# SPECTROPHOTOMETRY OF NEW SHORT-PERIOD VARIABLE STARS* 

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#### Abstract

The occurrence of short-period variability in a number of bright F-type stars was discovered during an extensive program of $U B V$ photoelectric observations made between October, 1965, and February, 1966. The amplitudes of the new variables are small and change in the manner shown by some of the variables of the $\delta$ Sct type Observations of the continua of the new variables were made by measuring the intensities at twenty-five discrete wavelengths between about 3300 and $10000 \AA$ using a photoelectric spectrum scanner Coudé spectra at $10 \AA / \mathrm{mm}$ were used to correct the intensities for line-blanketing effects, and the resulting continua were then compared with the model atmospheres of Mihalas to yield effective temperatures and surface gravities Some of the new variables have higher temperatures and gravities than the previously known variables Higher values of the projected rotational velocity $V \sin i$ also occur, and indicate that in this region of the H-R diagram pulsation is not incompatible with rapid rotation More than half the stars appear to be significantly undermassive for normal left-to-right evolution in the H-R diagram, for which masses $\sim 18 M \odot$ are typical Some of the variables, particularly $\rho$ Pup and HR 5017, have extremely low values of the pulsation constant $Q$ The stars of lowest mass are consistently cooler than those of higher mass, an effect that is observable in a correlation obtained between $\log g$ and $\log T_{e}$


## I. INTRODUCTION

The $\delta$ Scuti stars comprise a class of pulsating variable stars characterized by smallamplitude, short-period light and radial velocity variations. The existence of such a class named after the prototype star $\delta$ Sct was pointed out by Eggen (1957), who listed five known members of the class. They have amplitudes less than 0.3 mag , periods between 0 d 07 and $0 \mathrm{~d} 2, B-V$ colors between 0.3 and 0.4 mag , and are all reasonably sharp-lined stars. With absolute magnitudes between 0.0 and +2.2 (McNamara and Augason 1961) they occur in a region of the H-R diagram above the main sequence and roughly on an extension of the Cepheid instability strip. The space motions are low and appear to be typical of those of the dwarf A-type stars (Eggen 1960a) so that they are clearly stars of the disk population. In this paper photometric and spectrophotometric data are presented for a number of new short-period variables which probably belong to the $\delta$ Sct class.

## II. $U B V$ PHOTOMETRY OF NEW STARS

The stars included in the observing program were selected from the Strömgren-Perry catalogue (1962) of $u v b y$ photometry on the basis of their being in the same region of the H-R diagram as the known $\delta$ Sct stars.

The photoelectric observations to be described in this paper were made by one of us (R. J. D.) between October, 1965, and February, 1966, using the Mount Wilson 60 -inch and the Palomar 20 -inch reflectors with standard D.C. equipment. Each candidate selected for observation was observed continuously in three colors, $U, B$, and $V$ for up to about 4 hours, together with a nearby bright comparison star. During the night a number of Johnson-Morgan $U B V$ standards was also observed to determine the extinc-

[^0]tion and color transformations from the "natural" to the $U B V$ system. The magnitudes of the program stars were reduced to the $U B V$ system in the usual way. The differences between the derived values of $V, B-V$, and $U-B$ for each comparison star and the standard values (from Iriarte, Johnson, Mitchell, and Wisniewski 1965) were then applied to the derived magnitudes of the program star to give the final magnitudes referred to those of the comparison star.

The stars observed are listed in Table 1. Most of the columns are self-explanatory. The numbers given in the first column refer to the Yale Catalogue of Bright Stars (1964). The absolute magnitude $M_{v}$ (s) given in the seventh column was computed from the uvby photometry. A notation " 3 " is given in the tenth column against stars which are considered to have been observed sufficiently long to establish their short-term constancy at this epoch of observation. It should be noted, however, that, since many of the $\delta$ Sct variables exhibit beat phenomena in their light-curves which can result in little change in amplitude at some epochs (see the light-curves of 1 Mon, Fig. 1), the constant nature of all of these stars cannot be considered as firmly established.

It can be seen from Table 1 that of the sixty-nine stars observed, ten stars show some short-period variability and twenty-five are constant. The nature of the variability of HR 2539 and the close binary system HR 3889 is somewhat uncertain and these stars will not be discussed as variables in this paper. The individual observations of the eight variables are given as a function of heliocentric Julian Date in Table 2 and shown plotted in Figure 1. The values of $V, B-V$, and $U-B$ given in Table 2 are the magnitudes determined relative to the appropriate comparison stars whose magnitudes from Iriarte et al. (1965) are given at the head of the table.

## III. SCANNER OBSERVATIONS AND SPECTRA

Spectrophotometric observations of the continua of the variables were made with the Cassegrain photoelectric spectrum scanner (used with refrigerated 1P21 and 7102 photomultipliers) on the 60 -inch reflector at Mount Wilson. The intensities at sixteen discrete wavelengths in the visual-ultraviolet region were measured with the 1P21 tube, and at nine discrete wavelengths in the visual-infrared region with the 7102 tube. These two types of observation were made on different nights and the visual-ultraviolet observations, in particular, were repeated on several nights. The discrete wavelengths were selected by Oke (1964) to be relatively free of lines, and the intensities were measured with an exit slit of $50 \AA$ and corrected for atmospheric extinction by using mean extinction coefficients for Mount Wilson.

These measured intensities were tied to Oke's system of absolute standards by observing the stars $\gamma$ Gem, $\epsilon$ Ori, $\eta$ Hya, a Leo, $\theta$ Vir, and 109 Vir on the same night. No individual variations greater than 1 or 2 per cent or gross variations in extinction are to be expected. In Table 3 the measured magnitudes are given as a function of the inverse wavelength expressed in microns, together with the Julian Date for the observations. Also included in this table are the adopted line-blanketing corrections, which will be described below.

Coudé spectra of the new variables covering the blue and violet regions were taken on baked IIaO plates with the 32 -inch camera of the 100 -inch telescope at a dispersion of $10 \AA / \mathrm{mm}$. The line-blanketing corrections given in Table 3 for the blue-ultraviolet region were obtained by measuring the energy subtracted from a smooth continuum by the line spectrum in the manner given by Oke (1965). The small corrections for the wavelength region $\lambda \lambda 5000-5840$ were extrapolated from shorter wavelengths. No corrections are given at longer wavelengths since they are extremely small. The problem of measuring blanketing in the spectra of stars with rotational velocities $V \sin i>50 \mathrm{~km} / \mathrm{sec}$ cannot be overcome, since otherwise sharp lines are so smeared into one another that a realistic placing of the smooth continuum is impossible. Fortunately, blanketing corrections at all wavelengths have a systematic variation with temperature and luminosity. Therefore,

TABLE 1
STARS OBSERVED ON UBV SYSTEM OCTOBER 1965 - FEBRUARY 1966




Fig 1 -Continued
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Fig. 1.-Continued


TABLE 2 -continued


## NOTES TO TABLE 2

HR $1287 \equiv 44$ Tau: Beat phenomena indicated by irregularity of period between October, 1965, and January, 1966. Maximum light occurs at the following Julian Dates:

2439058874
67999
90902
138728
141667
HR $1706 \equiv 14$ Aur: Part of wide binary system. Companion has $V=7.99, B-V=047, U-B=$ 0.03 , which yields $M_{V}=+0.8$ for 14 Aur Companion has a small $U V$ excess. The adopted period of 0 d 122 is nearly a submultiple of the period of variation of the radial velocity, 3.789 days, obtained by Harper (1938), who discussed the star as a spectroscopic binary. Maximum light occurs at the following Julian Dates:

2439148677
49659
HR $2107 \equiv 1$ Mon: Minimum $B-V$ occurs about 0.04 of a period before minimum $V$ magnitude. The variations in light amplitude (Fig 1) indicate a strong secondary period Maximum light occurs at Julian Date 2439056.947.
HR 3265: Reported as showing 005 mag variation by Cape Observatory (Cape Mimeogram 1961). This star has a high $m_{1}$ index (Strömgren) The period is uncertain. Maximum light occurs at Julian Date 2439176730.

HR $3888 \equiv v$ UMa: Variations in light amplitude (Fig. 1) indicate a beat phenomena. The trigonometric parallax of $0 \prime \prime 036$ gives $M_{V}=+1.6$, in agreement with that derived from the uvby photometry. Maximum light occurs at Julian Date 2439149823.
HR $4715 \equiv 4 \mathrm{CVn}$ : Discovered to be variable in radial velocity, with a period of 0 d 17 , by Jones and Haslam (1966). Probable member of the Hyades moving group; the group parallax gives $M_{V}=$ +0.8 The variation in light amplitude (Fig 1) indicates beat phenomena. Maximum light occurs at the following Julian Dates:

2439147978
51023
HR 5005: Maximum light occurs at Julian Date 2439176981.
HR $5017 \equiv 20 \mathrm{CVn}$ : This star, like HR 3265, has a high $m_{1}$ index. The period is uncertain Probable member of the Hyades moving group, with a group parallax giving $M_{V}=+0.45$.
where stars have a $V \sin i>50 \mathrm{~km} / \mathrm{sec}(V \sin i$ discussed later and given in Table 4), the blanketing corrections were estimated for their known colors and luminosities by interpolating between the results for the sharp-lined variables, and also by using published results for the Hyades stars by Oke and Conti (1966).

## IV. EFFECTIVE TEMPERATURES, GRAVITIES, AND ROTATIONAL VELOCITIES

Effective temperatures and gravities were obtained from the scanner data by fitting the observed continua to those of theoretical model atmospheres. Two grids of model atmospheres were available; one, computed with a program written by Mihalas (1965), does not include the effects of line blanketing; the other, also by Mihalas (1966), does include the effect of blanketing by the Balmer lines of hydrogen. This latter grid has a lower temperature limit given by $\theta_{e}=0.70$ and, therefore, was only directly useful for a few of the stars. However, when the two grids were compared in the region of temperature overlap, it was found that near $\theta_{e} \sim 0.70$ the Balmer line-blanketed models gave $\theta_{e}$ approximately 0.02 cooler than the unblanketed models. Therefore, where it was necessary to use the unblanketed models, this correction factor was applied. It was found that the gravity obtained from the Balmer jump was independent of the grid of models used. These data are given in Table 4, together with other data to be discussed. Included in this table are data for other previously known $\delta$ Sct stars. Sources are referenced. The period of pulsation $P$ for some of the new variables is only approximate. $M_{v}$ represents the mean of available absolute magnitude estimates which have been derived by one or more of the following methods: (1) Strömgren's uvby photometry; (2) trigonometrical parallaxes greater than $0.030^{\prime \prime}$; (3) classification of a companion star; (4) by using Eggen's (1963) calibration of $U B V$ data; and (5) membership of a moving group. The
TABLE 3
monochromatic fluxes in magnitudes per unt frequency interval

relative merits of the different methods are arguable, but equal weight has been given to all methods.

The eighth column of Table 4 contains $V \sin i$, the rotational velocity of the star projected in the line of sight. It is immediately obvious that some of the stars have large rotational velocities. This demonstrates that Preston's (1965) conclusion that rapid rotation and pulsation are incompatible in Cepheid-type variables does not apply to $\delta$ Sct variables. The effective temperatures indicate that the region of pulsation extends to higher temperatures than indicated by the previously known $\delta$ Sct variables.

TABLE 4
Data for Known $\delta$ Sct Stars

| Star | $B-V$ | Spec Type | $\theta_{e}$ | $\log g$ | Period <br> (days) | $M_{V}$ | $\begin{gathered} V \sin i \\ (\mathrm{~km} / \mathrm{sec}) \end{gathered}$ | References* |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| HR 1287 | 034 | dF3 | 079 | 305 | 0132 | +18 | $\leq 10$ |  |
| 1706 | 25 | A9 | 69 | 353 | 12 | +10 | $\leq 15$ |  |
| 2107 | 29 | F2 II | 695 | 380 | 137 | +17 | 10-15 |  |
| 3265 | 30 | A5 | 76 | 317 | 12-14 | +19 | 20-25 |  |
| 3888 | 29 | F2 IV | 725 | 364 | 13 | +1.6 | 110 |  |
| 4715 | 33 | F0 | 77 | 307 | 17 | +09 | 85 |  |
| 5005 | 30 | F0 | 725 | 367 | 14 | +16 | 60 |  |
| 5017 | 32 | F0 II | 775 | 272 | 14 | +09 | $\leq 10$ |  |
| 5329 | 20 | A7 IV | 655 | 363 | 069 | +1.2 | $\leq 130$ | (1) |
| $\beta$ Cas | 34 | F2 IV |  |  | 104 | +15 | 70 | (1) |
| CC And | 33 | F3 IV-V |  |  | 125 | +22 | 20 | (2) |
| $\rho$ Pup | 39 | F6 III | 83 | 22 | 141 | +07 | 15 | (3) |
| $\delta \mathrm{Sct}$ | 35 | F3 III | 79 | 28 | 194 | +11 | 15 | (4) |
| $\delta$ Del | 30 | F2 IV | 73 | 32 | 135 | +13 | 20 | (4) |
| BS Aqr |  |  |  |  |  |  | 20 | (5) |
| DQ Cep | 030 | F1 IV | 075 | 332 | 0079 | +18 | 60 | (5) |

* (1) R. Millis, Comm 27 I.A U. Bull No 137 (Konkoloy Obs ) (1966); (2) D H. McNamara and G Augason, Ap J, 135, 64 (1961); (3) I. J Danziger and LV Kuhi, $A p J, 146,743$; (4) L V Kuhi and I J Danziger, $A p J$ (in press); (5) I J 'Danziger (unpublished)


## v. DISCUSSION

## a) The Color-Magnitude Diagram

The color-magnitude diagram is shown in Figure 2. The variables in Table 4 are shown as crosses, and non-variables from Table 2 shown as filled circles. Circled crosses denote variables for which at least two independent estimates of $M_{v}$ are available. The two open circles are the peculiar stars $\beta \mathrm{CrB}$ and $\gamma$ Equ, for which the absolute magnitudes are uncertain. The position of the Cepheid instability strip is indicated by broken lines. It is noticeable that the two possible Sirius group members, HR 5329 and $\delta \mathrm{Del}$, are bluer than the three possible Hyades group members, HR 4715, 5017, and $\delta$ Sct. This difference is in the same sense as the difference between the sequences of their member stars (Eggen, 1960b).

No clear separation between variable and non-variable stars is evident, although it is noticeable that very few non-variable stars occur brighter than $M_{v}=+1.5$. Since line blanketing and stellar rotation can affect the magnitude and color of these stars, some attention should be given to them. Differential line blanketing effects should be small because (1) the stars lie in a restricted range of temperature and luminosity, and (2), having small space motions, they all appear to be disk-population stars. However, the influence of a star's rotation on the observed colors may be considerable. Strittmatter (1966) gives the change in effective temperature and absolute magnitude found in various models of rotating stars. The theories indicate that a rotating star always appears cooler
than its non-rotating counterpart, irrespective of aspect. Also, a rotator viewed pole-on appears brighter than, and equator-on appears fainter than, its non-rotating counterpart. The largest effects are shown by the non-uniformly rotating, magnetic models of Roxburgh and Strittmatter (1966). We note in Figure 2 that, of the non-variables occurring in the region occupied by the variable stars, i.e., between about $M_{v}=+0.5$ and $M_{v}=$ +2.4 , six stars have observed values of $V \sin i$ available. For these six stars, $\langle V \sin i\rangle$ $\sim 130 \mathrm{~km} / \mathrm{sec}$; whereas, for sixteen known $\delta$ Sct stars, $\langle V \sin i\rangle \sim 40 \mathrm{~km} / \mathrm{sec}$.


Fig. 2.-Observed color-magnitude diagram showing all the known $\delta$ Scuti stars (crosses or circled crosses), together with stars from Table 1 that are probably non-variable (filled or open circles) The zero-age main sequence is shown as a solid line and the position of the Cepheid instability strip by two broken lines.

The mean equatorial rotational velocity $\langle V\rangle$ is given by (Chandrasekhar and Münch 1950) $\left\langle V^{2}\right\rangle=\frac{3}{2}\left\langle(V \sin i)^{2}\right\rangle$. For stars between $M_{v} \sim+0.5$ and $M_{v} \sim+2.4$, we find

$$
\left\langle V^{2}\right\rangle_{\text {variables }}=5000(\mathrm{~km} / \mathrm{s})^{2}, \quad\left\langle V^{2}\right\rangle_{\text {non-variables }}=26662(\mathrm{~km} / \mathrm{s})^{2} .
$$

Using these values in the models and adopting a mean mass for both groups of $M=$ $1.8 M \odot$, and a mean radius of $R=2.88 R \odot$ (from the $M_{\mathrm{bol}} / \log T_{e}$ diagram, Fig. 5 below), we find for an average inclination
$\Delta \log T_{e} \sim+0.009$ for the variables,$\quad \Delta \log T_{e} \sim+0.048$ for the non-variables, the effects on $M_{v}$ in both cases being negligible. The differential effect of +0.039 in log $T_{e}$ corresponds to $\sim 0.1$ in $B-V$.

Therefore, if in Figure 2 the non-variables are moved to the left by 01 in $B-V$
relative to the variables, a very noticeable separation is found between the variables and non-variables. Some overlap occurs, and indeed this must be expected in such a statistical calculation where the value of $V$ for individual stars is not known. Similarly, rotation would tend to separate variables from non-variables in the $U-B / B-V$ diagram ( $\S, \mathrm{V} b$ ). The largest effect again occurs in $B-V$, since, for the samples considered here, the separation in $U-B$ is only 0.02 mag , in the sense that a higher rotation increases the value of $U-B$.

However, we should be careful to point out that such a limited analysis does not prove that variables and non-variables occupy completely separate regions of the H-R diagram when their colors are normalized to zero rotation. A much more complete analysis of stars in the variable star region, and on both sides of it, should be made for temperature,


Fig. 3.-Observed two-color diagram for the same stars as in Fig 2. The solid line shows the position of the zero-age main sequence; the broken line is Eggen's calibration of the effect of a change of surface gravity of $\delta \log g=-1$
gravity, rotation, and variability. In this way selection effects should be avoided and greater weight can be given to a statistical analysis. The present conclusion to be drawn from these results seems to be that, if the colors of the stars are normalized to zero rotation, a high proportion of the stars in the range $B-V \sim 0.25-0.40 \mathrm{mag}$ and $M_{v} \sim$ +0.5 to +2.4 mag is variable.
b) The $U-B / B-V$ Diagram

The $U-B / B-V$ diagram is shown in Figure 3, in which variables are denoted by crosses and non-variables by filled or open circles, as in Figure 2. The zero-age main sequence ("ZAMS"; Eggen 1965) is shown as a solid line, and it can be seen that all the variable stars fall below this (luminosity class V ) line. The position of a star in the twocolor diagram is influenced by effective temperature, line blanketing, and surface gravity. If the stars in Figure 3 have compositions similar to the Hyades stars (ZAMS), the blanketing effects will be similar and the negative ultraviolet excesses of the variables in Figure 3 will be predominantly caused by their lower surface gravities. The broken line in Figure 3 indicates a calibration for $\Delta \log g=-1$ with respect to the Hyades, based on data from wide binaries by Eggen (1963).

In principle, effective temperatures and gravities can be obtained by interpolating the observed colors $U-B, B-V$ into this type of diagram, which has been calibrated with a grid of models. In practice, the scanner observations described previously have been used to obtain this information. It is worth noting that the dwarf Cepheids lie above the main-sequence line in the two-color diagram and generally appear bluer than the $\delta$ Sct stars. Since no abundance analyses exist for dwarf Cepheids, it is not yet clear how much of this difference is due to line-blanketing effects.

Figure 4 shows the paths traced out in the $U-B / B-V$ diagram by the variables HR 2107 and HR 4715 during one cycle of variation. Their behavior appears to be similar to that of RR Lyrae stars, such as SU Dra (Oke, Giver, and Searle 1962) in which the surface gravity reaches a maximum value during rising light. The maximum gravity change is about 0.4 in $\log g$ for HR 2107 and rather less for HR 4715.


Fig. 4 -Paths traced out in the two-color diagram by the variables HR 2107 (1 Mon) and HR 4715 ( 4 CVn ) during a cycle of light variation.

## c) The $M_{b o l} / \log T_{e}$ Diagram

By using the absolute magnitudes in Table 4 and the effective temperatures from the scanner observations, the variable stars have been plotted in an $M_{\text {bol }} / \log T_{e}$ diagram. This is given in Figure 5. The small bolometric corrections which have been applied were taken from Popper (1959). Also plotted in this diagram is the observational ZAMS from a combination of the work of Eggen (1965) and Sandage (1957). The theoretical $1.5 \mathrm{M} \odot$ evolutionary track of Iben (1967), moved -0.03 in $\log T_{e}$ to fit the ZAMS initially, is also shown. (This appears to be reasonable in the light of Iben's stated errors and the approximate nature of the model atmospheres used in his work.) The dashed horizontal lines are translated $1.5 M \odot$ tracks; the dashed vertical lines represent approximate isorotational contours assuming conservation of angular momentum in shells. The mean values of $V \sin i$ on the main sequence are taken from the work of Abt and Hunter (1962).

The estimated absolute magnitudes and the measured values of the gravities and effective temperatures allow the calculation of the masses for these variable stars. These calculated masses and the masses derived from their positions in the $M_{\mathrm{bol}} / \log T_{e}$ diagram of Figure 5 are shown in the second and third columns of Table 5. It can be seen that approximately one half of these stars have calculated masses which are compatible with their positions in the $M_{\mathrm{bol}} / \log T_{e}$ diagram when account is taken of the uncertainties
of some of the data. The remaining stars all appear to be under-massive for normal left-to-right evolution in the H-R diagram. It is also noticeable that the stars of lowest mass are consistently cooler than the stars of higher mass. This result is consistent with the observation of the sequence formed by plotting these stars in a $\log g$ versus $\log T_{e}$ diagram (discussed in § V $d$ ).

This effect might be explained if the determination of $\log g$ from model atmospheres was systematically increasing in error in going to lower temperatures. It is not obvious at the present time that this should be the case. ${ }^{1}$ At least it can be stated that the hotter
${ }^{1}$ See note added in proof.


Fig. 5.- $M_{\mathrm{bol}} / \log T_{e}$ diagram Variable stars (filled circles) are superposed on theoretical evolutionary tracks of Iben Rotational velocities are also marked

TABLE 5
Masses and Pulsation Constants

| Star | $\begin{gathered} \text { Computed } \\ \text { Mass } \\ \mathfrak{M} / \mathfrak{M}_{\odot} \end{gathered}$ | Mass from $M_{\text {bol }}-\log T_{e}$ Diagram $\mathfrak{M} / \mathfrak{M}_{\odot}$ | Pulsation <br> Constant Q |
| :---: | :---: | :---: | :---: |
| HR 1287 | 04 | 16 | 0015 |
| 1706 | 17 | 20 | 022 |
| 2107 | 17 | 17 | 041 |
| 3265 | 04 | 16 | 018 |
| 3888 | 14 | 17 | 030 |
| 4715 | 08 | 20 | 017 |
| 5005 | 15 | 17 | 034 |
| 5017 | 04 | 20 | 009 |
| 5329 | 14 | 19 | 016 |
| $\rho$ Pup | 02 | 21 | 005 |
| $\delta$ Sct | 04 | 19 | 015 |
| $\delta \mathrm{Del}$ | 07 | 18 | 018 |
| DQ Cep | 06 | 16 | 0013 |

variables have masses and luminosities which are consistent with their evolution from a ZAMS to the right in the H-R diagram. Further support for this idea is given by the fact that the mean rotational velocity $\langle V \sin i\rangle$ of this group of variables seems consistent with what would be expected for the former main-sequence positions of the variables in the group.

In the above discussion it has not been possible to take account of rotational effects on an individual star's position in the H-R diagram. This should not seriously affect the conclusions.

d) The $\log \mathrm{g} / \log T_{e}$ Relation

The location of the high-temperature boundary to the instability strip in the colormagnitude diagram has been predicted theoretically by Christy (1966a) on the basis of extensive calculations on RR Lyrae and Cepheid models. This is indicated by a solid


Fig $6-\log g / \log T_{e}$ diagram for short-period variables Theoretical boundaries of the instability region are marked
line in the $\log g / \log T_{e}$ diagram shown in Figure 6. The broken line shows the approximate location of the onset of deep convection from the work of several authors (see Christy 1966b). It has been suggested that this corresponds to the low-temperature boundary of the instability strip. The $\delta$ Sct variables are shown as points in Figure 6. They show a correlation between these two parameters which agrees well with the theoretical slope of the instability strip.

Only if a mass-luminosity relation were true for $\delta$ Scuti stars could we use the equation of the instability strip, together with the period-density law to predict a period-luminosity relation.
e) Pulsation

Since these stars are pulsating with reasonably regular periods it is of some interest to note whether they obey a period-density and period-luminosity law. Results for other short-period variables by Danziger and Kuhi (1966), Danziger and Oke (1967), and Kuhi and Danziger (1967) indicate that, to explain the observations, much lower values of the pulsation constant, $Q$, are required than the theory of RR Lyrae models indicates. In this paper a value of $Q$ is calculated for each star by combining the period-density law and the expressions for density and luminosity. The final expression is

$$
\log Q=\log P+\frac{1}{2} \log g / g \odot+\log T_{e} / T_{e} \odot+0.1\left(M_{\mathrm{bol}}-M_{\mathrm{bol} \odot}\right) .
$$

These calculated values of $Q$ are listed in the fourth column of Table 5. It can be seen that the values range from 0.005 for $\rho$ Pup to 0.041 for HR 2107. From theoretical calculations (Christy 1966a) we might expect values in the range $\sim 0.025$ to $\sim 0.040$ depending on the overtone, provided the pulsation is in either the fundamental or first overtone. The systematics of overtone $Q$ values (Schwarzschild 1941; Christy 1966b) indicate that $Q$ values less than 0.015 for spherical pulsation would require overtones higher than the fourth. Because of errors in the determination of $M_{v}$ and $\log g$, values of $Q$ as low as 0.015 cannot at the moment be considered to invalidate the idea of a fundamental or first overtone pulsation. However, the lowest values of $Q$ suggest higher-order overtones. It has in fact been suggested (Christy 1966b) that the modulation of about 3 or 4 periods that is common in $\delta$ Scuti stars could be the result of a mixture of higher overtones (such as the third and fourth for a low-gravity star).

These results imply that a unique period-luminosity law will not be obvious for these stars, and indeed this was found to be so.

## VI CONCLUSIONS

The results of this investigation show that the instability strip extends to fainter stars (or higher gravities) and higher temperatures than previously known. It is clear that many more stars in the region under discussion will be found to be variable, and there is an obvious need for closer investigation of stars in clusters. Established variables can now be used to study the relative phases of light, temperature, and radial velocity which are of some interest in theoretical models. It is planned to study in more detail the statistics of variability, rotation, gravity, and temperature in this region of the H-R diagram.

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Note added in proof: It has been suggested (M. S. Bessel 1967, private communication) that a systematic underestimate of line blanketing and an incorrect absolute calibration could cause an underestimate of the effective temperature and surface gravity (and hence the mass and $Q$ value), which increases as the absolute temperature decreases. If this is indeed correct, the stars plotted in Figure 6 would need to be compressed toward higher gravities and higher temperatures. However, at the present time the red scans given in this paper, which are essentially free of line-blanketing effects, can be arbitrarily fitted to the blue scans to give the same temperatures as those presented in Table 4. Because the relative phases of the red and blue scans are not known, we are not yet in a position to say whether it is a valid procedure or not.

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