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OBSERVATIONAL EVIDENCE FOR OVERTONE PULSATIONS IN CLASSICAL CEPHEIDS

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ABSTRACT

On the basis of a period-radius relation obtained from radius determinations for fifteen Cepheids by Wesselink's method, it is suggested that η Aql, W Sgr, β Dor, and X Cyg are pulsating in the first overtone, and that U Car may be pulsating in the third overtone.

No simple, decisive observational criterion has been found for distinguishing overtone pulsators from fundamental-mode pulsators, and since the former make up one-third of the fifteen stars examined, the determination of individual absolute magnitudes by the period-luminosity relation will frequently be liable to considerable error. Application of the period-luminosity relation for fundamental pulsators to first-overtone pulsators will underestimate their luminosities by about 0 4 mag, while in the case of third-overtone pulsators the luminosity will be underestimated by about 1.0 mag.

It is pointed out that where only a few Cepheids have been measured in an external galaxy, this effect could lead to a significant error in determining the slope of the *P-L* relation in that galaxy.

Unlike their counterparts among the RR Lyrae stars, overtone pulsators among classical Cepheids do not appear to be restricted to any particular period range.

I. INTRODUCTION

As the starting point for a detailed revision of the period-luminosity relation for classical Cepheids, an extensive program of radius determinations by Wesselink's method has been undertaken. The stars have been selected on the basis of their having well-determined light, color, and velocity curves; and further selected to give a good distribution of stars in period. Some practical precepts for applying Wesselink's method have been given by Fernie and Hube (1967), and these have been followed throughout.

In order to check the present determinations, radii were determined for a number of stars whose radii were already well known, such as δ Cep and η Aql. The agreement, as shown by Table 1, is satisfactory, except in the case of β Dor. The value of 105 $R_\odot$ for this star was obtained by Rodgers (1957), based on radial velocities by Stibbs (1955) and Applegate (1927) and photometry by Eggen, Gascoigne, and Burr (1957). However, further photometry by Irwin and by Walraven, Muller, and Oosterhoff (see Mitchell, Iriarte, Steinmetz, and Johnson 1964) agree in showing a difference in the shape of the color curve from that of Eggen *et al.* Because of the agreement between these other two independent sources I have chosen their photometry, which has led to the marked difference in the result for the radius. For the same reason I have chosen not to average my result with that of Rodgers in this particular case.

Table 1 lists the final results of the program, showing the present results, values determined by others, and the adopted radii. Detailed references to the determinations by others will be found in an earlier paper (Fernie 1964), except for the radius of U Sgr, which has been determined by Breger (1967).

II. PERIOD-RADIUS RELATION

It has been shown previously (Fernie 1965) that the period of a pulsating variable is related to its radius and mass by a function of the form

$$P \propto R^2\mathfrak{M}^{-1/2} .$$

In the case of just the limited class of stars, the classical Cepheids, simple evolutionary considerations indicate that the masses of the stars should be monotonically related to their periods, by virtue of the way in which B stars become red giants. (The effect of multiple crossings of the instability strip is discussed below.) Hence there should exist an *effective* period-radius relation of the form

$$P \propto R^n ,$$

where n will now be somewhat different from 2. This relation can be determined quite empirically by the data of Table 1; and this fact is important in allowing the period-luminosity-color relation to be determined without knowing the masses of the stars explicitly. The further development of this will form the subject of a later paper.

Figure 1 shows a plot of radius against period, based on the data of Table 1. The scatter appears unreasonably large, and one is led to search for possible causes.

TABLE 1

AVERAGE RADII OF CLASSICAL CEPHEIDS

Star	Period	Average Radius ($R_\odot$)		
		Present Determination	Other Determinations	Adopted
UX Car	3ᵈ68	37		37
AG Cru	3 84	37	. . . .	37
δ Cep.	5 37	50	53, 53, 40, 48	48
U Sgr	6 74		54	54
η Aql.	7 18	69	68, 67, 64	67
W Sgr	7 59	68	.	68
S Sge.	8 38	56	60	58
β Dor.	9 84	79	105	79
ζ Gem	10 15	70		70
X Cyg.	16 39	103	. .	103
T Mon	27 02	105	119	112
AQ Pup	29 86	124	. . .	124
l Car .	35 56	120	138	130
U Car.	38 76	220	. . .	220
SV Vul	45.10	165	142	154

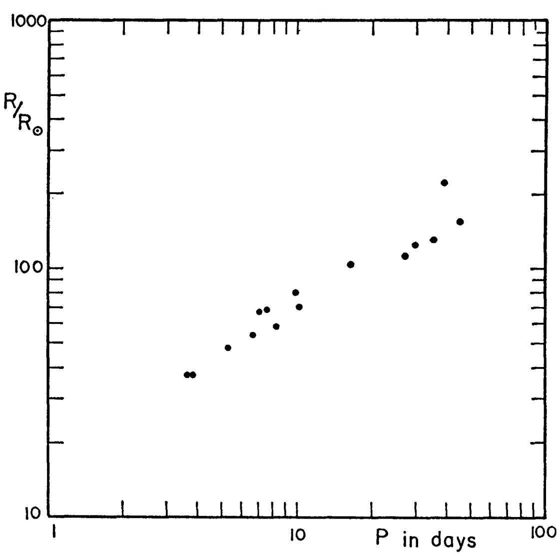

FIG. 1.—Period-radius relation as given by the data in Table 1

a) *Observational Error*

Inspection of Table 1, comparing the radius of a given star as determined by different workers, indicates that the adopted radii are probably correct to within 10 per cent. This seems to be a fairly reliable estimate of the uncertainty, since in some cases the original observational data came from quite different sources. The scatter in Figure 1, however, considerably exceeds 10 per cent; in the case of U Car it is six times as great.

b) *Multiple Crossings of the Instability Strip*

The theoretical work of Iben (see, e.g., Kraft 1966) and also of Kippenhahn and his co-workers (see, e.g., Hofmeister 1967) has shown that in the course of its evolution, as described on the H-R diagram, an early B star will cross the Cepheid instability strip to become a red giant, and will then return to cross the strip back and forth several more times. Since each crossing generally takes place at a different level of luminosity, this will cause a scatter in the period-radius (and also the period-luminosity) relation. This happens because a star observed at a given point in the instability strip (i.e., having a certain radius) may have arrived there directly from the B-star region, or it may already have been a red giant and now be making some higher-order crossing, in which case its mass will be different from that of a star coming directly from the B-star region. But the period depends on the mass as well as the radius, and therefore one can have more than one period present at a given point in the instability strip, depending on the star's evolutionary history. To put it another way, there can be a variety of radii corresponding to a given period. Fortunately, however, the period is largely determined by the radius and only slightly by the mass (R^2 as against $\sqrt{\mathfrak{M}}$), and thus minor variations in mass among stars of a given radius will cause only a small scatter in the period-radius relation. Detailed calculations based on the theoretical tracks show that the dispersion in radius at a given period should not exceed 5 per cent (whereas the observational error is already about 10 per cent). The multiple crossing effect becomes even less important when the relative speeds of crossing are considered. The second crossing is about one to two orders of magnitude slower than any of the other crossings, so the very great majority of stars observed to be in the instability strip must all be making the same crossing. We conclude that this effect cannot account for the observed dispersion in Figure 1.

c) *Overtone Pulsations*

If a star is pulsating in an overtone it will appear in Figure 1 with a period smaller than that appropriate to its radius, and since the overtone periods are definite fractions of the fundamental period, the effect should give rise to a number of parallel sequences in Figure 1. Inspection of this figure does indicate the possibility of there being two sequences present—a lower sequence, comprising two-thirds of the stars, and another sequence above it made up of the stars η Aql, W Sgr, β Dor, and X Cyg. The interpretation of the anomalous position of U Car in Figure 1 is uncertain. Although repeated determinations of the radius by more than one person agree in giving a result in the neighborhood of 220 $R_\odot$, this is known to be unsatisfactory because the individual results correlate with the color level at which they are evaluated. Evidently there is some discordance among the observations, which are radial velocities by Stibbs (1955) and photometry by Irwin and by Walraven *et al.* as listed by Mitchell *et al.* (1964). (Incidentally, the phases given by Stibbs for his velocity measures of this star contain a large zero-point error.) It seems clear that further observations, especially of radial velocity, will be necessary before a satisfactory result is obtained. For the moment, however, we may proceed with the value of 220 $R_\odot$. As shown below, it may be accounted for in terms of third overtone pulsation. If the interpretation is correct, then the scatter in Figure 1 should be removed when these five stars are replotted with their periods divided by the period ratios appropriate to the overtone.

The ratio in period of the first overtone to the fundamental in classical Cepheids is already quite well established on observational grounds from a few short-period Cepheids which exhibit two periods (Christy 1966*a*). It is 0.71. There are no known observational values for the period ratio of higher overtones (in particular the third) to the fundamental in Cepheids. However, examination of various theoretical calculations (Schwarzschild 1941; Stothers 1965) reveals that the period ratio of the third overtone to the first overtone is very nearly constant among a wide variety of models, and that it agrees in value (0.60) with the observed ratio in β Cephei stars as claimed by van Hoof (see Stothers 1965). Thus it may be taken that the period ratio of the third overtone to the fundamental in Cepheids is $0.60 \times 0.71 = 0.43$.

Figure 1 is now replotted with the observed periods of η Aql, W Sgr, β Dor, and X Cyg divided by 0.71, and the observed period of U Car divided by 0.43. The result is shown in Figure 2. The previous scatter is now seen to have been largely removed, the

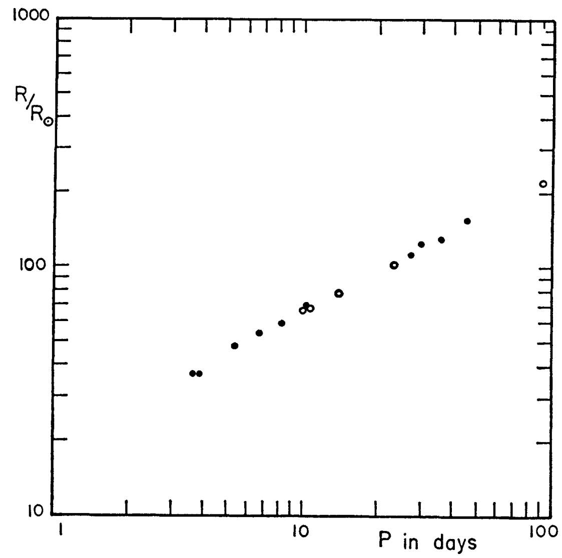

FIG 2.—Period-radius relation when the periods of five stars are adjusted for possible overtone pulsation. The stars so adjusted are indicated by open circles.

residual scatter being in accord with the likely observational error. For the scatter to be removed in this way without this interpretation of overtone pulsation being correct would be so remarkable a coincidence that this seems the most probable explanation. Further support to this interpretation is lent by Christy's (1966*b*) already having suggested on theoretical grounds that η Aql is pulsating in the first overtone. (Incidentally, if a radius of 105 $R_\odot$ is taken for β Dor, then it too appears as a third-overtone pulsator.)

It is concluded that η Aql, W Sgr, β Dor, and X Cyg are pulsating in the first overtone and that U Car may be pulsating in the third overtone.

For future reference, least-squares solutions for the period-radius relation have been made as follows:

For all stars, with the periods of the overtone pulsators adjusted as described above,

$$\log R/R_\odot = \underset{\pm 0.006}{0.553} \log P + \underset{\pm 0.007}{1.264} \text{ (p.e.)} .$$

For only those stars recognized as fundamental pulsators,

$$\log R/R_\odot = \underset{\pm 0.008}{0.560} \log P + \underset{\pm 0.009}{1.258} \text{ (p.e.)} .$$

Since the latter solution is free of any error in the assumed overtone period ratios, double weight is assigned to it. Thus the adopted relation is

$$\log R/R_\odot = \underset{\pm 0.005}{0.558} \log P + \underset{\pm 0.006}{1.260} \, .$$

III. CONSEQUENCES

If the above conclusion is correct, a number of disturbing consequences follow.

Since a third of the stars out of this essentially random selection are pulsating in an overtone mode, probably a similar fraction of all classical Cepheids are overtone pulsators. In this case, application of a period-luminosity relation derived for fundamental-mode pulsators will seriously underestimate the luminosities of the overtone pulsators. As derived above,

$$\log R = 0.558 \log P + \text{const.}$$

Hence

$$M_V = -5 \log R + \text{const.} = -2.79 \log P + \text{const.}$$

Thus for a first-overtone pulsator the error will be

$$\Delta M_V = -2.79 \log P + 2.79 \log (P/0.71) = 0.41 \text{ mag.}$$

Similarly for a third-overtone pulsator the error will be

$$\Delta M_V = 1.02 \text{ mag.}$$

It is therefore most important that some criterion be found to distinguish the mode in which a given Cepheid is pulsating. The five overtone pulsators found in the present investigation have been compared in detail with the remaining stars in Table 1. Comparison has been made with respect to the following: amplitude of light curve, amplitude ratio of light curve to color curve, presence of a hump in the light curve and the phase at which it occurs, shape of the light curve, and intrinsic color compared to average intrinsic color for the observed period (Fernie 1967). None of these has served to distinguish the overtone pulsators, much less to indicate which particular overtone is present. There is a possibility that a hump in the light curve around phase 0.2 or 0.3 after maximum light is present in all the overtone pulsators, although in the case of X Cyg this is not at all obvious. Only careful comparison of accurate photoelectric data obtained over several cycles indicates that there may be some slight instability of the light curve at this phase. On the other hand, U Sgr, S Sge, and S Nor also have distinct humps in their light curves and yet appear to be fundamental pulsators. Of course, a determination by Wesselink's method of the radius of a given star could be made, and its position in Figure 1 would then indicate the mode of pulsation, but the necessity for having first-class radial velocity data precludes this for the majority of Cepheids. In any case, the method cannot be applied where the light curve is almost sinusoidal, and it is also a very laborious technique. At the present stage of investigation no reasonable observational criterion for distinguishing the mode of pulsation is apparent.

It may also be pointed out that, where only a few Cepheids have been observed in an external galaxy, especially if the distribution in period is uneven, the presence of overtone pulsators might significantly bias the slope derived for "the" period-luminosity relation. Perhaps the variety of slopes among external galaxies which have been announced should be considered with some caution until more data are available.

Finally it is pointed out that by analogy with the RR Lyrae stars one might have expected to find those classical Cepheids which are overtone pulsators to be concentrated in some restricted period range, say, those of particularly short period. The present findings, however, seem to indicate that overtone pulsation can occur at almost any apparent period. This is supported by the observed period-luminosity relation in the Small Magellanic Cloud, where no sharp break is observed, as would be the case if all Cepheids below a certain period were overtone pulsators and all Cepheids above that period were fundamental pulsators.

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