THE ASTROPHYSICAL JOURNAL, Vol. 149, August 1967

THE EFFICIENCY OF THE BOWEN FLUORESCENT MECHANISM

JAMES B. KALER
University of Illinois Observatory
Received January 13, 1967; revised April 6, 1967

ABSTRACT

The efficiencies of the Bowen O III fluorescent mechanism are calculated for 17 planetary nebulae from a method developed by Burgess and Seaton. The efficiency of the process decreases with increasing electron temperature, with decreasing surface brightness, and with decreasing optical depth.

I. INTRODUCTION

Bowen (1934) discovered that a number of permitted O III lines are selectively produced in high-excitation planetary nebulae by a coincidence of He II Lyman-α at λ303.780 Å and the $2p^2\,{}^3P_2$–$2p3d\,{}^3P_2^o$ transition of O^{++} at λ303.799 Å. Following the excitation of the ${}^3P_2^o$ level there is a small chance (a probability of 0.0167 according to Burgess and Seaton 1960) of a cascade cycle of the electron through intermediate levels back down to the ground term, producing the lines of the *Bowen fluorescent mechanism.* Table 1 gives a list of the observed fluorescent lines and their transitions taken from Bowen (1935), the *Revised Multiplet Table* (Moore 1945), and recent spectrographic work.

Actually, the radiation observed in these lines is a combination of the above fluorescence and the usual recombination process, the two mechanisms playing a varying role from one multiplet to another. Burgess and Seaton (1960) have developed a method whereby the effects of recombination and fluorescence may be separated. The effective efficiency, which is defined as the ratio of the number of fluorescent cycles produced in the nebula to the number of He II Ly-α photons produced, may then be calculated.

II. CALCULATION OF EFFICIENCY

We denote by S the transitions of multiplet 3 (Table 1) and by D those of multiplet 2; f denotes fluorescence, r recombination, and q the quantum emission rate.

Burgess and Seaton (1960) show that for pure fluorescence

$$\frac{q_f(S)}{q_f(D)} = 14.9\,, \tag{1}$$

whereas for pure recombination

$$\frac{q_r(S)}{q_r(D)} = 0.18\,, \tag{2}$$

if we assume Baker-Menzel case B. (If case A is assumed the ratio of eq. [2] decreases markedly, but the resulting efficiencies are practically unchanged, so we will consider only case B).

Given the observed ratio

$$Q = \frac{q(S)}{q(D)} = \frac{q_f(S) + q_r(S)}{q_f(D) + q_r(D)}\,, \tag{3}$$

we may write

$$q_f(S) = 1.01\left(\frac{Q - 0.18}{Q}\right) q(S)\,, \tag{4}$$

and so derive the contribution of fluorescence to the $3p\,{}^3S$–$3s\,{}^3P^o$ multiplet.

Burgess and Seaton (1960) then define the efficiency as

$$\mathcal{R}=\frac{q_f(\text{O III})}{q(\text{He II Ly-}\alpha)}=\sum_L\frac{q_f(3d^3P_2{}^o-3p^3L)}{q(\text{He II Ly-}\alpha)}. \tag{5}$$

As the efficiency for one scattering is small (only 0.0167), there must be many scatterings before we would expect to see the fluorescent lines. If the nebula is optically thick to He II Ly-α, the Ly-α density will be very high, a large number of $3d^3P_2{}^o$ excitations will take place, and the efficiency should approach 100 per cent.

TABLE 1

OBSERVED LINES OF THE BOWEN FLUORESCENT MECHANISM

λ (Å)	Transition	RMT Multiplet No	λ (Å)	Transition	RMT Multiplet No.
3444 10..	$3d^3P_2{}^o$–$3p^3P_2$	15	3759 87	$3p^3D_3$–$3s^3P_2{}^o$	2
3428 67.	$3d^3P_2{}^o$–$3p^3P_1$		3754 67	$3p^3D_2$–$3s^3P_1{}^o$	
3047 13	$3p^3P_2$–$3s^3P_2{}^o$	4	3757 21	$3p^3D_1$–$3s^3P_0{}^o$	
3132 86	$3d^3P_2{}^o$–$3p^3S_1$	12	3791 26	$3p^3D_2$–$3s^3P_2{}^o$	
3340 74	$3p^3S_1$–$3s^3P_2{}^o$	3	3774 00 .	$3p^3D_1$–$3s^3P_1{}^o$	
3312 30 ..	$3p^3S_1$–$3s^3P_1{}^o$		3810 96 .	$3p^3D_1$–$3s^3P_2{}^o$	
3299 36	$3p^3S_1$–$3s^3P_0{}^o$				

According to Burgess and Seaton's cascade theory

$$q_f(\text{O III}) = 1.65\, q_f(3p^3S - 3s^3P^o)\,. \tag{6}$$

We also have

$$q(\text{He II Ly-}\alpha)=\frac{\alpha_{2p,1s}(\text{He}^+)}{\alpha_{4,3}(\text{He}^+)}\,q(\text{He II},\ \lambda 4686), \tag{7}$$

where the α's are effective recombination coefficients. If we combine equations (5)–(7) we obtain

$$\mathcal{R}=1.17\,\frac{I_f(S)}{I(4686)}\,\frac{\alpha_{4,3}(\text{He}^+)}{\alpha_{2p,1s}(\text{He}^+)}, \tag{8}$$

where I denotes the unreddened line intensities and $I_f(S)$ may be derived from the observations by equations (3) and (4). The values of $\alpha_{4,3}(\text{He}^+)$ are taken from Pengelly (1964) and $\alpha_{2p,1s}$ derived from Seaton (1960); the ratio shows a slow dependence on electron temperature.

In recent years observations have become available which extend accurate line intensities into the ultraviolet and make the calculation of the fluorescent efficiency possible for a number of nebulae. A total of seven nebulae have had the relevant line intensities measured, and are given in Table 2, together with the reddening constant (which is the log of the ratio of the true to observed Hβ intensity), electron temperature, Hβ surface brightness, and computed $\mathcal{R}$. The electron temperatures and reddening constants are taken from Kaler (1966); the reddening function is that of Seaton (1960), which is an average of those of Whitford (1948, 1958) and Divan (1954). The values of surface brightness are taken from Aller (1965) and O'Dell (1963) and are corrected for the reddening constants used in the present calculations.

In order to establish correlations between $\mathcal{R}$ and nebular parameters, seven nebulae are not sufficient. We need more objects, and must thus rely on other techniques to find $\mathcal{R}$ since the ultraviolet is not well observed for most nebulae.

Seaton (1960) suggested using the bright, easily observed λ3444 line of O III for this purpose. If we use Pengelly's (1964) recombination coefficient in Seaton's equation, we obtain

$$\mathcal{R}^* = 0.87 \frac{I(\lambda 3444)}{I(\lambda 4686)}. \tag{9}$$

No allowance is made for the effect of recombination on λ3444, however. Let us assume that the fraction of radiation supplied by recombination is the same for all nebulae, and compute

$$K = \frac{\mathcal{R}^*}{\mathcal{R}} \tag{10}$$

for the nebulae of Table 2. We then obtain

$$K = 2.4 \pm 0.25 \text{ (m.e.)}. \tag{11}$$

The low scatter indicates that the assumption is valid, and we can derive $\mathcal{R}$ from the measured $\mathcal{R}^*$ of those nebulae for which λ3444 has been observed. The results for an additional nine nebulae are thus presented in Table 3.

Because of its high electron temperature, it is interesting to determine $\mathcal{R}$ for NGC 2392. This is a well-observed nebula, but unfortunately the observations do not extend

TABLE 2

DATA AND EFFICIENCIES OF NEBULAE THAT ARE WELL OBSERVED IN THE ULTRAVIOLET

Nebula	Reference	c	log S(Hβ) (ergs cm^{-2} sec^{-1})	T_e (°K)	$\mathcal{R}$
VV 267	Aller and Walker (1965)	0 18	−2 43	17200	0 041
IC 2165	Aller and Kaler (unpublished)	0 16	−1 40	12200	.143
NGC 2440	Aller and Kaler (unpublished)	0 11	−2 25	13400	.103
3242	Czyzak, Aller, Kaler, and Faulkner (1966); Razmazde (1958)	0 31	−0 88	10900	32
7009	Aller and Kaler (1964)	0 82	−0 24	11300	266
7027	Aller, Bowen, and Wilson (1963)	1 02	−0 06	13900	187
7662	Aller, Kaler, and Bowen (1966)	0 28	−0 89	12800	0 186

TABLE 3

DATA AND EFFICIENCIES OF NEBULAE WITH LIMITED ULTRAVIOLET OBSERVATIONS

Nebula	Reference	c	log S(Hβ)	T_e	$\mathcal{R}$
IC 351.	Aller and Walker (1965)	0	− 81	12000	0 22
IC 2003	Aller (1951)	0	−1 30	10800	.18
IC 5217 .	Aller (1951)	0	−1 61	10800	.17
J 900	Aller (1951)	0 40	−1 60	11900	.19
NGC 2022	Aller and Walker (1965)	0 09	−1 48	14900	033
1535	Aller and Walker (1965)	0	−1.62	13700	.14
6741	Aller and Walker (1965)	0 25	−1 61	10800	.175
6818	Aller (1951)	0	−1 41	13100	.125
6886	Aller (1951)	0 68	−1 32	11000	.15
NGC 2392	Minkowski and Aller (1956)	0 11	−2 36	17300	0 020

shortward of λ3727. We can, however, make use of the $3p^3D$–$3s^3P^o$ multiplet, for which we find from equations (1)–(3) that

$$q_f(D) = \left(\frac{Q-0.18}{14.7}\right) q(D) . \tag{12}$$

Q cannot be found for this nebula. However, from the nebulae of Table 2, we find that

$$\left\langle \frac{q_f(D)}{q(D)} \right\rangle = 0.14 \pm 0.017 \,(\text{m.e.}) . \tag{13}$$

Again the scatter is low so that we should be able to apply the ratio to NGC 2392 and derive $q_f(S)$ from equation (1), and consequently find $\mathcal{R}$ from equation (5). The data for this nebula are given in the last row of Table 3.

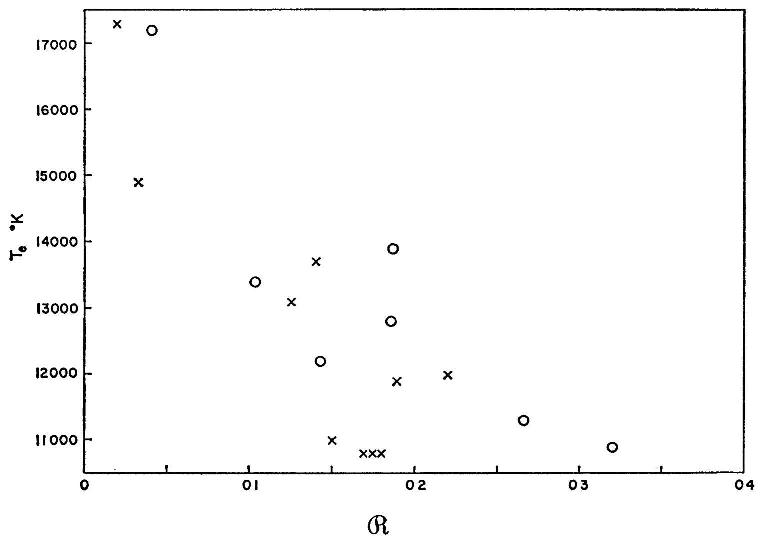

FIG. 1 —The efficiency of the O III fluorescent process, $\mathcal{R}$, as a function of electron temperature, T_e. The circles are the nebulae of Table 1, the crosses those of Table 2.

III. RESULTS

We first note that all the efficiencies are rather low, a fact pointed out by Burgess and Seaton (1960) in their study of NGC 7027.

The value of $\mathcal{R}$ taken from Tables 2 and 3 are plotted against electron temperature in Figure 1. There is a well-defined correlation, with $\mathcal{R}$ decreasing markedly as the electron temperature of the nebula increases. Note that for a given value of T_e that there is a large range in $\mathcal{R}$. We divide Figure 1 into two temperature regions; region 1 has $10000° \text{ K} < T_e \leq 12000° \text{ K}$, and region 2 has $12000° \text{ K} < T_e < 14000° \text{ K}$. The nebulae with $T_e > 14000° \text{ K}$ are excluded as there are so few of them. For each temperature region, $\mathcal{R}$ is plotted against $\log S(\text{H}\beta)$ (the Hβ surface brightness in ergs $\text{cm}^{-2}\text{ sec}^{-1}$) in Figure 2. We see that for each group there is a correlation, with the efficiency slowly decreasing with $\log S(\text{H}\beta)$. In addition, the $\log S(\text{H}\beta)$ dependency appears to be slightly different for each temperature region.

In Figure 3 we plot the efficiencies against the maximum and minimum characteristic optical depths given by Capriotti (1967). Again, there is a correlation with the efficiency increasing with optical depth. No discrimination is made here with regard to electron temperature. The nebula NGC 6818 appears to be an exception to the correlation, as $\tau_{\min} = 25$ and $\tau_{\max} = 40$.

1967ApJ...149..383K

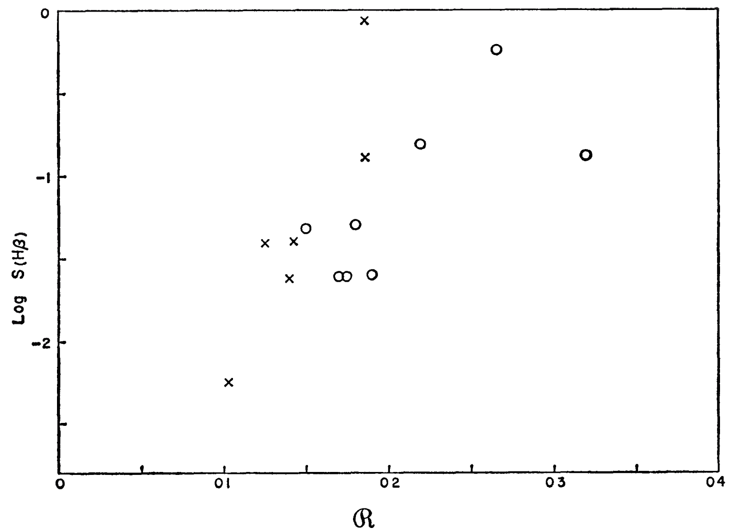

FIG. 2.—The efficiency of the O III fluorescent process, ℛ, as a function of Hβ surface brightness log $S(\mathrm{H}\beta)$ for two temperature regions. The circles denote those nebulae for which $10000° \mathrm{K} < T_e \leq 12000° \mathrm{K}$; the crosses those for which $12000° \mathrm{K} < T_e < 14000° \mathrm{K}$.

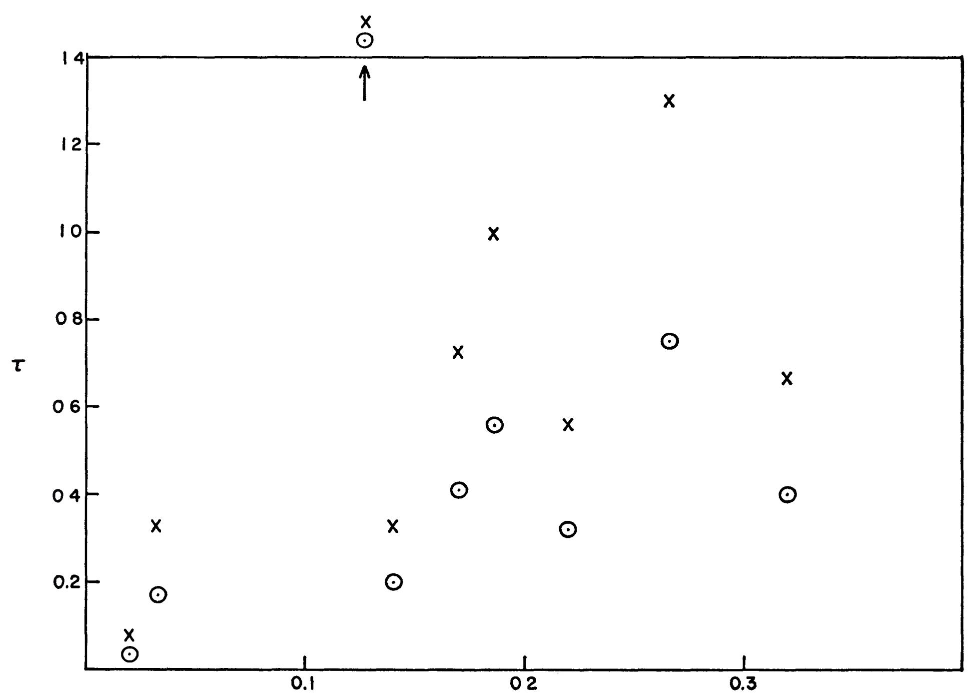

FIG. 3.—The efficiency of the O III fluorescent process, ℛ, as a function of the characteristic optical depth of the nebula. The circles represent Capriotti's (1967) minimum depths, and the crosses the maximum.

IV. INTERPRETATION

The quantitative reproduction of these graphs from theory is a difficult matter, primarily because of the complex He II Ly-α transfer problem. First we must consider the way Ly-α is scattered by the He^+ ion, a problem which has been treated by Osterbrock (1961). However, Hummer and Seaton (1964) contend that He II Ly-α plays an important role in ionizing hydrogen, which complicates matters considerably. For that matter Ly-α is quite capable of ionizing neutral helium. In order to predict $\mathcal{R}$ from theory we must also know the scattering relation that exists between O^{++} and the He II Ly-α line, considering the line shifted by an expanding nebula. Such a complete theory is not available.

We can, however, explain the results in a very qualitative manner. The separation of the resonance lines if 0.019 Å; at $T_e = 10000°$ K, the half-width of He II Ly-α is 0.011 Å, and that of the O^{++} resonance line is 0.0054 Å. Consequently, the overlap is not great. However, an expansion of the nebulae of 20 km sec^{-1} corresponds to a reddening of He II Ly-α of 0.02 Å, and we can consider the two lines to be in good coincidence. An increase of T_e will improve the coincidence only very slightly. The real effect of an increase of electron temperature is to allow more Ly-α photons to escape in the line wings (Osterbrock 1961) decreasing the number of scatterings a Ly-α photon undergoes, and also decreasing the Ly-α photon density. Consequently we will not have as many fluorescent cycles and $\mathcal{R}$ decreases. This conclusion is borne out by the correlation between the efficiency and Capriotti's (1967) optical depths.

For a constant temperature, it is obvious that we should have a dependence of $\mathcal{R}$ on $S(\mathrm{H}\beta)$. As $S(\mathrm{H}\beta)$ decreases, the optical depth of the nebula decreases, as does the number of Ly-α scatterings. A given Ly-α photon then spends less time in the nebula and the probability of producing a fluorescent cycle decreases.

The very low efficiencies of NGC 2392, NGC 2022, and VV 267 (which are not included in Fig. 2) are probably due to a combination of both high temperature and low optical depth.

The $3s^3\,P_1{}^o$–$2p^3P_2$ transition (the end product of the O III fluorescent cycle) at λ374.436 Å is coincident with the $2p^2P$–$3d^2D_{5/2,\,3/2}$ transitions of N III at 374.434 and λ374.442 Å, respectively (Bowen 1935). Seaton (1960) has worked out the formula for the N III efficiency which has been calculated for all available nebulae. The quantity $\mathcal{R}$ (N III) shows no clear-cut correlation with T_e, though this may be due to the fact that no account was taken of recombination.

At present no theoretical nebula exists which can explain Figures 1, 2, and 3 in detail. These results should provide a sensitive test for future theoretical efforts.

This work was supported by National Science Foundation grant GP 4928 to the University of Illinois.

REFERENCES

Aller, L. H. 1951, *Ap J.*, **113**, 125.
———. 1965, *Landolt-Börnstein Tables* (Berlin: Springer-Verlag), p. 566.
Aller, L. H., Bowen, I. S., and Wilson, O. C. 1963, *Ap. J.*, **138**, 1013.
Aller, L. H., and Kaler, J. B. 1964, *Ap. J.*, **139**, 1074.
Aller, L. H., Kaler, J. B., and Bowen, I. S. 1966, *Ap. J*, **144**, 291.
Aller, L. H., and Walker, M. F. 1965, *Ap. J.*, **141**, 1318.
Bowen, I. S. 1934, *Pub. A.S.P.*, **46**, 146.
———. 1935, *Ap. J.*, **81**, 1.
Burgess, A., and Seaton, M. J. 1960, *M.N.R.A.S.*, **121**, 76.
Capriotti, E. R. 1967, *Ap. J.*, **147**, 979.
Czyzak, S. J., Aller, L. H , Kaler, J. B., and Faulkner, D. J. 1966, *Ap. J.*, **143**, 327.
Divan, L. 1954, thèses, Université de Paris.
Hummer, D. G., and Seaton, M. J. 1964, *M.N.R.A.S.*, **127**, 217.
Kaler, J. B. 1966, *Ap. J.*, **143**, 722.

Minkowski, R , and Aller, L. H. 1956, *Ap. J* , **124,** 93.
Moore, C. 1945, *Revised Multiplet Table* (Princeton Contrib No. 22).
O'Dell, C. R. 1963, *Ap J.*, **138,** 293.
Osterbrock, D. E 1961, *Ap. J.*, **135,** 195.
Pengelly, R. M. 1964, *M.N.R A.S* , **127,** 145
Razmadze, N. A 1958, *Bull. Abastumani Ap. Obs* , No. 23.
Seaton, M. J. 1960, *Repts on Progr. Phys.*, **23,** 313.
Whitford, A. E. 1948, *Ap. J* , **107,** 102.
———. 1958, *A.J.*, **63,** 201.