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NEW LIMITS ON THE DIAMETERS OF RADIO SOURCES

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ABSTRACT

The interplanetary scintillations of seventy radio sources have been studied in detail The width of the spectrum of scintillation fluctuations gives a measure of source diameter Upper limits from 0".5 to 0".04 are obtained It is shown that most sources are not disks but probably consist of small-scale and large-scale components. The strength of scintillations is a measure of the strength of the small-scale component. A table is also given of sixty-eight sources which do not show scintillation.

There is excellent agreement between diameters (or limits thereto) deduced from scintillation observations and those obtained from interferometer observations Scintillating and non-scintillating galaxies have similar distributions of radio-frequency spectral index, but scintillating and non-scintillating quasars have different distributions. The former contain a predominance of curved and flat spectra; the latter are clustered between $\alpha = 0.8$ and 1 1.

I. INTRODUCTION

This article is one of a series on interplanetary scintillations. The first (Salpeter 1967; hereinafter referred to as "Paper I") discusses the theory of the phenomenon. The second (Cohen, Gundermann, Hardebeck, and Sharp 1967; hereinafter referred to as "Paper II") discusses the technique of the observations and some conclusions regarding the scale and strength of density fluctuations in the solar wind. The purpose of the present article is to present diameter information for a number of radio sources. The observations on which these results are based were made at the three frequencies 195, 430, and 611 MHz, with the 1000-foot reflector at the Arecibo Ionospheric Observatory, during June, 1965-October, 1966.

The first work on interplanetary scintillations (Hewish, Scott, and Wills 1964) pointed out that limits to the diameter of a radio source could be set by determining whether or not it scintillates when observed in favorable circumstances. A refinement was also suggested (Hewish and Okoye 1965; Little and Hewish 1966) which allows one to directly estimate the diameter by using the reduction in scintillation index which occurs when the source comes close to the Sun. In this paper, however, we base diameters on the width of the spectrum of the scintillation fluctuations rather than on the index, which is the integrated spectrum. In Papers I and II it was shown that the spectrum of the scintillations may contain a great deal of high-resolution information on the source. In principle, the brightness distribution itself (more accurately, a symmetrical version of the distribution) can be recovered from an ideal wavenumber spectrum. In practice, we have measured the frequency rather than the wavenumber spectrum and have assumed the two are related by $q = 2\pi f/u$, where u is the projected velocity of the solar wind (perpendicular to the line of sight). Since the irregularities move more or less radially outward from the Sun (Dennison and Hewish 1967), u is not a constant across the screen and this causes a smearing of the frequency spectrum. However, it appears that at least a reliable measure of the diameter of the source can be recovered.

Our study of the sizes of radio sources has been based on these spectrum techniques, which are discussed in detail in Papers I and II. In most cases we estimate an upper limit to "diameter." The limits range from 0".5 to 0".04. The higher limit is set by possible confusion with ionospheric scintillations; the lower limit is usually set by the closest approach of the source to the Sun.

Section II contains tables of the observed data. Section III is divided into four parts. The first gives evidence that about one-third of all extragalactic sources scintillate. Following that are comparisons between available interferometer measurements and the diameters estimated here. The third part discusses the structure of the sources derived from scintillation observations. Evidence is presented to show that most scintillating sources are composed of at least one very small-diameter component as well as a large-diameter component that contributes very little, if any, scintillation. The last part discusses differences between quasi-stellar objects (QSO's) and galaxies, and shows the correlation found for QSO's between the incidence of scintillation and a flat or curved radio spectrum. In the Appendix the thin screen theory is invoked to derive the equations used in the discussion of source structure.

II. LISTS OF SOURCES

a) *Scintillating Sources*

The date reduction to obtain the scintillation index, m, and the frequency (power) spectrum follows procedures described in Paper II. The index is the relative excess rms fluctuation level, generally averaged over 5 minutes. The spectrum is computed as the Fourier transform of the autocorrelation function. Examples of spectra are shown in Paper II. Table 1 lists the scintillating sources: the names in the first column are those commonly used. The second column lists β, the ecliptic latitude, which is the angle of closest approach to the Sun.

The third column lists μ, the fraction of the flux which scintillates. This can be interpreted as the fractional intensity of the small-scale component of the source (see § III*c*). The source CTA 21 is used as a standard, with $\mu \equiv 1$. The value of μ for any other source is the ratio of its scintillation index to that for the standard. CTA 21 was chosen as the standard since it is among that group of sources still unresolved by the longest base-line interferometers used to date (Palmer, Rowson, Anderson, Donaldson, Miley, Gent, Adgie, Slee, and Crowther 1967). It also displays effectively total scintillation; the intensity often drops very close to the "off-source" level. Since the 195 MHz data for CTA 21 are not reliable (the source is rather weak at the lower frequencies), 3C 138 has been used as a secondary standard at this frequency. The maximum scintillation index of 3C 138 is only 0.8 that of CTA 21 at 430 and 611 MHz. We have assumed this ratio holds at 195 MHz.

Figure 1 shows a plot of scintillation index versus solar elongation for CTA 21 and 3C 138 at the three frequencies. A portion of the CTA 21 curve has already been published (Sharp and Harris 1967); we are indebted to L. E. Sharp for the remainder. Corresponding plots were made for the other sources. The graphs are very similar, and can be superimposed by shifting the ordinate. The amount of scintillation relative to CTA 21 or 3C 138 can then be read off and is the value of μ. (In comparisons with 3C 138, μ is 0.8 times the relative scintillation.) Because of the variability of the index from day to day, it was judged that, except for a few of the stronger sources which were observed very often, the accuracy of the method was only sufficient to place the source in one of five categories: $\mu = 0.05$, 0.1, 0.3, 0.6, and 1.0.

The fourth column of Table 1 contains ψ, the diameter of the scintillating component. A circularly symmetric Gaussian model is used and ψ is the half-power diameter. The appropriate formula is (see Appendix)

$$\psi \leq \frac{u}{1.2\pi z f_2}, \tag{1}$$

where z is the distance to the irregularities, assumed to be 1 a.u.; f_2 is the measured (square-root) second moment of the frequency spectrum of scintillations; and u is the velocity of the diffraction pattern on the ground, assumed to be 350 km/sec. In nearly all cases f_2 appears to be set entirely by the screen; i.e., the spectrum is not affected by the source. Thus only upper limits to ψ can be estimated, and these are listed in Table 1.

TABLE 1—SCINTILLATING SOURCES

Source	β	μ	ψ	P A	Diam	θ	Ident	Spectrum	z	Notes*
3C 2.	−01°	0 6	~0″1	232°	A	<1″6	Q, V_0	S	..	1, 2, 3
23	+11	0 3	≤ 2	7	A	<2 4	.	S		4
33	06	0 05	5	261	B		G	S		5
43 .	13	0 3	1	320	C		Q, V_0	C		2
0128+03	−05	0 6	2	239	D			S		. ..
3C 44	−03	0 1	4	244	B	.	. .	S	. . .	6, 7
48	+21	0 6	1	341	A	0 4× <0 3	Q, V_0	C	0 367	3, 8, 9, 10, 11
49	03	0 6	1	263	A	1 6	Blank	S		1, 6
55 .	16	0 05	3	325	C	. . .	Blank	S		6
0202+14	02	0 6	1	268	A	<0 05	.	C		12, 14
3C 67 .	13	0 6	2	318	A	<3 5	G	S		4, 6, 7
71 .	−15	0 1	3	197	C	. . .	G	S	.	13
CTA 21...	−02	1 0	04	263	A	<0 05	Blank	C		14, 15
NRAO 140	+13	1 0	1	14	A	<0 05	V_r	F		14, 16
3C 93 1	14	0 6	2	23	D		G	S		6, 16
0347+05	−14	0 1	5	186	D		.	S		. ..
3C 99.	−20	0 6	2	182	D	. .	G	S		6
120	−16	0 3	2	227	A	≲3 5	G, V_r	C		4, 7, 12, 17
123	+07	0 05	.2	300	B		(G)	C		18, 19
124	−21	0 3	2	167	D	. .	G	S	. . . .	20
138	−07	0 8	~ 1	136	A	<0 2	Q	C	0 759	2, 3, 11, 14, 21
152	−03	0 6	≤ 1	109	A	<2 5	Obscured	S		3, 4, 5, 16, 22
158	−09	0 1	2	243	C		.	S	. . . .	6
181.	−07	0 3	2	300	C		Q	S	1 382	2, 12, 23
0735+17	−03	0 6	2	140	D	. .	.	(C)		24
3C 190	−06	0 3	2	117	A	<1 6	Q	S	. .	1, 7, 12
191	−10	0 3	3	126	A	<2 5	Q	S	1 946	2, 3, 4, 12, 25
208	−04	0 3	.3	106	B		Q	(S)	1 109	3, 26, 27
210	+10	0 6	2	301	B	.	G	S		6, 12, 16
222	−09	0 6	2	262	A	<1 6	G	S		1, 6
225	00	0 3	2	86	A	~0 6	Blank	S		3, 6, 19, 28
230	−12	0 3	2	245	A	<2 5	Q	C		3, 4, 6
237	−04	1 0	2	274	A	<0 4	(G)	C		3, 6, 19, 20, 28
238	−05	0 3	2	268	A	<3	Blank	C	.	3, 4, 6
241	+11	0 6	2	326	A	<3	Blank	S		3, 4, 6, 16
243	−03	0 05	3	104	B			S		12
1031+11	02	0 6	2	118	D	. . .	. .	S	. . . .	. .. ..
3C 245..	04	0 3	05	339	A	≤0 5	Q, (V_0)	F	1 029	3, 29, 30, 31, 32
1055+01	−05	0 6	.2	98	A	<0 05	Q	C		12, 14, 33
3C 249.	−07	0 3	.3	105	B		Blank	S		6, 12, 16
4C 10 31	04	0 6	2	125	D		.	.	. . . .	12
1116+12	07	0 6	2	135	D	.	Q	S	2 117	12, 34
3C 256	18	0 6	3	22	A	<3	G	S		1, 6, 12, 16
257	02	0 3	2	117	B		Blank	S		6, 12
263 1	19	0 3	2	147	C		(G)	S		6, 12, 16, 19
267	11	0 3	2	40	B	. . . .	. .	S	. . . .	6
273	05	0 3	04	346	A	<0 025	Q, V_0, V_r	C	0 158	3, 9, 14, 22, 32, 35
275.1	19	0 1	3	150	D		Q	S	0 557	2, 12, 21
1303+09	22	0 6	3	135	D		.	S		12
1318+11	18	0 1	3	195	D	. . .	(Q)	S	. . . .	7, 12
3C 287	32	0 8	2	352	A	<0 05	Q	F	1 055	11, 14, 26, 31
286	37	0 6	1	2	A	<0 05	Q	C	0 846	3, 9, 14, 36
1341+14	23	0 6	2	181	D		(Q)	S		7, 12
1345+12	15	1 0	2	155	D	.	G	F	. . .	7, 12
3C 298	19	0 6	2	358	A	<1	Q	C	1 439	2, 3, 21, 28
300 1	13	0 1	3	105	D	.	(G)	F		6, 12, 37
318.. .	37	0 6	2	348	A	<3	G	C		4, 6
1759+13	37	0 6	2	318	D			S		.
3C 368	35	0 1	4	322	B		. . .	S	.	6, 12
409	42	0 05	2	2	B	.	Obscured	C		6, 12
410.	48	0 3	4	4	A	3	Obscured	C	.	4, 6, 12
2127+04	19	1 0	2	326	A	<0 05	. .	C	.	14
2145+06	19	1 0	2	291	A	<0 05	(Q)	C	. . . .	14, 20
CTA 102	19	1 0	1	355	A	<0 05	Q	C	1 037	14, 26, 31
3C 454 .	24	1 0	.2	105	D	. . . .	Q	S	1 757	6, 23
454 3	21	1 0	.1	100	A	<0 025	Q, V_0, V_r	C	.	6, 14
455 .	19	0 3	1	340	C	. .	G	S	.	6
456	13	0 3	.2	337	A	<1 6	G	(S)	. .	1, 38
459 .	8	0 3	.1	351	A	<1 6	G	S	. . . .	1, 3, 18, 38
460 .	25	0 6	0 4	325	D		Blank	S		6

When a source is quite close to the Sun ($\epsilon \leq 10°$), the spectrum may be cut off and this behavior is ascribed to the source structure. In these cases, equation (1) becomes an equality and a diameter is estimated. This value is prefaced by the symbol $\sim$ in Table 1.

Values of ψ are to be regarded as estimates only. They are probably within a factor of 2 of being correct, but detailed comparison will have to be made with results of very long base-line interferometer observations before the accuracy can be estimated reliably.

The fifth column gives the position angle, defined by the vector from the Sun to the source measured east from north. Most sources were observed over a wide range of

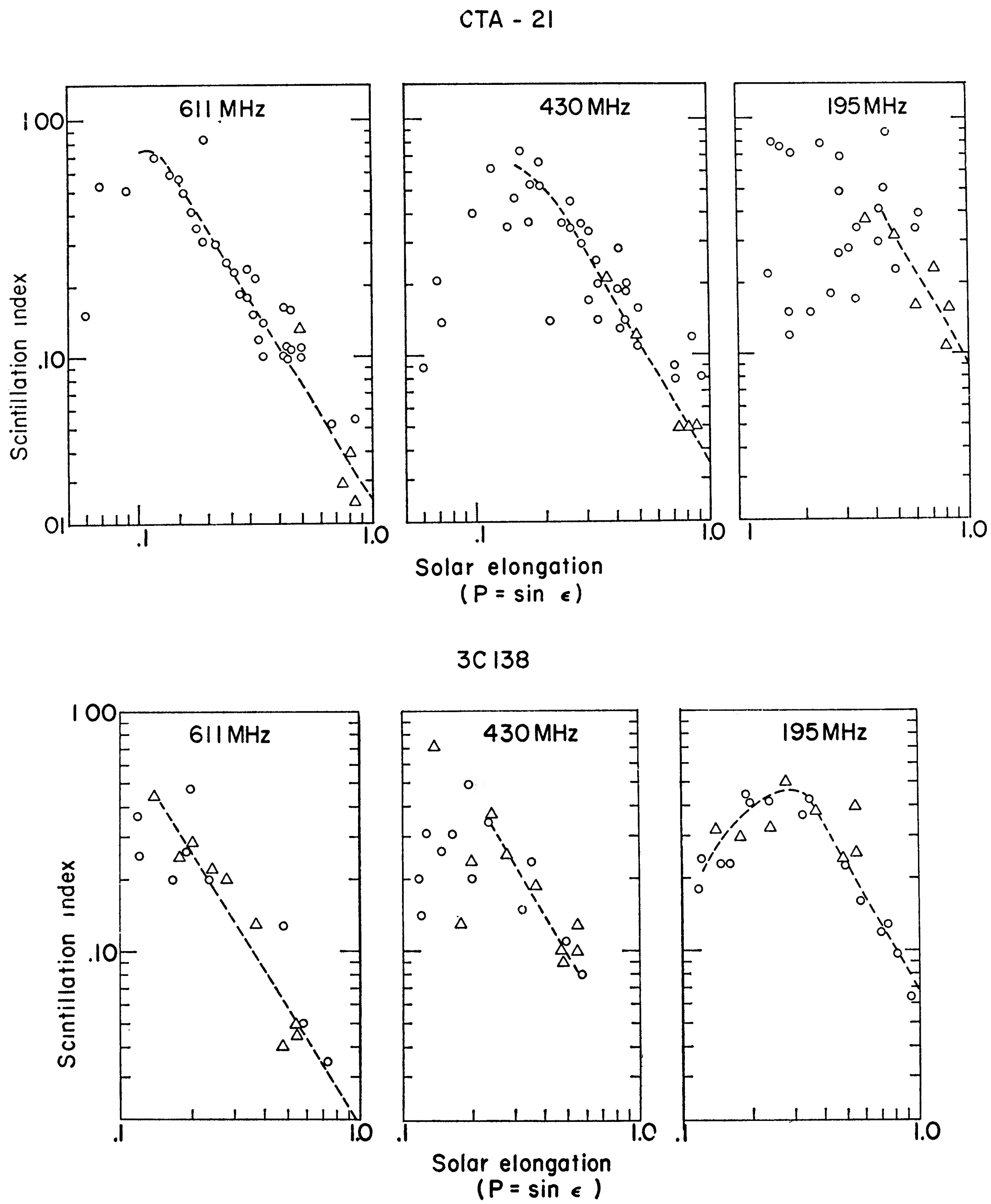

FIG. 1.—Scintillation index as a function of solar elongation, for CTA 21 and 3C 138. *Triangles*, May–August, 1965; *circles*, March–July, 1966.

position angles; the one listed applies to the date on which the least upper bound to the diameter was obtained. The diameter measurement pertains to the direction of the projected motion of the diffracting screen, and we assume that the screen (irregularities in electron density) moves radially outward from the Sun.

The sixth column gives a measure of the diameter of the source, as determined from interferometer or lunar occultation observations. Most of the values are taken from the Jodrell Bank survey (Allen, Anderson, Conway, Palmer, Reddish, and Rowson 1962) and the NRAO survey (Clark and Hogg 1966). Every source has been put into one of four categories:

A. There is evidence for structure $< 3\overset{''}{.}5$;
B. There is evidence that there is no substantial structure $< 3\overset{''}{.}5$;
C. There is an upper limit $\leq 10''$ to some structure (and A and B are not satisfied);
D. None of the above. (In most cases, this means there are no high-resolution observations.)

Where there was conflict between A and B, we put the source in A.

Each category A source has an "angular diameter" (generally an upper limit) listed in the seventh column. In most cases, this has come from fitting a model to limited interferometer data, so it is necessarily subjective. We have repeated an angle in cases where the interferometer observers have so interpreted their results. In other cases we have given $\frac{1}{4}$ of the fringe spacing, for the largest base line at which the source is still substantially unresolved.

Our diameter information is not homogeneous because we have not considered that source size may be a function of frequency or polarization. Although there occasionally may be some change in diameter with frequency (Clark and Hogg 1966), it is a minor effect in our study. This point is discussed again below.

The eighth, ninth, and tenth columns contain the identification of the optical object associated with the radio source, the *rf* spectrum, and, for QSO's, the redshift. The optical objects are separated only into quasi-stellar sources (Q) and galaxies (G); doubtful identifications are indicated in parentheses. In some cases a blank optical field has been reported for a radio source; these are so indicated. Observations of temporal variations in flux are indicated by V_0 (optical) and V_r (radio). The rf spectra have been separated into three classes: S (steep, index $\alpha \geq 0.6$), F (flat, $\alpha < 0.6$), and C (curved). Spectral information was obtained mainly from catalogues due to Kellermann (1964, 1966), Howard, Dennis, Maran, and Aller (1965), Dent and Haddock (1966), and the Parkes catalogue (Day, Shimmins, Ekers, and Cole 1966). No attempt was made to reconcile differences in the catalogues, and some of the assignments are doubtful. In many cases detailed spectrum studies were not available, and in most of these cases we used the two-point spectrum from the NRAO survey.

The eleventh column contains notes for a few sources, and references for the angular diameters given in the seventh column, and for the optical identifications and redshifts. No attempt was made to be complete in listing the references; we apologize for cases where we have not given the original material.

b) Non-scintillating Sources

Sources that come close to the Sun were observed at least twice near the elongation that would produce the maximum scintillation (15°–20° at 195 MHz, 10° at 430 MHz). An upper limit of $m \leq 0.1$ could generally be assigned to these sources, except for the very weakest. Sources which do not pass this close to the Sun were observed at closest approach. Figure 1 was then used to determine the limit on the possible fraction of flux in a small-scale component.

Table 2 lists the non-scintillating sources. The quantities are the same as in Table 1 except that ψ and P.A. have been omitted and μ is replaced by the upper limit of μ.

TABLE 2

NON-SCINTILLATING SOURCES

Source	β	Lim μ	Diam	Ident	Spectrum	z	Notes*
3C 9. .	+12°	≤0 1	B	Q	S	2 012	30, 31
14 .	14	.15	B	(G)	S	.	6, 19
15	−05	1	A(θ∼2″)	G	S		16, 29
16	+09	15	B	(G)	S		6, 19
18	05	1	B	G	(C)		6
28	19	1	B	G	S		38
34	22	1	B	(G)	S	.	6, 16, 19
0118+03	−04	.15	B	(Q)	S		7, 29, 39
3C 40	−10	05	B	G	S		5
41	+22	1	C	(G)	F		6, 19
42	18	1	D	Blank	S	.	6, 16
46 .	26	15	B	G	(C)	. ..	6
47	10	05	C	Q, V_0	S	0 425	40, 41
0207+09	−03	1	D	G	S		7
3C 64	−05	1	D	G	S		20
68 1	+19	1	D	(G)	S		6, 16
75	−10	1	B	G	S		13
76 1	−01	1	D	G	S		6
78 .	+13	1	B	G	F		13
88	−16	05	B	G	S		5
98.	−10	05	B	G	S		13
105	−17	05	B	Blank	S		6
109	−10	1	B	G	S		18
114	−04	15	B	G	S		6, 16
118	−21	15	D	.	S		
131	+09	05	C	Obscured	S		6
132	00	1	B	G	S		6
135 .	−22	2	B	G	S		6
142 1	−17	05	B	(G)	S		6, 19
165	00	15	D	.. ..	S	. .	6, 16
172	02	1	B	Q	(C)	. . .	6
175 .	−11	1	B	Q	S	0 768	6, 42
272 1	+14	1	B	G	(S)	.	5
274 1	23	2	C	G	S		6, 16
277 3	31	25	B	G	(S)		38
281	13	15	B	Q	S		2
284..	32	2	B	G	S		6, 16
1326+06	15	3	D	(Q)	..		7
1346+09	20	3	D	(Q)	.		7
3C 293	40	3	A(θ<3″)	G	S		4, 16, 43
296..	21	15	B	G	C		6
1434+03	18	2	D		S		. .
3C 310	41	15	B	G	(C)		5
313	25	15	B	G	S		20
315 .	42	15	B	G	S		5
318 1	25	2	D		S		. .
320 .	52	6	B	G	S	. . .	6, 16
323 1	40	.3	D	Q	S	0 264	6, 16, 44
324	40	.5	B	. .	S	.	43
326 .	39	15	D	G	....	.	6
326 1	39	3	D	.	S		6, 16
327 .	22	1	B	G	S		5
327.1	22	1	C	(G)	S		6
CTD 93	47	7	D	..	C	. .	40
3C 332	53	4	D	G	S	. ...	43
334 .	38	3	B	Q	S	0 555	43, 45
336 .	45	3	B	Q	S	0 927	2, 16, 21
340 .	45	4	D	(G)	S		6, 16, 19
341	49	3	D	. .. .	S	. ..	6, 19
345	61	5	A(θ<0″.1)	Q, V_0, V_r	C	0 594	26, 46
347.	35	.2	B	. .	S	. .	. ...
382.	56	4	B	G	(S)	... .	5
411 .	29	2	B	... .	C	.. ...	6, 19
432.	31	.5	B	Q	S	1 805	6, 23
435 .	21	.2	B	Q	S	.....	6
436 ..	39	.35	B	G	S	.... ..	6
441.. . .	38	45	B	(G)	S	....	19, 43
458........	10	0 15	B	G	S	....	6

* See Notes following Table 2.

NOTES TO TABLES 1 AND 2

1. Allen *et al.* 1962.
2. Sandage, Véron, and Wyndham 1965.
3. Scintillations reported at 178 MHz (Hewish *et al.* 1964).
4. Clark and Hogg 1966.
5. Matthews, Morgan, and Schmidt 1964.
6. Wyndham 1966.
7. Clarke, Bolton, and Shimmins 1966.
8. Adgie, Gent, Slee, Frost, Palmer, and Rowson 1965.
9. Matthews and Sandage 1963.
10. Sandage 1964; Greenstein and Matthews 1963.
11. Strong signal and many observations allow μ to be determined more precisely than for the other sources.
12. Value of μ based on three or less observations.
13. Maltby, Matthews, and Moffet 1963.
14. Palmer, Rowson, Anderson, Donaldson, Miley, Gent, Adgie, Slee, and Crowther 1967.
15. Clarke and Batchelor 1965.
16. Spectrum based on two points, 750 and 1400 MHz (Pauliny-Toth, Wade, and Heeschen 1966).
17. 3C 120 = 0430+05.
18. Longair 1965.
19. Véron 1966.
20. Bolton and Ekers 1966*a*.
21. Lynds, Hill, Heere, and Stockton 1966.
22. Values of μ are for 195 MHz. See Table 5 for variation with frequency.
23. Schmidt 1966.
24. Value of μ is for 611 MHz.
25. Burbidge, Lynds, and Burbidge 1966.
26 Sandage and Wyndham 1965.
27. Burbidge 1966.
28. Anderson, Donaldson, Palmer, and Rowson 1965.
29. Hazard, Mackey, and Nicholson 1964.
30. Ryle and Sandage 1964.
31. Schmidt 1965*b*.
32. For consistency, value of ψ is computed from second moment of fluctuation spectrum. Smaller limits, based on spectrum cutoff, have been reported by Cohen *et al.* (1966).
33. Bolton, Shimmins, Ekers, Kinman, Lamla, and Wirtanen 1966.
34. Bolton, Clarke, Sandage, and Véron 1965; Lynds and Stockton 1966.
35. Schmidt 1963.
36. Oke 1965.
37. Bolton and Ekers 1966*b*.
38. Schmidt 1965*a*.
39 0118+03 = 3C 39.
40. Schmidt and Matthews 1964.
41. Sandage 1966.
42. Observations by Lynds (1967); reported by Burbidge (1967).
43. Wyndham 1965.
44. Observations by Schmidt; reported by Burbidge (1967).
45. Burbidge 1965.
46. Lynds, Stockton, and Livingston 1965; Barber, Donaldson, Miley and Smith 1966.

III. DISCUSSION OF THE OBSERVATIONAL RESULTS

a) The Percentage of Sources That Scintillate

From Tables 1 and 2 we find that 70 out of 138 sources scintillate. This ratio is not significant, however, since a selection was made in favor of sources with known small angular diameters. In an attempt to avoid this selection effect, two days were spent in March, 1966, observing twenty-seven sources in the Parkes catalogue. These observations were made at 430 MHz with the radar line feed, which illuminates a larger area of the reflector and hence gives a better signal than the usual radio astronomy feeds. The sources were chosen at random with the following limitations: (1) flux density at 408 MHz greater than 2 flux units; (2) no other listed sources closer than 45′; (3) distance from the Sun to the source between 6° and 18°.

Of the twenty-seven sources observed, ten (37 per cent) scintillated and three were questionable. The other fourteen displayed no scintillation greater than 10 per cent. Table 3 lists these sources.

The figure of 37 per cent for sources that scintillate seems to be fairly indicative of all sources. For example, we have tried to minimize the selection effects inherent in our main list by choosing a sample of sources whose minimum distance to the Sun $\beta \leq 30°$. Of these sources, 41 per cent scintillated. To check the presence of a possible intensity effect, the sources were then divided into three intensity groups of about forty-five sources each. No trend in the percentage of sources that scintillate was discernible—the

TABLE 3

SOURCE LIST FOR THE 430 MHz MARCH, 1966, SURVEY

Scintillate	Non-Scintillate	Possible
2313+03	2308+07	2310+05
2335+03	2313+01	0030+06
2337+13	2313+10	0109+14
2338+03	2334+08	
2338+04	0002+12	
2344+09	0003+15	
2354+14	0007+12	
0040+06	0035+13	
0114+07	0036+03	
0116+08	0037+04	
	0038+09	
	0038+08	
	0042+13	
	0057+07	

TABLE 4

COMPARISON OF SCINTILLATION AND INTERFEROMETER DATA

	A	B	C	D
Scintillators	34	11	7	18
Non-Scintillators	3	42	5	18

small percentage increase for weak sources could easily be explained by other unavoidable selection effects. We find no evidence that diameter decreases with decreasing flux. Thus we may give a preliminary value of about 40 per cent for sources with $S_{408} \geq 2$ f.u. The above discussion refers exclusively to extragalactic sources

b) Comparison with Interferometer Data

Table 4 shows how the scintillating and non-scintillating sources separate into the four size categories. In general the agreement is very good: nearly all sources known to have structure on the order of a second of arc or less do scintillate, and most sources known to be large do not scintillate. Six of the category B scintillators have $\mu = 0.05$ or 0.1, so their small-scale component could have been missed in the interferometer studies. Four other sources (3C 208, 249, 257, 267) have $\mu = 0.3$, which is roughly consistent with the limits on visibility given by Allen *et al.* (1962). More extensive observations on these sources will probably show that about a third of the flux comes from a small-scale component. The μ of 0.6 for 3C 210 is poorly determined.

Three category A sources do not scintillate. One of these, 3C 345, has the largest value of β, and nearly the largest limit for μ, of any source, and is included mainly because it is a well-known QSO of very small angular diameter (Palmer *et al.* 1967). Although the reliability for this source is low, it suggests that μ increases with frequency, since the 21-cm interferometer observation would give $\mu = 1.0$. Similar conclusions hold for 3C 293 where $\mu \leq 0.3$; we expect $\mu \sim 0.6$ from 11-cm interferometer observations. This variation with frequency is discussed further below.

3C 15 has $\mu \leq 0.1$, and the size of its components, determined by occultation observations at 410 MHz, is $\approx 2''$. A circular Gaussian source of half-power width $2''$ should give $\mu \approx 0.1$ (see Appendix). Thus, the two observations are not in conflict.

c) Structure of Sources

It was stated in § II that μ can be interpreted as that fraction of the source intensity contained within a very small diameter. The alternate explanation for values of $\mu < 1$ is that the source intensity is more or less uniform with an intermediate diameter such that the scintillation is partially reduced. By utilizing the information available from the frequency spectrum of scintillations, it is possible to show that the first interpretation is probably correct in most cases.

In ideal circumstances the wavenumber spectrum of scintillations allows one to compute a brightness distribution (see Appendix). However, our situation is not ideal because the conversion from wavenumber to frequency spectrum assumes the screen moves as a rigid body, and also because the screen is not infinitely thin. Nevertheless, we conjecture that two measures of the spectrum: its width (second moment), and its integrated intensity (scintillation index), are indeed representative of the source. The second moment and the index are the measured quantities; they have been converted into ψ and μ for Table 1.

The thin screen theory makes specific predictions concerning the relation between μ and ψ. In the Appendix we calculate the appropriate formulae for μ as a function of ψ, for three source models: (*a*) a Gaussian line source of half-length ψ; (*b*) a circular Gaussian source of half-width ψ; and (*c*) a double source with two equal point components of separation ψ. In Figure 2 we show the (μ,ψ) plane, with lines corresponding to the three source models. In each case it is assumed that the screen is 1 a.u. distant from the observer, and that a point source would have given a Gaussian wavenumber spectrum of reciprocal width $a = 110$ km.

The scintillating sources in Table 1 have been assigned values of μ according to their strength of scintillation and values of ψ according to the width of their fluctuation spectra. These pairs of points (μ,ψ) are plotted in Figure 2. Most of the ψ values are upper limits and therefore the true distribution would be squeezed more to the left. There may also be some sources with $\psi > 0''.5$ that scintillate weakly. These have been omitted, as stated earlier, due to possible confusion with ionospheric scintillations.

It will be seen that most of the observational points are far from the calculated lines. It appears, therefore, that the models are poor representations of the sources. The simplest interpretation is that most sources are a mixture of small- and large-scale components; μ is an estimate of the fraction of the flux from the small-scale component, and ψ is its diameter. It is known in a few cases (3C 245, 273) that this is correct. For these two sources both μ and ψ agree roughly with lunar occultation observations.

If these conclusions are correct, then a measure of μ as a function of frequency can give the radio-frequency spectrum of the components separately. Table 5 lists a few sources for which μ is appreciably different at the three frequencies of observation. These values should be considered as preliminary. As discussed in § III*b* above, there are also two sources, 3C 345 and 3C 293, which scintillate less at meter wavelengths than the centimeter-wavelength interferometer observations would have suggested. It appears as though these sources may be like 3C 273: the small-scale component has a flatter spectrum than the large-scale component.

d) QSO's and Galaxies

The division of sources between scintillators and non-scintillators, according to Tables 1 and 2, is somewhat misleading. The lists are not mutually exclusive since a few non-scintillators have very high upper limits to the degree of scintillation. In Table 6, therefore, we show the distribution of optical identifications, after exclusion of the weak scintillators ($\mu < 0.3$) and the non-scintillators with lim $\mu > 0.3$. The categories in Table 6 are now mutually exclusive, with the separation at sources which have 0.3 of their flux coming from a small-scale component.

Although we have examined almost all QSO's within our declination limits, our selection of galaxies and unidentified sources has favored those known to be narrow. Thus, from Table 6 we may say that two-thirds of the QSO's but only one-third or less of the galaxies scintillate. The ratios in Table 6 suggest that most of the unidentified sources are QSO's. However, this is very weak evidence, since the sources were not chosen at random.

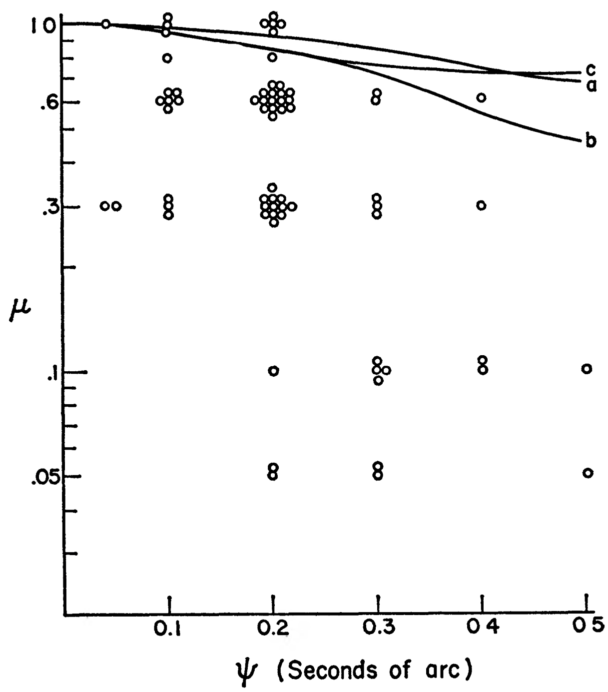

FIG. 2.—Strength of scintillation, μ, versus ψ, limit to diameter from width of fluctuation spectrum. Three lines are for (*a*) Gaussian line source of half-length ψ, (*b*) circular Gaussian source of half-width ψ, and (*c*) double source with two equal point components separated by ψ. Calculations assume a point source would give a circular Gaussian pattern on the ground with $a = 110$ km; $z = 1$ a.u.

TABLE 5

STRENGTH OF SCINTILLATIONS AS A FUNCTION OF FREQUENCY

	μ_{195}	μ_{430}	μ_{611}
3C 152 ..	0 4	0 5	0 5
245 ...	0 3	0 5	0 55
273 ...	0 25	0 35	0 45

Figure 3 shows histograms of μ for QSO's, galaxies, and unidentified objects. There is some difference between the distributions for QSO's and galaxies: the former includes most of the 100 per cent scintillators while the latter contains most of the very weak scintillators. The two galaxies with $\mu = 1.0$ must have essentially all their radio radiation coming from a region less than about 0"2 in size. One of these, 3C 237, has a doubtful identification; further work on this source is clearly indicated. The other, 1345 + 12, is identified as a S0 galaxy with $m_{pg} = 17$.

TABLE 6

OPTICAL IDENTIFICATIONS (INCLUDING DOUBTFUL IDENTIFICATIONS)

	Optical Identification		
	QSO's	Galaxies	Uniden-tified
Scintillators ($\mu \geq 0$ 3)	21	15	21
Non-scintillators (lim $\mu \leq 0$ 3)	9	35	11

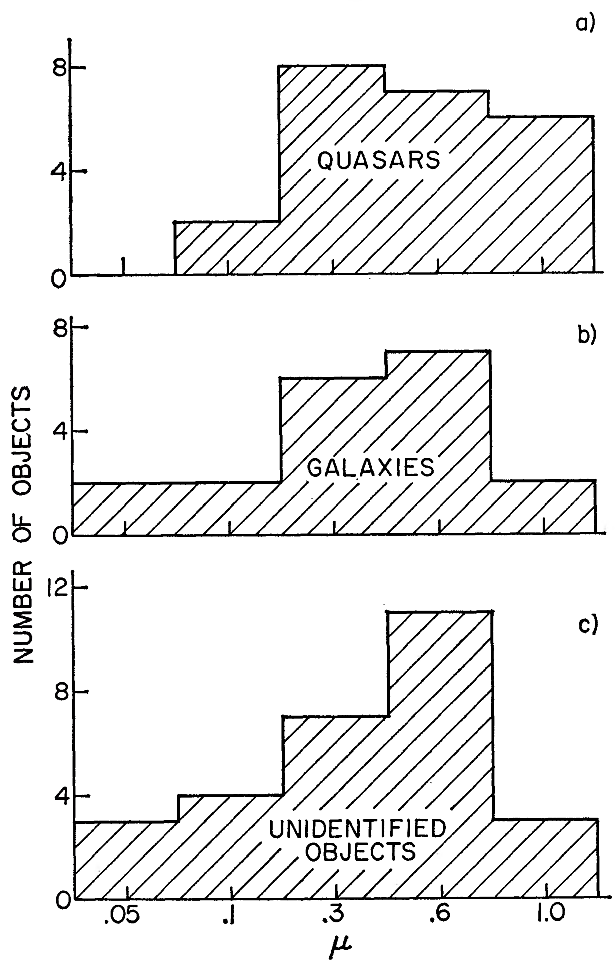

FIG. 3.—Histograms of μ, the strength of scintillation. (a) QSO's, (b) galaxies, (c) unidentified objects

The scintillating and non-scintillating QSO's have different distributions of rf spectrum, as shown in Table 7. The two non-scintillating QSO's with curved spectra are 3C 345, which was discussed above, and 3C 172, for which there is a disagreement between the spectral assignments given by Kellermann (1964) and Howard *et al.* (1965). Bolton (1966) has suggested that QSO's should be separated into two classes, one with flat or curved spectra and the other with steep spectra. This only partially agrees with

TABLE 7

DISTRIBUTIONS OF RF SPECTRUM FOR QSO'S
(INCLUDING DOUBTFUL VALUES)

	Steep	Flat	Curved
Scintillators .. .	10	2	11
Non-scintillators..	10	....	2

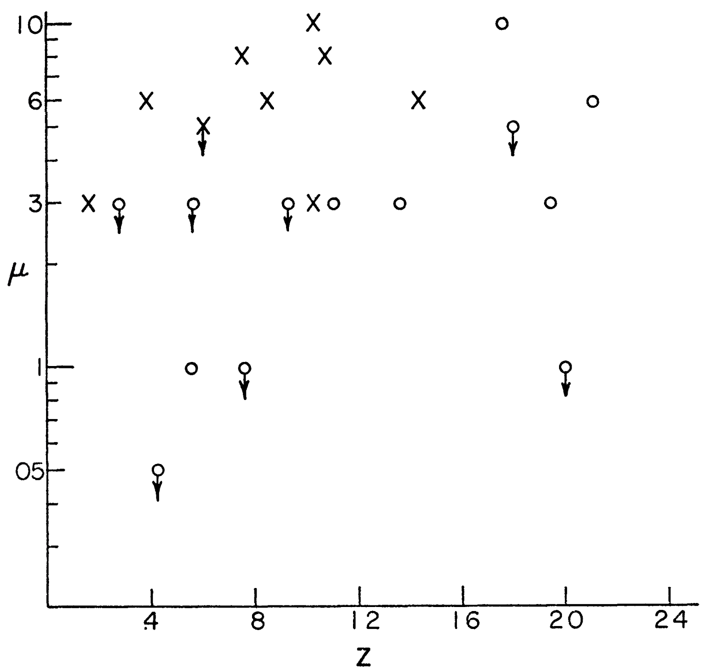

FIG. 4.—Strength of scintillation (μ) versus redshift (z) for QSO's *Crosses*, objects with flat or curved radio-frequency spectrum; *circles*, objects with steep spectrum.

our separation into scintillators (narrow) and non-scintillators (wide). Flat and curved spectrum QSO's are nearly all narrow, but steep-spectrum QSO's are as likely to be narrow as wide.

Twenty-two QSO's have a measured redshift. Figure 4 shows a plot of μ versus z for these sources. The crosses show the scintillating quasars with curved or flat ($\alpha < 0.6$) spectra, the circles show the scintillators with steep spectra ($\alpha \geq 0.6$), and the arrows show the limits for the non-scintillating QSO's. This graph shows some concentration of curved and flat spectrum objects to high values of μ. It does not show a concentration of values of high μ with high values of z.

APPENDIX

Consider an extended radio source shining through a random, thin, phase-changing screen. The diffraction pattern on the ground has fluctuations in intensity. In the one-dimensional case (Paper I) the wavenumber spectrum of the fluctuations $M_{\text{ext}}(q)$ is given by

$$M_{\text{ext}}(q) = M_0(q)\,|V(q)|^2\,, \tag{2}$$

where $M_0(q)$ is the spectrum a point source would produce under the same circumstances, and $V(q)$ is the visibility function of the source.

In two dimensions similar arguments may be used, and the result is

$$M_{\text{ext}}(q_1,q_2) = M_0(q_1,q_2)\,|V(q_1,q_2)|^2\,, \tag{3}$$

where $M_0(q_1,q_2)$ is again the spectrum produced by a point source, $V(q_1,q_2)$ is the visibility function

$$V(q_1,q_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} b(\zeta_1,\zeta_2)\, e^{iq_1 z\zeta_1} e^{iq_2 z\zeta_2} d\zeta_1 d\zeta_2\,; \tag{4}$$

z is the distance to the screen, and $b(\zeta_1,\zeta_2)$ is the angular brightness distribution of the source, normalized so that $V(0,0) = 1$.

The scintillation index, m, is the relative rms fluctuation level, and it is given by

$$m^2\langle I\rangle^2 = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} M_{\text{ext}}(q_1,q_2)\, dq_1 dq_2\,, \tag{5}$$

where $\langle I\rangle$ is the mean intensity of the source.

In principle, the spectrum $M(q_1,q_2)$ could be determined by observing the diffraction pattern at a sufficient number of points on the ground. In this study, however, the effective set of points used is all on one line, since we observe with one antenna and record the pattern as the solar wind blows it past us. The spectrum we observe is, therefore, the two-dimensional spectrum collapsed onto the line of observation. The situation is the same as that obtaining in (*a*) one-dimensional interferometry and (*b*) lunar occultations. The analogy can be extended to synthesis techniques: scintillation observations on different days, with the solar wind variously projected across the source, are analogous to (*a*) synthesizing different base lines by observing at various hour angles, and (*b*) observing occultations at different position angles on the Moon's limb. An analogy to lunar occultations (and a major advantage over interferometry) is that many base lines are effectively sampled simultaneously. It should be noted that the analogies are to an intensity interferometer, since the scintillations give no information on the phase of the visibility function.

Note that the spectrum may be different for various one-dimensional cuts but the index is invariant. (In special, non-random cases, the index may also depend on the direction of the cut.)

In using equation (3) to estimate properties of the source, we must first know $M_0(q_1,q_2)$. One-dimensional observations (Paper II) have shown that the shape is often Gaussian (in the weak-scattering case). We have little knowledge of the isotropy of the function, although, by analogy with the ionosphere, we might expect the irregularities to be elongated substantially. For simplicity, however, we have assumed an isotropic Gaussian function for M_0:

$$M_0(q_1,q_2) = 2\pi a^2 m_0^2 \langle I\rangle^2 \exp\left[-a^2(q_1^2+q_2^2)/2\right]\,, \tag{6}$$

where m_0 is the scintillation index a point source displays, and a is the scale of the diffraction pattern.

For reasons discussed in § III*c*, we have not attempted to compute brightness distributions from equation (3). We have confined ourselves to estimating diameters only in § II; and in § III*c* to comparing the measured index and the diameter to those expected for three simple source models. The models are (*a*) Gaussian line source, (*b*) circular Gaussian source, and (*c*) double source of two equal point components. The brightness distributions are

$$(a)\ b(\zeta_1,\zeta_2) = \delta(\zeta_2)(2\pi\gamma^2)^{-1/2} \exp\left[-\zeta_1^2/(2\gamma^2)\right], \tag{7}$$

$$(b)\ b(\zeta_1,\zeta_2) = (2\pi\gamma^2)^{-1} \exp\left[-(\zeta_1^2+\zeta_2^2)/(2\gamma^2)\right], \tag{8}$$

$$(c)\ b(\zeta_1,\zeta_2) = \tfrac{1}{2}\delta(\zeta_2)\left[\delta(\zeta_1-\tfrac{1}{2}\psi)+\delta(\zeta_1+\tfrac{1}{2}\psi)\right]. \tag{9}$$

In (*a*) and (*c*) δ is the unit impulse function, and for (*a*) and (*b*) we define ψ as the half-power width: $\psi = 2.35\ \gamma$. Use of these brightness distributions, and the isotropic Gaussian model for M_0, equation (6), gives the following formulae for μ as a function of ψ:

$$(a)\ \mu^4 = [1+0.36(z\psi/a)^2]^{-1},$$

$$(b)\ \mu^2 = [1+0.36(z\psi/a)^2]^{-1},$$

$$(c)\ \mu^2 = 0.5+0.5\exp[-\tfrac{1}{2}(z\psi/a)^2].$$

These formulae have also been derived by Little and Hewish (1966).

The circular Gaussian source (eq. [8]), when used with formula (6) for M_0, gives the following spectrum:

$$M_{\text{ext}}(q_1,q_2) = 2\pi a^2 m_0^2 \langle I\rangle^2 \exp\left\{-\tfrac{1}{2}a^2(q_1^2+q_2^2)[1+0.36(z\psi/a)^2]\right\}. \tag{10}$$

We assume the pattern is convected with the solar-wind velocity, so that $q = 2\pi f/u$, and the frequency spectrum is identical (apart from scaling) to the one-dimensional integral of $M_{\text{ext}}(q_1,q_2)$:

$$M_{\text{ext}}(f) \sim \exp\left\{-\tfrac{1}{2}(f/f_0)^2[1+0.36(z\psi/a)^2]\right\}, \tag{11}$$

where $f_0 = u/(2\pi a)$. The second moment of the spectrum is

$$f_2^2 = f_0^2[1+0.36(z\psi/a)^2]^{-1}, \tag{12}$$

so that

$$\psi = \frac{u}{1.2\pi f_2 z}\left[1-\left(\frac{f_2}{f_0}\right)^2\right]^{1/2}. \tag{13}$$

Caution must be taken in using this model of the source and the solar wind to explain an observed spectrum. There is an unknown amount of smearing within the wavenumber spectrum, and this presumably is further smeared somewhat by the drifting of the pattern over the ground. We assume, however, that the second moment of the spectrum is a rather stable feature which does not change much during any smearing.

In using equation (13) we might measure f_2, take $z = 1$ a.u. (or somewhat less, depending on the model for the screen), assume $u = 350$ km/sec, and make a guess for f_0. Observations reported in Paper II show that f_0 is variable and can change by a factor of 2 in a few days. (It is not known whether this is mainly due to variations in a or in u; the latter is known to vary substantially near Earth.) Since the variability in f_0 is so great we usually make no attempt to compute ψ directly, but rather compute only the upper limit

$$\psi \leq \frac{u}{1.2\pi f_2 z}. \tag{14}$$

In a few cases the spectrum appears to be decidedly narrower than the range of f_0 would allow for a point source. In these cases we estimate ψ by using equation (14) as an equality rather than an inequality.

Formula (14) can also be derived in an intuitive way: the diffraction pattern is smeared over distances on the order of ψz, so that frequencies higher than about $u/(\psi z)$ must be missing from the fluctuation spectrum (Cohen, Gundermann, Hardebeck, Harris, Salpeter, and Sharp 1966). The argument is simple but powerful. It does not depend on the screen's being thin, insofar as equation (14) is an inequality and not an equality. It also does not depend on the scale, a, to the extent that the screen is rigid. If it is not, but each scale still has a characteristic velocity, then equation (14) still holds provided the velocity appropriate to the cutoff frequency is used.

When observations are close to the Sun ϕ_0 gets bigger than unity, and $M_0(f)$ becomes wider and more nearly exponential than Gaussian (Paper II). $M_{\text{ext}}(f)$ then becomes the product of an exponential and a Gaussian (still assuming a circular Gaussian source). In the case of 3C 138 at elongation 7° this general form seems to be recognizable; i.e., the spectrum appears to be cut off. The diameter ψ may then directly be computed from the cutoff frequency. The spectrum for this case has already been published by Cohen *et al.* (1966).

Far from the Sun, f_0 is about 0.6 c/s (Paper II). The resolution attainable in this case is about $0\rlap{.}''2$. The spectrum gets much wider close to the Sun; values of f_0 of several c/s have been seen at 430 MHz. At a wavelength of 11 cm the Sun can be approached very closely, and values of f_0 between 5 and 10 c/s have been seen (Cohen and Gundermann 1967). It appears as though a resolution of perhaps $0\rlap{.}''001$ will be attainable with this technique.

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