THE INTERACTION OF THE SOLAR WIND WITH A COMET

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Abstract. The flow of plasma on the sunward side of a comet is investigated by means of an axial-symmetric model based on hydrodynamics modified by source terms. The model assumes a given curvature of the isobaric surfaces, which corresponds to paraboloids around the nucleus of the comet. The flow on the axis can be represented by a solution of a system of seven ordinary differential equations (respectively five in case of pure photo-ionization). The flow pattern always contains a widely detached bow shock and a contact discontinuity separating a cavity with purely cometary plasma from the transition region containing also solar wind ions. The model is applied to the special case where the cometary gas is ionized by the solar UV radiation only. Numerical solutions are integrated for five levels of production of neutral gas by the comet and for seven typical situations in the undisturbed solar wind. The results imply standoff distances of the stagnation point from the nucleus of the order of 10 000 km or more, and distances of the bow shock of the order of 10^6–10^7 km.

1. Introduction

Ionized gas is visible in the inner part of the head and in the long, straight type-I tails of many comets. There is no doubt, that every gas molecule released from the surface of the comet's nucleus is ultimately ionized, and the same is true for every atom produced by its dissociation. However, for well-known reasons, only those molecules are visible in the spectral range accessible from the ground which have resonance transitions in that range. The total gas production of a comet should therefore be considerably larger than the quantities observed in the forms of neutral molecules like CN and C_2, the main constituents of the head, or of ionized molecules like CO^+. While those quantities for medium-bright comets are typically of the order of 10^{26}–10^{27} respectively 10^{28}–10^{29} molecules per sec, the total gas production for such comets appears to be of the order of 10^{30}–10^{31} molecules per sec (BIERMANN and TREFFTZ, 1964; HUEBNER, 1965). It is this latter figure which determines the source strength of the comet considered as a source of interplanetary plasma.

The general aspects of this situation were qualitatively discussed elsewhere (BIERMANN, BROSOWSKI, and SCHMIDT, 1962; BIERMANN and TREFFTZ, 1964; BIERMANN and LÜST, 1966; BIERMANN, 1966). The interaction between the ionized particles, which form a plasma, is ordinarily much stronger than that between neutral particles or between the ionized and neutral particles; this is especially true in the presence of magnetic fields, which are usually found to be present in cosmic plasmas and which actually pervade the interplanetary plasma – most probably also the type-I tails of comets – such that outside the region of relatively high density near the nucleus of a comet the plasma may also be considered as a fluid for itself; the neutral molecules of cometary origin, on the other hand, behave essentially as independent particles until they are also ionized (after 10^6–10^7 seconds, in which distances of some 10^{11} cm are reached). The CO^+ molecules which contribute more than all others ions to the visible light and – unlike probably most other ionized molecules – seem to originate near the nucleus, should be considered as tracers of the flow of the plasma, the larger part of which is invisible. Since the CO^+ is observed only in certain parts of a comet, the information it provides is necessarily incomplete; it can be used, however, to check the correctness of the overall assumptions of a theory of this flow.

While at small distances from the cometary nucleus the field of flow will be dominated by the distribution of the sources, that is that of the neutral gas, from which the cometary ions are assumed to originate, at large distance the flow field must be controlled by the interaction with the magnetized solar wind moving with hypersonic velocity (Mach number 5–10). The mass emitted from the nucleus of, say, 10^8 gr/sec is comparable to that carried by the solar wind through a circle with a radius of a few 10^6 km; this is therefore, by order of magnitude, the size of the flow pattern produced in interplanetary space by the comet. In this large volume photo-ionization competes with ionization by charge transfer and possibly, at least at times, by energetic electrons (BEARD, 1966). In the dense inner regions, on the other hand, shock waves and chemical reactions will be important in the often rather fast appearance of CO^+ features (BIERMANN and TREFFTZ, 1964; MAROCHNIK, 1963a).

In the plasma flow (see Figure 1) we expect under these circumstances a contact discontinuity which separates streamlines originating in the nucleus from those

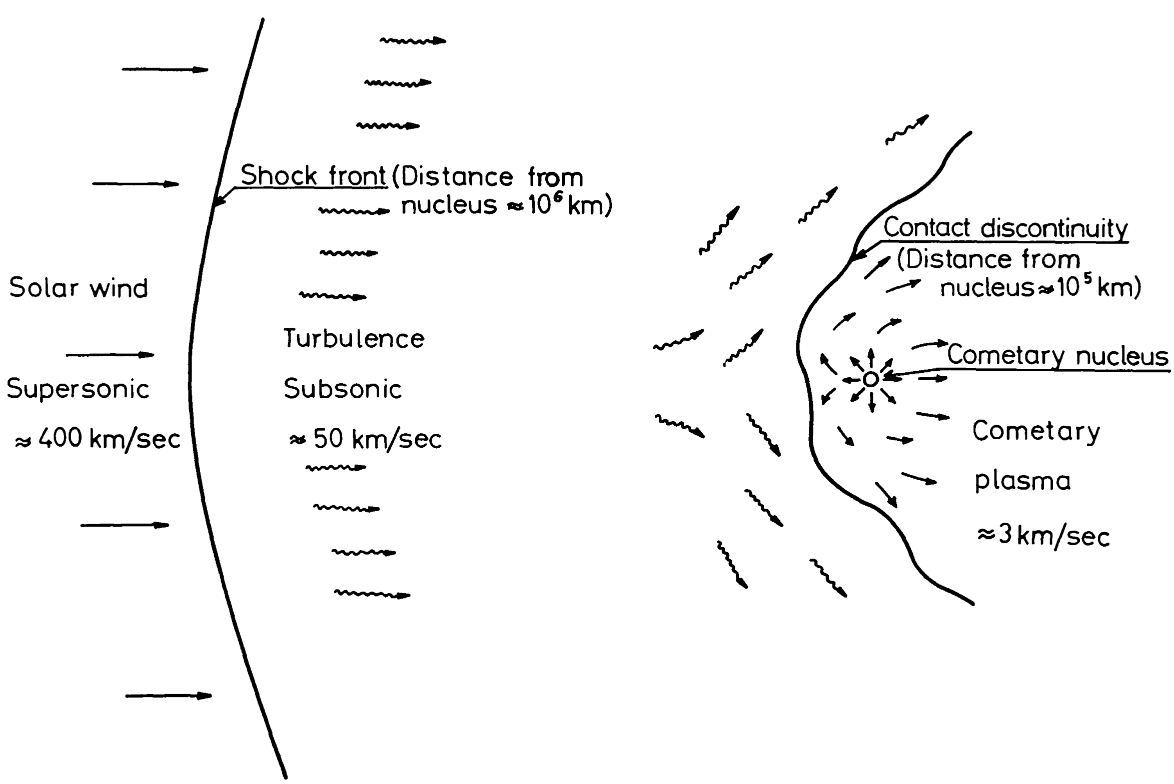

Fig. 1. Plasma flow near to a comet.

originating in the sun, and ionized matter of purely cometary origin from the solar wind containing some cometary admixtures. Farther upstream the hypersonic solar wind will undergo a shock-transition similar to the bow-shock upstream of the terrestrial magnetosphere (AXFORD, 1964; BIERMANN, 1965, 1966; IOFFE, 1966; KOVAR and KERN, 1966). Since a complete thermalization is not possible due to the large mean free paths, the flow between these discontinuities will be rather turbulent as in the terrestrial case (HEPPNER *et al.*, 1963; NESS, 1965). But the actual structure and dimensions of this region must be different in a comet due to the complex dynamic effects of the heavier cometary ions added to the solar particles, which cause a drastic change in the mean molecular weight of the plasma (BIERMANN, BROSOWSKI, and SCHMIDT, 1962; HARWIT and HOYLE, 1961). When the cometary matter leaves this region towards the leeside of the comet, it is drawn out into the long tail rays, probably by magnetohydrodynamic effects (ALFVÉN, 1957; BEARD, 1966; BIERMANN, 1951, 1953, 1960; BIERMANN and LÜST, 1963; HARWIT and HOYLE, 1961; HOYLE and HARWIT, 1961; KOVAR and KERN, 1966; LÜST, 1961, 1963). To describe the plasma flow in the head of a comet a purely hydrodynamical model with point source in the nucleus has been discussed (IOFFE, 1966). Other authors have considered the penetration of the neutral gas into the solar wind (KOVAR and KERN, 1966).

In this paper* we investigate the macroscopic features of the plasma flow on the sunward side of a comet by means of stationary quasi-hydrodynamic models. We use hydrodynamic equations which are modified by source terms describing the influence of the added molecules of cometary origin. These equations are given in the following section. In Section 3 a simplified one-dimensional model based on these equations is constructed and discussed. In the following sections we deal with a more realistic two-dimensional model assuming rotational symmetry around the axis connecting the sun with the comet. After specifying the corresponding equations with their proper source terms in Section 4, we derive the limiting form of these equations for the vicinity of the axis in Sections 5 and 6 for the plasma and the neutral cometary component respectively. The method which we use to solve these equations is described in Section 7, and the numerical results are discussed in the last section.

2. Modified Hydrodynamic Equations

It has been supposed that over distances, which are large compared to the gyroradii ($\lesssim$ 1000 km), the interplanetary plasma forms a fluid in the sense of fluid dynamics despite the fact that the mean free paths of the particles are larger by many orders of magnitude (AXFORD, 1962; KELLOGG, 1962). The assumption is made that the fluctuations of the magnetic fields in the solar wind and plasma instabilities (MOISSEV and

* Preliminary partial results of this work were presented to the Bayerische Akademie der Wissenschaften (BIERMANN, BROSOWSKI, and SCHMIDT, 1962, 1966), to the Pasadena Conference on the Solar Wind (BIERMANN, 1966), and to the 1966 meeting of the Astronomische Gesellschaft (BROSOWSKI, 1966; SCHMIDT, 1966). In the time between 1962 and present work a linearized approximation of the two-dimensional problem was formulated by G. R. Watts, the discussions with whom we greatly appreciated.

SAGDEEV, 1962) provide for a nearly scalar pressure and a well-defined mass-motion on a macroscopic scale. Theoretical predictions for the location of discontinuities in the plasma flow around the earth based on this supposition have been rather successful (SPREITER, SUMMERS, and ALKSNE, 1966). Steep shocklike structures have been observed in outer space (SONNET, *et al.*, 1964) and recent data from satellites VELA 2 and 3 (GREENSTADT *et al.*, 1966), OGO 1 (MCLEOD *et al.*, 1966), and MARINER 4 (SISCOE *et al.*, 1966) show that in the transition zone behind the bow shock of the earth an essential fraction of the solar wind's kinetic energy is readily transformed into that of waves of various types (FREDERICKS and SCARF, 1966; HEPPNER *et al.*, 1966). Though we cannot expect a complete thermalization of the kinetic energy, these results seem to justify the application of rather simple hydrodynamic methods to predict the gross features of the interplanetary plasma.

In such an approach the pressure tensor is replaced by a scalar, which means that we assume equal diagonal components. But it should be mentioned that the actual pressure tensor of the plasma also contains the magnetic stress tensor. If the magnetic field H is essentially perpendicular to the flow relative to the comet, we can include the magnetic pressure $H^2/8\pi$ in a scalar pressure term if we use a ratio of specific heats $\gamma = 2$. This choice, as far as the gas pressure is concerned, allows for an exchange of kinetic energy with the two degrees of freedom connected with the gyration of the particles in the magnetic field. For $\gamma = 2$ the compression of the magnetic field frozen into the plasma is correctly accounted for in the hydrodynamic equations. It is for this reason that we deliberately choose $\gamma = 2$ in the numerical computations described in later sections.

We write the inviscid hydrodynamic equations as conservation laws modified by source terms on the right-hand side:

$$\frac{\partial}{\partial t} n + \nabla \cdot n\mathbf{u} = A\,, \tag{2.1}$$

$$\frac{\partial}{\partial t} \rho + \nabla \cdot \rho\mathbf{u} = B\,, \tag{2.2}$$

$$\frac{\partial}{\partial t} \rho\mathbf{u} + (\mathbf{u} \cdot \nabla)\, \rho\mathbf{u} + \rho\mathbf{u}(\nabla \cdot \mathbf{u}) + \nabla p = \mathbf{C}\,, \tag{2.3}$$

$$\frac{\partial}{\partial t}\left(\rho \frac{\mathbf{u}^2}{2} + \frac{p}{\gamma - 1}\right) + \nabla \cdot \mathbf{u}\left(\rho \frac{\mathbf{u}^2}{2} + \frac{\gamma}{\gamma - 1}\, p\right) = D\,. \tag{2.4}$$

n, ρ, p represent the number density (ions plus electrons), the mass density and the pressure of the plasma, $\mathbf{u}$ its mass velocity vector, and γ stands for the ratio of specific heats. These equations describe the total change of particle number (2.1), mass (2.2), momentum (2.3), and energy (2.4) of the plasma at any one point in space. The source terms A through D describe the local gains which the plasma undergoes in the corresponding quantities due to the exchange of particles with the neutral cometary gas.

A is therefore twice the local rate of ionizations per unit time and volume, B equals the local rate of ionizations times the mass of the cometary molecule involved plus the local rate of charge exchange processes times the mass difference between the cometary molecule and the solar proton involved in the charge exchange; the solar wind is assumed to consist of ionized hydrogen only. It is the change of composition in the plasma, manifest in a change of the mean molecular weight, which makes it necessary to deal with the conservation of particle number and mass separately in the two continuity Equations (2.1) and (2.3). The source term C contains the gain in momentum due to the addition of the (small) momentum of newly formed cometary ions and the loss of momentum due to extraction of the momentum of neutralized solar protons. The energy source term D describes the corresponding gains and losses for the energy including the changes of the kinetic energy of the particles caused by the ionization itself. We will give a quantitative description of these source terms in Section 4. But already at this stage it can readily be shown that in a steady supersonic flow the source term B for the mass is the predominant right-hand side term in Equations (2.1) through (2.4). To estimate the relative importance of the source terms we compare them with the largest term on the left-hand side. We assume l to be the characteristic length of change in the space derivatives and we neglect the internal energy of the supersonic solar wind.

If we write $\bar{m}$ for the mean mass per particle in the plasma ($\bar{m}=\rho/n$), m_p for the proton mass, m_C and u_C for the mass and velocity of the added cometary ions, and χ for the energy changes connected with the ionization, we have the following comparisons:

$$\begin{aligned} nu/l &: A\,, \\ nu\bar{m}/l &: B \geqslant A m_\mathrm{C}\,, \\ nu^2\bar{m}/l &: |C| \leqslant B\left(u_\mathrm{C} + \frac{m_\mathrm{p}}{m_\mathrm{C}}\,u\right), \\ nu^2\bar{m}/l &: |D| \leqslant B\left(u_\mathrm{C}^2 + \frac{m_\mathrm{p}}{m_\mathrm{C}}\,u^2 + \frac{\chi}{m_\mathrm{C}}\right). \end{aligned}$$

In the undisturbed solar wind we have $\bar{m}=\frac{1}{2}m_\mathrm{p}$ and $u=300$ km/sec, whereas typical values for the cometary parameters give

$$\begin{aligned} m_\mathrm{C} &\approx 30\, m_\mathrm{p}\,, \\ u_\mathrm{C} &\approx 1 \text{ km/sec}\,, \\ \chi &\approx 10 \text{ eV}, \end{aligned}$$

the average excess energy of the ionizing quanta.

Inserting these values we see that $B\!:\!nu\bar{m}/l$ is larger than all the other ratios by a factor $m_\mathrm{C}/m_\mathrm{p}$ or more as long as $\bar{m}$ is of the order of m_p. The change of the molecular weight enforced by the source term B is therefore the predominant mechanism of interaction between the cometary matter and the supersonic solar wind at large distance from the contact surface.

3. A Simplified One-Dimensional Model

To study the physical consequences of the admixture of heavy cometary ions into the supersonic solar wind flow near a comet, we use first a stationary one-dimensional model in which all the variables depend only on a coordinate z parallel to the undisturbed solar wind velocity relative to the comet (BIERMANN, BROSOWSKI, and SCHMIDT, 1963; BIERMANN, 1966). In this model, the velocities have to be parallel everywhere. Consistent with the discussion of the source terms in the last section, we neglect all other source terms besides the mass production term B, which increases the mean molecular weight. From Equations (2.1) through (2.4) we get for this model using again the mean mass $\bar{m}=\rho/n$:

$$\frac{\mathrm{d}}{\mathrm{d}z}(n\cdot u) = 0\,, \tag{3.1}$$

$$\frac{\mathrm{d}}{\mathrm{d}z}(n\bar{m}u) = B\,, \tag{3.2}$$

$$\frac{\mathrm{d}}{\mathrm{d}z}(n\bar{m}u^2 + p) = 0\,, \tag{3.3}$$

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(u\left(n\bar{m}\frac{u^2}{2} + \frac{\gamma}{\gamma-1}p\right)\right) = 0\,. \tag{3.4}$$

We can normalize n, u, $\bar{m}$ by their values at large distance from the comet ($|z|\approx\infty$), n_∞, u_∞, $\bar{m}_\infty$ for the extreme hypersonic case with $p_\infty=0$. We normalize p by $n_\infty\bar{m}_\infty u_\infty^2$ and we write the solution in implicit form:

$$\frac{n}{n_\infty} = \frac{u_\infty}{u} = \frac{\gamma+1}{\gamma-1}\cdot\frac{p}{n_\infty\bar{m}_\infty u_\infty^2} + 1 = \frac{\gamma}{\gamma-1}\left(1 \pm \sqrt{1 - \frac{\gamma^2-1}{\gamma^2}\cdot\frac{\bar{m}}{\bar{m}_\infty}}\right) \tag{3.5}$$

With

$$\frac{\bar{m}}{\bar{m}_\infty} \equiv m^* \quad \text{and} \quad \frac{p}{n_\infty\bar{m}_\infty u_\infty^2} \equiv p^*$$

we have

$$m^* = (1-p^*)\left(1 + \frac{\gamma+1}{\gamma-1}p^*\right)$$

and $m^*_{\max}=\gamma^2/(\gamma^2-1)$ at $p^*=1/(\gamma+1)$.

This solution is shown in Figure 2. It has two branches for the supersonic and the subsonic flow. The supersonic branch which is of main interest to us corresponds to the negative sign of the root. It exists therefore only in a narrow range for the mean molecular weight:

$$1 \leqslant \bar{m}^* \leqslant \frac{\gamma^2}{\gamma^2-1} = \frac{4}{3} \quad \text{for} \quad \gamma = 2\,.$$

An increase in the mean molecular weight enforces a strong deceleration in the supersonic flow, such that for $\gamma=2$ initially $u\sim(\bar{m})^{-3/2}$; since the relative change of n is much smaller, also $n\bar{m}u^{3/2}$; $\approx$const. and $n\bar{m}u$ is $\bar{m}^{-1/2}$ (BIERMANN, BROSOWSKI, and SCHMIDT, 1962).

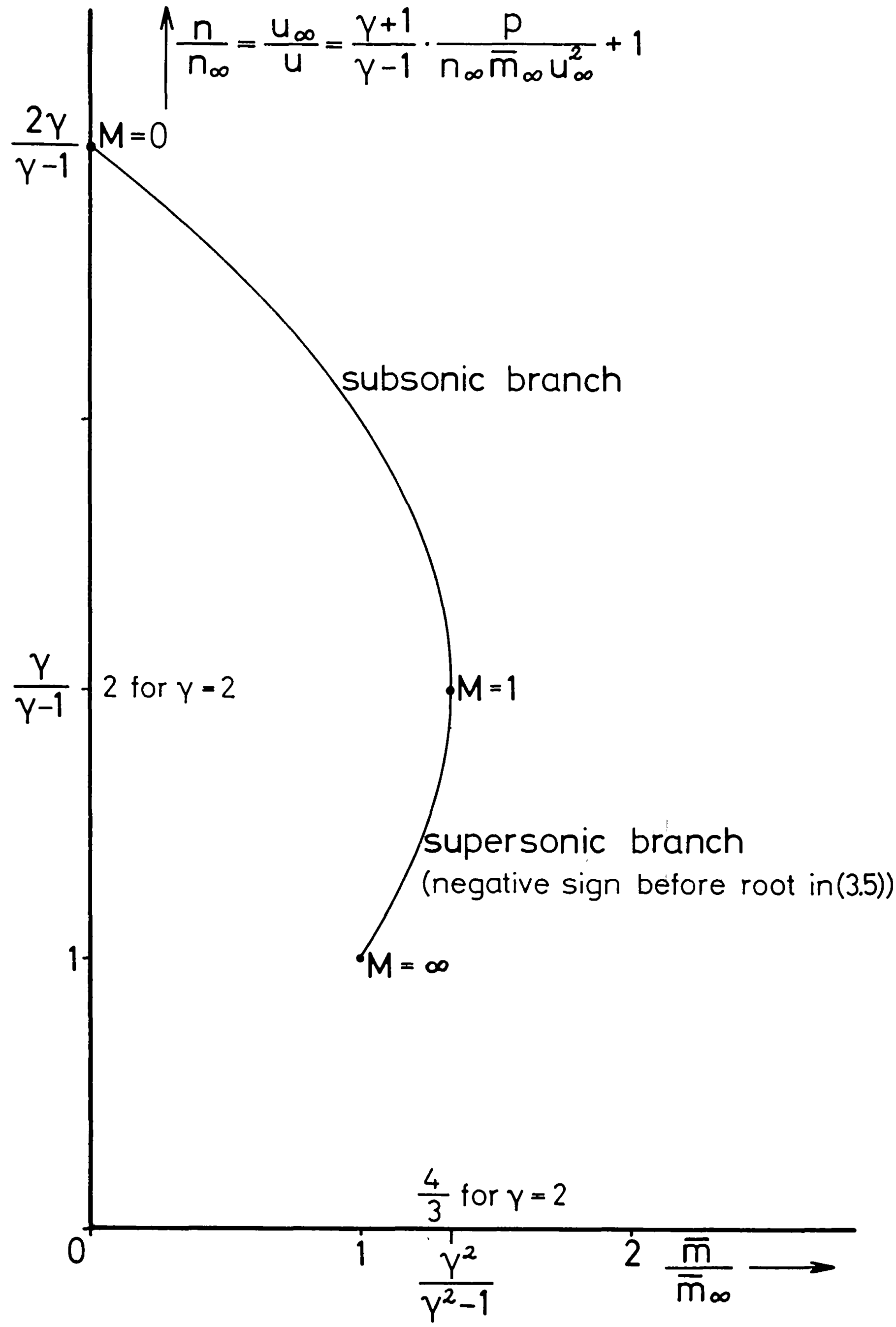

Fig. 2. Plasma flow in the one-dimensional model.

This deceleration is stronger than the one predicted by HARWIT and HOYLE (1961). In the subsonic flow we get an acceleration by the increase in the mean molecular weight. In short, such an increase acts on the flow like a nozzle with a diminishing cross section, which tends to make the flow more sonic, i.e. to shift the Mach number $M = \sqrt{(n\bar{m}u^2/\gamma p)}$ towards unity. The two branches end in a common singular point where

$$m^*_{\max} = \frac{\gamma^2}{\gamma^2 - 1} = \frac{4}{3} \quad \text{for} \quad \gamma = 2\,. \tag{3.6}$$

This singularity represents a self reversal of the flow at Mach number one, which is not realistic. A real flow will develop a shock before such a self reversal is encountered and the flow must diverge, since no stationary flow along only one coordinate is possible beyond the singular point. Upstream from a comet we therefore expect a shock transition in the solar wind at such a distance from the nucleus, that the mean molecular weight of the solar wind is increased due to the added cometary ions by less than a factor $\gamma^2/(\gamma^2-1)$ (Biermann, Brosowski, and Schmidt, 1963; Biermann, 1966). This limit corresponds to an admixture of only about 1% of cometary CO^+ or N_2^+ ions to a solar wind for $\gamma=2$, applicable to the case of a strong transverse magnetic field, and less than 2% for $\gamma=\frac{5}{3}$. We therefore expect a rather large transition region between the bow shock and the contact discontinuity of a comet. For a reliable estimate of the standoff distance of the bow shock one has to introduce the divergence of the flow field explicitly. In our one-dimensional model, the shock front could not be stationary in that coordinate system, in which the comet, i.e. the source of the added heavy ions, is at rest. The shock front would rather proceed upstream as in the case where it is driven by a piston. We will consider a more realistic two-dimensional model in the following sections, which permits stationary solutions and represents the true geometry of the situation much more closely.

4. Equations for an Axial-Symmetric Model

We consider a flow which obeys rotational symmetry with respect to an axis connecting the sun with the comet. We choose cylindrical coordinates z, r with their origin located in the nucleus of the comet so that the negative z-axis points upstream in the solar wind encountering the comet. r is then the distance of a point from the z-axis. Let u and v be the velocity components of the plasma parallel and perpendicular to the z-axis, respectively. We can now rewrite the modified hydrodynamic Equations (2.1) through (2.4) in cylindrical coordinates:

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial z}(n \cdot u) + \frac{1}{r}\frac{\partial}{\partial r}(r \cdot nv) \qquad = A, \tag{4.1}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z}(\rho u) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v) \qquad = B, \tag{4.2}$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial z}(\rho u^2) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho uv) + \frac{\partial p}{\partial z} = C_z, \tag{4.3}$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial}{\partial z}(\rho vu) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v^2) + \frac{\partial p}{\partial r} = C_r \tag{4.4}$$

$$\frac{\partial}{\partial t}\left(\rho\frac{u^2+v^2}{2} + \frac{p}{\gamma-1}\right) + \frac{\partial}{\partial z}\left[u\left(\rho\frac{u^2+v^2}{2} + \frac{\gamma}{\gamma-1}p\right)\right] + \frac{1}{r}\frac{\partial}{\partial r}\left[r \cdot v\left(\rho\frac{u^2+v^2}{2} + \frac{\gamma}{\gamma-1}p\right)\right] = D. \tag{4.5}$$

The vector Equation (2.3) for the changes in the momentum is now split into the two components parallel and perpendicular to the z-axis, (4.3) and (4.4). To describe the interaction between the plasma and the neutral cometary gas in the approximation aimed at here, we also need the equation of continuity for the neutral cometary molecules which eventually are added to the plasma by ionization. Since the cross-sections for transfer of momentum between the plasma and the neutral molecules are of the order of some 10^{-16} cm^2, we assume that the latter freely penetrate the solar wind plasma in a highly supersonic radial motion away from the nucleus. We will regard them as uniform molecules moving as independent particles under the influence of the small radiative acceleration g (which is unobservable even for C_2 out to distances of some 10^5 km), until they happen to become ionized; their velocity vector is therefore a given function of position once the initial speed, with which they leave the collision-dominated zone around the nucleus, is given. Let n_{Cn} be their density and w_z and w_r their velocity components parallel and perpendicular to the z-axis, respectively and let A_{Cn} be the local rate of ionization processes, which these particles undergo per unit volume and time. Their equation of continuity may then be written as:

$$\frac{\partial}{\partial t} n_{\mathrm{Cn}} + \frac{\partial}{\partial z}(n_{\mathrm{Cn}} w_z) + \frac{1}{r}\frac{\partial}{\partial r}(r \cdot n_{\mathrm{Cn}} \cdot w_r) = A_{\mathrm{Cn}} . \tag{4.6}$$

The proper boundary conditions for the six Equations (4.1)–(4.6) follow from a fit of the plasma flow to the undisturbed solar wind at large distances from the nucleus and from a fit of the flow of neutral particles to a radially symmetric outflow near to the nucleus. We assume then that the plasma contains only solar protons, uniform singly ionized cometary ions and electrons, and we will neglect the electron mass wherever it competes with ion masses.

To describe the source terms in detail we use the following symbols. Let m_{e}, m_{p} and m_{C} be the masses of an electron, a proton and a cometary molecule, respectively; R_{C} the probability of photo-ionization for a neutral molecule per unit time; Q_{pn} the cross-section of the neutral molecules for charge exchange with solar protons; Q_{en} the cross-section of the neutral molecules for ionization by electrons (a function of their velocity); χ the potential of ionization of the molecules; and n_{p} the number density of the solar protons. Further we define v_{pn} and v_{en}, two kinds of average relative velocities between the neutrals and the protons or electrons respectively:

$$v_{\mathrm{pn}}^2 := \frac{p}{m_{\mathrm{p}} \cdot n} + (u - w_z)^2 + (v - w_r)^2 , \tag{4.7}$$

$$v_{\mathrm{en}}^2 := \frac{p}{m_{\mathrm{e}} n} + (u - w_z)^2 + (v - w_r)^2 . \tag{4.8}$$

These definitions imply an equipartition of the thermal energy over protons as well as electrons. This is a rather crude assumption for the electrons, since in the transition zone behind the bow shock of the earth they seem to acquire their proper share of energy only in occasional bursts (ASBRIDGE *et al.*, 1966; VASYLINUAS, 1966), where under normal conditions they seem to have only 5–10% of that energy. For the rates of ionizing processes per unit volume and time we use the following abbreviations:

$$P_{\mathrm{n}} := n_{\mathrm{Cn}} \cdot R_{\mathrm{C}} \quad \text{for photo-ionization,} \tag{4.9}$$

$$P_{\mathrm{p}} := n_{\mathrm{Cn}} \cdot n_{\mathrm{p}} \cdot v_{\mathrm{pn}} \cdot Q_{\mathrm{pn}} \quad \text{for charge exchange,} \tag{4.10}$$

$$P_{\mathrm{e}} := n_{\mathrm{Cn}} \cdot \frac{n}{2} \cdot v_{\mathrm{en}} \cdot Q_{\mathrm{en}}(v_{\mathrm{en}}) \quad \text{for ionization by electron impact.} \tag{4.11}$$

With these definitions we can now specify the source terms in Equations (4.1) through (4.6):

$$A = 2(P_{\mathrm{n}} + P_{\mathrm{e}}), \tag{4.12}$$

$$B = P_{\mathrm{n}} \cdot m_{\mathrm{C}} + P_{\mathrm{e}} m_{\mathrm{C}} + P_{\mathrm{p}}(m_{\mathrm{C}} - m_{\mathrm{p}}), \tag{4.13}$$

$$C_z = P_{\mathrm{n}} m_{\mathrm{C}} \cdot w_z + P_{\mathrm{e}} m_{\mathrm{C}} w_z + P_{\mathrm{p}}(m_{\mathrm{C}} w_z - m_{\mathrm{p}} u), \tag{4.14}$$

$$C_r = P_{\mathrm{n}} m_{\mathrm{C}} w_r + P_{\mathrm{e}} m_{\mathrm{C}} w_r + P_{\mathrm{p}}(m_{\mathrm{C}} w_r - m_{\mathrm{p}} v), \tag{4.15}$$

$$\begin{aligned} D = {} & P_{\mathrm{n}} \cdot \left(m_{\mathrm{C}} \cdot \frac{w_z^2 + w_r^2}{2} + \chi^* \right) + P_{\mathrm{e}} \cdot \left(m_{\mathrm{C}} \cdot \frac{w_z^2 + w_r^2}{2} - \chi \right) \\ & + P_{\mathrm{p}} \cdot \left(m_{\mathrm{C}} \cdot \frac{w_z^2 + w_r^2}{2} - m_{\mathrm{p}} \cdot \frac{u^2 + v^2}{2} - \frac{p}{n(\gamma - 1)} \right), \end{aligned} \tag{4.16}$$

$$A_{\mathrm{Cn}} = - P_{\mathrm{p}} - P_{\mathrm{e}} - P_{\mathrm{n}} . \tag{4.17}$$

The source terms for particle numbers ((4.12) and (4.17)), for mass ((4.13)) and for momentum ((4.14) and (4.15)) interprete themselves, but the energy source term ((4,16)) needs further comment. Common to all ionizing processes is the gain of the kinetic energy of the ionized molecule $m_{\mathrm{C}} \cdot (w_z^2 + w_r^2)/2$.

In a photo-ionization we have an additional gain χ^* due to the surplus energy of the photon, which must be transferred mainly to the electron. We have determined this gain numerically, using observed solar UV flux data at 1 a.u. (HINTEREGGER, HALL and SCHMIDTKE, 1965) and suitable experimental cross-sections (NICOLET, 1961). From these data we got a surplus energy of 14.5 eV, which resembles the earlier estimate $\gtrsim$ 10 eV for the most common molecules (BIERMANN and TREFFTZ, 1964). It is obvious that the plasma suffers a loss χ in an ionization by electron impact and a loss of the proton's share of the kinetic and thermal energy in a charge exchange process (BIERMANN and TREFFTZ, 1960). It should be mentioned that we neglect all energy losses of the plasma electrons due to collision processes.

We assumed that the neutrals move under the influence of the acceleration g by solar radiation. The orbits of these particles can therefore easily be integrated. For a particle leaving the nucleus at time $t = 0$ we have

$$z(t) = \frac{g}{2} t^2 - wt \cos \vartheta \tag{4.18}$$

and

$$r(t) = wt \sin \vartheta , \tag{4.19}$$

where ϑ represents the angle between the initial velocity vector and the z-axis, and w is the initial velocity itself. With these orbits we can integrate the equation of continuity. But it is necessary to divide the number density into two parts:

$$n_{\mathrm{Cn}} = n_{\mathrm{Cn}}^{(1)} + n_{\mathrm{Cn}}^{(2)} . \tag{4.20}$$

$n_{\mathrm{Cn}}^{(1)}(z, r)$ counts the particles which have not yet reached the common envelope of their orbits, whereas $n_{\mathrm{Cn}}^{(2)}(z, r)$ counts those particles which have already passed it. Although $n_{\mathrm{Cn}}^{(2)}$ in the case considered subsequently is usually $\ll n_{\mathrm{Cn}}^{(1)}$, this subdivision is interesting, because the two kinds of particles have different velocities: $w_z^{(i)}$, $w_z^{(i)}$, $i=1, 2$. Correspondingly, we must subdivide the rate of change in the particle number, which is the negative rate of ionizations:

$$A_{\mathrm{Cn}} = A_{\mathrm{Cn}}^{(1)} + A_{\mathrm{Cn}}^{(2)} . \tag{4.21}$$

We now define probabilities for these two kinds of particles to become ionized during unit time by any of the three processes:

$$R_{\mathrm{C}}^{(i)} = - \frac{A_{\mathrm{Cn}}^{(i)}}{n_{\mathrm{Cn}}^{(i)}} , \quad i = 1, 2 . \tag{4.22}$$

These probabilities are independent of the number densities themselves, but they depend on the velocities $w_z^{(i)}$, $w_r^{(i)}$ as can be verified from Equations (4.17), (4.9), (4.10), (4.11). It should be mentioned here, that the same subdivision has to be made for all the source terms, since they all depend on the velocities $w_z^{(i)}$ and $w_r^{(i)}$.

For stationary conditions Equation (4.6) is solved by:

$$n_{\mathrm{Cn}}^{(i)}(z, r) = \frac{G}{4\pi w^2} \cdot \frac{\exp - \left[\int_0^{(z, r)} R_{\mathrm{C}}^{(i)} \cdot \frac{\mathrm{d}s}{w_s^{(i)}} \right]}{t_i^2 (g \cdot t_i \cos \vartheta_i - w)} , \tag{4.23}$$

$i=1, 2$, where G is the total of the neutrals leaving the nucleus per unit time, t_i and ϑ_i are determined for given z and r by the Equations (4.18) and (4.19). The exponential represents the probability that a given neutral leaving the nuclear regions under the proper angle ϑ_i actually reaches the position (z, r) without being ionized. The integral

$$\int_0^{(z, r)} R_{\mathrm{C}}^{(i)} \frac{\mathrm{d}s}{w_s^{(i)}} , \quad i = 1, 2 , \tag{4.24}$$

sums up the probability of ionization over the flight time on a possible orbit from the nucleus to the position (z, r). The integration therefore proceeds along such an orbit with ds being the differential of the length of arc and $w_s^{(i)}$ being the instantaneous velocity tangential to the orbit.

5. A Closed Description for the Stationary Flow on the Axis

We now consider the stationary flow on the axis connecting the sun and the comet. For any function $\alpha(r)$ which has a continuous derivative with respect to r the following identity holds:

$$\lim_{r\to 0} \frac{1}{r} \frac{\partial}{\partial r}\big(r\cdot\alpha(r)\big) = 2\cdot\frac{\partial\alpha(r)}{\partial r}. \tag{5.1}$$

With this equation we can derive from Equations (4.1) through (4.5) the corresponding equations in the limit $r\to 0$, i.e. on the axis. For the stationary case we get for the balance of particle number, mass, momentum parallel to the axis and energy from Equations (4.1), (4.2), (4.3), and (4.5):

$$\frac{\partial}{\partial z}(n\cdot u) + 2n\cdot\frac{\partial v}{\partial r} = A, \tag{5.2}$$

$$\frac{\partial}{\partial z}(\rho\cdot u) + 2\rho\cdot\frac{\partial v}{\partial r} = B, \tag{5.3}$$

$$\frac{\partial}{\partial z}(\rho u^2 + p) + 2\rho u\frac{\partial v}{\partial r} = C_z, \tag{5.4}$$

$$\frac{\partial}{\partial z}\left[u\cdot\left(\rho\frac{u^2}{2} + \frac{\gamma}{\gamma-1}p\right)\right] + 2\left(\rho\frac{u^2}{2} + \frac{\gamma}{\gamma-1}p\right)\frac{\partial v}{\partial r} = D. \tag{5.5}$$

The equation for the transport of momentum perpendicular to the axis (4.4) becomes trivial on the axis. Therefore we first differentiate (4.4) with respect to r and then go to limit $r\to 0$:

$$\frac{\partial}{\partial z}\left(\rho u\frac{\partial v}{\partial r}\right) + 3\rho\cdot\left(\frac{\partial v}{\partial r}\right)^2 + \frac{\partial^2 p}{\partial r^2} = \frac{\partial C_r}{\partial r}. \tag{5.6}$$

We now define the quantity $R(z)$ as the radius of curvature of the isobaric surface, which cuts through the z-axis at the location z. With this definition we have

$$\frac{\partial^2 p}{\partial r^2} = -\frac{1}{R(z)}\cdot\frac{\partial p}{\partial z} \tag{5.7}$$

assuming $\partial^2 p/\partial r^2 < 0$. Inserting (5.7) into Equation (5.6) we get an equation for $\partial v/\partial r$, which contains no other explicit derivatives with respect to r on the left hand side.

$$\frac{\partial}{\partial z}\left(\rho u\frac{\partial v}{\partial r}\right) + 3\rho\left(\frac{\partial v}{\partial r}\right)^2 - \frac{1}{R(z)}\cdot\frac{\partial p}{\partial z} = \frac{\partial C_r}{\partial r}. \tag{5.8}$$

We can now resolve Equations (5.3), (5.4), (5.5) for the derivatives $\partial u/\partial z$, $\partial\rho/\partial z$ and $\partial p/\partial z$. Using the Machnumber $M^2 = \rho u^2/\gamma p$ for abbreviation we get:

$$\frac{\partial u}{\partial z} = \frac{\left(\frac{\gamma+1}{2\gamma}Bu^2 - C_z u + \frac{\gamma-1}{\gamma}D\right)\cdot\frac{1}{p} - 2\cdot\frac{\partial v}{\partial r}}{1-M^2}, \tag{5.9}$$

$$\frac{\partial\rho}{\partial z} = \frac{1}{u}\cdot\frac{\left[\left(1-\frac{\gamma+3}{2}M^2\right)Bu^2 + \gamma M^2 C_z u - (\gamma-1)\,M^2 D\right]\cdot\frac{1}{u^2} + 2\rho M^2\frac{\partial v}{\partial r}}{1-M^2}, \tag{5.10}$$

$$\frac{\partial p}{\partial z}=\frac{\left[-\left(1+\frac{\gamma-1}{2}M^2\right)Bu^2+\left(1+(\gamma-1)M^2\right)C_z u-(\gamma-1)M^2D\right]\cdot\frac{1}{u}+2\rho u\frac{\partial v}{\partial r}}{1-M^2}, \tag{5.11}$$

From Equations (5.2) and (5.8) we get

$$\frac{\partial n}{\partial z}=+\frac{1}{u}\left\{-n\cdot\frac{\partial u}{\partial z}+A-2n\cdot\frac{\partial v}{\partial r}\right\} \tag{5.12}$$

and

$$\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial r}\right)=-\frac{1}{\rho u}\frac{\partial v}{\partial r}\frac{\partial}{\partial z}(\rho u)-\frac{3}{u}\left(\frac{\partial v}{\partial r}\right)^2+\frac{1}{R(z)\,\rho u}\frac{\partial p}{\partial z}+\frac{1}{\rho u}\frac{\partial C_r}{\partial r}. \tag{5.13}$$

The five Equations (5.9) through (5.13) form a system of ordinary differential equations for the functions u, ρ, p, n and $\partial v/\partial r$, which is closed if the function $R(z)$ is given.

For the discussion of the boundary conditions and of the conditions to be met at the flow discontinuities, we refer to Figure 3.

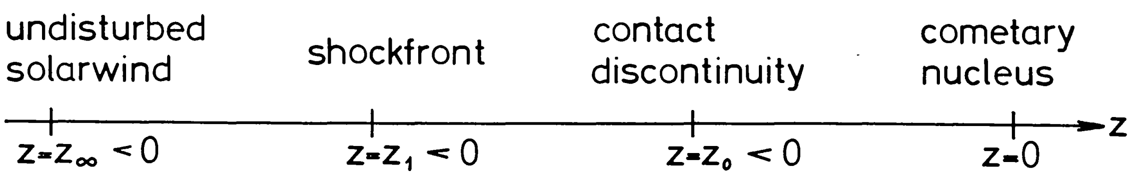

Fig. 3. Characteristic distances along the radius vector sun–comet.

The following boundary conditions have to be met at $z=z_\infty$:

$$\begin{aligned}
&(\mathrm{B1}) \quad u(z_\infty) &&= u_\infty,\\
&(\mathrm{B2}) \quad \rho(z_\infty) &&= \rho_\infty,\\
&(\mathrm{B3}) \quad p(z_\infty) &&= p_\infty,\\
&(\mathrm{B4}) \quad n(z_\infty) &&= n_\infty,\\
&(\mathrm{B5}) \quad \left.\frac{\partial v}{\partial r}\right|_{z=z_\infty} &&= 0,
\end{aligned}$$

where the subscript ∞ refers to the values in the undisturbed solar wind.

At the position of the shock $z=z_1$ the Rankine–Hugoniot conditions are applicable (Courant and Friedrichs, 1948). Using the subscripts 1 and 2 for the values of the variables immediately before and behind the shock and writing $\tilde{R}(z_1)$ for the radius of curvature of the shock on the axis we get

$$u_2=u_1\left[1-\frac{2}{\gamma+1}\left(1-\frac{1}{M_1^2}\right)\right], \tag{5.14}$$

$$\rho_2=\rho_1\cdot\frac{1}{1-\frac{2}{\gamma+1}\left(1-\frac{1}{M_1^2}\right)}, \tag{5.15}$$

$$p_2 = p_1 \cdot \left[1 + \frac{2\gamma}{\gamma + 1} \left(M_1^2 - 1 \right) \right], \tag{5.16}$$

$$n_2 = n_1 \cdot \frac{1}{1 - \dfrac{2}{\gamma + 1} \left(1 - \dfrac{1}{M_1^2} \right)}, \tag{5.17}$$

$$\left(\frac{\partial v}{\partial r} \right)_2 = \left(\frac{\partial v}{\partial r} \right)_1 + \frac{u_1 - u_2}{\tilde{R}(z_1)}. \tag{5.18}$$

To illustrate condition (5.18) we use Figure 4. Let $\mathbf{u}_1(z, r)$ and $\mathbf{u}_2(z, r)$ be the velocity vectors immediately before and behind the shock, $\tilde{R}(z, r)$ the radius of curvature of the shock front, and $\tilde{u}_i(z, r)$ and $\tilde{v}_i(z, r)$ the components of $\mathbf{u}_i(z, r)$, $i = 1, 2$, which are tangential and perpendicular to the shock front, whereas u_i and v_i are, as before, the components parallel and perpendicular to the z-axis.

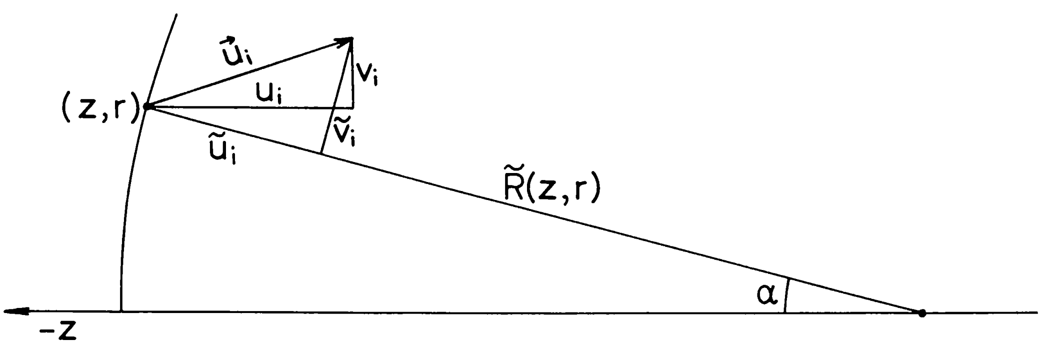

Fig. 4. Geometrical relations near the shock front.

From Figure 4 we read:

$$\left. \begin{aligned} \tilde{v}_1 &= |\mathbf{u}_1| \cdot \sin \left(\alpha + \operatorname{artcg} \frac{v_1}{u_1} \right), \\ \tilde{v}_2 &= |\mathbf{u}_2| \cdot \sin \left(\alpha + \operatorname{artcg} \frac{v_2}{u_2} \right). \end{aligned} \right\} \tag{5.19}$$

Since $\tilde{v}_1 = \tilde{v}_2$, we get from (5.19) for small values of $r/\tilde{R}$:

$$v_2 - v_1 = (u_1 - u_2) \cdot \frac{r}{\tilde{R}}. \tag{5.20}$$

From the derivative of (5.20) with respect to r we get (5.18) in the limit $r \to 0$.

At $z = z_0$, where the contact discontinuity intersects the axis, we have a stagnation point. Therefore the velocity must vanish at this point, and the pressure of the plasma has to balance the pressure p_{Ci} of the purely cometary plasma inside the contact discontinuity:

$$\text{(B6)} \quad u(z_0) = 0,$$

$$\text{(B7)} \quad p(z_0) = p_{Ci}(z_0).$$

We now have to estimate the pressure $p_{Ci}(z)$. The purely cometary plasma inside the contact surface is produced by photo-ionization only (neglecting secondary ionization by photo-electrons). We therefore assume, in analogy to the treatment of the source term D ((4.16)), that this plasma carries a kinetic energy per molecule χ^*, which near the contact surface is nearly microscopic, neglecting in this region changes in the velocity w due to radiative acceleration and neglecting also any energy losses of the electrons. We estimate the density of cometary molecular ions n_{Ci} from the density of all cometary molecules n_C multiplied by the probability that they might be ionized at the respective position. On the axis we therefore get:

$$n_{Cn}(0 > z > z_0) = \frac{G}{4\pi z^2 w} \tag{5.21}$$

and, in a less satisfactory approximation for the ions (index j):

$$n_{Ci}(0 > z > z_0) = \frac{G}{4\pi z^2 w_j}\left(1 - e^{-R_C \frac{-z}{w}}\right) \tag{5.22}$$

in analogy to (4.23). We get the following estimate for the stagnation point:

$$p_{Ci} = (\gamma - 1)\cdot\frac{G}{4\pi z_0^2 w_j}\left(1 - e^{-R_C \cdot \frac{-z_0}{w}}\right)\cdot\chi^* \tag{5.23}$$

in which the bracket is $\approx R_C \cdot z_0 / w$. The last term is the average energy per ion near the contact surface, which has been estimated to be $\gtrsim 10$ eV. This leads to

$$p_{Ci}(z_0) = \frac{(\gamma - 1)\cdot\chi^*}{w_j w}\cdot\frac{G\cdot R_C}{4\pi\,|z_0|}. \tag{5.24}$$

Our task is to determine the flow of plasma on the z-axis. For this purpose we have to solve the five differential Equations (5.9) through (5.13) in such a way that the seven boundary conditions (B1), through (B7), which involve the two unknowns z_0 and z_1, are met. For arbitrary z_0 and z_1 there will in general be no solution. Therefore we can reformulate the problem: One has to determine z_0 and z_1 in such a way that for given boundary conditions (B1) through (B7) the system (5.9) through (5.13) has a solution. To solve this problem we have to supply the density of the neutrals $n_{Cn}(z)$ and the radius of curvature $R(z)$ of the isobaric surfaces, both on the z-axis. In the next section we will derive two differential equations for the density of the neutrals. These equations must be integrated together with the system (5.9) through (5.13). For the case of photo-ionization we can derive $n_{Cn}(z)$ in closed form. But as far as the radius of curvature $R(z)$ is concerned we have to introduce an additional assumption. We assume $R(z)$ to be proportional to the distance of the isobare from the nucleus. For the numerical calculations discussed in Section 8 we choose

$$R(z) = 2\,|z|\,. \tag{5.25}$$

This formula implies that the isobaric surfaces near to the axis form a family of confocal paraboloids, which have their focus in the nucleus. The assumption seems to be a reasonable one for all z values not too close to the stagnation point, where the isobars gradually transform into spheres centered at the stagnation point. But since the Mach number goes to zero, $M \to 0$, the pressure is predominant in this region, and it must therefore be very nearly constant. The flow is then, to a good approximation, determined by equations of continuity (5.10) and (5.12), taking $M=0$ and using (5.9), so that our assumption about $\partial p^2/\partial r^2$ cannot introduce an essential error. To check this supposition, we have subsequently to confirm that the pressure becomes constant near the stagnation point in any model constructed on the basis of our assumption (5.25).

6. The Density of the Neutrals on the Axis

We rewrite Equations (4.20) and (4.23) for the density of neutrals in the limit $r \to 0$:

$$n_{\mathrm{Cn}}^{(1)}(z) = \frac{G}{4\pi w^2} \cdot \frac{\exp - \left[\int_0^z R_{\mathrm{C}}^{(1)} \frac{\mathrm{d}z}{w_z^{(1)}}\right]}{t_1^2 (g t_1 - w)} \tag{6.1}$$

and

$$n_{\mathrm{Cn}}^{(2)}(z) = \frac{G}{4\pi w^2} \cdot \frac{\exp - \left[\int_0^{z_t} R_{\mathrm{C}}^{(1)} \frac{\mathrm{d}z}{w_z^{(1)}} + \int_{z_t}^{z} R_{\mathrm{C}}^{(2)} \frac{\mathrm{d}z}{w_z^{(2)}}\right]}{t_2^2 (g t_2 - w)}, \tag{6.2}$$

where z_t is the location of the turning point of the neutrals on the z-axis:

$$z_t = -\frac{1}{2} \cdot \frac{w^2}{g} \tag{6.3}$$

t_1 and t_2 can be computed from

$$z = \frac{g}{2} t^2 - wt, \tag{6.4}$$

once z is given:

$$\left.\begin{aligned} t_1 &= \frac{w}{g} \cdot (1 - H(z)), \\ t_2 &= \frac{w}{g} \cdot (1 + H(z)). \end{aligned}\right\} \tag{6.5}$$

where we have used the abbreviation

$$H(z) = \sqrt{1 + \frac{2zg}{w_2}}, \quad (z < 0). \tag{6.6}$$

The velocities $w_z^{(i)}$ follow from the equations in (6.5):

$$w_z^{(1)} = - w \cdot H(z), \tag{6.7}$$

$$w_z^{(2)} = w \cdot H(z). \tag{6.8}$$

Using these denotations we get

$$n_{\mathrm{Cn}}^{(1)}(z) = \frac{G \cdot g^2}{4\pi w^5} \cdot \frac{\exp - \left[\int_0^z R_{\mathrm{C}}^{(1)} \cdot \frac{\mathrm{d}z}{- wH(z)}\right]}{(1 - H(z))^2 \cdot H(z)}. \tag{6.9}$$

and

$$n_{\mathrm{Cn}}^{(2)}(z) = \frac{G \cdot g^2}{4\pi w^5} \cdot \frac{\exp - \left[\int_0^{z_t} R_{\mathrm{C}}^{(1)} \frac{\mathrm{d}z}{- wH(z)} + \int_{z_t}^z R_{\mathrm{C}}^{(2)} \frac{\mathrm{d}z}{wH(z)}\right]}{(1 + H(z))^2 \cdot H(z)}. \tag{6.10}$$

Using

$$\frac{\mathrm{d}H(z)}{\mathrm{d}z} = \frac{g}{w^2} \cdot \frac{1}{H(z)} \tag{6.11}$$

and

$$H(z_t) = 0, \tag{6.12}$$

we obtain after integration by parts

$$\frac{1}{w}\int_{z0}^{z} R_{\mathrm{C}}^{(1)} \frac{\mathrm{d}z}{H(z)} = \frac{w}{g}\left[R_{\mathrm{C}}^{(1)}(z)\, H(z) - R_{\mathrm{C}}^{(1)}(z_0)\, H(z_0)\right] + J_1(z) \tag{6.13}$$

and

$$\frac{1}{w}\int_{z_t}^{z} R_{\mathrm{C}}^{(2)} \frac{\mathrm{d}z}{H(z)} = \frac{w}{g}\left[R_{\mathrm{C}}^{(2)}(z) \cdot H(z)\right] + J_2(z), \tag{6.14}$$

where we have used

$$J_1(z) = - \frac{w}{g}\int_{z_0}^{z} H(z) \frac{\mathrm{d}}{\mathrm{d}z}(R_{\mathrm{C}}^{(1)}(z))\, \mathrm{d}z \tag{6.15}$$

and

$$J_2(z) = - \frac{w}{g}\int_{z_t}^{z} H(z) \frac{\mathrm{d}}{\mathrm{d}z}(R_{\mathrm{C}}^{(2)}(z))\, \mathrm{d}z \tag{6.16}$$

for abbreviation.

The functions $J_1(z)$ and $J_2(z)$ obey the differential equations:

$$\frac{\mathrm{d}\ J_1(z)}{\mathrm{d}z} = - \frac{w}{g} H(z) \frac{\mathrm{d}}{\mathrm{d}z} R_{\mathrm{C}}^{(1)}(z) \tag{6.17}$$

with the initial value $J_1(z_0) = 0$ and

$$\frac{\mathrm{d}\ J_2(z)}{\mathrm{d}z} = - \frac{w}{g} H(z) \frac{\mathrm{d}}{\mathrm{d}z} R_{\mathrm{C}}^{(2)}(z) \tag{6.18}$$

with the initial value $J_2(z_t) = 0$. Defining

$$I_0 = \int_0^{z_0} R_C^{(1)} \cdot \frac{dz}{wH(z)} \tag{6.19}$$

and

$$I(z) = \int_{z_0}^{z} R_C^{(1)} \cdot \frac{dz}{w \cdot H(z)} \tag{6.20}$$

we derive from (6.9) and (6.10):

$$n_{Cn}^{(1)}(z) = \frac{G \cdot g^2}{4\pi w^5} \cdot \frac{\exp\left[I_0 + \frac{w}{g}\left(R_C^{(1)}(z) \cdot H(z) - R_C^{(1)}(z_0) \cdot H(z_0)\right) + J_1(z)\right]}{(1 - H(z))^2 \cdot H(z)} \tag{6.21}$$

and

$$n_{Cn}^{(2)}(z) = \frac{Gg^2}{4\pi w^5} \cdot \frac{\exp - \left[I_0 + I(z_t) - \frac{w}{g} R_C^{(2)}(z) \cdot H(z) - J_2(z)\right]}{(1 + H(z))^2 \cdot H(z)} \tag{6.22}$$

In the special case where all ions are due only to photo-ionization, i.e. $R_C^{(i)} = R_{Cn}$, we can simplify Equations (6.21) and (6.22):

$$n_{Cn}^{(1)}(z) = \frac{G \cdot g^2}{4\pi w^5} \cdot \frac{\exp - \left[\frac{R_{Cn} w}{g} (1 - H(z)\right]}{(1 - H(z))^2 \cdot H(z)} \tag{6.23}$$

and

$$n_{Cn}^{(2)}(z) = \frac{G \cdot g^2}{4\pi w^5} \cdot \frac{\exp - \left[\frac{R_{Cn} w}{g} (1 - H(z)\right]}{(1 + H(z))^2 \cdot H(z)}. \tag{6.24}$$

To determine the density of neutrals, which we need to determine the source terms defined in (4.12) through (4.16) and used in the system (5.9) through (5.13), we generally use Equations (6.21) and (6.22) together with the differential Equations (6.17) and (6.18) for the functions $J_i(z)$. For pure photo-ionization we can replace (6.21) and (6.22) by the closed expressions (6.23) and (6.24).

7. Numerical Method of Solution

We now comprise the results of the last two sections. To determine the plasma flow on the z-axis we have to find values z_0 and z_1 for the location of the discontinuities such that the following system of ordinary differential equations has a solution fitting boundary conditions to be specified:

$$\frac{\mathrm{d}u}{\mathrm{d}z}=\frac{\left(\frac{\gamma+1}{2\gamma}u^2\cdot B-u\cdot C_z+\frac{\gamma-1}{\gamma}D\right)\cdot\frac{1}{p}-2\cdot\frac{\partial v}{\partial r}}{1-M^2}, \tag{7.1}$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}z}=\frac{1}{u}\cdot\frac{\left[\left(1-\frac{\gamma+3}{2}M^2\right)Bu^2+\gamma M^2uC_z-(\gamma-1)\,M^2D\right]\cdot\frac{1}{u^2}+2\rho M^2\frac{\partial v}{\partial r}}{1-M^2}, \tag{7.2}$$

$$\frac{\mathrm{d}p}{\mathrm{d}z}=\frac{\left[-\left(1+\frac{\gamma-1}{2}M^2\right)Bu^2+\left(1+(\gamma-1)M^2\right)C_z\cdot u-(\gamma-1)\,M^2\cdot D\right]\cdot\frac{1}{u}+2\rho u\cdot\frac{\partial v}{\partial r}}{1-M^2}, \tag{7.3}$$

$$\frac{\mathrm{d}n}{\mathrm{d}z}=+\frac{1}{u}\left\{-n\cdot\frac{\mathrm{d}u}{\mathrm{d}z}+A-2n\frac{\partial v}{\partial r}\right\}, \tag{7.4}$$

$$\frac{\mathrm{d}\frac{\partial v}{\partial r}}{\mathrm{d}z}=-\frac{1}{\rho u}\frac{\partial v}{\partial r}\frac{\mathrm{d}\rho u}{\mathrm{d}z}-\frac{3}{u}\left(\frac{\partial v}{\partial r}\right)^2+\frac{1}{R(z)\,\rho u}\frac{\mathrm{d}p}{\mathrm{d}z}+\frac{1}{\rho u}\cdot\frac{\partial C_r}{\partial r} \tag{7.5}$$

$$\frac{\mathrm{d}\ J_1}{\mathrm{d}z}=-\frac{w}{g}H(z)\cdot\frac{\mathrm{d}}{\mathrm{d}z}\left(R_{\mathrm{C}}^{(1)}(z)\right), \tag{7.6}$$

$$\frac{\mathrm{d}\ J_2}{\mathrm{d}z}=-\frac{w}{g}H(z)\cdot\frac{\mathrm{d}}{\mathrm{d}z}\left(R_{\mathrm{C}}^{(2)}(z)\right). \tag{7.7}$$

These are the Equations (5.9) through (5.13), (6.17), and (6.18) rewritten as ordinary equations for the variation along the z-axis. $\partial v/\partial r$ is handled as an ordinary variable. For the definition of the variables $u, \rho, p, n, \partial v/\partial r$, of the source terms $A, B, C_z, \partial C_r/\partial r, D$, of the functions $R_{\mathrm{C}}^i(z)$, and of the parameters w, g we refer to Section 4; for the Mach-number M and the function $R(z)$ to Section 5; and for the variables J_i and the function $H(z)$ to Section 6. The right-hand sides of the system (7.1) through (7.7) are coupled explicitly by the variables themselves and implicitly by the source terms and the functions $R_{\mathrm{C}}^i/(z)$, which are closely connected with the source terms. The source terms can be computed from the variables and the density of the neutral particles, which in turn can be determined from the variables using Equations (6.21) and (6.22).

At the shock position $z=z_1$ the variables $u, \rho, p, n, \partial v/\partial r$ have to fulfill conditions (5.14) through (5.18). The boundary conditions result from the fit to the undisturbed solar wind at $z=z_\infty$:

(B1) $u(z_\infty)=u_\infty$,

(B2) $\rho(z_\infty)=\rho_\infty$,

(B3) $p(z_\infty)=p_\infty$,

(B4) $n(z_\infty)=n_\infty$,

$$\text{(B5)} \quad \left.\frac{\partial v}{\partial r}\right|_{z=z_\infty} = 0\,,$$

from the fit to the purely cometary plasma at the stagnation point $z = z_0$:

$$\text{(B6)} \quad u(z_0) = 0\,,$$

$$\text{(B7)} \quad p(z_0) = p_{Ci}(z_0)\,,$$

where p_{Ci} is given in (5.24), and from the fit of the functions J_i (see (6.15), (6.16)) at stagnation point $z = z_0$ and at the turning point $z = z_t$:

$$\text{(B8)} \quad J_1(z_0) = 0\,,$$

$$\text{(B9)} \quad J_2(z_t) = 0\,,$$

where the turning point is given in (6.3).

To determine the positions z_0 and z_1 which refer to the stagnation point and the shock front we proceed as follows. We choose trial values for z_1 and $J_1(z_t)$ and integrate the system (7.1) through (7.7) beginning at $z = z_\infty$ using (B1) through (B5) and (B9) as initial values. We proceed until we reach a point $\tilde{z}$, for which we get $u(\tilde{z}) = 0$. Here we determine $J_1(\tilde{z})$ and $p(\tilde{z})$. If $J_1(\tilde{z}) = 0$ and if $p(\tilde{z}) = p_{Ci}(\tilde{z})$, we have a solution, since the boundary conditions (B6), (B7) and (B8) are met at $z = \tilde{z} = z_0$. If this is not the case, we derive corrections for the initial guesses of z_1 and $J_1(z_t)$ from the errors $p(\tilde{z}) - p_{Ci}(\tilde{z})$ and $J_1(\tilde{z})$ and repeat the integration. For the case of pure photo-ionization the procedure is simpler, since we do not need to integrate Equations (7.6) and (7.7). To determine the density of neutrals we use instead the explicit formulae (6.23) and (6.24). Therefore we have to choose trial values for z_1 only, until the subsequent integration of the remaining five differential equations with the initial values (B1) through (B5) meets the boundary conditions (B6) and (B7) at the same location $\tilde{z} = z_0$. It should be mentioned, however, that for a given trial value there is not necessarily a solution of the initial value problem for which the velocity u has a root, $u(\tilde{z}) = 0$ (BROSOWSKI, 1966).

8. Numerical Results and Discussion

With the method just described, 35 solutions of the system (7.1) through (7.5) were integrated for the case of photo-ionization. These 35 models refer to different values of solar and cometary parameters. For the total cometary gas production we choose five levels, which seem to be in the range of recent estimates for a typical comet at 1 a.u. (BIERMANN and TREFFTZ, 1964; HUEBNER, 1965):

$$G = 5 \cdot 10^{29},\ 10^{30},\ 2 \cdot 10^{30},\ 5 \cdot 10^{30},\ 10^{31}\,[\mathrm{sec}^{-1}]\,.$$

For all other cometary parameters we have used fixed values:

$$w = w_j = 1\ \mathrm{km/sec},\ g = 0.01\ \mathrm{cm/sec}^2,\ m_C = 30 m_p\,,$$
$$\chi = \chi^* = 14.5\ \mathrm{eV},\ R_C = 10^{-6}\ \mathrm{sec}^{-1}\,,$$

for the initial expansion velocity of the neutrals, the radiative acceleration, the mass, the ionization potential and the probability of photo-ionization of a cometary molecule, respectively. It may be noted that R_C and particularly $\chi^*/w_j w$ are likely to be on the high side (BIERMANN and TREFFTZ, 1964). Concerning the value of g we note that the large majority of the neutral molecules does not contribute to the spectrum above $\lambda = 3000$ Å and that for this reason the average acceleration by solar light pressure must be exceedingly small, the actual value depending of course on the unknown chemical composition; in the earlier work the radiative acceleration had been neglected altogether. For the solar wind we have chosen seven typical situations from the Mariner 2 data for the densities and velocities at 1 a.u. The corresponding values for n_∞ and u_∞ are given in Table I. The boundary values for the pressure p_∞ and the mass density ρ_∞ are adjusted so that the Mach number is always $M_\infty = 10$ and the average mass per particle $\rho_\infty/n_\infty = m_p/2$, corresponding to a pure hydrogen plasma. The choice of the Mach number is rather arbitrary, but any Mach number above 5 can change the solution only by less than 4%. The ratio of the specific heats γ was set equal 2 (see Section 2).

TABLE I

No.	n_∞ [cm^{-3}]	u_∞ [km/sec]
1	1	400
2	1	650
3	1	750
4	3	400
5	3	650
6	10	400
7	30	400

For all 35 cases the integration yielded a shock and a stagnation point in pressure equilibrium such that the seven boundary conditions were fulfilled. The pressure at the stagnation point was always between 80% and 90% of the momentum flow at infinity. This makes a fairly accurate estimate of the position of the stagnation point as a function of the other parameters relatively easy. The solution for $G = 10^{30}$ sec^{-1}, $n_\infty = 3$ cm^{-3} and $u_\infty = 400$ km/sec is shown in Figures 5 and 6 to serve as an example. We find a shock at 3.9×10^6 km and a stagnation point at 380000 km from the nucleus, so that the transition zone is rather extended. The molecular weight begins to rise at 2×10^6 km. This causes an inversion in the Mach number around 1.5×10^6 km, which is finally overcome by the divergence in the flow. The pressure rises to a plateau which confirms the supposition raised at the end of Section 5. Also the density forms a plateau until the cometary particles begin to change the composition, as can be seen in the molecular weight. This increase in the density is entirely due to the molecules, the density of solar protons stays constant until the stagnation point is approached. Near the stagnation point the mean molecular weight is saturated since only about one percent of the particles are still of solar origin. These features are typical for all the solutions.

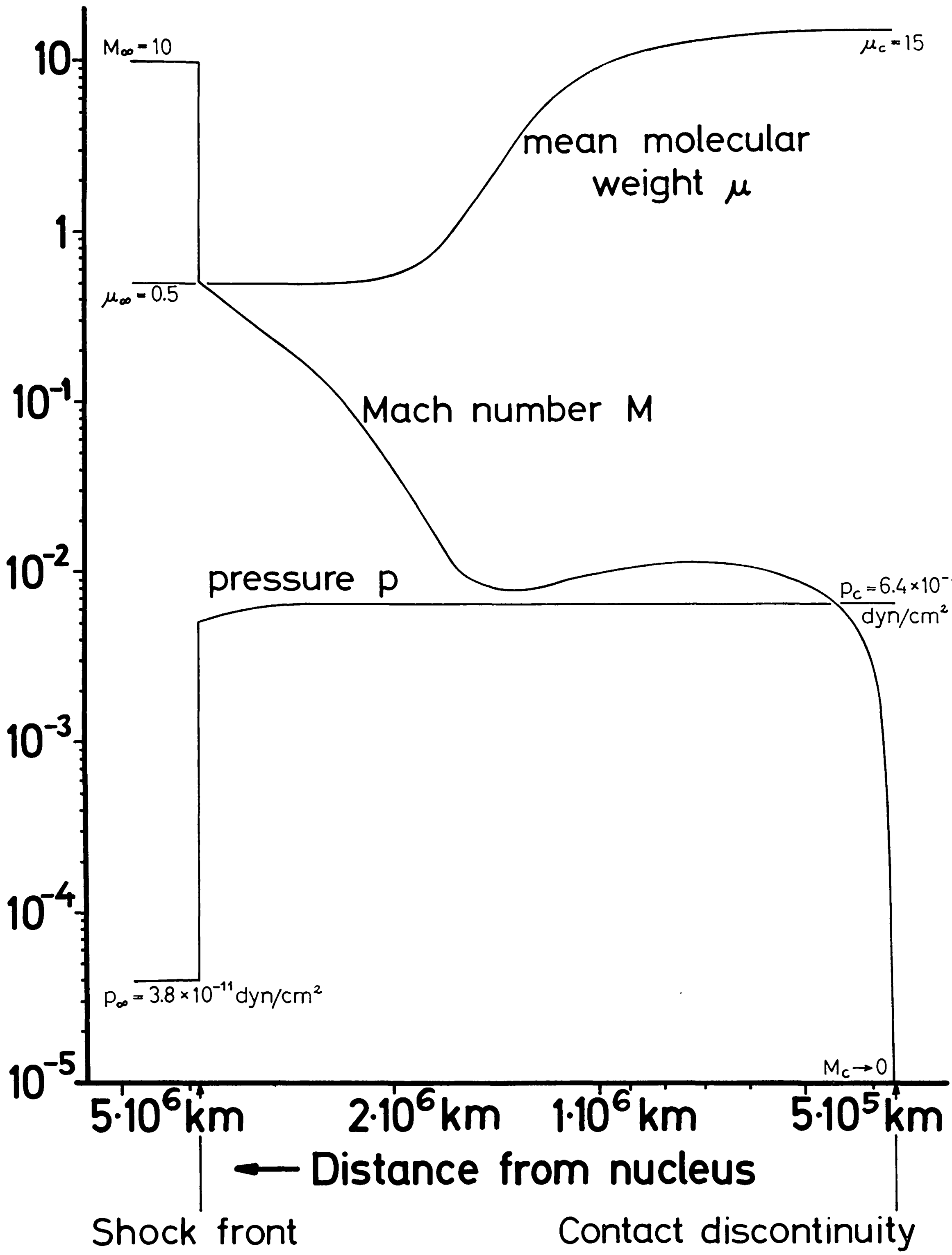

Fig. 5. Model-solution for $G = 10^{30}$ molecules/sec.

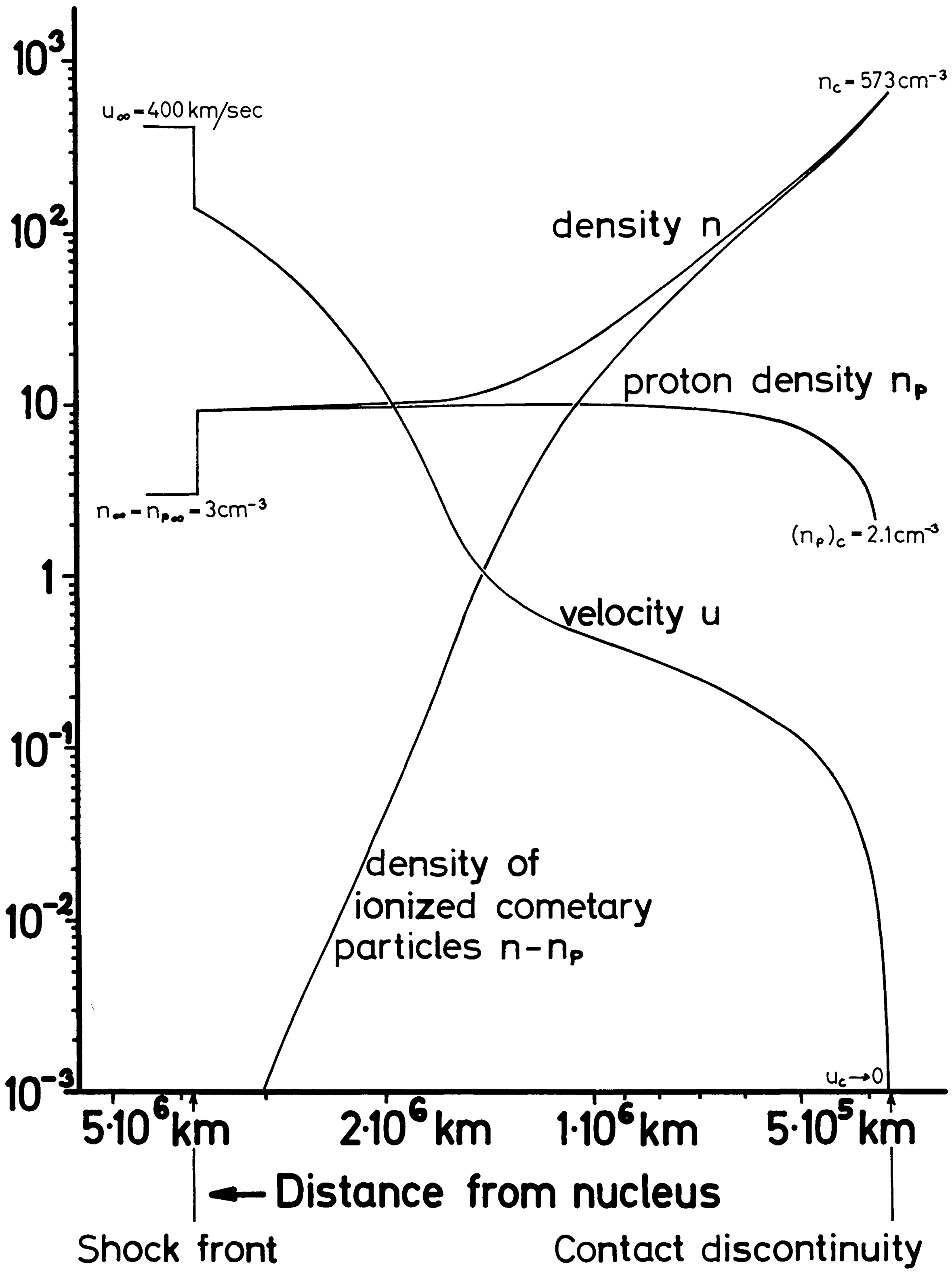

Fig. 6. Model-solution for $G = 10^{30}$ molecules/sec.

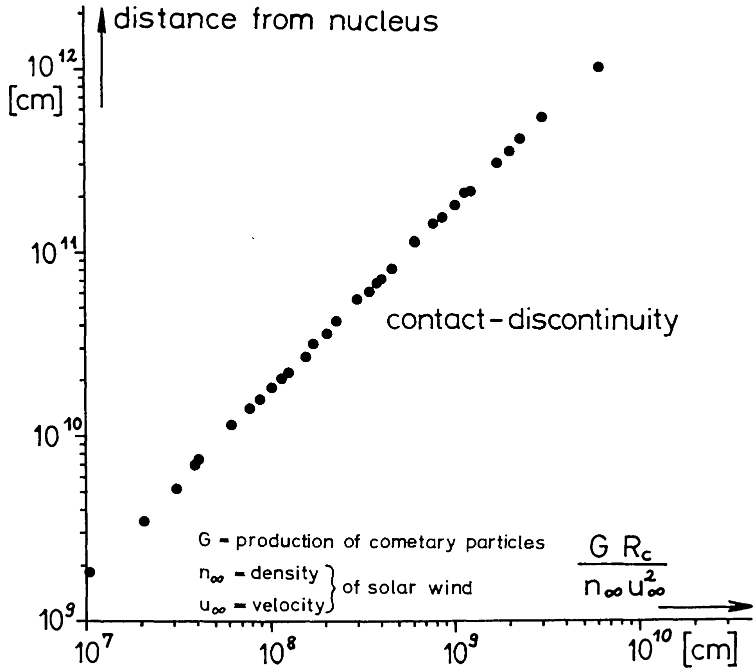

Fig. 7. z_0 as a function of $G^1 \cdot R_c$ for all models.

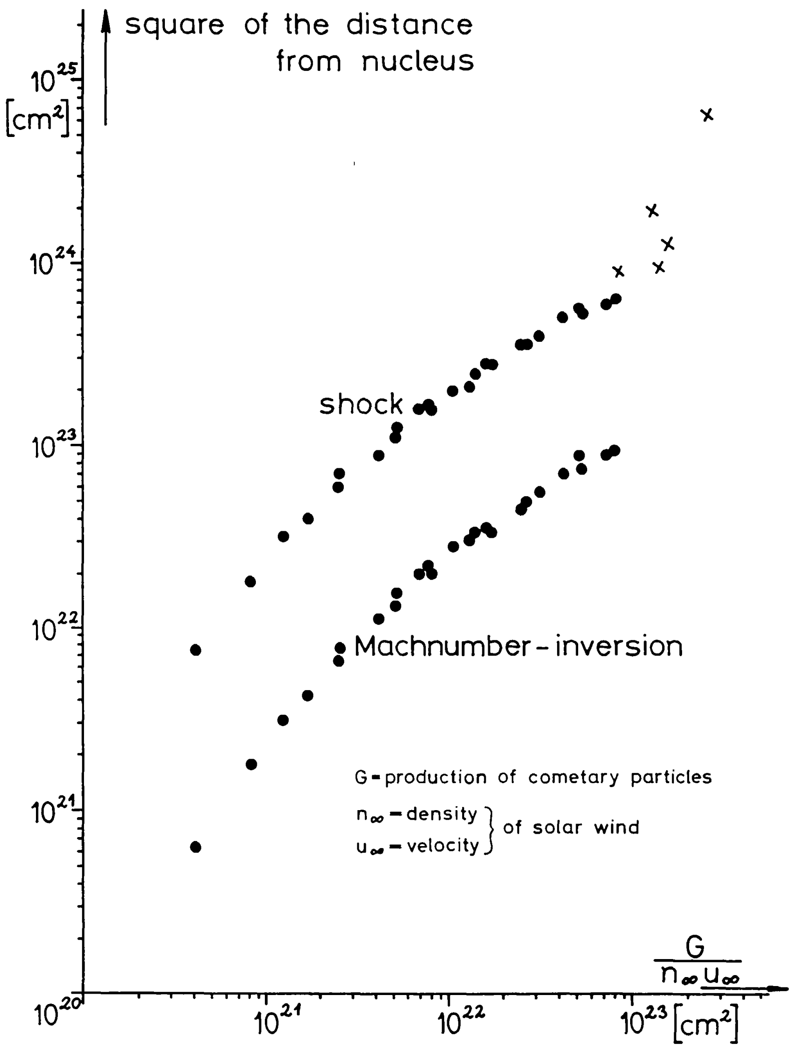

Fig. 8. z_1^2 as a function of $G/(n_\infty \cdot u_\infty)$ for all models.

We have tried to describe the numerical values for the distances of the flow discontinuities from the nucleus as functions of single parameters (SCHMIDT, 1966). For the contact discontinuity it seemed logical to choose the ratio of the undisturbed flows of momentum in the solar wind and in the purely cometary plasma. Using (5.23) we get for this ratio approximately

$$\frac{4\pi n_\infty u_\infty}{G \cdot R_C} \cdot \frac{m_p}{\chi^*/w_j w} \cdot z_0 \, .$$

From Figure 7 we see that for all our models this parameter – taking account of the fact that with the values assumed above $\chi^*/w_j \cdot w$ is $\approx 140\, m_p$ – is about equal to 1 at the stagnation point, so that

$$z_0 \approx \frac{G \cdot R_C}{4\pi n_\infty u_\infty^2} \cdot \frac{\chi^*/w_j \cdot w}{m_p} \, . \tag{8.1}$$

For the shock discontinuity we choose the ratio of the undisturbed mass flow in the neutral cometary gas and in the solar wind, since the strong interaction between the cometary gas and the solar wind is due to the continuous transfer of heavy molecules from the neutral gas into the plasma. Assuming $z_1 \lesssim z_t (\approx -\frac{1}{2}\, 10^{12}$ cm) we get for

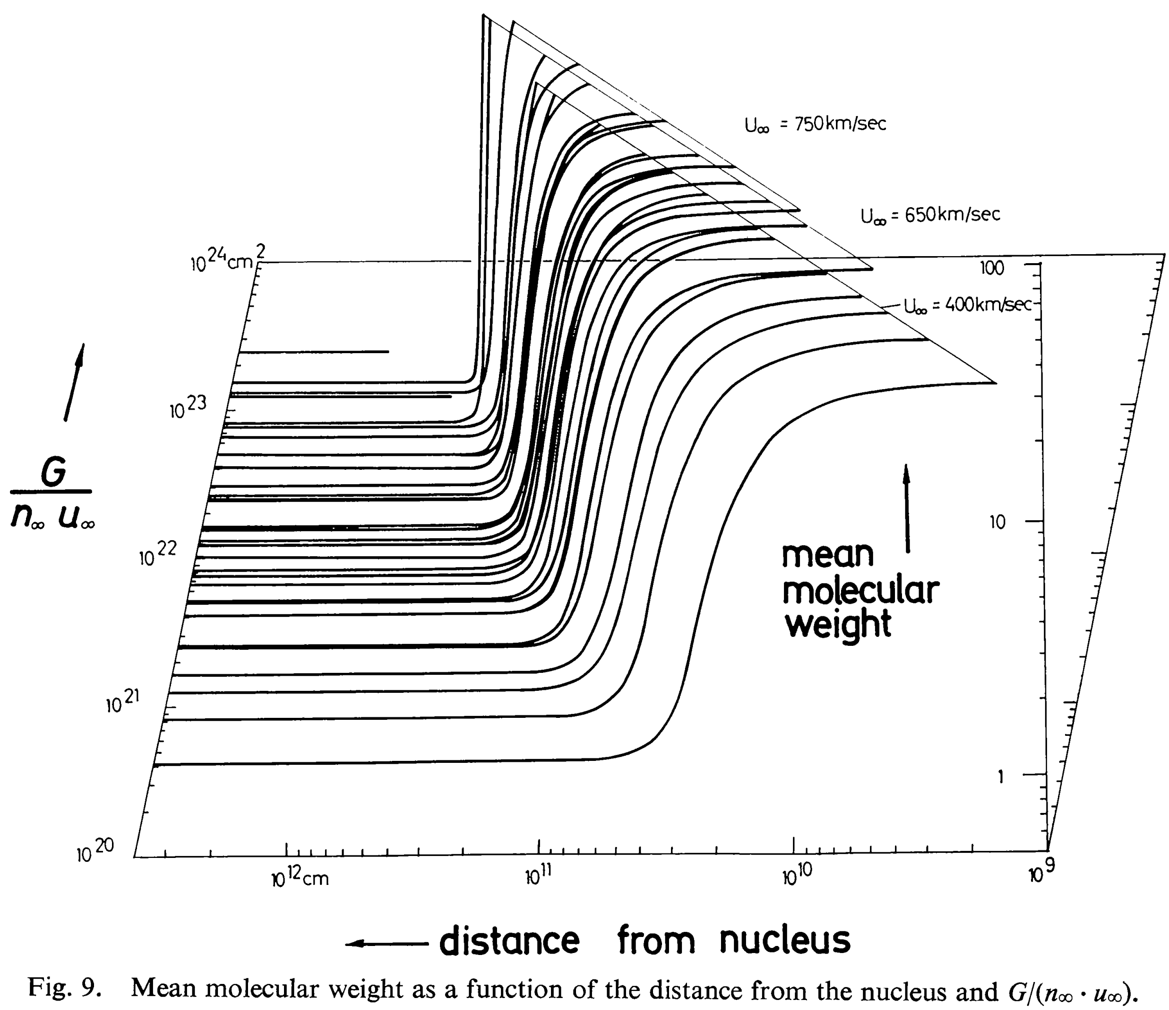

Fig. 9. Mean molecular weight as a function of the distance from the nucleus and $G/(n_\infty \cdot u_\infty)$.

this ratio:

$$\frac{Gm_C}{4\pi z_1^2} \cdot \frac{1}{n_\infty \cdot u_\infty \cdot m_p} \tag{8.2}$$

Figure 8 shows that $G/(n_\infty \cdot u_\infty)$ is actually a useful parameter to describe the location of the shock z_1 for most of our models. For small values of $G/(n_\infty \cdot u_\infty)$ the ratio (8.2) approaches a constant value of order $\frac{1}{10}$. For larger values of $G/(n_\infty \cdot u_\infty)$ the curve bends over in the sense that it lies below the straight line corresponding to (8.2), which should be due to the fact that the location of the shock is near the turning point $z_t = 5 \times 10^6$ km.

For very large values of $G/(n_\infty \cdot u_\infty)$ we have five solutions which show a different behaviour, since the stagnation point has moved so far out that it also begins to shift the location of the shock outwards. Here we begin to approach the case where the source terms become unimportant so that ordinary hydrodynamics become applicable. In Figure 8 we have indicated by crosses those solutions for which $|z_1| < 3|z_0|$. They also have no Mach inversion which circumstance points in the same direction. From Figure 8 we also infer that the location of the Mach inversion is well described by the

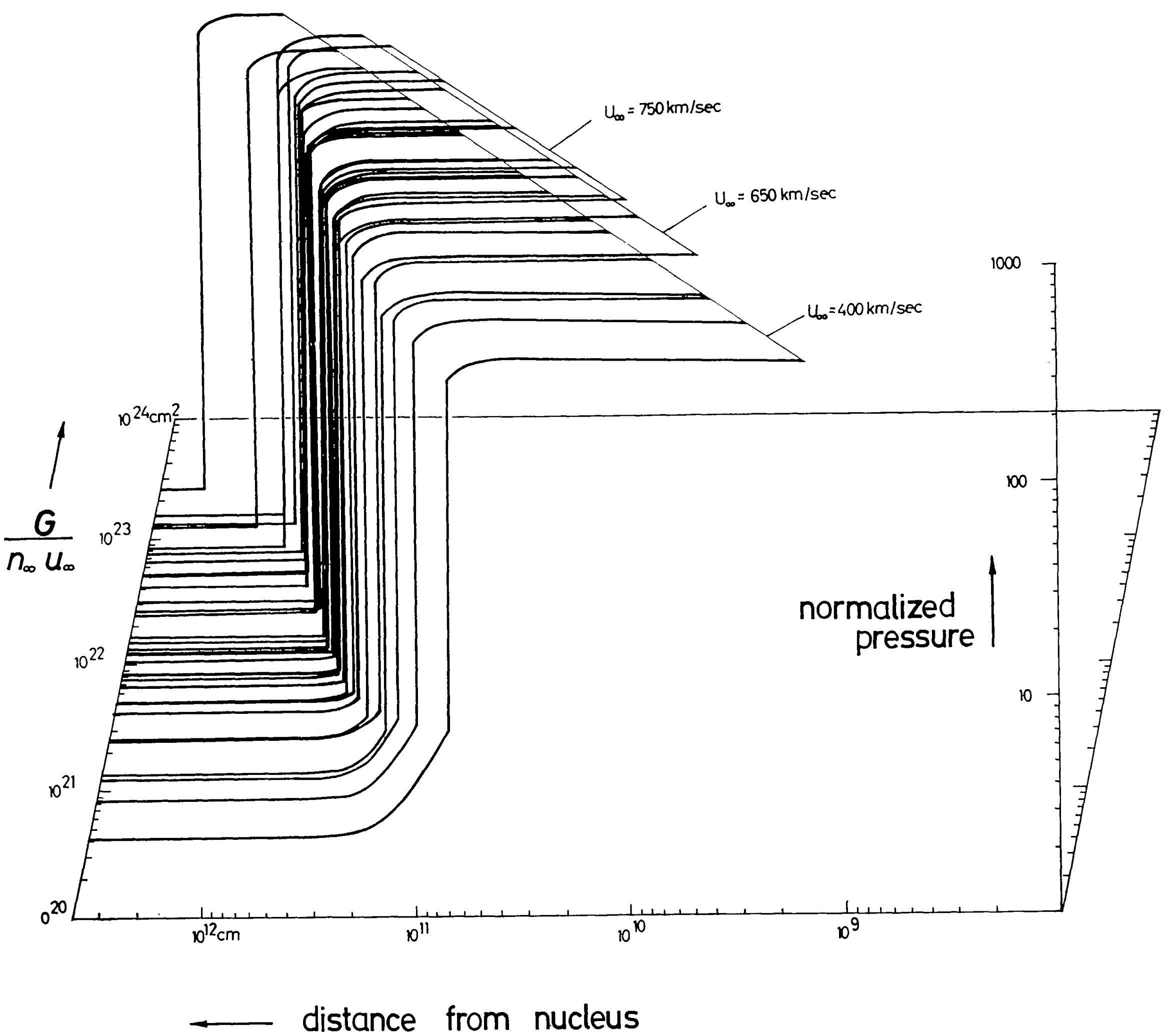

Fig. 10. Normalized pressure (p/p_∞) as a function of the distance from the nucleus and $G/(n_\infty \cdot u_\infty)$.

parameter $G/(n_\infty \cdot u_\infty)$. We therefore have chosen it as group parameter for the representation of the solutions. Figures 9 through 13 show the mean molecular weight, the pressure, the density, the Mach number and the velocity as functions of the distance from the nucleus and of the group parameter $G/(n_\infty \cdot u_\infty)$. The mean molecular weight shows a saturation near the stagnation point. Its value at the beginning of the Mach inversion is always about 3. The pressure forms a plateau nearly equal to the momentum flow of the undisturbed solar wind, which implies that the source terms for momentum and energy are not very influential. This is consistent with the esti-

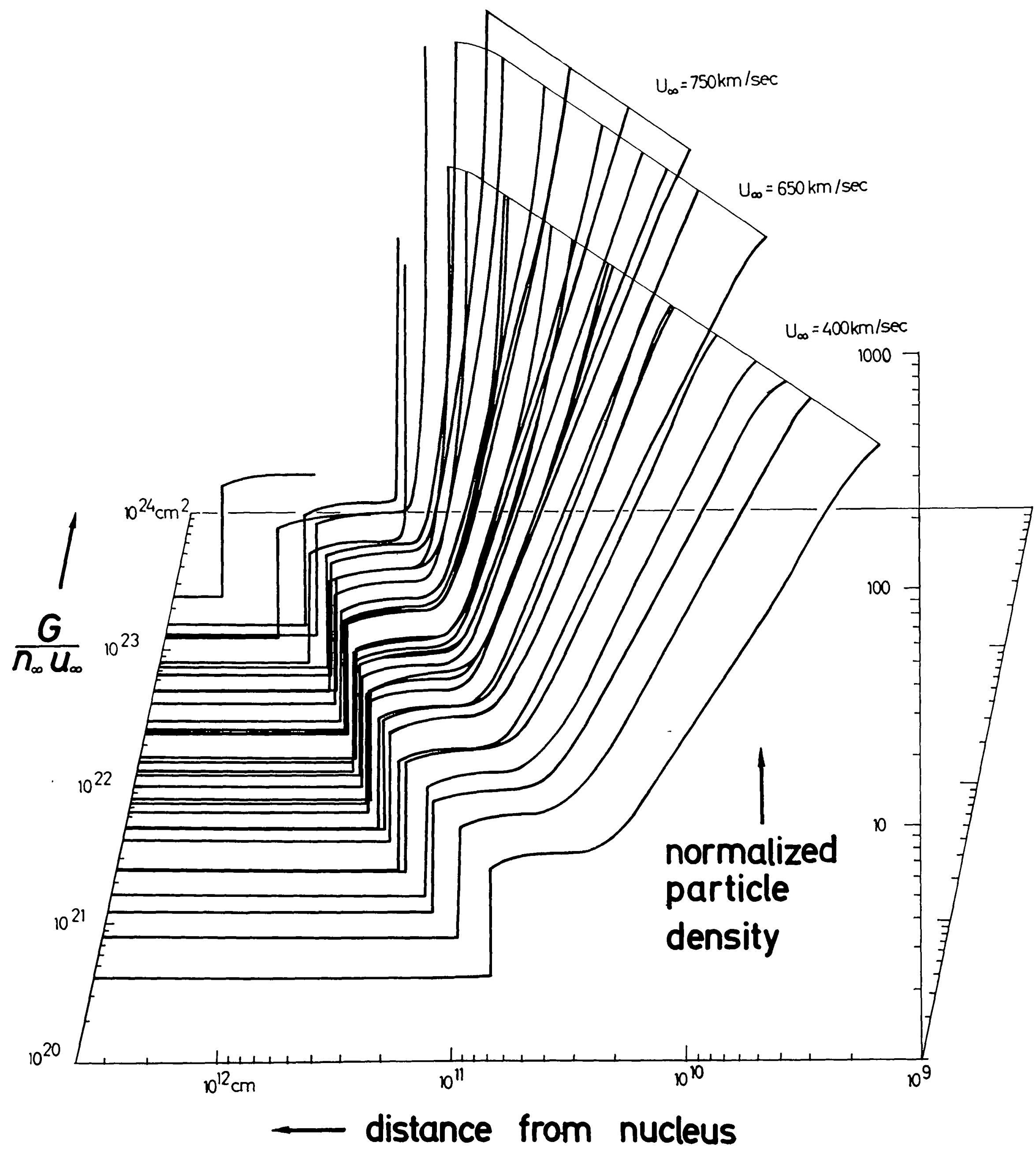

Fig. 11. Normalized particle density (n/n_∞) as a function of the distance from the nucleus and $G/(n_\infty \cdot u_\infty)$.

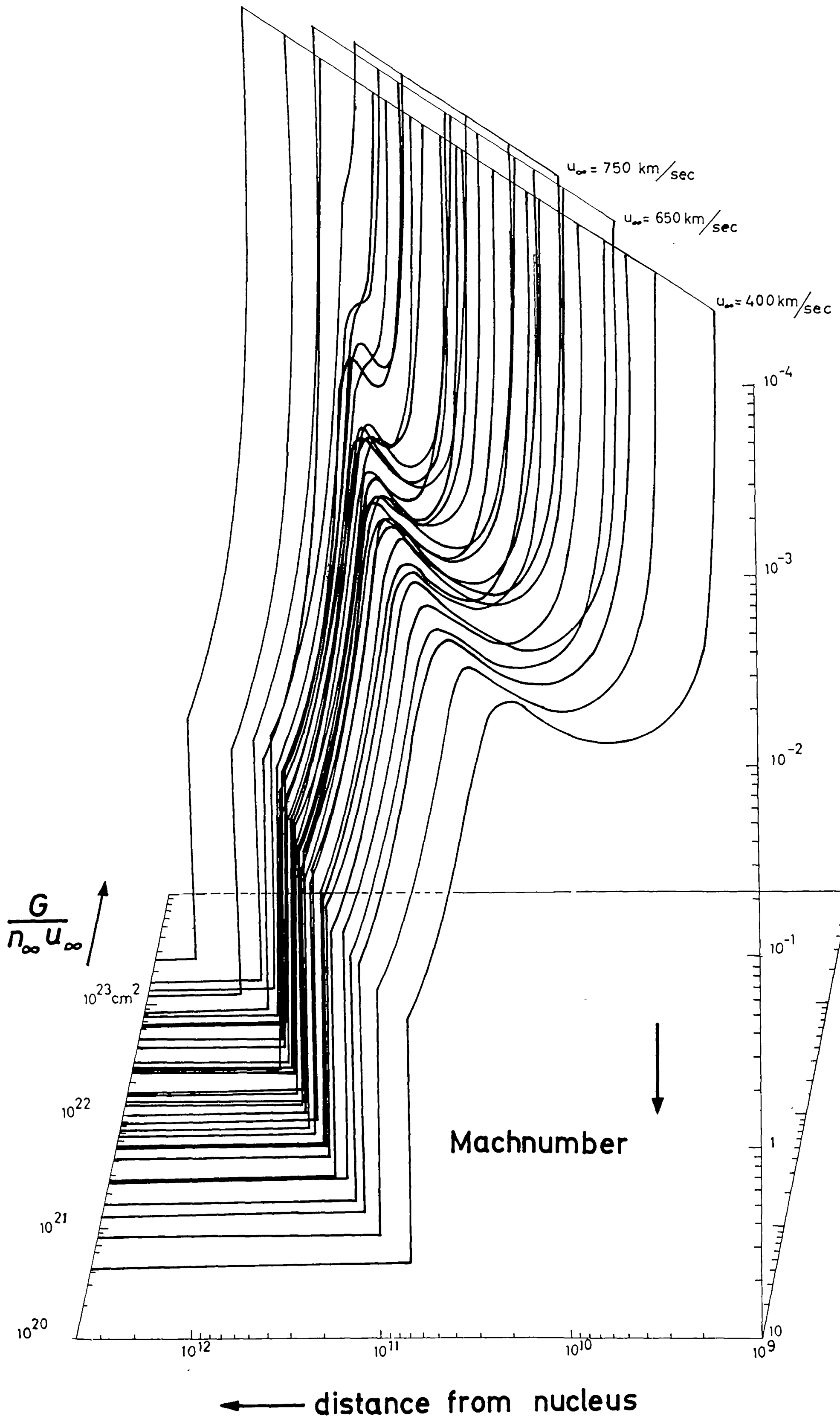

Fig. 12. Mach number as a function of the distance from the nucleus and $G/(n_\infty \cdot u_\infty)$.

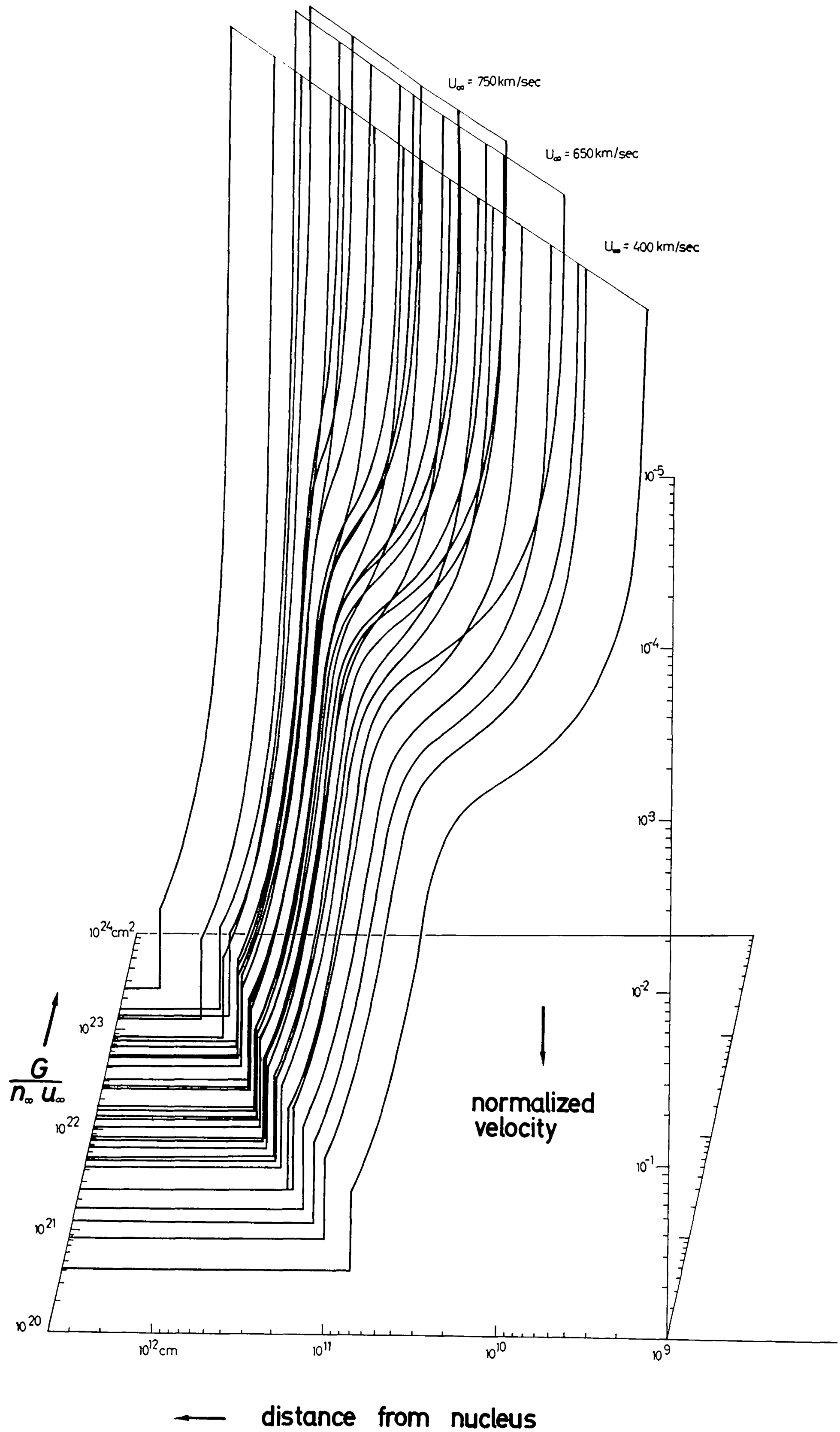

Fig. 13. Normalized velocity u/u_∞ as a function of the distance from the nucleus and $G/(n_\infty \cdot u_\infty)$.

mates made in Section 2. Also the density forms a plateau which is terminated by a steep rise due to the cometary ions. Their density is larger than the density of the protons beginning at the Mach inversion.

We conclude that a comet interferes with the flow of the solar wind over a much larger region than the dimensions of the visible cometary head would suggest. The size of the cavity filled with purely cometary plasma should coincide roughly with the dimensions of the visible cometary head (WURM, 1963). The distances of the stagnation point from the nucleus, which in comets Halley and Morehouse appear to have been of the order of some 10 000 km, are rather large in our models, which must be due to the fact that our estimate of $\chi^*/w_j w$ is likely to be considerably too high and also R_C may be somewhat lower (cf. Equation (8.1) above and subsequent remarks); furthermore we have accounted only for photo-ionization and we have not included any energy losses of the cometary plasma. On the balance these circumstances tend to inflate the cometary cavity in the solar wind. Allowance for ionization by charge exchange (BIERMANN and TREFFTZ, 1960; TSCHEREDNIDSCHENKO, 1960) and by electrons (AXFORD, 1964; BEARD, 1966) as well as for energy losses in the cavity would improve the model representation of the comet; further integrations (which are in progress) should indicate whether the simple relations found in the present investigations still hold, if some of the more complicated conditions actually present are taken into account.

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