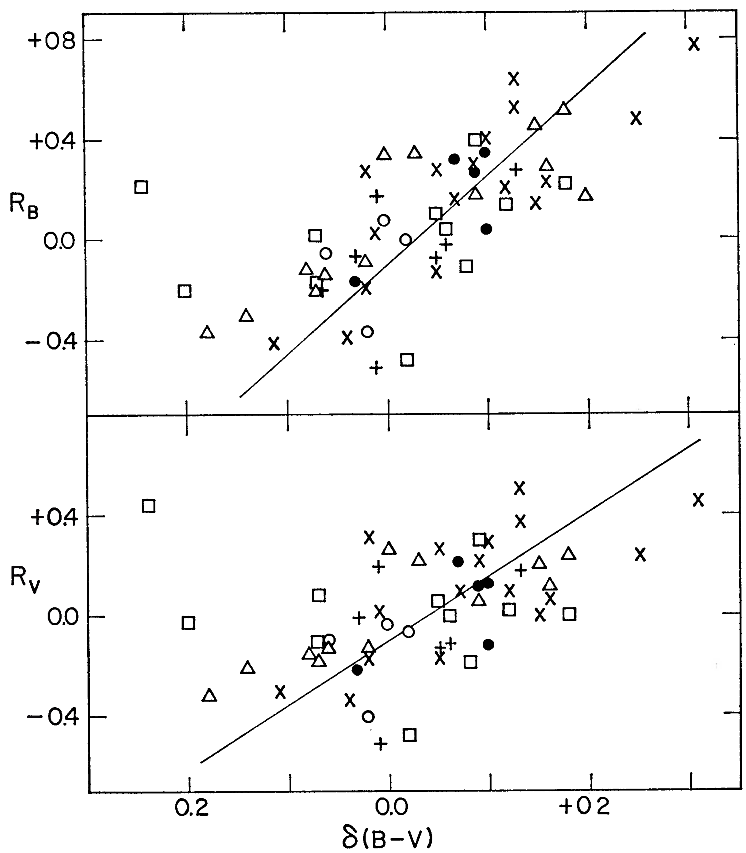

FIG. 3.—Same as Fig 2 for the non-photoelectric data of Fig 1 with the restriction that stars with log $P < 1.10$ are excluded in both the SMC and LMC. The coding is the same as in Fig. 1.

It is now of interest to see if the remaining photographic stars conform with the predictions. Figure 3 shows the deviation curve in B and V for all stars with adequate data. There are thirteen available Cepheids in M31 (Baade and Swope 1963; *open triangles*), thirteen in NGC 6822 (Kayser 1967; *open squares*), seventeen in the SMC (Arp 1960, restricted to log $P > 1.10$; *crosses*), and seven in the LMC (Woolley *et al.* 1962; Dickens 1966, restricted to log $P > 1.10$; *plus signs*). The period restriction for the photographic data in the SMC and LMC guards against inclusion of material of possibly lower weight which could mask the delicate correlations sought. By this restriction we have excluded stars fainter than about $B \simeq 15$ mag on the assumption that the somewhat anomalous run of observed colors, relative to the photoelectrically determined Gascoigne and Kron color-period relation, for stars fainter than $B \simeq 15$ mag reflects systematic errors in the photographic photometry. This is reasonable in view of the extreme difficulty of obtain ing the requisite accuracy in photographic photometry at this magnitude level with the equipment used.

In Figure 3, R_B and R_V are again the magnitude differences from the central lines of Figure 1. The color differences for stars in NGC 6822, the LMC, and the SMC are again relative to equation (6). However, the color differences for M31 pose a problem. The extensive and virtually definitive work of Kron and Svolopoulos (1959); Kraft (1961); Bahner, Hiltner, and Kraft (1962); Fernie (1963, 1967); Johnson (1964); and Nikolov (1967) on the normal colors of luminosity class I stars and of galactic Cepheids leaves little doubt that the period-color relation for Cepheids in the galactic system differs from that for Cepheids in the Clouds studied by Gascoigne and Kron (1965, Fig. 10). Results for the galactic Cepheids were derived in the following way. New photoelectric photometry compiled by Mitchell, Iriarte, Steinmetz, and Johnson (1964) in what is hereafter called the "Tonantzintla Catalogue," together with Kraft's original spectral classifications and some additional unpublished spectroscopic and Γ-photometric observations which Kraft very kindly put at our disposal, permitted a rediscussion of the $E(B - V)$ values and the intrinsic colors of the Cepheids. To use Kraft's method (1961), knowledge of the intrinsic colors of supergiants is necessary. Fernie's (1963, 1967) intrinsic colors are in the average 0.04 to 0.06 mag redder than Kraft's (1961) or Kraft and Hiltner's (1961) values. However, Johnson (1964), using a independent method, found no systematic deviations from Kraft's colors from F5 to G2, but suggested that G-type supergiants were somewhat bluer than Kraft's values. Nikolov's colors (1967) average only 0.02 redder than those of Kraft from F5 to G2, but become progressively redder for stars later than G3, reaching a difference of 0.08 mag at G8. Considering the *opposite sign* and the smallness of the Fernie and Nikolov corrections compared with the Johnson values, we have used only Kraft's results, which are close to the mean of all present determinations.

The rediscussion of the Cepheids, whose principal innovation is the use of the Tonantzintla Catalogue, leads practically to the same values as previously given (Kraft 1961; Bahner, Hiltner, and Kraft 1962). We have, however, added a few galactic Cepheids with $P > 10$ days from other sources (Kron and Svolopoulous 1959; Mianes 1963; Fernie, Hiltner and Kraft 1966). Two stars are included from Kron and Svolopoulos and three from Mianes, although there are systematic differences of 0.04 and 0.15 mag, respectively (both in the sense of redder intrinsic colors), for other stars in common. Of the longer-period Cepheids, RW Cam and CH Cas were omitted because the former is double (Mianes 1963) and the latter is of Population II (Kraft 1963).

The adopted values for the galactic Cepheids at mean and at maximum light are compiled in Table A2 (see Appendix). At mean light the data are satisfied by the average relation

$$\langle B\rangle_0 - \langle V\rangle_0 = 0.264 \log P + 0.37 . \quad (7)$$

This relation intersects equation (6) at $\log P = 1.78$, but is 0.20 mag bluer at $\log P = 0.6$.

Additional evidence suggests the difference may be real. The colors determined by Baade and Swope (1963) for fourteen Cepheids in Field IV of M31 follow equation (7) very well after correction is made for galactic reddening of $E(B - V) = 0.16$. On the other hand, Kayser's recent work on NGC 6822 (1967, Fig. 7) follows equation (6) after applying $E(B - V) = 0.27$.

Although the reason for this difference is not understood, two routes are open in the present context:

1. One can assume that the color difference represents a real difference between galaxies of the ridge-line temperature for the instability strip. This would be most serious because the parent expression leading to equation (1) would require that the slope of the P-L relation should be considerably less steep for the LMC, SMC, and NGC 6822 than

for M31 and the galactic Cepheids. This means that if the separate *P-L* relations for galaxies with different period-color relations are normalized at log $P = 1.78$ where the colors agree, the *P-L* functions should diverge by $\Delta B = 0.28$ mag, $\Delta V = 0.20$ mag at log $P = 1.3$ where the colors differ by 0.08 mag; and $\Delta B = 0.70$, $\Delta V = 0.50$ at log $P = 0.6$ where the color difference reaches 0.2 mag. These results follow from the coefficients of equations (5). The sense of the difference is that the M31 and galactic Cepheids should be *fainter* than those in the Magellanic Cloud–type galaxies.

This interpretation may be checked by looking for differences of slope of the *P-L* relation between $1.3 > \log P > 0.6$ where the data for the two types of galaxies overlap. The expected magnitude difference between these two limiting periods is $\Delta M_B = 0.42$, and $\Delta M_V = 0.30$. Inspection of Figure 1, *a* and *b*, shows that no such systematic divergence exists of the proper sign. A slight effect seems to exist in M_B, with the M31 data having a smaller slope, but this is the wrong sense. No divergence exists in M_V.

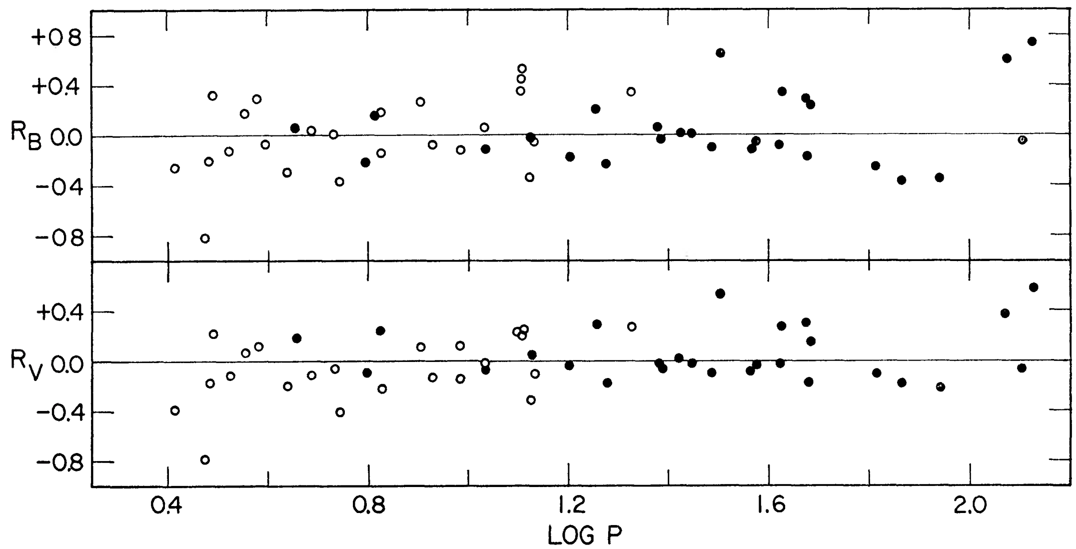

FIG. 4.—Correlation of the magnitude residuals at mean light with log *P* for the photoelectric data of Gascoigne and Kron in the Clouds (*closed circles*) and for the nine calibrating Cepheids of the galactic system plus the variables in M31 (*open circles*).

The absence of a slope-difference is most clearly seen in Figure 4 where the residuals from the central curves of Figure 1 are plotted against log *P* for the two types of systems (M31 plus galactic Cepheids as open circles; SMC plus LMC as closed circles). That there is no difference of slope of the *P-L* relation between these galaxies is proved by the absence of a systematic variation of the residuals with period. We therefore reject the possibility that the color differences signal a temperature difference and therefore a systematic change in the form of the *P-L* relation from galaxy to galaxy.

2. The second route is to turn a blind eye to the color difference because it apparently has no effect on the slope. With this point of view, equation (6) can be used as before to derive $\delta(B - V)$ for stars in the LMC, SMC, and NGC 6822, and equation (7) for stars in M31 together with the nine Cepheids in the galactic system. Figure 3 is constructed on this basis. Lines with the theoretical slopes are superposed, and again the agreement is satisfactory.

Although the tests just described provide justification for the view that Figure 1 is systematically correct and does not vary from galaxy to galaxy, there does remain the question of the previously reported differences of slope of the *P-L* relation. Arp (1960) derived $\Delta M_B/\Delta \log P = -2.23$ for the SMC, while Woolley *et al.* (1962) obtained -2.85

for the LMC—a value which is close to the slope of the linear portion of Figure 1. Examination of the data shows that, although the seven brightest stars of Arp with $1.7 > \log P > 1.4$, which are important in fixing his slope, are all fainter than the ridge lines of Figure 1, *they also are redder than expected from equation* (*6*) *by the amount required by equations* (*5*). These seven stars, therefore, follow the deviation pattern of Figure 3. One might, therefore, interpret part of the apparent difference of slope as a result of small-sample statistics in the presence of a relative large intrinsic dispersion. If this is so, the seven brightest stars would not fill the instability strip uniformly but would all be accidentally fainter than average. Although the fainter stars of Arp with $\log P < 0.5$ also contribute to his slope determination, these variables, not plotted in Figure 1, average only about 0.10 mag brighter than the mean line in B and about 0.15 mag brighter in V. These differences are quite small and show that accuracies of ± 0.2 mag must be maintained at faint levels if true differences in slope are eventually to be found. Although we do not know if this explanation for the discrepancy between Arp's work and the present results is correct, it is true that no incontrovertible evidence exists supporting a difference of slope between the LMC and SMC in view of the agreement of slope obtained by Gascoigne and Kron.[1]

Finally, Figure 1 can be compared with P-L relation for the SMC derived by the Gaposchkins (1966, Fig. 6). The detailed agreement is only moderately good, and the most serious qualitative difference is for the longest-period Cepheids with $\log P > 1.7$, where no evidence of the bright turn-over point is present in the Gaposchkin data. The difference is mainly due to the very large corrections for internal absorption which the Gaposchkins have applied to individual stars. If these corrections are removed, the agreement between $\langle m \rangle$ and $\langle B \rangle$ is good.

The decision to avoid such corrections in the present data rests on three points. (1) The colors of Gascoigne and Kron for the relevant SMC variables with $\log P > 1.7$ are already bluer than the average relation defined by equation (6). Absorption corrections of 0.8 mag would make them even bluer by $\Delta(B - V) \simeq 0.2$ mag, putting them at $\langle B \rangle - \langle V \rangle \simeq 0.55$ which appears to be impossible for Cepheids in this period range. (2) Such corrections to the longest-period stars would virtually remove any dependence of $\langle B \rangle - \langle V \rangle$ on period at mean light. This would be serious in view of the well-established relation between spectral type and period at mean light for galactic Cepheids. (3) The observed dispersion in Figure 1 is already close to the value expected from the known width of $\Delta(B - V) = 0.30$ mag for the instability strip of galactic Cepheids (Kraft 1961), and no large additional spread is available for internal absorption among the Cepheids in the present material. The most that can be permitted is an additional $\Delta M_B = 1.2 - 3.52\Delta(B - V) = 0.15$ mag, and $\Delta M_V = 1.0 - 2.52\Delta(B - V) = 0.25$ mag, where the value of $\Delta(B - V) = 0.30$ mag and the observed dispersions of Figure 1 are used. These values are so small that they would seem to provide strong evidence for the absence of internal absorption for the particular Cepheids studied here.

In view of the arguments of this section, we adopt the composite relation of Figure 1 as a standard to be subsequently applied to other galaxies in the field. The mean relation is listed in Table A1 of the Appendix.

III. THE ABSOLUTE CALIBRATION

Table 1 gives the adopted data for the nine calibrating Cepheids, five of which are from the well-known galactic clusters, and four are taken from Eggen's (1965) discussion of the h and χ Per association. The adopted moduli of the clusters are given in the ninth

[1] *Note added October 16, 1967:* Two independent studies reported at the 1967 Prague meeting of the International Astronomical Union, one by Andrews working at Pretoria, and the other comprising work at Boyden as summarized by Wayman, showed that Arp's photometric scale could not be reproduced fainter than $V \simeq 14$. The sign and the size of the deviation from a Pogson scale is very close to that needed here. We consider this to be strong evidence for our present point of view.

TABLE 1

PARAMETERS FOR CEPHEIDS IN GALACTIC CLUSTERS AND IN THE h AND χ PER ASSOCIATION

Star	Cluster	log P	$E(B-V)$	$\langle B\rangle_0$	$\langle V\rangle_0$	B_0(max)	V_0(max)	$(m-M)_0$	$M_{\langle B\rangle_0}$	$M_{\langle V\rangle_0}$	M_{B_0}(max)	M_{V_0}(max)
S Nor.	NGC 6087	0.989	0.23	6.46	5.73	6.02	5.42	9 76	−3.30	−4.03	−3 74	−4.34
DL Cas.	NGC 129	0.908	0.50	8.14	7 44	7 74	7.19	11.28	−3.14	−3.84	−3.54	−4.09
U Sgr..	M25	0.828	0.55	5.61	5.05	5.07	4.71	8.98	−3.37	−3.93	−3.91	−4.27
CF Cas..	NGC 7790	0.687	0.555	10 09	9.44	9.65	9.18	12.90	−2.81	−3.46	−3.25	−3.73
EV Sct	NGC 6664	0.490	0.58	8.98	8.41	8.64	8.16	11.03	−2.05	−2.62	−2.39	−2.87
SZ Cas	h+χ Per	1.134	0.88	7.80	7.19	7 46	6.97	11.90	−4.10	−4.71	−4.44	−4.93
VX Per	h+χ Per	1.037	0.58	8.20	7.56	7.69	7.22	11.90	−3 70	−4.34	−4.21	−4.68
VY Per	h+χ Per	0.743	1.06	8.54	7.99	7.99	7.60	11.90	−3.36	−3.91	−3.91	−4.30
UY Per	h+χ Per	0.730	0.98	8.94	8.36	8.28	7.90	11.90	−2.96	−3.54	−3.62	−4.00

column and are from Kraft's rediscussion (1961) of the color-magnitude diagrams using Johnson's (1960) evolutionary deviation curve method with the reddening values of the fourth column. The modulus of the h and χ Per association is 11.9 after Wildey (1964), which is close to the mean of the separate values of 11.99 and 11.66 favored by Schild (1967). The mean and maximum B and V magnitudes, corrected for reddening, have been taken from the Tonantzintla Catalogue. The resulting absolute luminosities are given in the last four columns. It should be noted that, although RS Pup (Westerlund 1964), with $M\langle_V\rangle = -6.1$, $\log P = 1.617$, and $\langle B - V\rangle_0 = 0.87$, agrees with our final calibration, we have not used this star pending a rediscussion of the modulus of the association using a fit to the age-zero main sequence in $(U - B)_0$.

The absolute calibration of Figure 1 was accomplished by minimizing by eye the systematic differences in Figures 2 and 3 between the stars in Table 1 and the extragalactic Cepheids. Any change in Table 1 will cause a corresponding change in the calibration. The distance to the Hyades is crucial here because it sets the zero point of the age-zero main sequence from which the last five columns were obtained. Hodge and Wallerstein (1966) have recently advocated a change of 0.4 mag in this calibration, and although there has been no general acceptance of this view (see Wilson 1967; Wayman 1967; Demarque 1967; Eggen 1967), the discussion is not yet complete. For example, the trigonometric parallaxes of high weight with $\pi \geq 0\overset{''}{.}070$ for stars with $0.8 > B - V > 0.4$ might support a correction of about +0.2 mag to the moving cluster modulus, even though the solution by this method has very high internal accuracy (Wayman, Symms, and Blackwell 1965). But other evidence, such as Eggen's recent $R - I$ photometry of parallax stars with $\pi \geq 0\overset{''}{.}125$ in the range $1.5 > B - V > 0.8$, strongly supports the presently accepted age-zero main sequence. We have accepted the current calibration (Sandage 1957; Johnson and Iriarte 1958; Blaauw 1963; and Eggen 1965) and therefore adopt Kraft's moduli for the relevant open clusters without correction.

IV. THE P-L RELATION AT MAXIMUM LIGHT

The analogous relation to Figure 1 is needed at maximum light if Cepheids are to be used as distance indicators in the range $28 > (m - M) > 25$. The present material defines such a relation which is shown in Figure 5. In view of the statistical nature of the period-amplitude function, it is remarkable and fortunate that the scatter in Figure 5 is so small. Most of this scatter is intrinsic for the same reason as in Figure 1, as can be proved by the strong correlation between the magnitude residuals at mean and at maximum light. Furthermore, the residuals R_B(max) and R_V(max) are again correlated with the color residuals $\delta(B - V)_{\max}$ obtained from the center line of the $\log P = f(B - V)_{\max}$ relation.

This color-period function at maximum light is shown in Figure 6 where the data for fifty-eight galactic Cepheids, combined with the twenty-seven stars of Gascoigne and Kron from the Clouds, are shown. The color data for the galactic Cepheids are from Kraft (1961); Bahner, Hiltner, and Kraft (1961); and from our values for a few additional stars as already mentioned. These data are summarized in Tables A2 and A3 of the Appendix.

There is no evidence from Figure 6 that the period-color relation at maximum light is different for different galaxies, as is the case at mean light. Although this may be a result of the small population and an accidentally favorable sampling of the color-amplitude distribution, we accept the result and obtain the $\delta(B - V)_{\max}$ residuals from the single central curve.

Figure 7 shows the correlation of the magnitude and color residuals at maximum light. Although the scatter is somewhat greater than in Figures 2 and 3, the general form of the correlation is similar to that at mean light. The lines, put through by eye, have slopes of $\Delta R_B/\Delta\delta(B - V) = 2.58$, and $\Delta R_V/\Delta\delta(B - V) = 1.66$ which differ from the corresponding slopes at mean light. This difference can be taken as additional evidence that the dispersions in Figures 1 and 5 are not caused primarily by patchy reddening, because if such were the case, the slopes of the lines in Figures 3 and 7 would have values of 4 and 3, respectively, which is not the case.

Figure 7 provides additional evidence for the correctness of the present absolute calibration, providing that the values of Table 1 are accepted. The relation between the magnitude and color residuals of the nine calibrating Cepheids agrees well with the general trend of the other residuals, which would not be the case if the calibration was incorrect. But even if the absolute calibration of Table 1 is eventually changed, for example, by a proof that the presently adopted Hyades modulus is incorrect, the *relative* distances to the Clouds, M31, and NGC 6822 appear to be well determined using the Cepheids alone. The maximum error is about $\Delta(m - M) = \pm 0.3$ mag, again assuming the uniqueness of the *P-L* relation. This uncertainty arises from two causes. (1) A change of $\Delta(m - M) = \pm 0.2$ mag in the modulus for any galaxy listed at the lower right of Figures 1 and

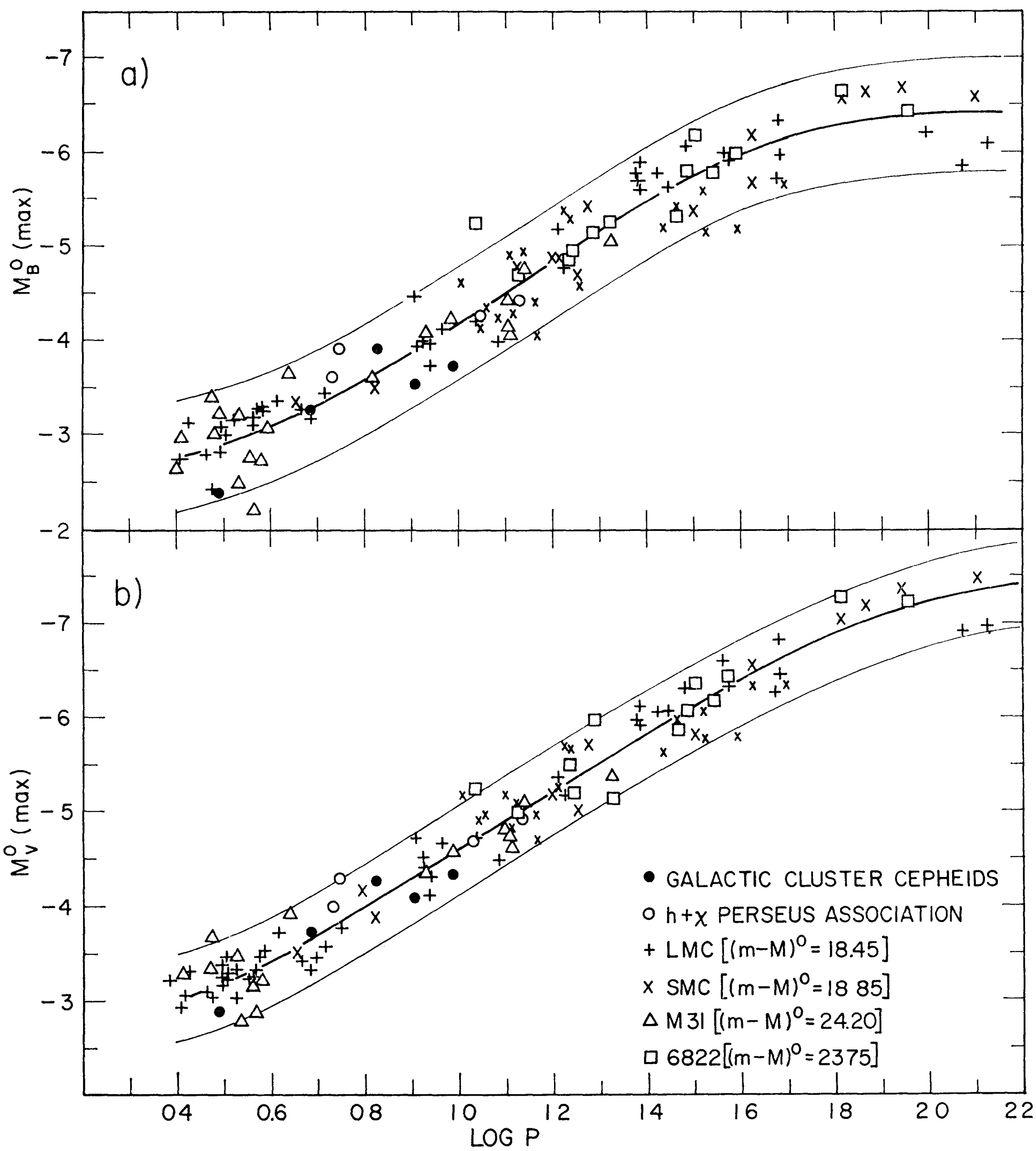

FIG. 5.—The period-luminosity relation at maximum light for the same stars shown in Fig 1

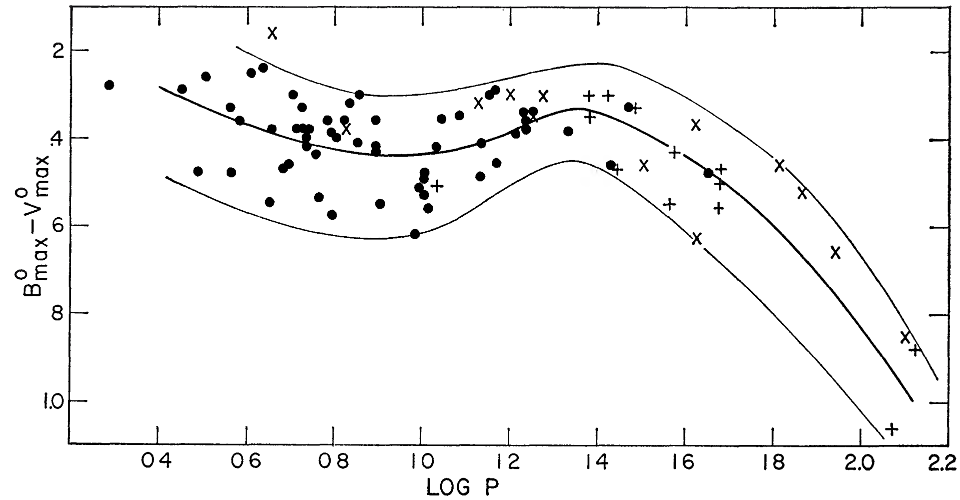

FIG. 6.—The adopted relation between period and color at maximum light. The galactic Cepheids from Table A2 of the Appendix are plotted as closed circles. The photoelectric data for stars in the Clouds are shown by crosses (SMC) and plus signs (LMC). The plotted function is listed in Table A3 of the Appendix.

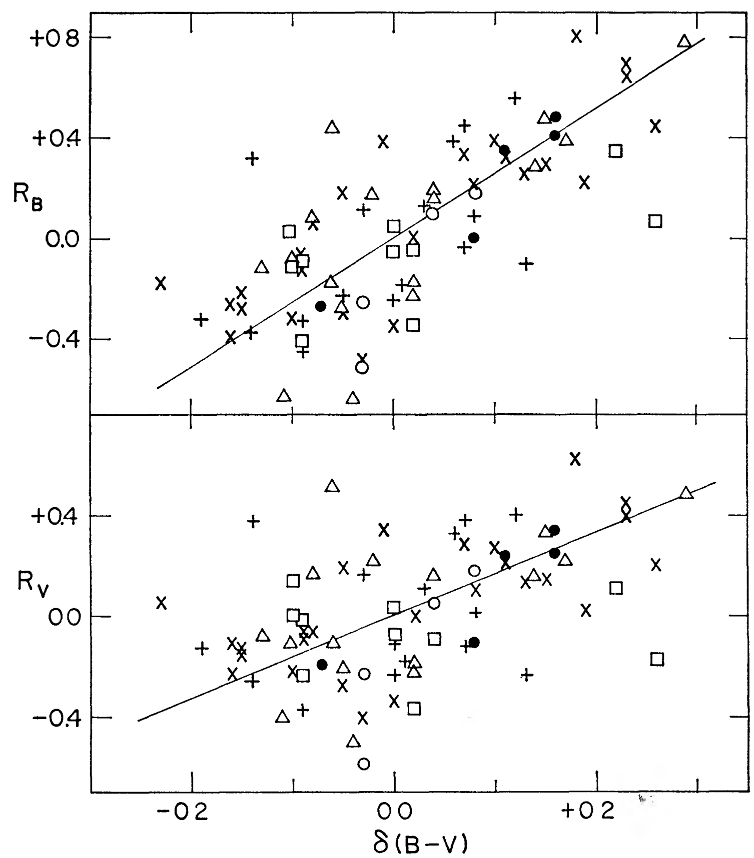

FIG 7.—The correlation at maximum light of the magnitude residuals from the central lines of Fig. 5 with the color residuals read from Fig 6. The same stars as plotted in Figs 2 and 3. The coding is the same as Fig. 1 The lines, drawn through the data by eye, have appreciably different slopes than either (1) the lines in Figs. 2 and 3; or (2) lines of slopes 4 and 3 expected if the dispersion of Fig 5 were largely due to variable absorption.

5 will produce an appreciable increase in the scatter of the *P-L* relation, and this sets an upper limit to the allowed fitting error. (2) The uncertainty of the galactic reddening may be as large as $\Delta E(B - V) = \pm 0.03$ mag, which, with the normal value of extinction to reddening, gives errors of $\Delta(m - M)_B = \pm 0.12$ mag and $\Delta(m - M)_V = 0.09$ mag due to this cause alone.

However, aside from the question of the systematic error of the Hyades modulus, the error in the final absolute calibration of Figures 1 and 5 should be smaller than the $\Delta(m - M)$ error because it depends only on the uncertainty in the fitting of the nine calibrating Cepheids into Figures 2, 3, and 7. We estimate the calibration error to be as small as $\Delta M_B \simeq \Delta M_V \simeq \pm 0.1$ mag.

The *P-L* function of Figure 5, listed in Table A1 of the Appendix, is used in a following paper as the principal relation for the distance determination of NGC 2403.

V. SUMMARY

The composite *P-L* relations at mean and at maximum light for stars in the Magellanic Clouds, M31, and NGC 6822 suggest that the form of the function does not vary from galaxy to galaxy. Although some evidence is present for differences in mean color between Cepheids in different types of galaxies, such differences do not appear to affect the *P-L* relation. Much of the dispersion of this relation appears to be intrinsic and follows the predictions of the theory of the instability strip.

The absolute calibration of the *P-L* relation rests on the assumptions that (1) the galactic Cepheids of Table 1 are typical; (2) the calibration of the age-zero main sequence via the Hyades moving cluster method is correct; and (3) the properties of Cepheids do not vary from galaxy to galaxy. Although any of these assumptions may be incorrect, they can eventually be checked by searching for inconsistencies among the several distance indicators, in various galaxies, calibrated with these single *P-L* relations on the assumption that they are unique.

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APPENDIX

Table A1 lists the adopted *P-L* relations at mean and at maximum light in *B* and *V* wavelengths as plotted in Figures 1 and 5. The upper and lower envelope lines can be found from the tabulated values by adding the dispersion half-widths listed at the bottom of this table.

TABLE A1

ADOPTED RIDGE-LINE *P-L* RELATIONS*

log *P*	$M_{\langle B \rangle}$	$M_{\langle V \rangle}$	M_B(max)	M_V(max)	log *P*	$M_{\langle B \rangle}$	$M_{\langle V \rangle}$	M_B(max)	M_V(max)
0 4	−2 18	−2 60	−2 75	−3 02	1 3	−4 44	−5 06	−5 14	−5 49
0 5	−2 40	−2 85	−2 90	−3 23	1 4	−4 67	−5 35	−5 45	−5 79
0 6	−2 65	−3 12	−3 08	−3 45	1 5	−4 91	−5 64	−5 73	−6 11
0 7	−2 89	−3 38	−3 28	−3 68	1 6	−5 16	−5 92	−5 96	−6 40
0 8	−3 12	−3 64	−3 57	−4 00	1 7	−5 41	−6 22	−6 14	−6 65
0 9	−3 38	−3 92	−3 86	−4 30	1 8	−5 64	−6 49	−6 29	−6 89
1 0	−3 64	−4 20	−4 18	−4 61	1 9	−5 82	−6.72	−6 36	−7 06
1 1	−3 92	−4 50	−4 50	−4 92	2 0	−5 96	−6 90	−6 39	−7 22
1 2	−4 18	−4 78	−4 82	−5 21	2 1	−6.05	−7 02	−6 39	−7 36

* The upper and lower envelope lines can be obtained by adding ±0 60 mag to the tabulated $M_{\langle B \rangle}$ and M_B(max) relations, ±0.50 mag to $M_{\langle V \rangle}$, and ±0 45 mag to M_V(max).

Table A2 gives the adopted colors, corrected for reddening, for the galactic Cepheids used to define the color-period relations at mean and at maximum light. The ridge line at mean light for the period-color relation is expressed by equation (7) as defined by these data.

The relation between period and color at maximum light is listed in Table A3 as plotted in Figure 6.

TABLE A2

GALACTIC CEPHEIDS USED TO DEFINE THE PERIOD-COLOR RELATIONS

Star	log P	$E(B-V)$	$\langle B\rangle_0 - \langle V\rangle_0$	$(B_M - V_M)_0$	Star	log P	$E(B-V)$	$\langle B\rangle_0 - \langle V\rangle_0$	$(B_M - V_M)_0$
SU Cas	0 290	0 33	0 38	0 28	AK Cep	0 859	0 84	0 47	0 30
AY Cas	458	0 79	48	29	CD Cas	0 892	0 83	62	42
EV Sct ..	490	0 58	57	48	VY Cyg .	0 895	0 69	57	36
BY Cas	508	0 95	33	26	RX Cam	0 898	0 62	.62	43
DW Per	562	0 65	51	33	DL Cas .	0 903	0 50	.70	55
S Sct .	565	0 33	64	48	S Nor .	0 989	0 21	75	62
DF Cas	583	0 60	55	36	DD Cas	0 992	0 56	66	.51
SY Cas	610	0 53	45	25	BZ Cyg	1 006	1 00	.60	48
CG Cas	640	0 76	47	.24	SY Aur	1 006	0 44	61	48
XY Cas	653	0 43	70	55	ζ Gem	1 007	0 15	67	53
V482 Sco	655	0 43	56	38	Y Sct .	1 015	0 80	80	56
CF Cas	687	0 56	65	47	Z Lac	1 037	0 48	67	42
DW Cas	699	0 84	62	46	SV Per	1 046	0 44	59	36
AP Sgr	703	0 29	53	30	RY Cas	1 084	0 76	.60	35
V386 Cyg	720	1 02	54	38	SZ Cas	1 134	0 88	.61	.49
δ Cep .	728	0 11	54	.33	TT Aql	1 138	0 55	.74	41
UY Per	730	0 98	57	38	CY Cas	1 158	1 10	.59	30:
SW Cas ..	736	0 54	58	40	TX Cyg	1 167	1 25	62	.29:
X Lac	736	0 40	53	42	RW Cas	1 170	0 41	82	.46
VY Per	743	1 06	.55	39	X Cyg .	1 214	0 45	72	39
CZ Cas	.753	0 78	62	.44	CD Cyg	1 233	0 64	67	.34
FM Cas	764	0 32	71	54	Y Oph	1 234	0 85	52	39
RV Sco	781	0 40	58	36	SZ Aql .	1 234	0 68	74	.36
VV Cas	793	0 54	59	39	CP Cep	1 252	1 08	56	34:
CR Cep	795	0 78	68	58	WZ Sgr	1 339	0 68	73	.38
RR Lac	808	0 31	61	.40	T Mon .	1 431	0 43	78	.46
U Sgr	828	0 55	56	36	AQ Pup	1 474	0 62	78	33
AP Cas	836	0 91	49	32	SV Vul .	1 654	0 64	0 81	0 48
η Aql	0 856	0 18	0 62	0 41					

TABLE A3

COORDINATES OF THE CENTER AND THE UPPER AND LOWER ENVELOPE PERIOD-COLOR RELATION AT MAXIMUM LIGHT

LOG P	$B_{max} - V_{max}$			LOG P	$B_{max} - V_{max}$		
	Center	Upper	Lower		Center	Upper	Lower
0 4	0 27		. ..	1 3	0 35	0 24	0 46
0 5	33	. ..	0 53	1 4 .	34	23	0 46
0 6	37	0 21	.57	1 5	38	25	0 53
0 7	40	25	60	1 6	44	30	0 61
0 8	43	28	62	1 7	51	37	0 70
0 9	44	30	63	1 8	60	44	0 79
1 0	.43	.29	.62	1 9	70	.54	0 90
1 1	42	28	58	2 0	83	.67	1 01
1 2	0 39	0 26	0 51	2 1	0.97	0 81	

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