THE ANOMALOUS COLOR-MAGNITUDE DIAGRAM OF THE REMOTE GLOBULAR CLUSTER NGC 7006

Allan Sandage and Robert Wildey*

Mount Wilson and Palomar Observatories, Carnegie Institution of Washington, California Institute of Technology Received A pril 4, 1967

ABSTRACT

The color-magnitude diagram for NGC 7006 has been obtained to V = 19.3 by a combination of pho-tographic and photoelectric measurements. The horizontal branch occurs at V = 18.80. Interstellar reddening is small. Adopting $M_V = +0.5$ for the RR Lyrae variables gives a true modulus of $(m - M)_0 = 18.3$, a distance of 45 kpc from the Sun, and 15 kpc from the galactic plane. The metal abundance is low, as judged by $\Delta V = 2.6$ mag and by the spectroscopic and photometric criteria of Morgan and van den Bergh, and yet the distribution of stars along the horizontal branch imi-tates that of much higher metal-abundance clusters. This is the first clear violation of the heretofore unique correlation of metal abundance and density gradient along the horizontal branch. It shows that

unique correlation of metal abundance and density gradient along the horizontal branch. It shows that

at least one additional parameter besides Z controls the stellar distribution along the branch. Models of Faulkner and of Faulkner and Iben suggest that the second parameter may be the abundance of helium. The NGC 7006 anomaly is present in many intergalactic globular clusters and in the clusters of the Small Magellanic Cloud, suggesting a difference in the second parameter between objects that have or have not partaken in the chemical evolution of the galaxy. NGC 7006 may not be gravita-tionally bound to the galaxy, in which case it was formed in the Local Group but outside the galactic halo. If future observations and more extensive model calculations can establish an abundance difference, A V, between objects inside and outside the Galaxy, then we would be able to conclude that some helium was made in the early history of the galactic system. This additional helium is then added to the pristine helium produced, for example, in the primeval fireball of a Friedman-type universe or in any other event that manufactured helium which preceded the formation of the Local Group of galaxies.

I. INTRODUCTION

NGC 7006 ($a = 20^{h}59^{m}1$, $\delta = +16^{\circ}00'$ (1950); $l^{II} = 64^{\circ}$, $b^{II} = -19^{\circ}$) is a remote globular cluster far into the halo of the galactic system. Its distance from the Sun is exceeded only by NGC 2419 (Baade 1935) and by a few loose "intergalactic" clusters discovered during the Palomar Sky Survey (Abell 1955; Burbidge and Sandage 1958; Arp and van den Bergh 1960). Early work by Shapley and Mayberry (1921), Shapley (1920, 1930), and Hubble (quoted by Baade 1935) placed the cluster on the far side of the Galaxy at a distance of about 50 kpc.

Besides helping to outline the extent of the system of globular clusters, NGC 7006 is of even greater importance in modern studies of the chemical evolution of our Galaxy. If the cluster was associated with the galactic system at the time of its collapse toward the plane, it should be among the most metal-poor aggregates known, because the data for other globular clusters (Kinman 1959a; Morgan 1959; Böhm-Vitense, Holweger, and Kohl 1963) and for field subdwarfs and RR Lyrae stars (Preston 1959; Eggen, Lynden-Bell, and Sandage 1962, Fig. 5; Eggen 1965, Fig. 1, p. 39) indicate that the upper bound of metallicity is a function of the distance from the galactic plane. If the correlations (Sandage and Wallerstein 1960; Wildey 1961) between the morphology of globular cluster color-magnitude diagrams and metallicity are correct with no second parameter involved, then some information on this point was expected once the C-M diagram for NGC 7006 was known. Our results have been a surprise in this regard, and suggest that at least one other parameter does indeed exist.

* Now at the Center of Astrogeology, U.S. Geological Survey, Flagstaff, Arizona.

470

II. THE DATA

The photoelectric sequence upon which the photographic photometry is based is listed in Table 1 as determined in 1958 and 1963 with the Hale reflector. The observations were tied to the UBV system in the usual manner, with mean errors smaller than ± 0.03 mag in each wavelength band. The standards are identified in Figure 1 (Plate 3), where many of the known variable stars are also marked.

The cluster is relatively rich in variables. The first eleven were discussed by Shapley and Mayberry (1921). Hubble (Sandage 1954) discovered thirty-two new variables, including the long-period red star V19. Rosino and Mannino (1955) and Mannino (1957) discovered nine additional variables and numbered these V44 through V52. We have identified two new variables (V53 and V54), one of which is a red star on the giant branch of the C-M diagram, similar to V19. With the exception of V4 and V7 (which Hubble found to be constant), and V25, 26, 27, 28, 30, 32, 36, 37, 40, 41, and 42, which are too crowded for easy identification here (but see Sandage 1954), all suspected variables are marked in Figure 1 (Plate 3) as reproduced from a 103aD plate of 7-min exposure taken with the 200-inch reflector.

No.	v	B-V	U-B	n
III-11 III-32 III-57 III-17 III-17 III-33 III-33 III-35 III-45 III-38 III-60 III-61 III-101 III-8 I.	$\begin{array}{c} 13 \ 95 \\ 14 \ 04 \\ 15 \ 51 \\ 16 \ 13 \\ 16 \ 43 \\ 16 \ 77 \\ 17 \ 31 \\ 17 \ 95 \\ 17 \ 96 \\ 18 \ 02 \\ 18 \ 58 \\ 18 \ 68 \\ 18 \ 69 \\ 18 \ 69 \\ 18 \ 69 \\ 41 \\ 20 \ 89 \end{array}$	$\begin{array}{c} 0 & 53 \\ 1 & 12 \\ 0 & 76 \\ 0 & 83 \\ 0 & 68 \\ 1 & 10 \\ 0 & 70 \\ 0 & 93 \\ 0 & 80 \\ 0 & 64 \\ 0 & 85 \\ 0 & 62 \\ 0 & 80 \\ 1 & 01: \\ \dots \end{array}$	0 03 1 05 0 32 0 37 0 10 0 78 0 13	3 3 1 1 1 1 1 1 1 1 1 1 1 1 1

TABLE 1Photoelectric Sequence

To obtain the C-M diagram, a series of photographic plates of differing exposure times were taken and measured with an iris photometer. The stars are identified in Figure 2 (Plate 4) and are generally numbered within each zone in progressive order of right ascension, with the numbers placed in the upper right corner where possible. Because it was apparent that crowding difficulties were severe for stars closer than about 30" from the center, we retain and list in Table 2 data for those stars more distant than 28" from the nucleus—a distance defined by the inner circle of Figure 2 (Plate 4).

Study of the residuals of the derived magnitudes from three plates in each color indicates that the probable error of a tabulated value is ± 0.03 mag with, of course, no indication of the systematic error involved. It is likely that photometric errors due to crowding have not been completely eliminated for stars near the inner boundary of zone II, but such errors as remain do not affect the characteristics of the derived C-M diagram.

III. THE ANOMALOUS C-M DIAGRAM

The color-magnitude diagram is shown in Figure 3, where all stars in Table 2 are plotted. The known RR Lyrae stars have been excluded and the characteristic gap is



PLATE 3



PLATE 4

FIG. 2.— Identification chart for the program stars of Table 2. The angular radii of the circles separating the zones are 28 and 78 arc seconds. NGC 7006 has nearly the same distance modulus as the Magellanic Clouds. Globular clusters in the Clouds would be resolved with this clarity if a 200-inch reflector existed in the southern hemisphere.

SANDAGE AND WILDEY (see page 470)

TABLE 2

PHOTOGRAPHIC PHOTOMETRY OF PROGRAM STARS

No.	v	B — V	No.	V	B - V	No.	V	B – V	No.	V	B - V
Ring II											
$\begin{array}{c} 1. \\ 2. \\ 3. \\ 3. \\ 4. \\ 5. \\ 6. \\ 7. \\ 8. \\ 9. \\ 10. \\ 11. \\ 10. \\ 11. \\ 13. \\ 14. \\ 15. \\ 15. \\ 16. \\ 17. \\ 18. \\ 19. \\ 20. \\ 21. \\ 22. \\ 23. \\ 23. \\ 24. \\ 25. \\ 26. \\ 27. \\ \end{array}$	18. 91 18. 77 18. 77 18. 52 16. 62 16. 94 18. 83 18. 42 18. 05 18. 90 18. 90 18. 90 18. 90 18. 58 17. 55 18. 11 18. 75 18. 31 18. 75 18. 32 17. 05 19. 02 18. 39 18. 39 18. 39 18. 29 18. 28 18. 28	$\begin{array}{c} 0.07\\ 0.92\\ 0.80\\ 1.21\\ 0.59\\ 0.98\\ 1.04\\ 0.48\\ 0.72\\ 0.90\\ 0.81\\ 0.79\\ 0.57\\ 0.79\\ 1.01\\ 1.10\\ 1.33\\ 1.00\\ 0.77\\ 0.53\\ 0.81\\ 0.86\\ 0.47\\ 1.17\\ 1.06\\ 0.05; \end{array}$	$\begin{array}{c} 28. \\ 29. \\ 30. \\ 31. \\ 32. \\ 33. \\ 35. \\ 36. \\ 35. \\ 36. \\ 37. \\ 38. \\ 39. \\ 40. \\ 41. \\ 42. \\ 43. \\ 44. \\ 44. \\ 44. \\ 45. \\ 46. \\ 47. \\ 48. \\ 49. \\ 49. \\ 50. \\ 51. \\ 52. \\ 53. \\ 54. \\ \end{array}$	16, 77 18, 51 16, 75 18, 96 18, 16 17, 27 17, 47 18, 64 18, 59 18, 33 16, 90 18, 16; 18, 07 18, 51 18, 07 18, 51 18, 76 19, 04 18, 61 18, 87 18, 97 18, 55	$\begin{array}{c} 1.24\\ 0.85\\ 1.13\\ 0.69\\ 0.94\\ 0.53\\ 0.94\\ 0.53\\ 0.40\\ 0.84\\ 0.84\\ 1.08\\ 0.22\\ 0.61\\ 1.38\\ 0.63\\ 0.22\\ 0.61\\ 1.38\\ 0.60\\ 0.83\\ 0.60\\ 0.80\\ 0.60\\ 0.87\\ \end{array}$	$\begin{array}{c} 55. \\ 56. \\ 57. \\ 58. \\ 59. \\ 60. \\ 61. \\ 62. \\ 63. \\ 64. \\ 65. \\ 66. \\ 67. \\ 68. \\ 66. \\ 67. \\ 70. \\ 71. \\ 71. \\ 72. \\ 73. \\ 74. \\ 75. \\ 76. \\ 77. \\ 78. \\ 79. \\ 79. \\ 79. \\ 78. \\ 79. \\ 80. \\ 81. \\ \end{array}$	18, 56 17, 59 15, 51 17, 72 18, 77 18, 19 19, 00 18, 36 18, 31 18, 66 19, 09 17, 59 18, 33 16, 43 16, 43 18, 31 18, 66 19, 09 17, 59 18, 33 16, 43 18, 70 18, 90 18, 33 16, 43 18, 66 18, 60 18, 21	$\begin{array}{c} +0.\ 62\\ +1.\ 20\\ +0.\ 76\\ +1.\ 27\\ +0.\ 06\\ +0.\ 61\\ -0.\ 02\\ +0.\ 54\\ +0.\ 54\\ +0.\ 51\\ +0.\ 91\\ +0.\ 46\\ +0.\ 53\\ +0.\ 68\\ +1.\ 07\\ +0.\ 86\\ +0.\ 61\\ +0.\ 64\\ +0.\ 62\\ +0.\ 46\\ +0.\ 46\\ +0.\ 70\\ +0.\ 70\\ +0.\ 80\\ \end{array}$	82	$\begin{array}{c} 18,43\\ 18,69\\ 16,18,75\\ 16,57\\ 16,57\\ 16,57\\ 16,57\\ 16,57\\ 16,57\\ 16,57\\ 16,57\\ 16,57\\ 16,57\\ 16,57\\ 16,71\\ 17,92\\ 16,71\\ 17,91\\ 18,54\\ 17,63\\ 18,69\\ 17,34\\ 16,23\\ 18,71\\ 17,93\\ 18,57\\ 19,03\\ 18,67\\ \end{array}$	$\begin{array}{c} +0.69\\ -0.17\\ +1.46\\ +1.20\\ +0.57\\ +0.55\\ +0.81\\ +1.21\\ -0.01\\ +0.63\\ +1.41\\ +0.66\\ +0.53\\ +1.10\\ +0.84\\ +0.72\\ +0.64\\ +0.74\\ +1.07\\ +1.41\\ +0.53\\ +0.74\\ +1.07\\ +1.41\\ +0.53\\ +0.78\\ -0.01\\ \end{array}$
		·	· · · · · · · · · · · · · · · ·		Rin	gIII		·	••••••••••••••••••••••••••••••••••••••		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 16.44\\ 18.96\\ 18.37\\ 18.61\\ 16.73\\ 17.56\\ 17.97\\ 19.41\\ 18.77\\ 18.79\\ 13.95\\ 18.51\\ 18.51\\ 18.84\\ 18.64\\ \end{array}$	$\begin{array}{c} 1.37\\ 0.76\\ 0.63\\ 0.62\\ 1.01\\ 0.95\\ 0.95\\ 0.67\\ 0.54\\ 0.53\\ 0.69\\ 0.76\\ 0.60\\ 0.64 \end{array}$	16 17 18 20 21 22 23 24 25 26 27 28 29 30	18.59 16.13 18.53 18.68 18.90 18.78 19.29 17.70 18.77 19.23 19.03 18.19 16.49 17.37	$\begin{array}{c} 0 & 70 \\ 0 & 83 \\ 1 & 00 \\ 0 & 86 \\ 0 & 40 \\ 0 & 62 \\ 0 & 56 \\ 0 & 77 \\ 0 & 99 \\ 0 & 58 \\ 0 & 77 \\ 0 & 99 \\ 0 & 58 \\ 0 & 77 \\ 0 & 97 \\ 0 & 67 \\ 0 & 97 \end{array}$	$\begin{array}{c} 31. \\ 32. \\ 33. \\ 35. \\ 36. \\ 37. \\ 38. \\ 39. \\ 40. \\ 41. \\ 42. \\ 43. \\ 44. \\ 45. \\ \end{array}$	$\begin{array}{c} 13.85\\ 14.04\\ 16.77\\ 18.44\\ 18.46\\ 18.76\\ 17.31\\ 18.02\\ 16.64\\ 16.96\\ 18.74\\ 17.33\\ 18.68\\ 18.55\\ 17.96 \end{array}$	$\begin{array}{c} 0.81\\ 1.12\\ 1.10\\ 0.87\\ 0.64\\ 0.70\\ 0.64\\ 0.72\\ 1.12\\ 0.13\\ 0.70\\ 0.68\\ 0.78\\ 0.80\\ \end{array}$	$\begin{array}{c} 46. \\ 47. \\ 48. \\ 50 \\ 51. \\ 52. \\ 53. \\ 54. \\ 55. \\ 56. \\ 56. \\ 57. \\ 58. \\ 59. \\ 60. \\ 61. \\ \end{array}$	$\begin{array}{c} 16, 78\\ 18, 71\\ 17, 62\\ 19, 20\\ 18, 79\\ 16, 62\\ 18, 59\\ 18, 82\\ 17, 90\\ 17, 95\\ 17, 19\\ 18, 17\\ 15, 08\\ 18, 76\\ 18, 58\\ 18, 09\\ \end{array}$	$\begin{array}{c} \textbf{1.24}\\ \textbf{0.62}\\ \textbf{0.87}\\ \textbf{0.86}\\ \textbf{0.97}\\ \textbf{0.66}\\ \textbf{0.59}\\ \textbf{0.56}\\ \textbf{0.93}\\ \textbf{0.65}\\ \textbf{0.62}\\ \textbf{0.62}\\ \textbf{0.62}\\ \textbf{0.80}\\ \textbf{0.76} \end{array}$

evident. Variables 19 and 54 are shown as crosses on the giant branch as derived in the Appendix.

The most striking feature of the diagram is the density gradient along the horizontal branch, with many more stars on the red side of the variable-star gap than on the blue side. This characteristic has previously been considered as a sure indicator of relatively *high* metal abundance because of the unique correlation of the density distribution with independent spectroscopic data. Until now there have been no clear exceptions to the rule that a cluster with a strong concentration of stars redward of the variable-star gap is metal rich relative to other globular clusters. [It should, however, be noted here that the Draco System (Baade and Swope 1961) and NGC 121 in the Small Magellanic



FIG. 3.—The C-M diagram of NGC 7006 obtained from the data of Table 2. Stars which fall off the sequences and very likely belong to the field are shown as small dots. The reddest non-variable star, I-1, not listed in Table 2, has V = 15.69, B - V = +1.70.

Cloud (Tifft 1963*a*) have anomalous C-M diagrams like Figure 3, and, as such, they were not considered at this point of the argument to be metal-poor—as we now know them to be by the ΔV parameter discussed in § V. No direct spectroscopic information is available on their metal content.]

The available strong-lined clusters which lead to this conclusion include 47 Tuc (Wildey 1961; Tifft 1963b), whose spectral type is G3 (Kinman 1959a) and whose Q value as defined by van den Bergh (1967) is quite low at -0.26; NGC 6356 (Sandage and Wallerstein 1960), of spectral type G5 (Kinman 1959a) or G4 (Kron and Mayall 1960), with a Morgan (1959) metal class of VI, with Q = -0.24, with the van den Bergh and Henry (1962) photometric parameters of $\Delta = 0.48$, $\Phi = 0.39$, and $\Psi = 0.88$, and with a Gascoigne-Koehler (1963) G-band index of $\gamma = 3.50$ —all indicating metal richness; NGC 6723 (Gascoigne and Ogston 1963), of spectral type G2 (Kinman 1959a) or G4 (Kron and Mayall 1960), with Q = -0.28 and with $\gamma = 3.37$ —suggesting a Morgan metal class V; NGC 6171 (Sandage and Katem 1964) with a spectral class G3

472

(Kron and Mayall 1960) or G0-1 (Kinman 1959*a*), and Q = -0.35; and NGC 6712 (Sandage and Smith 1966), of Morgan class V, spectral type G4 or 5, Q = -0.31, and $\gamma = 3.36$. All five clusters are metal-rich, and each has its horizontal branch concentrated toward the red end.

NGC 7006 violates the rule because all spectrographic data clearly show that the cluster is metal poor. Morgan (1959) places the cluster in his Class II. He recently reexamined his material and, if anything, would place the cluster slightly weaker than Class II but not quite as weak as Class I, which includes the most metal-poor clusters known. The spectral type of NGC 7006 is F3-4 (Kinman 1959a) or F2 (Kron and Mayall 1960). The photometric parameters are Q = -0.40 (van den Bergh 1967), $\Delta = 0.33$, $\Phi = 0.15$, and $\Psi = 0.46$ (van den Bergh and Henry 1962)—all of which are consistent with the early Morgan class and a metal-poor spectral type.

We then have a clear discrepancy whose explanation requires that at least one other parameter besides the metal abundance controls the distribution of stars along the horizontal branch. This is not an entirely new result, but had been suspected in a less clear way from the case of M13 (Sandage and Wallerstein 1960; van den Bergh 1967), where the metal abundance is as high or higher than in M3 as judged by (1) the spectral type of F5 or F6, (2) Morgan's Class III, (3) Deutsch's metallic-strength classification of individual stars (see Kinman 1959a), and (4) the values of Δ , Φ , and Ψ . Yet the horizontal branch of M13 is confined entirely to the extreme blue edge of the RR Lyrae gap, as in M15 and M92 and in contrast to M3.

The M13 anomaly is therefore in the opposite sense from that in NGC 7006, which suggests that these two clusters are at the extremes of the variation of the second parameter of the clusters studied so far.

Similar, but much less convincing, anomalies may occur in M22 (Arp and Melbourne 1959) and M53 (Cuffey 1958), in that their spectra are weak but not extremely weak (Deutsch type B as summarized by Kinman 1959*a*), but their C-M diagrams resemble the type C prototypes of M15 and M92.

IV. POSSIBLE IDENTIFICATION OF THE SECOND PARAMETER AS THE HELIUM ABUNDANCE

A consistent case for the possible identification of the second parameter as the helium abundance, as suggested by van den Bergh (1967), appears to be available from recent theoretical work of Faulkner (1966) and Faulkner and Iben (1966). These authors suggest that the horizontal branch may consist of unmixed stars that have passed through the helium flash near the top of the giant branch. The stars are burning helium into carbon in a helium-rich core that contains about half the total mass, surrounded by a hydrogen-burning shell operating on the CNO cycle. Faulkner showed that stellar models with the same Z, Y_{e} , and core mass M_c , but with different total mass M_s , fell at different places along the horizontal branch, but are located in such a way that there is a critical turning point redward of which an initial, post-helium-flash star of given Z, Y_{e} , and M_c cannot exist. This turning point corresponds to a particular value of the total mass and is tentatively identified by Faulkner as the red end of the horizontal branch. The turning point itself was shown to be a strong function of Z, occurring far to the blue (as observed in M15 and M92) for very low Z values (10^{-5}) , and far to the red, abutting onto the nearly vertical subgiant sequence as observed in 47 Tuc, NGC 6356, NGC 6171, and NGC 6712 for higher Z values $(10^{-2} to 10^{-3})$. The dependence on Z is so pronounced and agrees so well with the observations of most clusters that it seems likely that Faulkner's work has application to this problem.

There is of course no guaranty that stars in a given cluster will initially position themselves *at* the critical turning point just after the helium flash, since this demands that they be at the critical total mass—which occurs when $M_s \simeq 2 M_c$. But Faulkner shows that, if clusters exist that are older or younger than the prototypes (say M92 on the blue side, and 47 Tuc on the red side), in which case $M_s \leq 2 M_c$ or $M_s \geq 2 M_c$, then 1967ApJ...150..469S

the observed gap¹ between the red end of the horizontal branch and the subgiants can only widen. Thus, until the properties of NGC 7006 were found, the second parameter could have been the variation of M_s (i.e., age difference), which would anomalously widen the gap as in M13. Now, however, the anomalous red position of the branch in NGC 7006 shows that variations of M_s cannot be the explanation—at least within the framework of Faulkner's theory—because the gap in NGC 7006 is narrower than is expected for its metal abundance.

The only remaining parameter in the case of NGC 7006 is the helium abundance, and, indeed, Faulkner's theory shows a dependence of the position of the critical turning point on Y, in the sense that small Y narrows the gap if Z is kept constant (cf. Faulkner 1966, Figs. 1 and 2). The sensitivity to changes in Y was very great in Faulkner's first models, but was considerably reduced when Faulkner and Iben included electron degeneracy in the core. However, the effect is still large enough to explain the observations, even for small changes in Y, as can be seen by a comparison of NGC 7006 and M3, which are clusters of about the same metal abundance as judged from the ΔV parameter discussed in the next section. The red side of the horizontal branch of M3 occurs at $B - V \simeq 0.62$ (Johnson and Sandage 1956, Fig. 5) Its position in NGC 7006 occurs at $B - V \simeq 0.75$. From an uncertain calibration, these values correspond to log $T_e \simeq$ 3.76 and $\log_e T \simeq 3.72$, where the *difference*, $\Delta \log T_e$, is reliable even if the zero point is uncertain due to low surface gravity of horizontal-branch stars. Without an extensive grid of models, and with no certain knowledge of Z, M_c , and Y, it is not possible to enter Faulkner and Iben's diagrams uniquely to estimate ΔY , but the principle can be illustrated. For a starting model at the critical turning point defined by $Z = 10^{-4}$, Y = 0.35, and $M_c = 0.5$, one obtains log $T_e = 3.76$ from Figure 1 of Faulkner and Iben. A change of helium abundance to Y = 0.1 gives log $T_e = 3.695$ at the turning for $Z = 10^{-4}$ and $M_c = 0.6$ from their Figure 2. This temperature difference gives $\Delta(B - V) = 0.24$ mag, which is considerably more than enough to explain the observed value of $\Delta(B - A)$ V) = 0.13 between the two clusters. A different combination of parameters will give different results, so it is not yet possible to interpolate linearly the $\Delta Y/\Delta(B-V)$ calibration to estimate ΔY from the observations of $\Delta(B-V)$, but the point here is that *small* values of ΔY can evidently give the required large effect.

An interesting feature of the present models that must be taken into account in the calibration of $\Delta Y/\Delta(B-V)$ is that $\Delta(B-V)$ for a given ΔY is itself a function of Z, if the relatively few models of Faulkner and Iben are representative. A comparison of their Figures 1 and 2 shows that $\Delta \log T_e$ for $\Delta Y = 0.25$ varies from 0.15 to 0.00 as Z changes from 10^{-5} to 10^{-3} . This dependence of the gap width on Z for a fixed ΔY suggests that changes in the helium abundance will not greatly affect the horizontal branches of very metal-rich clusters ($Z = 10^{-3}$) but will have an increasing effect as the metallicity decreases. This is in agreement with the observations that no extremely metal-rich clusters (Morgan class $\geq V$) have blue horizontal branches, while metal-poor clusters, such as NGC 7006, have red branches.

It seems to us that the Faulkner-Iben theory is so illuminating, and has passed the observational tests now available, that a large grid of models with small increments of Y and Z should be calculated so as to calibrate the observed C-M diagrams for an eventual determination of the second parameter, if indeed it is the helium abundance.

V. SEPARATION OF THE FIRST AND SECOND PARAMETERS

If the variation of helium from cluster to cluster can be proved, the implications for the early chemical evolution of the Galaxy and of the universe are far reaching. But the theoretical information from stellar models cannot be properly used to evaluate ΔY

¹ The term "gap" in this connection should not be confused with the variable-star gap. We mean here the difference in log T_e (or B - V) between the red end of the horizontal branch and the subgiant sequence near $M_V \simeq +0.5$.

unless we can separate the effects of Z and Y observationally. One parameter which has consistently correlated with metal abundance is ΔV —the V-magnitude difference between the horizontal branch and the giant branch read at $(B - V)_0 = +1.4$ (Arp 1955; Kinman 1959*a*; Sandage and Wallerstein 1960, Table 4). Theoretical justification for this parameter came originally from the work of Hoyle and Schwarzschild (1955, Fig. 4), and more recently from Demarque and Geisler (1963, Fig. 6), who showed that ΔV is strongly dependent on Z but is very nearly independent of changes in Y—an ideal situation in the present context.

These theoretical results show that the ΔV parameter can be used to separate clusters into classes of similar Z, after which the nature of the horizontal branch can be used to divide the clusters of each Z group into a helium-rich and helium-poor class. For such a program we need not only observations of many more globular clusters, but also an extension of the Faulkner-Iben theory, assuming that it correctly describes the situation. From such a program one might eventually determine not only ΔY within each group of similar ΔV , but also obtain the absolute Y values of various clusters, which is crucial for the age-dating problem.

The value of ΔV for NGC 7006 is 2.60 mag if we adopt V = 18.80 for the horizontal branch, as given by zone III stars alone, and if we read the level of the giant branch at an observed B - V of +1.4 as if the reddening, E(B - V), were zero. Our data are meager on the reddening value, although the following three methods are available.

1. The RR Lyrae-star gap occurs between B - V = 0.15 and B - V = 0.39, which is to be compared with 0.17 and 0.42 for M3, indicating $E(B - V) \simeq 0.0$. The problem here is that our color scale for NGC 7006 at V = 18.8 is not definitive and, further, that the color of the blue edge of the RR Lyrae domain is itself a function of the helium abundance (Christy 1966a).

2. The two-color diagram for the field stars of the photoelectric sequence of Table 1 gives a maximum reddening of $E(B - V) \simeq 0.1$. The true value is probably smaller because the sequence stars may have an intrinsic ultraviolet excess, $\delta(U - B)$, due to a metal abundance which may be smaller than for stars in the Hyades.

3. The absorption, A_B , cannot be high because of the large number of background galaxies seen on direct plates. Although we plan to strengthen the reddening value, no significant error in ΔV results from our present assumption that $B - V = (B - V)_0 = +1.4$ in the definition of ΔV . The value $\Delta V = 2.60$ mag puts NGC 7006 in the intermediate metal-abundance group composed of NGC 4147, M3, M5, M13, and M22 (Sandage and Wallerstein 1960, Table 4).

VI. DEPENDENCE OF THE SECOND PARAMETER ON MEMBERSHIP IN THE GALAXY

If the preceding precepts are correct, NGC 7006 has an abnormally low helium abundance compared to other clusters studied so far in our Galaxy. As mentioned previously, it is not yet possible to determine a reliable value of ΔY between a "normal" cluster, such as M3, and NGC 7006, although the incomplete calibration discussed in § IV shows that ΔY can be quite small compared with Y itself to produce the observed effect.

Although Figure 3 did come as a surprise to us, the abnormal features are not unique. The present conclusions could have been drawn earlier from the $\Delta V = 3.0$ value for the Draco System (Wilson 1955; Baade and Swope 1961)—a value that indicates extremely low metal content. In this system the horizontal branch is stubby toward the red. The two intergalactic clusters, Palomar 3 and Palomar 4 (Wilson 1955; Abell 1955; Burbidge and Sandage 1958), also have red horizontal branches and have ΔV values which indicate moderately low metal abundance. A most striking example is the Leo II intergalactic system (Wilson 1955) now under study by Miss Swope (1967). It has an exceedingly stubby red horizontal branch and a near absence of blue stars. NGC 121 in the Small Magellanic Cloud (Tifft 1963*a*; Gascoigne 1966) shows the same anomaly, as does every other cluster in the SMC for which data are available (NGC 339, Lindsay 1, and Kron

1967ApJ...150..469S

3 by Gascoigne 1966; NGC 419 by Arp 1958). On the other hand, all globular clusters studied so far in the LMC, such as NGC 1466, NGC 1841, and NGC 2257 (Gascoigne 1966) do not show the anomaly.

Except for NGC 7006, all of these anomalous systems are outside the Galaxy, are presumably not gravitationally bound to the galactic system, and, therefore, have not partaken in its chemical evolution. It may even be that NGC 7006 is itself unbound, as was first suggested by Kinman (1959b, Fig. 5), but this result depends critically on the rotation constants assumed.

Any cluster will be unbound if the total energy, E, satisfies

$$E = \frac{1}{2}V^2 - \frac{GM}{D} \ge 0,$$
 (1)

where V is the space motion relative to the galactic center, M is the mass of the Galaxy, and D is the distance of the cluster from the center, assuming a point galactic potential. We know only the radial component of the space motion of any cluster, and equation (1) cannot be tested in general. Only in those cases where

$$V_{\rm rad}^2 \ge \frac{2GM}{D} \tag{2}$$

can we be certain that any given cluster is free. Equation (2) is nearly satisfied by NGC 7006. The observed radial velocity is -348 ± 33 (A.D.) km/sec based on four plates by Mayall (1946) and one plate of Humason (1934). The component of solar motion about the galactic center toward the cluster is +212 km/sec if we adopt $V\odot = 250$ km/sec toward $l^{II} = 90^{\circ}$, $b^{II} = 0$, giving a net radial velocity of the cluster relative to the galactic center of -136 km/sec. The distance modulus of NGC 7006 is $(m - M)_0 \simeq$ 18.3, according to our present data, if $\overline{M}_{v} = +0.5$ for the RR Lyrae stars, giving a distance of 45.7 kpc from the Sun. The coordinates of the cluster are $l^{11} = 64^{\circ}$, $b^{11} =$ -19°, which, together with $R\odot = 10$ kpc, give D = 42.6 kpc. The kinetic energy in the radial component alone is a factor of 1.6 less than the potential energy, but the cluster would have $E \ge 0$ if the transverse velocity were 104 km/sec or larger. Moreover, only a slight reduction of $V \odot$ is sufficient to make NGC 7006 unbound, using the radial component of the kinetic energy alone. A consistent calculation taking into account the variation of GM as $V \odot$ changes, and using $V \odot / R \odot = A - B = 25$, where V is in km/sec, R is in kpc, and A and B are the Oort constants, shows that a reduction of $V \odot$ by only 20 km/sec to 230 km/sec is sufficient for the cluster to satisfy equation (2). The difference between our result and that of Kinman (1959b, Fig. 5) rests entirely on the value of $V \odot$.

It seems possible, therefore, that NGC 7006 may not be gravitationally bound, and was formed within the Local Group but outside the Galaxy at the time the Local Group collapsed from the intergalactic medium.

We cannot, of course, generalize these results with certainty because we lack data on C-M diagrams for the majority of globular clusters which can be studied. But the case does appear strong enough to suggest, as a working hypothesis, that any cluster that was formed outside the Galaxy and that, therefore, has not partaken in the chemical evolution of either the galactic system, the LMC, and presumably M31 and M33 may have a characteristically different horizontal branch than the majority of clusters within the Galaxy *if only two parameters* are acting, and this anomaly may reflect a difference in the helium abundance inside and outside the Galaxy.

Although not proven, this hypothesis does form the basis for future work. Several globular clusters are known which are clearly unbound because their data satisfy equation (2). The most striking case is NGC 5694 (Kinman 1959b, Fig. 5). No observations

are yet available for this cluster, but, if the working hypothesis is correct, its C-M diagram should be similar to that of NGC 7006, unless a third parameter, such as a variation of M_s , is also involved. As explained in § IV, the variation of M_s will widen the gap even in those clusters where Y is lower than in "standard" clusters within the Galaxy. There is indeed some evidence for the existence of a third parameter from de Kort's observation's (1965) of the Ursa Minor System. This is an intergalactic aggregate where the gap is wide contrary to the simple two-parameter supposition based on the NGC 7006 anomaly alone. We are forced to involve a third parameter for Ursa Minor if we maintain our working hypothesis that all intergalactic aggregates have a low helium abundance relative to galactic globular clusters. The following, independent, set of data appear to strengthen this working hypothesis, and thereby strengthens the supposition that a more complicated three-parameter model may exist for the Ursa Minor System.

Recent work on the variable stars with periods greater than 1 day in the intergalactic systems of Draco (Baade and Swope 1961), Sculptor (Baade and Swope 1961), Ursa Minor (de Kort 1965), and Leo II (Swope 1967) shows that the period-luminosity relation defined by these stars is considerably different than that defined by the type II Cepheids in the galactic globular clusters. The available information to 1965 is summarized by de Kort (1965, Fig. 1), to which Variable 6-31 in NGC 121 in the SMC (Tifft 1963a) can be added with $\bar{P} = 1.4300$ days, and at least two variables in Leo II (Swope 1967), which also show the effect. This is strong evidence that there is a difference in at least one parameter controlling M_V for a given period for variable stars in aggregates inside and outside the Galaxy, including the Ursa Minor System. This is the same conclusion we previously reached on the basis of the character of the horizontal branch. That this parameter is helium is not yet certain because Christy has not performed calculations on the $M_V = f(P, Y)$ relation of W Vir stars following his initial work (1966b) on such stars. If Christy's work in progress can confirm the observed effect as due to variations in Y, the case for ΔY inside and outside the Galaxy will certainly be strengthened.

One final point should be emphasized. If the manufacture of helium within the Galaxy was spotty, we should expect that some gravitationally bound clusters will show the NGC 7006 anomaly. It may well be that only partial mixing of the manufactured helium occurred in the halo at the time of cluster formation. Such inhomogeneities would produce a helium-poor cluster even though its total energy were negative. NGC 362 (Menzies 1967) may be such a case.

VII. THE MEANING OF ΔY INSIDE AND OUTSIDE THE GALAXY

An average difference of the helium abundance inside and outside the Galaxy has important consequences for cosmology because it has a direct bearing on the problem of the pristine helium content to which the galactic helium production is added. If we can eventually determine ΔV by use of C-M diagrams and by a more complete theory of the horizontal branch, we could determine the primeval value of Y, providing that the present Y values of globular clusters can be found.

The direct observational determination of helium in globular clusters is remarkably difficult. The abnormally weak helium lines of blue horizontal-branch stars observed by Searle and Rogers (1966) in NGC 6397, by Sargent and Searle (1966) for halo blue stars, and by Greenstein and Münch (1966) for globular clusters and field stars does not necessarily mean low helium abundance in Population II stars, although this would be the most natural interpretation. There exists evidence that certain Population I stars of apparently normal helium abundance exhibit abnormally weak helium lines (Sargent and Strittmatter 1966, and further references therein), the abnormality apparently correlating with rotational velocity (Deeming and Walker 1967).

Direct determination of Y exists for two objects in galactic globular clusters themselves. Traving (1962) has analyzed Barnard 29 in M13, with the result that the number ratio of He to H is 0.5 of that in τ Sco. If we adopt N(He)/N(H) = 0.16 for τ Sco (Aller 1961), then Y = 0.25 for Barnard 29 from Traving's analysis. The planetary nebula K648 in M15 has been analyzed by O'Dell, Peimbert, and Kinman (1964) with the result that $N(\text{He})/N(\text{H}) = 0.18 \pm 0.03$, or $Y = 0.42^{+0.03}_{-0.04}$.

Interpretation of these high Y values is complicated by the unknown effects of the chemical evolution of Barnard 29 and K648 themselves, and we cannot be certain that the observed Y values represent the initial helium abundance in these objects. But, even disregarding these direct determinations, the anomalous behavior of the He strengths in the normal Population I stars discussed by Sargent and Strittmatter, as previously mentioned, shows that some mechanism appears to operate that can weaken He lines in objects where the helium abundance is normal. Such a mechanism has recently been proposed by Greenstein, Turan, and Cameron (1967).

It seems that we are not yet forced by any observational evidence available so far to conclude that the helium abundance is low in globular clusters. Indeed, Christy (1966a) favors relatively high helium abundance in the atmospheres of globular-cluster RR Lyrae stars on the basis of his pulsation calculations.

The importance of a high Y value for the ideas in §§ IV and V lies in the direction of evolution along the horizontal branch. If evolution proceeds from right to left, the gap between the initial-model-position and the subgiant branch will be preserved; whereas, if the evolution is from left to right, the initial gap will be destroyed, thus eliminating the present interpretation. Faulkner and Iben (1966) show that for high Ythe evolution is indeed from right to left, which preserves the argument.

We finally wish to comment on the problem of the manufacture of He, in amount ΔY , by chemical evolution within the Galaxy. This He must have been made at the earliest epoch of star formation within the Galaxy so as to contaminate the globular clusters, which are the oldest galactic structures known. The contamination must take place within the short-time interval of about 2×10^8 years during the collapse of the Galaxy to the plane (Eggen *et al.* 1962). If all Population II stars are so contaminated, the Galaxy must have manufactured of the order of $\Delta YM_{\rm II}$ grams of He, where $M_{\rm II}$ is the total mass of Population II stars. Rough calculation, adopting, with Oort (1958, Table 2), that $M_{\rm II} = 10^{10} M_{\odot}$, shows that no order of magnitude discrepancy exists for the inferred luminosity of the Galaxy due to He production if we adopt the time scale quoted above and $\Delta Y \simeq 0.1$, which is a rough guess. The absolute bolometric luminosity of the Galaxy at these early epochs due to the events that led to the manufacture of galactic He is $M_{\rm bol} \simeq -24.5$, which does not appear to be unreasonable. The nature of the event itself is quite unknown, but may possibly be connected with the formation of supermassive objects such as postulated by Wagoner, Fowler, and Hoyle (1967), following earlier work by Hoyle and Tayler (1964) and others referenced therein.

These ideas are obviously speculative and rest on the tentative identification of the second parameter as He. It should be strongly emphasized that the only point which we believe has been definitely proved by our present data is that at least a second parameter *is* involved along the horizontal branch, but that the subsequent conclusions drawn from its identification with He are certainly not established.

VIII. SUMMARY

1. NGC 7006 has a horizontal branch that is anomalous for a cluster of its low metal abundance. This anomaly, together with its counterpart in M13, shows that at least one parameter in addition to Z controls the position of stars along the branch.

2. The theory of Faulkner and Iben (1966) points to a variation of the helium abundance as this parameter. Only a slight variation of Y appears to explain the observed discrepancy, if the Faulkner-Iben models are representative. An accurate determination of ΔY depends on calculations of more horizontal-branch models.

3. The Y/Z ratio varies from cluster to cluster. There is some evidence that intergalactic clusters that are not gravitationally bound to the Galaxy have a *lower* helium

478

abundance than do clusters formed as part of the Galaxy. The intergalactic aggregates that show the NGC 7006 anomaly are the Draco, Leo II, Palomar 3, Palomar 4, and possibly the Sculptor System.

4. The NGC 7006 anomaly is present in all clusters studied so far in the Small Magellanic Cloud, suggesting that the SMC did not manufacture much helium in its *early* evolution at the time its globular clusters were formed. The clusters in the LMC appear to be normal.

5. If points 2 and 3 are correct, helium must have been made in the Galaxy in its earliest phase. This abundance is added to any pristine helium made in the primeval fireball of a Friedman-type universe, or by any other early event which manufactured helium before the formation of the Local Group.

We wish to thank many people who read a first draft of the manuscript and suggested changes. Especially helpful were H. W. Babcock, J. Faulkner, W. A Fowler, P W. Hodge, J. Kristian, G. Münch, J. Peach, W. L. W. Sargent, M. Schwarzschild, R. V. Wagoner and G. Wallerstein. One of us (R. W.) wishes to acknowledge his affiliation, first with the Division of Geological Sciences, California Institute of Technology, and later with the United States Geological Survey, for intermediate and terminal portions of his participation in this research.

APPENDIX

PROPERTIES OF THE TWO RED VARIABLES V19 AND V54

Variable 19, discovered by Hubble, belongs to the class of red variables of small amplitude at the end of the giant branches of globular clusters. Hubble obtained an uncertain period of 252 days, which would have made V19 the longest-period star of this class by a wide margin.



FIG. 4.—The light-curve in B - V wavelengths for the semiregular red variable V19, phased with a period of 92.21 days.

© American Astronomical Society • Provided by the NASA Astrophysics Data System

TA	BL	\mathbf{E}	3
----	----	--------------	---

B MAGNITUDES AND PHASES FOR V19

H.J.D.	Year	В	Phase	H.J.D.	Year	В	Phase
2422876 96	1021	17 61	0 709	2427333 64	1933	16.9	0.041
2426531 93	1031	17 09	346	335 64	1933	17 0	062
592 72	1031	16.83	.006	361 62	1933	17 6	.344
593 76	1931	16 73	017	2428021 62	1935	17.5	.501
595 66	1931	16 70	037	313 96	1936	17.3	.672
619.64	1931	17.18	297	368.77	1936	17.8	.266
625 66	1931	17.09	363	392.78	1936	17.4	527
626.64	1931	17.07	.373	689.97	1937	17.3	.750
626.72	1931	17.23	374	2429550.76	1939	16 9	.085
627 67	1931	17 27	385	851.69	1940	17.4	.348
632 64	1931	17.3	.438	2434544.93	1953	17.8	.246
869.96	1932	16.88	.012	689.68	1953	17.15	.816
888 92	1932	16.86	.218	978.74	1954	16 96	.950
891.95	1932	16.94	.251	978.74	1954	16.97	.950
893.89.	1932	16.94	.272	978.74	1954	16.88	.950
918.70	1932	17.54	.541	2435009.75	1954	17.0	.287
918.77.	1932	17.60	.541	017.63	1954	17.5	.372
918.87.	1932	17.56	.542	017.66	1954	17.2	.372
919.70	1932	17.63	.552	017.68	1954	17.3	.372
919.81.	1932	17.71	.553	018.68	1954	17.5	.383
919.98.	1932	17.53	.555	018.71	1954	17.5	.384
920.80.	1932	17.69	.563	342.72	1955	16 85	.898
928.72.	1932	17.30	.649	345.72 .	1955	16.86	.930
928.98.	1932	17.32	.652	374 69 .	1955	17.3	.244
975.65	1932	17.01	.158	396.64	1955	17 44	.482
977 63.	1932	16.91	.180	403 64 .	1955	17.49	.558
2427033.63	1932	17.03	.787	456.59 .	1955	16.84	.132
033.64	1932	17.10	.787	630.93	1956	16 8	.023
039 62	1932	16.95	.852	657.93	1956	17.2	.316
2427209.94	1933	17.32	.699	697.80	1956	17.3	.748
274.86.	1933	17.55	.403	780 64	1956	17.8	.647
300.67	1933	17.24	.683	2436162.62	1957	17.15	.789
303.77	1933	17.4	.717	426.86	1958	17.4	.655
2427330.65	1933	16.9	0.008	2436456.75	1958	17.0	0.979
	1			11			1

H.J.D.	Year	V	Phase
2426595.70	1931	15 19	0.038
599.64	1931	15 2	.081
919.92	1932	15.88	.554
977.64.	1932	15 5	.180
2434544.94	1953	15 9	.246
689.69	1953	15 17	.816
2435009.75.	1954	15.6	.287
018 69	1954	15.6	.384
342.73	1955	15 2	.898
345.73	1955	15.12	.930
374.69	1955	15.27	.244
403 64	1955	15 9	. 558
456.60.	1955	15 21	.133
630 94	1956	15 08	.023
657.95	1956	15 49	.316
696.87	1956	15.28	.738
696.91	1956	15.39	.739
698.82	1956	15.38	.759
698.85	1956	15.36	.760
698.86	1956	15.30	.760
780.65	1956	15.8	.647
2436161_61	1957	15.44	778
456.75	1958	15.2	0.979

TABLE 4

V MAGNITUDES AND PHASES FOR V19

Rosino (1965) quotes a period of 92 days based on extensive new plate material obtained at Asiago and at Mount Wilson. We confirm Rosino's period.

Table 3 lists magnitudes and phases for seventy-one blue plates taken with the 60-, 100-, and 200-inch reflectors between 1921 and 1958, reduced with a period of 92.21 days. The magnitudes were determined by iris photometry for those entries given to two decimals, and by eye estimate for the others. Table 4 lists similar data for yellow plates.

The light-curve is shown in Figure 4 and indicates that, except for two observations near phase 0.25 (one in 1936 and one in 1953), all data satisfy the adopted period. The elements of the light variation are

H.J.D.(max) = 2,426,500.00 + 92.21 E,

where the epoch of maximum has an uncertainty of perhaps ± 10 days. The amplitude of variation is $A_B = 0.75$, with $B(\max) = 16.85$ and $B(\min) = 17.60$ for a median value of $B_{med} =$ 17.22; $A_V = 0.80$, with $V(\max) = 15.10$ and $V(\min) = 15.90$ for $V_{med} = 15.50$, giving $(B-V)_{med} = 1.72$, which places the star at the tip of the giant branch near star I-1, which has V = 15.69 and B - V = 1.70.

We have not been able to derive a period for V54, but our data show it to be a red variable similar to V19. The amplitude of light variation is small, with B ranging from 16.9–17.5 and V from 15.6-16.1, giving $V_{\text{med}} = 15.85$, $(B - V)_{\text{med}} \simeq 1.35$. These values put V54 slightly above the interpolated giant sequence of Figure 3, but not beyond the uncertainty of our values.

REFERENCES

- Abell, G. O. 1955, Pub. A.S.P., 67, 258.
- Aller, L. H. 1961, The Abundances of the Elements (New York: Interscience Publishers). Arp, H. C. 1955, Ap. J., 60, 317. ——. 1958, ibid., 63, 273. Arp, H. C., and Bergh, S. van den. 1960, Pub. A.S.P., 72, 48. Arp, H. C. and Malbourne, W. C. 1950, A.J. 64, 28

- Arp, H. C., and Bergh, S. van den. 1960, Pub. A.S.P., 72, 48.
 Arp, H. C., and Melbourne, W. G. 1959, A.J., 64, 28.
 Baade, W. 1935, Ap. J., 82, 396.
 Baade, W., and Swope, H. H. 1961, A.J., 66, 300.
 Bergh, S. van den. 1967, A.J., 72, 70.
 Bergh, S. van den, and Henry, R. C. 1962, Pub. David Dunlap Obs., 11, No. 10, p. 281.
 Böhm-Vitense, E., Holweger, H., and Kohl, K. 1963, Ap. J., 138, 604.
 Burbidge, G. R. 1958, Pub. A.S.P., 70, 83.
 Christy, R. F. 1966a, Ap. J., 144, 108.
 ——. 1966b. ibid., 145, 337.

- . 1966b, ibid., 145, 337

- Cuffey, J. 1958, Ap. J., 128, 219. Deeming, T. J., and Walker, G. A. H. 1967, Nature, 212, 479. Demarque, P., and Geisler, J. E. 1963, Ap. J., 137, 1102. Eggen, O. J. 1965, First Conference on Faint Blue Stars, ed. W. J. Luyten (Minneapolis: Observatory, University of Minnesota). p. 39. Eggen, O. J., Lynden-Bell, D., and Sandage, A. R. 1962, Ap. J., 136, 748. Faulkner, J. 1966, Ap. J., 144, 978. Faulkner, J., and Iben, I. 1966, Ap. J., 144, 995. Gascoigne, S. C. B. 1966, M.N.R.A.S., 134, 59. Gascoigne, S. C. B., and Koehler, J. A. 1963, Observatory, 83, 66. Gascoigne, S. C. B., and Ogston, F. A. 1963, Observatory, 83, 66. Greenstein, G. S., Turan, J. W., and Cameron, A. G. W. 1967, Nature, 213, 871. Greenstein, J. L., and Münch, G. 1966, Ap. J., 146, 618. Hoyle, F., and Schwarzschild, M. 1955, Ap. J. Suppl., 2, 1. Hoyle, F., and Tayler, R. J. 1964, Nature, 203, 1108. Humason, M. L. 1934, Pub. A.S.P., 46, 357. Johnson, H. L., and Sandage, A. R. 1956, Ap. J., 124, 379. Kinman, T. D. 1959a, M.N.R.A.S., 119, 538. ———. 1959b, ibid., 119, 559. Kort, J. de. 1965, Kleine Veröff. Remeis-Sternw., Bamberg, 4, No. 30, p. 136. University of Minnesota). p. 39

- Kort, J. de. 1965, Kleine Veröff. Remeis-Sternw., Bamberg, 4, No. 30, p. 136. Kron, G. E., and Mayall, N. U. 1960, A.J., 65, 581.
- Mannino, G. 1957, Contr. Oss. Astrofis. Asiago, No. 84, or Mem. Soc. Astr. Italiana, 28, 185.

482

- Mayall, N. U. 1946, Ap. J., 104, 290. Menzies, J. W. 1967 (preprint). Morgan, W. W. 1959, A. J., 64, 432. O'Dell, C. R., Peimbert, M., and Kinman, T. D. 1964, Ap. J., 140, 119. Oort, J. H. 1958, Stellar Populations, ed. D. J. K. O'Connell, Ric. Astr. Specola Vaticana, 5, p. 415.
- Preston, G. W. 1959, Ap. J., 130, 507.
- Rosino, L. 1965, communication at the Herstmonceux Summer School.
- Rosino, K., and Mannino, G. 1955, Contr. Oss. Astrofis. Asiago, No. 59, or Mem. Soc. Astr. Italiana, 26. 275.
- Sandage, A. R. 1954, Pub. A.S.P., 66, 324.
- Sandage, A., and Katem, B. 1964, Ap. J., 139, 1088. Sandage, A., and Smith, L. L. 1966, Ap. J., 144, 886.

- Sandage, A., and Wallerstein, G. 1960, Ap. J., 131, 598. Sargent, W. L. W., and Searle, L. 1966, Ap. J., 145, 652. Sargent, W. L. W., and Strittmatter, P. A. 1966, Ap. J., 145, 938.

- Sargent, W. L. W., and Strittmatter, P. A. 1966, Ap. J., 145, 938.
 Searle, L., and Rogers, A. W. 1966, Ap. J., 143, 809.
 Shapley, H. 1920, Ap. J., 52, 73.
 ——. 1930, Star Clusters (New York: McGraw-Hill Publishing Co.), p. 227.
 Shapley, H., and Mayberry, B. W. 1921, Proc. Nat. Acad. Sci., 7, 152.
 Swope, H. H. 1967 meeting of Astr. Soc. Pac., Pasadena, June 29.
 Tifft, W. A. 1963a, M.N.R.A.S., 125, 199.
 ——. 1963b, ibid. 126, 209.
 Traving, G. 1962, Ap. J., 135, 439.
 Wagoner, R. V., Fowler, W. A., and Hoyle, F. 1967, Ap. J., 148, 3.
 Wildey, R. L. 1961, Ap. J., 133, 430.
 Wilson, A. G. 1955, Pub. A.S.P., 67, 27.