Orbit of Neptune and the Mass of Pluto

R. L. Duncombe, W. J. Klepczynski, and P. K. Seidelmann
U. S. Naval Observatory, Washington, D. C.
(Received 29 August 1968)

Several simultaneous numerical integrations of the orbits of the five outer planets were generated, utilizing different values for the disturbing mass of Pluto. It is found that the observations of Neptune from 1846 to 1938 and 1960 to 1968 are best satisfied by a reciprocal mass of 1 812 000 for Pluto (0.18 earth mass).

I. INTRODUCTION

A SIGNIFICANT characteristic of the past theories of the motion of Neptune has been the apparent failure to represent observations very far removed in time from the observations to which the constants of the theory were adjusted. Newcomb's theory (1899), amended to include the effect of Pluto with reciprocal mass 360 000, was adjusted to observations in the period 1795 and 1846–1896. By 1938 this theory failed to represent observations in orbital longitude by over 5 arc sec. The present numerical theory of the motion of Neptune (Eckert, Brouwer, and Clemence 1950), incorporating the reciprocal mass 360 000 for Pluto and fitted to observations 1795, 1846–1938, fails to represent the observed longitude of Neptune at the present epoch by nearly 4 arc sec as shown by 158 meridian-circle observations in the period 1960 to 1968 (Adams and Scott 1964, 1967, 1968). The apparent failure of these two theories to represent the observations of Neptune's longitude over extended periods indicated the possibility that an adjustment to the mass of Pluto incorporated in the theories might be required.

II. OBSERVATIONAL MATERIAL

At the present epoch the observations of Neptune since discovery encompass more than 70% of the orbit. The previous discussions of Newcomb (1899), of Wylie (1942), and of Eckert, Brouwer, and Clemence (1951) incorporated the prediscovery observations of 1795 due to Lalande. An examination of the reduction of these observations (Newcomb 1867) indicates that the uncertainty in the true value of several of the instrumental constants can cause a variation in the derived longitude as great as 7 arc secs. Because of this uncertainty, and since the postdiscovery observations cover such an extensive orbital arc, the 1795 observations have been omitted from this discussion.

The residuals of the 1960–1968 observations in orbital longitude and latitude are shown in Table I as a continuation of the Eckert, Brouwer, and Clemence (1951) Table IV. During this analysis they were given weights that are comparable to the other modern observations on the weighting system of Wylie (1942).

Table I. Residuals (O−Int.) in orbital longitude and latitude for the observations 1960–1968.

Date	Longitude (O−Int.)	Latitude (O−Int.)
1960.29	−2.″28	0.″30
1961.39	−2.36	0.19
1962.37	−2.77	0.08
1963.36	−2.89	0.25
1964.34	−3.12	0.16
1965.33	−3.19	0.25
1966.42	−3.27	0.00
1967.41	−3.88	0.16
1968.29	−3.75	···

III. PROCEDURE

The procedure adopted in this investigation was to assume a range of values for the mass of Pluto. For each discrete value of the mass, a simultaneous numerical integration of the orbits of the five outer planets was generated. An elliptic adjustment was then made to the observations of Neptune (1846–1938), and the integration was repeated to obtain a final orbit of Neptune consistent with that particular mass of Pluto. Each final orbit of Neptune, thus adjusted to observations in the period 1846 to 1938, was then extrapolated to compare against the observations in the period 1960 to 1968. Figure 1 shows the $(O-C)$'s in orbital longitude and latitude with respect to these final integrations which utilized the following disturbing masses of Pluto: 360 000, 930 000, 1 500 000, and 2 640 000. The criterion was adopted that the best mass of Pluto is that one which, when fitted to observations from 1846–1938, best represents the longitude observations in the period 1960–1968.

Table II. Σv^2 in Neptune's longitude and latitude for test values of the reciprocal mass of Pluto.

Reciprocal mass	Σv^2 1846–1938 $\Delta\lambda$	$\Delta\beta$	1960–1968 $\Delta\lambda$	$\Delta\beta$	Total $\Delta\lambda$	$\Delta\beta$
360 000	32.″01	21.″87	64.″47	0.″33	96.″48	22.″20
930 000	30.72	24.00	3.71	0.06	34.43	24.06
1 500 000	30.42	25.39	0.29	0.09	30.71	25.48
2 640 000	30.22	26.51	0.51	0.14	30.73	26.65
1 812 000	30.20	25.77	0.12	0.11	30.32	25.88

IV. RESULTS

Table II exhibits the presently adopted reciprocal mass of Pluto and the three test values of the reciprocal mass. Opposite each are the sums of the squares of the residuals (Σv^2) in Neptune's longitude and latitude resulting from each of these orbits for the observations in the interval 1846 to 1938 against which the orbits were fitted. Shown also in the table are the Σv^2 for the observations 1960–1968 which were compared against these orbits and finally the total Σv^2 of the previous columns. Differentiation of the parabola fitted through the Σv^2 of the longitude (1960–1968) for the reciprocal masses 930 000, 1 500 000, and 2 640 000 indicated the reciprocal mass of 1 812 000 as the value best representing the observations. A final orbit of Neptune using this mass of Pluto and fitted to observations 1846 to 1938 represents the observations 1960–1968 as shown in the last line of Table II. It is noteworthy that this orbit also fits the observations 1846–1938 in longitude better than the other orbits.

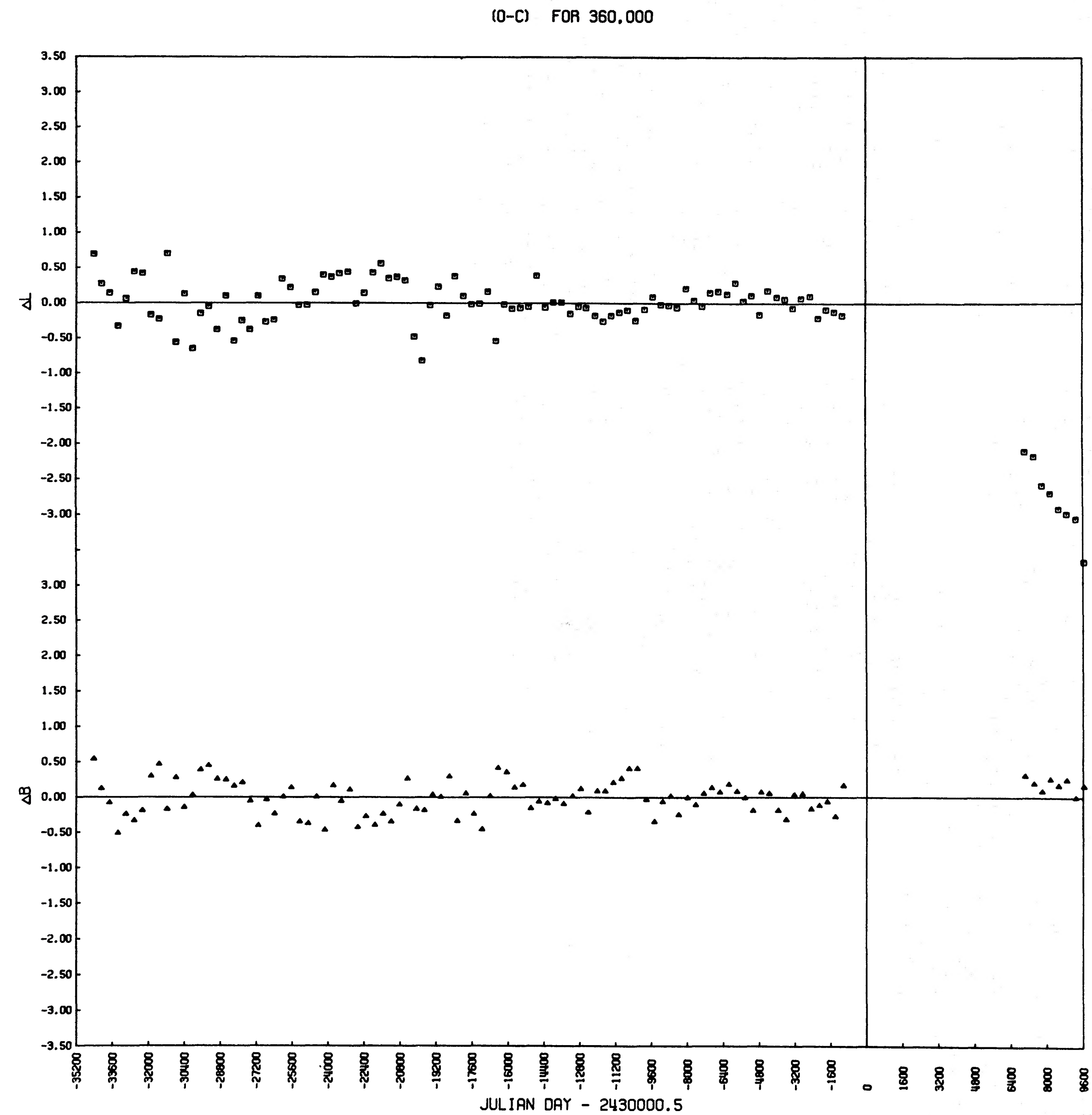

FIG. 1. $(O-C)$'s of Neptune with respect to integrations utilizing reciprocal masses of Pluto of 360 000, 930 000, 1 500 000, and 2 640 000.

TABLE III. Osculating elliptic elements of Neptune, epoch JD 2430000.5.

	Elements	Change
M	133°44′ 9″.783	−2′49″.353
$\tilde{\omega}$	270° 3′30″.833	+2′48″.149
Ω	131°16′41″.893	− 2″.598
i	1°46′33″.651	+ 0″.244
e	0.0118570458	+0.0000024223
a	29.9871290465	+0.0003408578

TABLE IV. Osculating position and velocity vectors of Neptune, epoch JD 2430000.5.

	Coordinates	Velocities (40 day)
x	−30.155 409 792 079 8	−0.009 620 284 318 442 42
y	1.657 039 655 120 15	−0.115 063 710 657 455
z	1.437 916 474 400 08	−0.046 887 657 786 994 9

The representation of the observations of Neptune utilizing this value of the mass of Pluto is exhibited in Fig. 2.

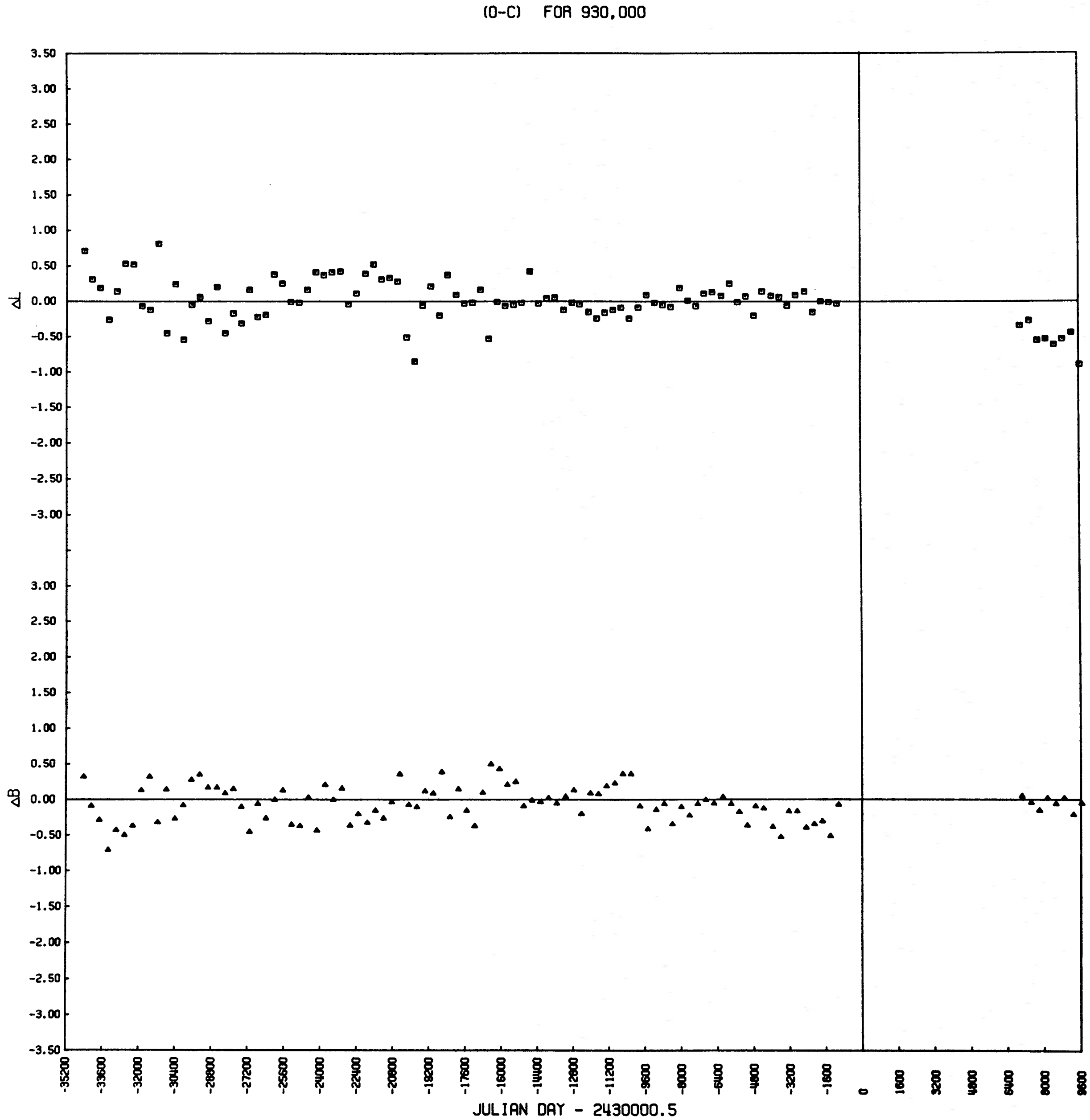

FIG. 1 (*continued*)

An elliptic adjustment of the Eckert, Brouwer, and Clemence orbit to all of the observations 1846 through 1968 resulted in Σv^2 in longitude of 37″.76 and in latitude of 22″.06. It is evident that an orbit incorporating a reciprocal mass of Pluto of 360 000 is not capable of satisfying the observations in longitude over this arc as well as the new orbit with the reciprocal mass of 1 812 000.

A slight increase in the Σv^2 in orbital latitude for reciprocal mass 1 812 000 as compared to that for 360 000 is apparent. Brouwer (1955), in an analysis of the observations of Uranus, showed the presence of an unexplained systematic trend in the latitude residuals which could not be removed by either an adjustment of the orbital elements or the mass of Pluto. It is doubtful, therefore, if any significance should be attached to the slight degradation in representation of the latitude observations of Neptune with the new mass of Pluto.

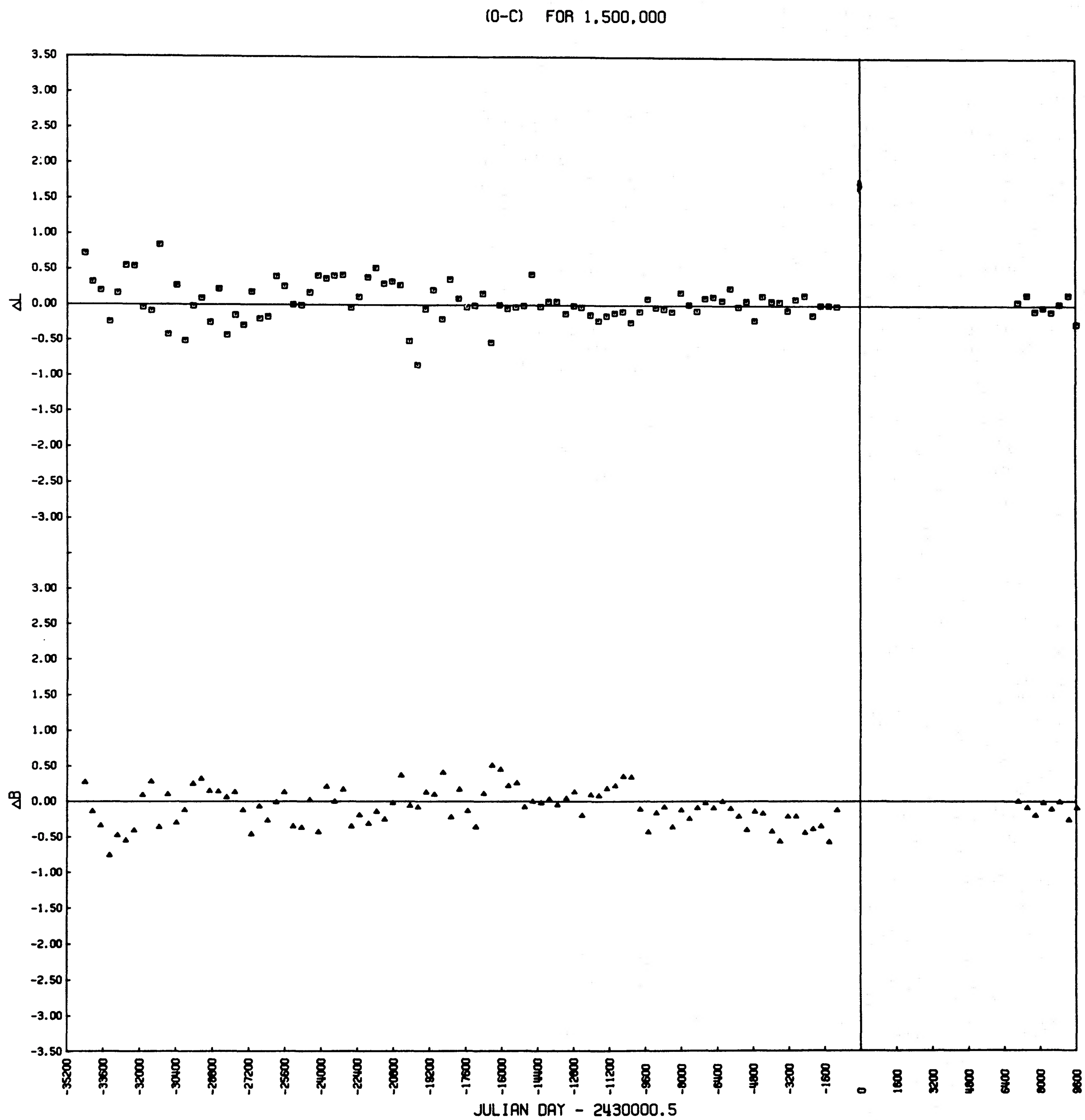

Fig. 1 (*continued*)

V. DISCUSSION

The gravitationally determined value of the mass of Pluto (Wylie 1942) of 0.91 earth mass (reciprocal solar mass 360 000) when combined with the direct measure of the diameter of 5928 km (Kuiper 1950) or the inferred upper limit of the diameter of 6400 km (Halliday, Hardie, Franz, and Priser 1965) yields an unacceptably large value of the mean density of Pluto of at least 40 g/cm^3.

If Pluto is assumed to have the same density as the earth, then the new determination of its mass (0.18 earth mass) indicates a diameter of 7200 km. On the other hand, accepting 6400 km as the upper limit of the diameter of Pluto (Halliday, Hardie, Franz, and Priser 1965), then Pluto must be at least 1.4 times as dense as the earth.

The osculating elliptic elements (epoch JD 2 430 000.5) representing the new numerical theory of the motion of Neptune are given in Table III along with the changes from the elements of the Eckert, Brouwer, and Clemence theory. Table IV gives the osculating coordinates and velocities for the epoch.

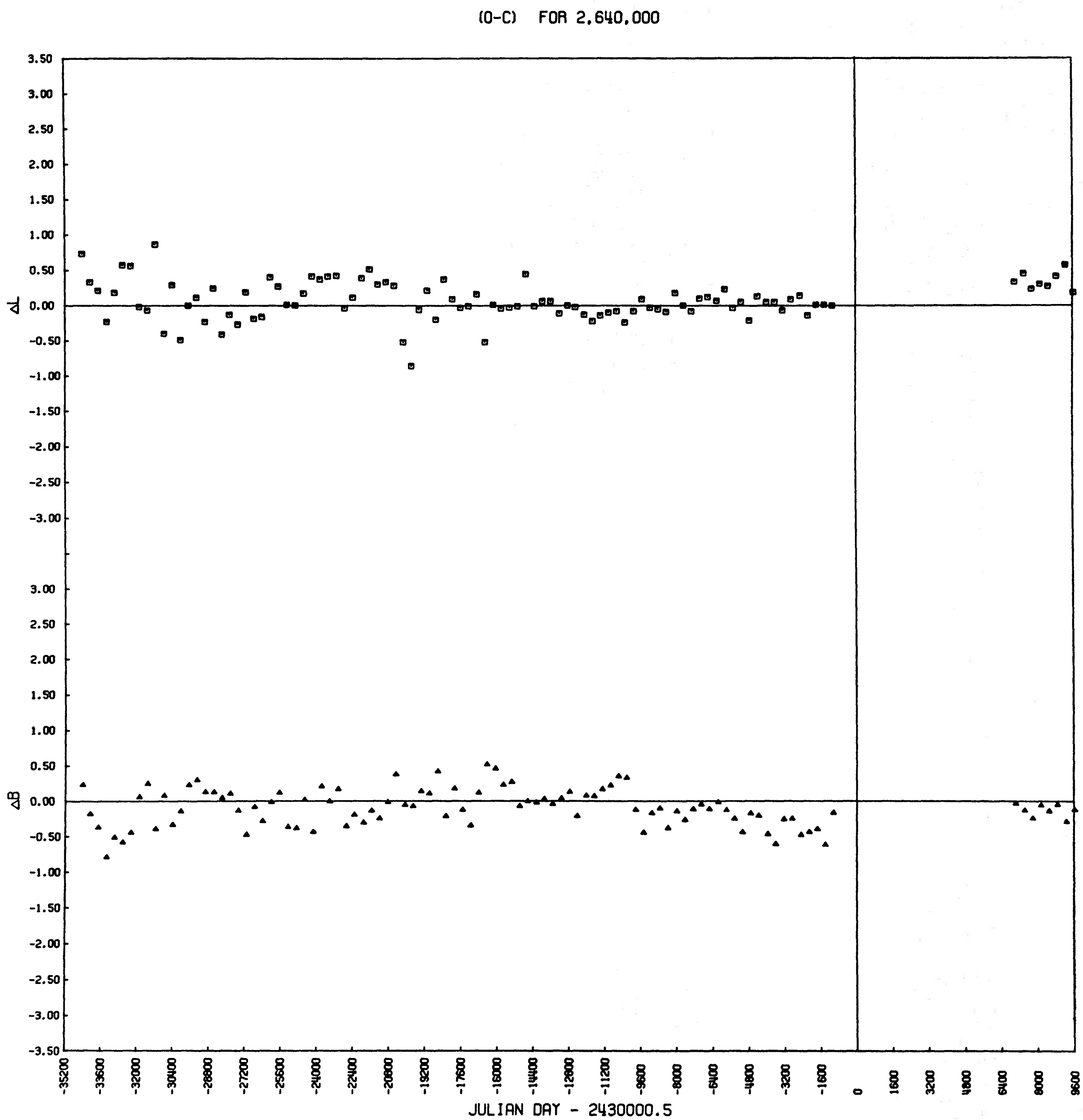

FIG. 1 (*continued*)

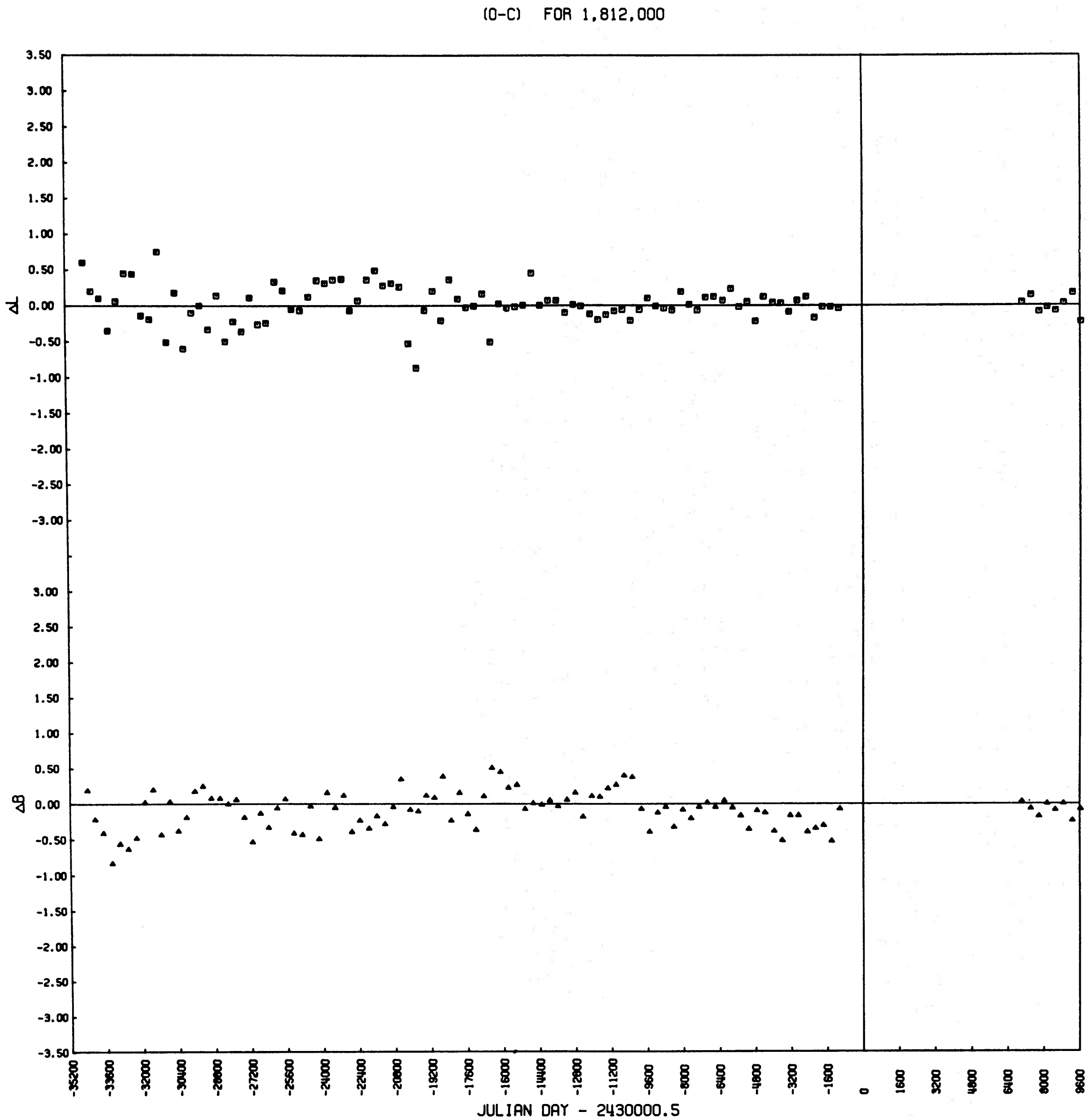

FIG. 2. $(O-C)$'s of Neptune with respect to integration utilizing a reciprocal mass of Pluto of 1 812 000.

Further improvement of the orbit of Neptune and the mass of Pluto will await completion of a systematic discussion of the observations of Neptune 1846–1968 currently being made at the U. S. Naval Observatory.

REFERENCES

Adams, A. N., and Scott, D. K. 1964–1968, *U. S. Naval Obs. Circ.* 103, 105, 115, 118.

Brouwer, D. 1955, *Monthly Notices, Royal Astron. Soc.* **115**, 221.

Eckert, W. J., Brouwer, D., and Clemence, G. M. 1951, *Astron. Papers Am. Ephemeris* **XII**.

Halliday, I., Hardie, R. H., Franz, O., and Priser, J. B. 1965, *Astron. J.* **70**, 676.

Kuiper, G. P. 1950, *Publ. Astron. Soc. Pacific* **62**, 133.

Newcomb, S. 1899, *Astron. Papers Am. Ephemeris* **VI** Pt. 4.

——. 1867, *Smithsonian Contrib. Knowledge* **II** Art. II.

Wylie, L. R. 1942, *Publ. U. S. Naval Obs. Ser. 2* **XV**, Pt. **1**.