HYDROGEN AND HELIUM SPECTRA OF GASEOUS NEBULAE

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ABSTRACT

The intensities of the high members of various series of hydrogen and neutral and ionized helium are observed in some nebulae to be brighter than would be expected from recombination theory. Detailed results of a comparison between theory and observation are presented for 33 planetary nebulae and the Orion Nebula. New values for the reddening constants, electron temperatures, and electron densities have been computed for the nebulae considered.

I. INTRODUCTION

It has been shown, as reported in earlier papers (Aller, Bowen, and Minkowski 1955; Kaler 1964*a*, *b*), that the decrements of hydrogen and neutral and ionized helium in gaseous nebulae do not always agree with those predicted by recombination theory. For example, the observations of NGC 7027 of Aller, Bowen, and Wilson (1963) show that the ratio of $I(\mathrm{H}n)/I(\mathrm{H}\beta)$, where I denotes intensity, is over 6 times too large in the neighborhood of H30. At the present time, Balmer-line intensities have been measured to H30 for seven nebulae, and to greater than H20 for several more, so that a detailed comparison between theory and observation is possible. Kaler (1964*b*) gives the principal conclusions drawn from a study of thirty-three planetary nebulae and the Orion Nebula. The purpose of this paper is to give the detailed results for the individual objects. The data have been revised quite recently in the light of an improved interstellar-reddening function, better electron temperatures, and extended theoretical He I intensities. The principal results and essential conclusions, however, remain unchanged.

The present paper gives a detailed comparison of observations of the Balmer and Paschen series of hydrogen and the Pickering and Pfund series of ionized helium with the theoretical intensities of Seaton (1959*a*, *b*) and Pengelly (1964) and a comparison of the neutral helium nd–$2p$ series with the intensities of Pengelly (private communication). For this purpose, line-intensity data have been assembled from several sources.

Section II gives the nebulae considered, the references to the observations used, and any previously unpublished line intensities. Section III discusses the calculation of the reddening constants, and the new, self-consistent determinations of electron temperatures and densities for the nebulae involved. Section IV discusses the theoretical intensities and how they are to be applied, and § V gives the actual comparison and the observed results. Finally, § VI discusses the results in the light of theoretical interpretations.

II. THE OBSERVATIONS

The nebulae used in the study are listed in Table 1, together with their appropriate references, and are divided into two groups. Group I consists of those nebulae for which we have accurate line intensities of hydrogen to at least H15. In general, this group includes newer observations, and various series of helium are observed. Group II, on the other hand, contains observations of lower accuracy, generally older, for which hydrogen is observed from H12 to H15. Helium is not included in this group. In many cases, line-intensity data were taken from several sources; if so, the principal reference is listed first in Table 1.

Table 2 gives the line intensities of those nebulae for which detailed data have not

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yet been published. The number "n" in the first column denotes the principal quantum number of the upper level of the appropriate transition, the second column gives the wavelength of the line, and the succeeding columns list the intensities relative to Hβ of the various nebulae. These observations were made with the 120-inch telescope, either with the prime-focus spectrograph or with the Lallemand image tube at coudé focus. We estimate the accuracy of these intensities as between 20 and 40 per cent for the higher lines, which is quite sufficient for the present study.

III. NEBULAR PARAMETERS

Before we can compare theory with observation, we must first correct the observations for the effect of interstellar reddening. Since the theoretical decrements are functions of electron temperature, we must know the electron temperatures of the nebulae so that we

TABLE 1

THE NEBULAE AND REFERENCES TO OBSERVATIONS

Nebula	References
Group I:	
Anon 21^h31^m = VV 267	Aller and Walker (1965)
IC 351	Aller and Walker (1965)
IC 418	Aller and Kaler (1964*c*); O'Dell (1963)
IC 2165	Aller and Kaler (unpublished); Aller (1951); Minkowski and Aller (1956)
IC 4997	Aller and Kaler (1964*b*); O'Dell (1963)
NGC 1535	Aller and Walker (1965); Aller (1951)
NGC 1976 = Orion Nebula	Kaler, Aller, and Bowen (1965); Aller and Liller (1959)
NGC 2392	Minkowski and Aller (1956); O'Dell (1963)
NGC 2440	Aller and Kaler (unpublished); Aller (1951); Liller and Aller (1963)
NGC 3242	Czyzak, Aller, Kaler, and Faulkner (1965); Aller (1951)
NGC 6572	Aller and Kaler (1964*b*); Aller (1965); O'Dell (1963)
NGC 6741	Aller and Walker (1965); Aller (1951)
NGC 7009	Aller and Kaler (1964*a*); O'Dell (1963); Liller and Aller (1963); Aller and Faulkner (private communication)
NGC 7027	Aller, Bowen, and Wilson (1963); O'Dell (1963); Aller, Bowen, and Minkowski (1955)
NGC 7662	Aller, Kaler, and Bowen (1966); Aller (1965); O'Dell (1963); Minkowski and Aller (1956); Chopinet (1963)
Group II:	
Anon 18^h15^m = VV 171	Aller (1951)
Anon 18^h47^m = VV 208	Aller (1951)
Anon 22^h29^m	Aller (1951)
Anon 23^h24^m = VV 286	Aller (1951)
IC 2149	O'Dell (1963); Aller (1951)
IC 4846	Aller (1951)
IC 5117	Aller (1951)
IC 5217	Aller (1951)
J900	Aller (1951)
NGC 40	Aller (1965)
NGC 4361	Aller and Walker (private communication)
NGC 6309	Aller and Walker (1965); Aller (1951)
NGC 6720	Aller and Walker (1965)
NGC 6790	Aller (1951)
NGC 6803	Aller (1951)
NGC 6807	Aller (1951)
NGC 6833	Aller (1951)
NGC 6884	Aller (1951)
NGC 6886	Aller (1951)

might choose the correct decrement. We also compute electron densities from published [O II] ratios. All the above parameters are used to test for possible correlations between them and the degree of deviation of observation from theory.

The reddening constant c is defined by

$$\log I_c(\lambda) = \log I_o(\lambda) + cf'(\lambda) , \tag{1}$$

where $I_o(\lambda)$ is the intensity of the line of wavelength λ observed just outside the Earth's atmosphere, $I_c(\lambda)$ is the true intensity corrected for interstellar reddening, and $f'(\lambda)$ is the interstellar-reddening function scaled such that $f'(\mathrm{H}\beta) = 1, f'(\infty) = 0$. Then,

$$c = \log \frac{I_c(\mathrm{H}\beta)}{I_o(\mathrm{H}\beta)} . \tag{2}$$

TABLE 2

HYDROGEN AND HELIUM LINE INTENSITIES FOR NEBULAE FOR WHICH PUBLISHED DATA ARE NOT AVAILABLE

n	λ	IC 2165	NGC 2440	NGC 4361	n	λ	IC 2165	NGC 2440	NGC 4361
	H (Balmer)					H (Balmer)			
4...	4861	100	100	100	15....	3712	2.15	1.89	0.65
5...	4340	46	43	39.0	16....	3703	1.9	....	....
6...	4101	22.8	26.3	19.7	17....	3697	1.4	1.47	....
8...	3889	....	....	9.55	18....	3691	1.3	1.12	....
9...	3835	6.1	6.8	7.16	19....	3686	1.1	1.10	....
10...	3797	4.8	4.9	5.63	20....	3682	1.0	1.04	....
11...	3771	4.1	3.5	2.98	21....	3679	0.8	1.03	....
12...	3750	3.32	3.02	2.48	22....	3676	0.8	0.88	....
13...	3734	2.65	2.45	1.87	23....	3673	0.8	0.59	....
14...	3721	3.74	....	0.82					

n	λ	IC 2165	NGC 2440	n	λ	IC 2165	NGC 2440
	He I (n^3D–2^3P)				He II (Paschen)		
3.........	5876	17.4	....	4.........	4686	60	71.5
4.........	4471	3.8	3.65	5.........	3203	19	22.4
5.........	4026	1.9	2.14				
6.........	3819	1.3	0.94		He II (Pickering)		
	He I (n^1D–2^1P)			7.........	5411	7.2	....
				9.........	4541	2.4	3.29
3.........	6678	6.4	....	11.........	4199	0.97	1.81
5.........	4387	0.50	0.38	14.........	3968	....	....
6.........	4143	0.35	.27	15.........	3923	0.72	0.71
7.........	4009	0.38	.15	16.........	3887	0.70	....
8.........	3926	0.27	0.11	17.........	3858	0.47	0.50
				18.........	3833	0.4	....
				19.........	3813	0.55	0.46
				21.........	3781	0.3	0.26

Since all line intensities are measured relative to Hβ, we define another function $f(\lambda)$ such that $f(\mathrm{H}\beta) = 0, f(\infty) = -1$, where c retains the definition of equation (2). Then,

$$f(\lambda) = f'(\lambda) - 1 . \tag{3}$$

The function $f(\lambda)$ given by Seaton (1960) from an average of observed values is used throughout.

The methods available to us for the determination of the reddening constant fall into two general categories. In the first, we compare the observed intensity ratios of lines of different series of the same ion which arise from a common upper level with the theoretically predicted ratio. In the second, the comparison involves only intensity ratios of a given series. As regards methods of the former category, the ratios depend primarily upon atomic constants (transition probabilities) so that we should not expect deviations from theory. When we use one of these methods, we find that the first few members of a given series agree with theory, so that the second category is valid as long as we do not use the higher members of a series. This is equivalent to saying that the reddening constants determined by methods of both categories are the same. Table 3 demonstrates this fact, using methods 1 and 2 below. The agreement is well within the error that one might expect from random errors in the line intensities. We also see that, even when the higher members of a series deviate from theory, the line ratios involving those higher members do not (§ V). Thus we can use theory to correct observation even while using observation as a check on the theory. Further justification arises from the fact (see § VI) that there is no correlation between the reddening constant and the degree of deviation.

TABLE 3

COMPARISON OF REDDENING CONSTANTS COMPUTED BY DIFFERENT METHODS

NEBULA	METHOD (SEE TEXT)			
	1	2	3	4
IC 418	0.35	0.46	....	....
IC 2149	0.63	0.49	....	....
IC 4997	0.61	0.40	....	....
NGC 1976	0.32	0.23	....	....
NGC 6572	0.34	0.40	....	....
NGC 7027	1.10	1.19	0.82	1.14
NGC 7662	0.28	0.23	0.00	0.15

Specifically, four methods have been used in this study to determine c, where the theoretical intensities of Pengelly (1964), which are valid for low n (§ IV), are used. These methods are:

1. Hydrogen Paschen/Balmer ratios. The principal method of the first category, this procedure is by far the most accurate. In addition to its inherent accuracy, we have the advantage of a large base line in wavelength. We use ratios of transitions from $n = 6$, 9, and 10, where possible.

2. When the Paschen lines are not observed, we must, with one exception, use the relative intensities of Hα, Hβ, Hγ, and Hδ; even Hα is not often observed, so we are forced to use only the last three lines. The above is the principal method of the second category. As was said before, these intensities should fit the theory well. The next unblended Balmer line is H9 at which point deviations from theory become apparent. The biggest disadvantage of the method is the small base line in wavelength.

3. He II Paschen-β/Paschen-α ratio. When He II is observed, we may use the ratio $I(\lambda\ 3203)/I(\lambda\ 4686)$. The theoretical ratio is from Pengelly (private communication).

4. He II Pickering-α/Paschen-β ratio, $I(\lambda\ 10123)/I(\lambda\ 3203)$. Both lines arise from the $n = 5$ level. Unfortunately, only one actual ratio is available, as the Paschen series is cut off shortward of Paschen-β by the atmosphere, but we have the advantage of the longest possible base line in wavelength.

Methods 3 and 4 can be used only in high-excitation nebulae, and have the disadvantage that $I(\lambda\ 3203)$ is very hard to measure accurately; thus they are not often available for use.

TABLE 4

REDDENING CONSTANTS

Nebula	Group I		Nebula	Group II	
	c	Method		c	Method
Anon 21^h31^m	0.18	2	Anon 18^h15^m*	0.00	2
IC 351*	0.00	2	Anon 18^h47^m	0.06	2
IC 418	0.35	1	Anon 22^h29^m†	0.00	2
IC 2165	0.16	2	Anon 23^h24^m	0.88	2
IC 4997	0.61	1	IC 2149	0.63	1
NGC 1535†	0.00	2	IC 4846†	0.00	2
NGC 1976	0.32	1	IC 5117	0.30	2
NGC 2392	0.11	2	IC 5217†	0.00	2
NGC 2440‡	0.12	2	J900	0.40	2
NGC 3242	0.31	2	NGC 40	0.31	2
NGC 6572	0.37	1, 2	NGC 4361	0.57	2
NGC 6741	0.25	2	NGC 6309‡	0.09	2
NGC 7009	0.82	3	NGC 6720‡	0.10	2
NGC 7027	1.02	1, 3, 4	NGC 6790	0.60	2
NGC 7662	0.28	1	NGC 6803	0.16	2
			NGC 6807†	0.00	2
			NGC 6833	0.76	2
			NGC 6884	2.14	2
			NGC 6886	0.68	2

* $H\gamma/H\beta$ gives negative c.
† c predicted negative.
‡ $H\delta/H\beta$ gives negative c.

Table 3 gives the comparison of all four methods for NGC 7027 and NGC 7662. The agreement is generally quite good. When the line intensities are available, these methods are certainly better than method 2.

Table 4 gives the adopted values of c and the method(s) used. In some cases, negative reddening constants are computed. This is to be expected when the constants are small, simply by the random errors in the line intensities; it only occurs when method 2 is used, and we have a small base in wavelength. Individual negative values are averaged, and if the average is negative, c is set equal to zero. The above methods appear to give reddening constants which are quite accurate enough for the analysis to follow in later sections.

The electron temperature may be calculated from the intensity ratio of the green nebular lines, λ 4959 and λ 5007 of [O III] to λ 4363 of [O III] through an expression of the form (cf. Menzel, Aller, and Hebb 1941)

$$\frac{I(\lambda 4959)+I(\lambda 5007)}{I(\lambda 4363)}=\frac{a\exp(33000/T_e)}{1+bN_e/\sqrt{T_e}}, \tag{4}$$

where T_e and N_e denote electron temperature and density, respectively, and where we have adopted numerical values of the coefficients a and b from Seaton (1960), namely, $a = 7.1$, $b = 3.8 \times 10^{-4}$. In order to apply this formula, we need to correct the intensities for the effect of interstellar reddening, and to have values of N_e. The latter were taken from Aller (1965) after the appropriate corrections due to different reddening constants. Values of T_e and c must be determined by an iterational procedure, which converges rapidly, as very rough estimates of T_e yield good values of c.

The observations of the [O III] lines were taken from many sources, and means were adopted. In several cases, improved values have come from recent work. Table 5 lists in successive columns the nebula, adopted values of $I(\lambda\, 4959) + I(\lambda\, 5007)$ with references, values of $I(\lambda\, 4363)$ with references, and the resulting value of T_e.

We also derive electron densities from the [O II] $I(\lambda\, 3729)/I(\lambda\, 3726)$ ratio in accordance with the formula given by Seaton and Osterbrock (1957),

$$r = \frac{I(\lambda 3729)}{I(\lambda 3726)} = 1.5\, \frac{1 + 0.33\epsilon + 2.30x(1 + 0.75\epsilon + 0.14\epsilon^2)}{1 + 0.40\epsilon + 9.9x(1 + 0.84\epsilon + 0.17\epsilon^2)}, \tag{5}$$

where

$$\epsilon = \exp(-1.96 \times 10^4/T_e) \qquad \text{and} \qquad x = 10^{-2}N_e/\sqrt{T_e}\,.$$

The result is only weakly dependent on T_e, and we assume the values given in Table 5 are typical of the nebula. Table 6 gives the results; the second and third columns give the values of r and the references; the fourth column gives the computed log N_e; and, for comparison, the fifth column gives log N_e as derived from Aller (1965), computed from surface-brightness data. If the nebula is filamentary, we might expect the value as derived from [O II] data to be higher than that derived from surface brightness, since the former gives a true value in the regions in which O^+ is radiating, and the latter an rms value, averaging over empty space. Table 6 shows more or less of a random scatter between the two values, and generally good agreement, NGC 2392 being the biggest exception.

IV. THE THEORETICAL INTENSITIES

The predicted theoretical intensities of the lines of hydrogen and helium are the result of application of recombination theory, i.e., the atoms are photo-ionized from the ground state, the free electrons are recaptured on higher levels and give the observed spectrum as they cascade to lower levels in returning to the ground state. The H and He II spectra are entirely similar and can be discussed together, as both are one-electron spectra. Neutral helium is more complicated, and the spectrum is discussed separately. We assume Baker-Menzel case B, i.e., the nebula is optically thick in the Lyman lines, throughout. This is indicated by the relatively high surface brightnesses and densities of most of the nebulae considered, and by the work of Osterbrock (1962) which demonstrated the validity of case B.

Early calculations based on the ideas of Zanstra (1927) which enabled one to predict the intensities of the lines of H and He II were carried out by Plaskett (1928), Carroll (1930), Cillié (1932, 1936), and Baker and Menzel (1938), all of whom assumed degenerate angular-momentum states and considered the principal levels only. This procedure tacitly assumes a Boltzmann distribution among the l-states for a given n, i.e., $N_{nl} = N_n(2l + 1)/n^2$. The most extensive of these calculations were by Baker and Menzel who were the first to assume an atom with an infinite number of levels. The resulting intensities agreed fairly well with the first few members of the Balmer series, however, even with the assumption of degeneracy. These computations have been redone by Seaton (1959b), who corrected errors in the Baker-Menzel calculations and extended the temperature range. His results are in the form of factors which give the departure from

TABLE 5

VALUES OF ADOPTED [O III] LINE INTENSITIES WITH REFERENCES, AND ELECTRON TEMPERATURES

(I[Hβ] = 100)

Nebula	$I(\lambda$ 5007+ λ 4959)	References*	$I(\lambda$ 4363)	References	T_e(° K)
	Group I				
Anon 21h31m	1040	6	20	6	17200
IC 351	1814	1, 5, 6	16.8	1, 6, 8, 12	12000
IC 418	186	1, 4, 5, 7	1.30	7, 13	11200
IC 2165	2050	5, 6	18.4	6, 8	12200
IC 4997	845	1, 7, 10	29.8	7, 14	22700
NGC 1535	1518	1, 4, 8	19.4	1, 6, 8, 12	13700
NGC 1976	455	9	1.60	9, 15	9200
NGC 2392	1396	1, 3, 7, 8	27.9	1, 7, 8	17300
NGC 2440	2374	1, 6	27.4	1, 6	13400
NGC 3242	1871	2, 3, 6, 11	11.8	11	10900
NGC 6572	1471	1, 3, 4, 5, 21	8.8	1, 7, 14	10800
NGC 6741	2148	1, 2, 6	13	6	10800
NGC 7009	1482	1, 3, 4, 21	8.7	1, 16, 21	11300
NGC 7027	2059	1, 3, 4, 5, 7	18.9	1, 7, 17, 18	13900
NGC 7662	1704	1, 3, 4, 5, 7, 8	16.9	1, 7, 8, 19	12800
	Group II				
Anon 18h15m	20.9	6			
Anon 18h47m	510	6	2.2	6	9500
Anon 22h29m	800	6	8.3	6	12600
Anon 23h24m	440	6	22	6	41000
IC 2149	595	1, 4, 5, 6, 7	5.2	4, 6, 7	12900
IC 4846	1330	6	14	6	12600
IC 5117	2210	6	19	6	12200
IC 5217	1789	1, 2, 6	12	1, 6	10800
J900	1728	1, 5, 6	13	6	11900
NGC 40	43	3, 8	....	....	....
NGC 4361	462	10	3.5	20	12200
NGC 6309	1735	2, 6	....	....	....
NGC 6720	1310	1, 2, 3	14.4	1, 12	13100
NGC 6790	1559	6, 10	14	6	12800
NGC 6803	1475	1, 6	9.5	1, 4, 6	10900
NGC 6807	1600	6	11	6	10700
NGC 6833	760	6	11	6	15100
NGC 6884	2190	2, 6	5.2	6	9900
NGC 6886	2760	2, 6	15	6	11000

* References:
1. Liller and Aller (1963)
2. Collins, Daub, and O'Dell (1961)
3. Liller (1955)
4. Liller and Aller (1954)
5. Capriotti and Daub (1960)
6. Aller (1951)
7. O'Dell (1963)
8. Minkowski and Aller (1956)
9. Aller and Liller (1959)
10. O'Dell (1962)
11. Czyzak *et al.* (1965)
12. Aller and Walker (1965)
13. Aller and Kaler (1964*c*)
14. Aller and Kaler (1964*b*)
15. Kaler *et al.* (1965)
16. Aller and Kaler (1964*a*)
17. Aller *et al.* (1955)
18. Aller *et al.* (1963)
19. Aller *et al.* (1966)
20. Aller and Walker (private communication)
21. O'Dell (private communication)

thermodynamic equilibrium for a given energy level. From these, intensities for the various series of H and He II have been calculated for $T_e = 10000°$, $15000°$, $20000°$, $25000°$, and $40000°$ K.

Mathis (1957) was the first to attempt to allow for non-degenerate energy levels in this type of calculation, whereas Burgess (1958) and Searle (1958) were the first to calculate the intensities of the hydrogen lines, allowing for the discrete angular-momentum states. The solutions were not carried to high-energy levels, however. More recently, Pengelly (1964) has published results, essentially considering an atom with an infinite number of energy levels; he has published decrements of various series of H and He II for a wide range of electron temperature to $n = 20$.

TABLE 6

ELECTRON DENSITIES FROM [O II] RATIOS

$r = I(\lambda\,3729)/I(\lambda\,3726)$

Nebula	r	References*	log N_e	log N_e†
IC 351	0.51	2	3.79	3.90
IC 418	0.37	1	4.62	4.33
IC 2149	0.69	1	3.38	3.87
IC 2165	0.75	3	3.26	3.39
NGC 40	0.78	1	3.19	2.92
NGC 1535	0.56	2	3.66	3.51
NGC 2392	0.47	1	3.95	3.16
NGC 2440	0.64	1	3.47	....
NGC 3242	0.65	1, 4	3.43	3.95
NGC 6309	0.58	2	3.56	2.90
NGC 6572	0.46	1	3.94	4.05
NGC 6720	1.01	1	2.87	2.88
NGC 6741	0.61	2	3.49	3.58
NGC 6803	0.57	1	3.59	3.58
NGC 7009	0.50	1	3.79	4.23
NGC 7027	0.50	1, 5	3.82	3.99
NGC 7662	0.57	6	3.62	3.98

* References:
1. Osterbrock (1960)
2. Aller and Walker (1965)
3. Aller and Kaler (unpublished)
4. Czyzak *et al.* (1965)
5. Aller *et al.* (1963)
6. Aller *et al.* (1965)

† Surface brightness; Aller (1965).

We thus have two sets of theoretical intensities, the choice of which depends on the assumption regarding the existence of discrete angular-momentum states. The Pengelly decrements are considerably steeper than Seaton's decrements calculated under the assumption of degeneracy.

In recent papers, Pengelly and Seaton (1964) and Seaton (1964) discuss various collisional processes in gaseous nebulae. They show that at densities typical of planetary nebulae (10^4 cm^{-3}), collisions of the type $nl \rightarrow nl \pm 1$ become important for high n, and occur at the same rate as radiative transitions at $n = 15$ for H and $n = 22$ for He II; above these values, collisions predominate. We might then say that above $n = 15$ or 20, the collisions establish a Boltzmann distribution among the l-states, and the degenerate (Seaton-Baker-Menzel) approach is appropriate. Below $n = 10$ or 15, we must take the discrete l-states into account, and the situation is more complex. Pengelly's intensities are not strictly appropriate as he assumes that no reshuffling of electrons takes place anywhere. The actual intensities of the lines below $n = 15$ will, however, depend somewhat upon radiative transitions from the high levels where collisional redistribution occurs. We can show from the work of Seaton (1959*a*) that the recombination rate to all levels with $n > 15$ is much less than the rate to all levels with $n < 15$. Thus in this latter region, especially for the low members of the series used for reddening calculations, Pengelly's intensities should be a very good approximation.

The H-line intensities are expressed relative to Hβ, and the He II line intensities relative to λ 4686 (Paschen-α). Pengelly's (1964) paper shows that the absolute energy in Hβ radiated by a cubic centimeter of nebular material is nearly the same for both calculation schemes. That is, if b_n denotes the factor of departure of the population of level n from thermodynamic equilibrium, b_4 (Seaton) $= \langle b_4 \rangle$ (Pengelly). It is easily seen that the same is true for λ 4686, as this line also deals with $n = 4$. Thus even though Hβ radiates according to the scheme of discrete angular-momentum states, and a high line of a series (say, $n > 20$) radiates according to the assumption of degeneracy, the $I(\mathrm{H}n)/I(\mathrm{H}\beta)$ ratio will be given by Seaton's degenerate type of calculation, according to which Hβ radiates as if the l-states were degenerate. The same argument holds for the He II spectrum. We can thus directly compare the observations with Pengelly's or Seaton's intensities, where the former should be good at low levels, and the latter at high. (Recall that Pengelly's intensities were used to compute the reddening constant, for which we always used lines such that $n \leq 10$.)

The spectrum of He I is different, since the effect of discrete l-states *must* be included, because of the large separation in their energies. Pengelly (private communication) has computed the data necessary to enable one to calculate intensities for the relatively hydrogenic n^3D–2^3P and n^1D–2^1P series, for a range of electron temperatures, to $n = 10$. These intensities have been extrapolated by a simple $I \sim 1/n^3$ rule to $n = 21$ (the upper limit of observation), a procedure sufficiently accurate as compared to the error of observation for the high members of the He I series.

Pengelly (private communication) has also provided He II Paschen-line intensities ($I[\lambda\ 3203]/I[\lambda\ 4686]$) under the assumption of discrete l-states.

V. COMPARISON OF OBSERVATION WITH THEORY

The theoretical decrements have been computed at intervals of 5000° K in electron temperature. The temperatures of Table 5 were rounded off to the nearest 5000° K, and the appropriate theoretical decrement chosen. The intensity ratios change so slowly with temperature that an interval of 5000° K is small enough. The ratio of theoretical line intensity to observed line intensity corrected for interstellar reddening was then formed for each line.

Figures 1–15 show the comparison between observation and theory for H, He I, and He II for the nebulae of Group I. All H-lines are normalized to $I(\mathrm{H}\beta) = 100$, all He I lines to $I(\lambda\ 5876) = 100$, and all He II lines to $I(\lambda\ 4686) = 100$. The normalization factors are derived from the observations, and depend on relative abundances and excitation levels.

With regard to H and He II, the upper plot in each figure is that of log $I(P)/I_c$ versus n, where $I(P)$ is Pengelly's intensity (non-degenerate solution), I_c is the observed intensity corrected for interstellar reddening, and n is the principal quantum number of the upper level of the transition. The lower plot is that of log $I(S)/I_c$ versus n, where $I(S)$ is Seaton's degenerate-type intensity. The values of log $I(T)/I_c$ versus n, where $I(T)$ is the theoretical He I intensity of Pengelly, are graphed on both upper and lower plots.

The comparison for Group II is not graphed, and the results are presented later in this section.

The results for Group I are presented in Table 7. The measured parameter is the deviation at a given principal quantum number for hydrogen. Since the Balmer lines are the only series observed for *all* nebulae, they alone are used in determining the deviation for a given nebula. In order to measure the deviation for Group I nebulae, we use log $I(S)/I_c$ since the H-lines are observed at high enough n so that the degenerate theory holds. To measure this quantity at a given n, we draw a smooth line through the points

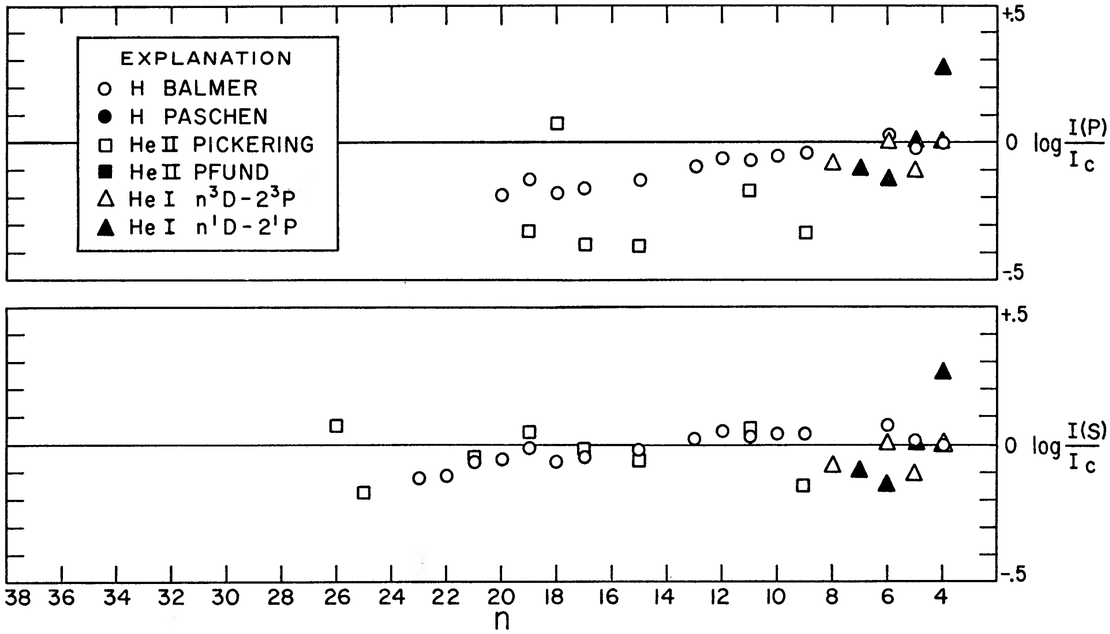

FIG. 1.—Comparison between theory and observation for Anon 21^h31^m. The upper graph plots the comparison between observation and Pengelly's intensities, the lower Seaton's degenerate intensities. Values of $\log I(T)/I_c$ for He I are plotted on each graph.

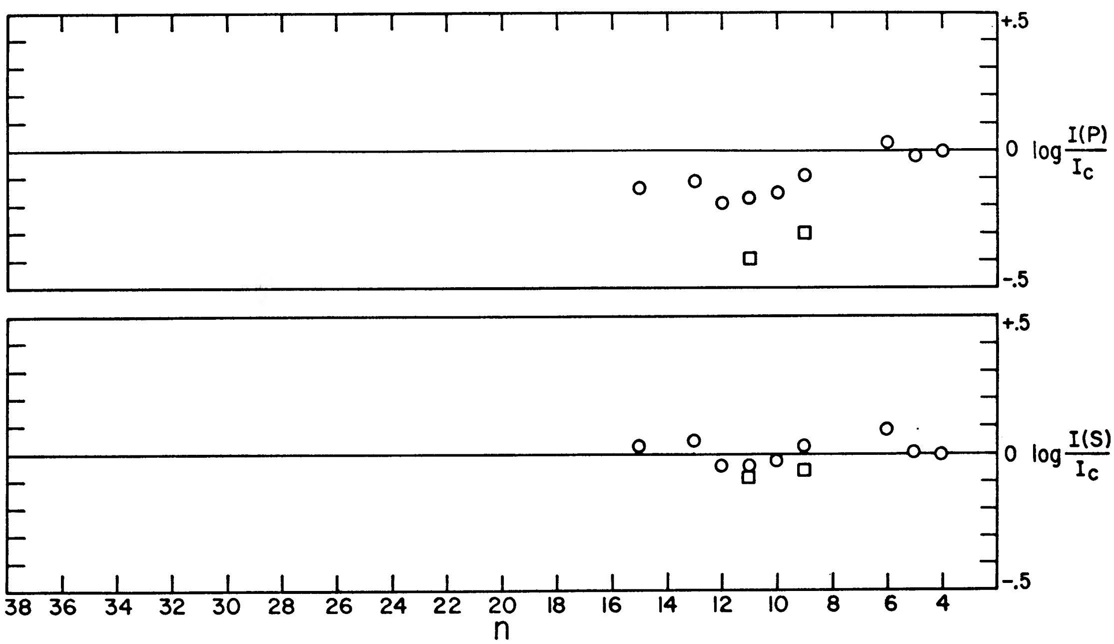

FIG. 2.—IC 351

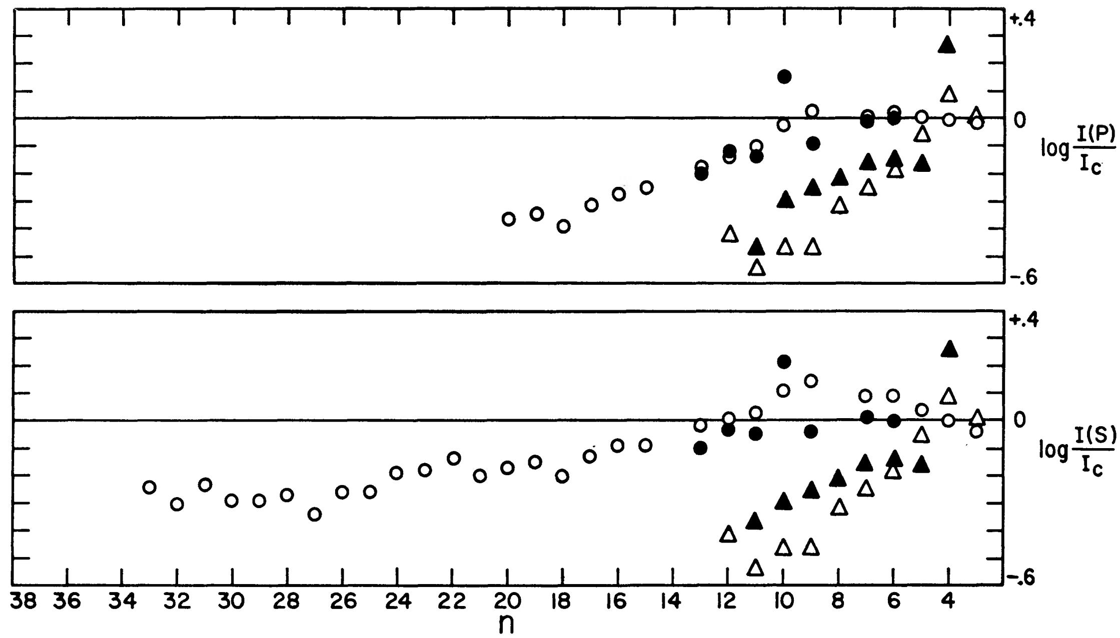

FIG. 3.—IC 418

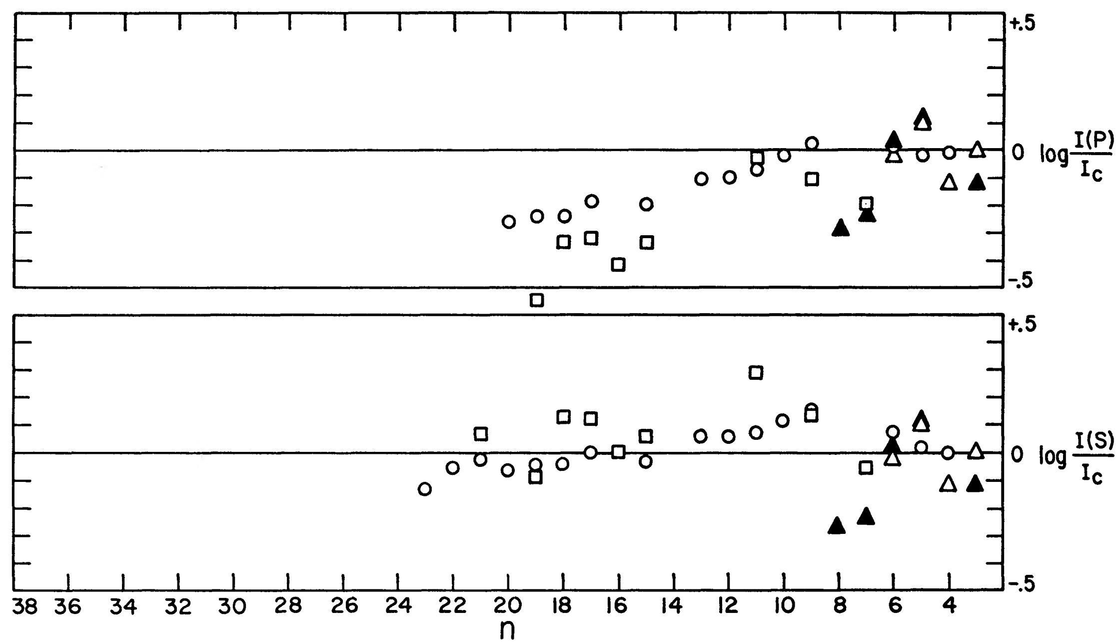

FIG. 4.—IC 2165

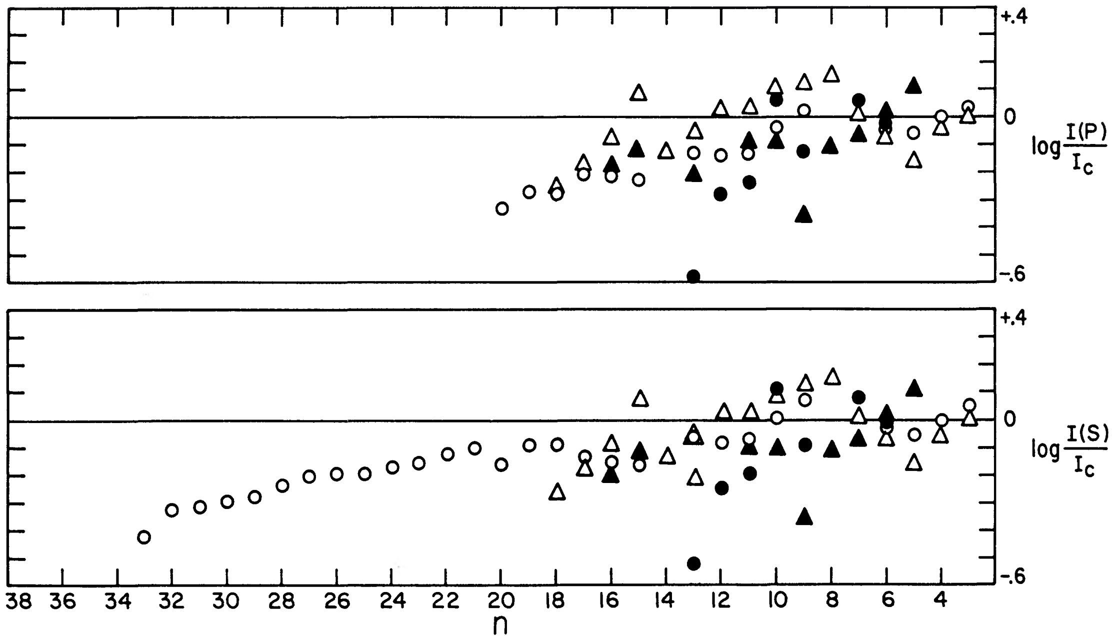

FIG. 5.—IC 4997

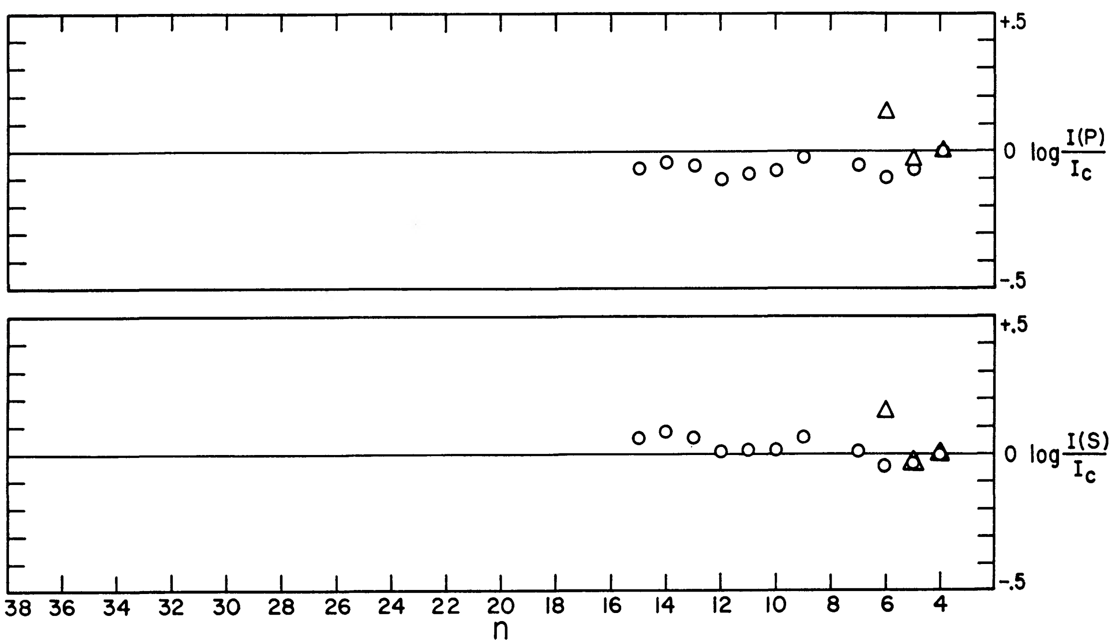

FIG. 6.—NGC 1535

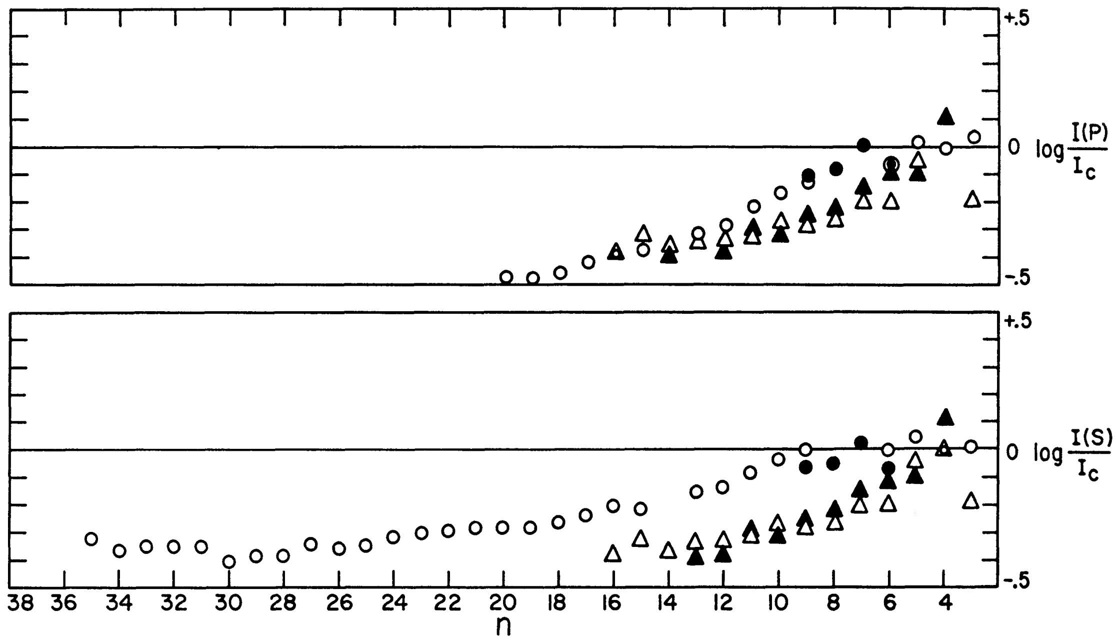

FIG. 7.—NGC 1976 = Orion Nebula. The bright central region

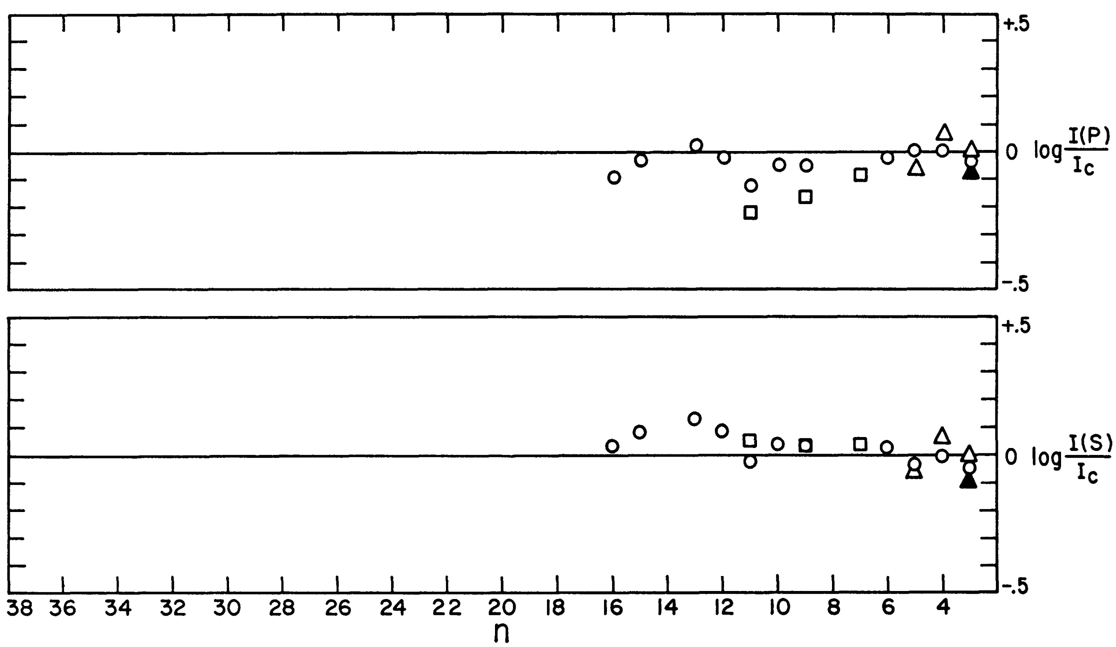

FIG. 8.—NGC 2392

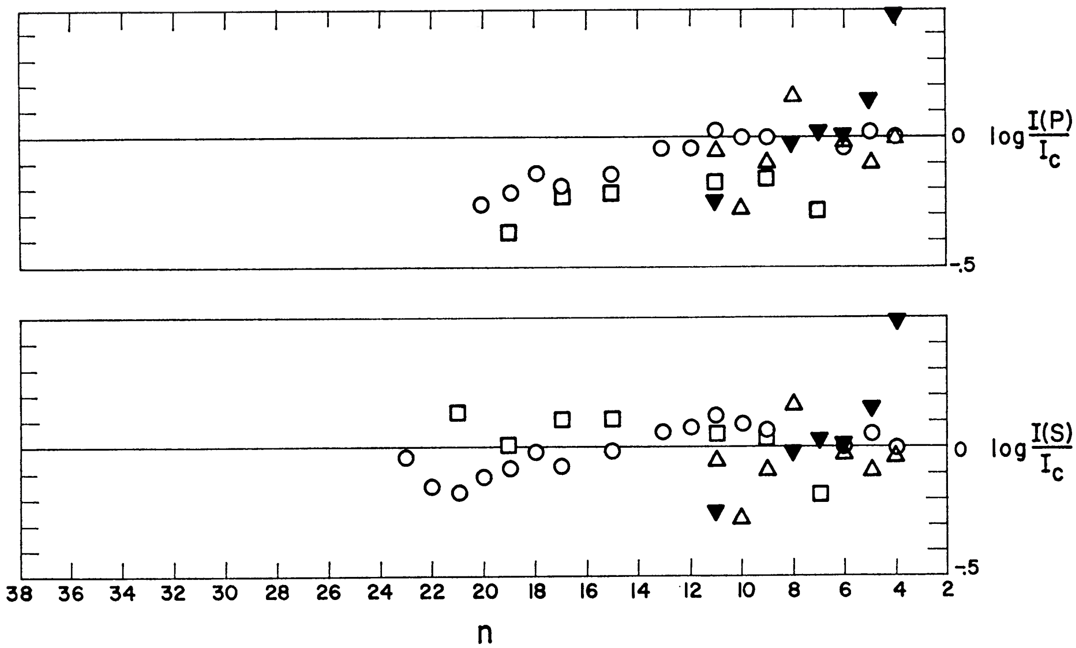

FIG. 9.—NGC 2440

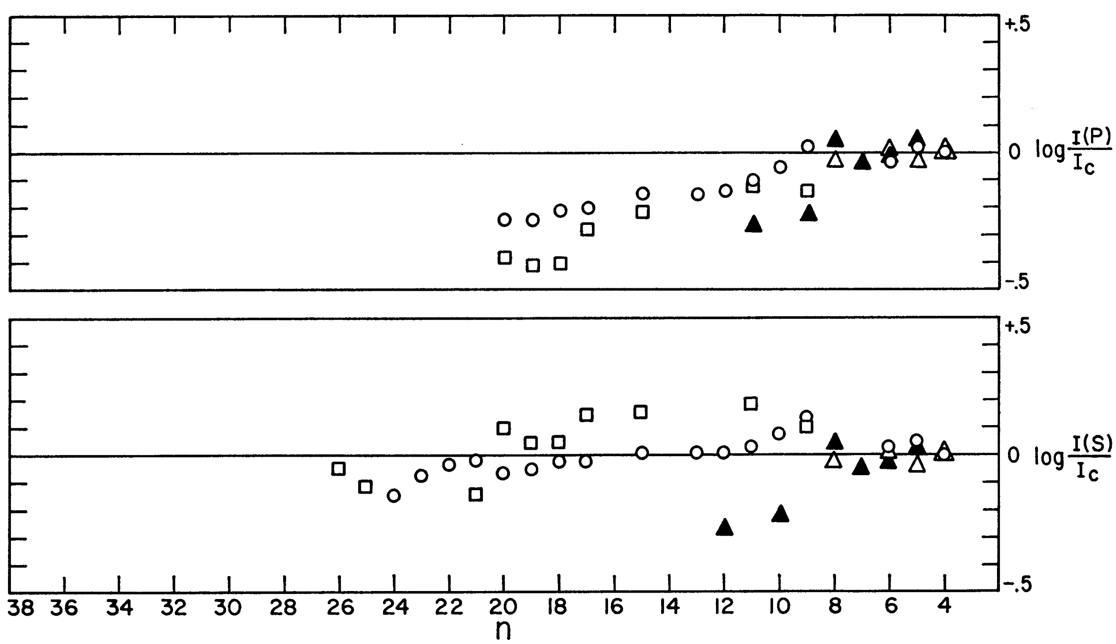

FIG. 10.—NGC 3242

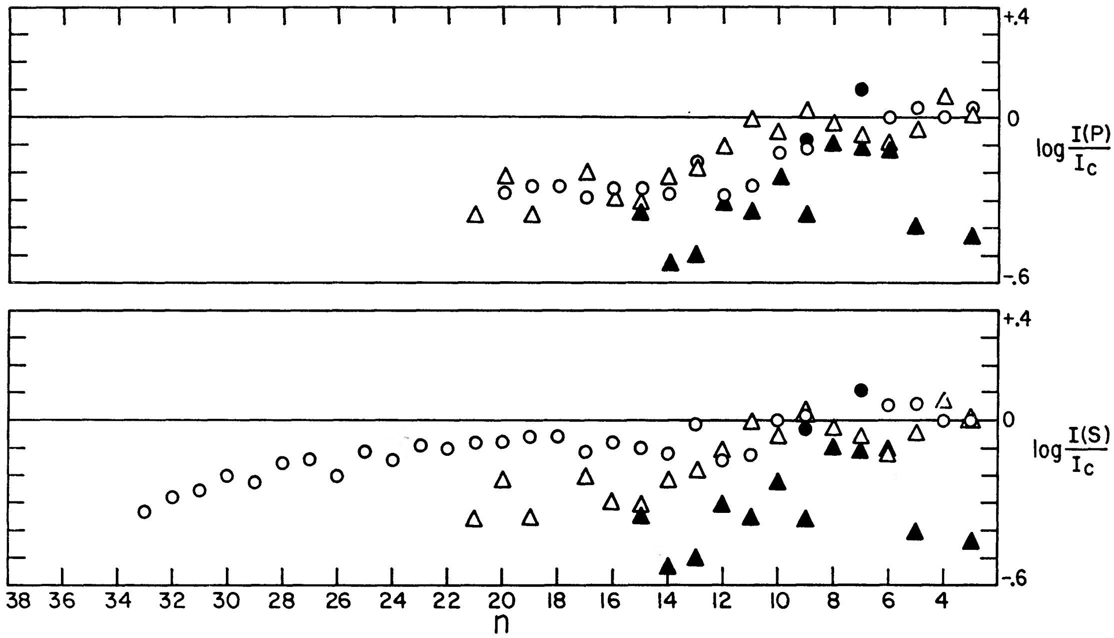

FIG. 11.—NGC 6572

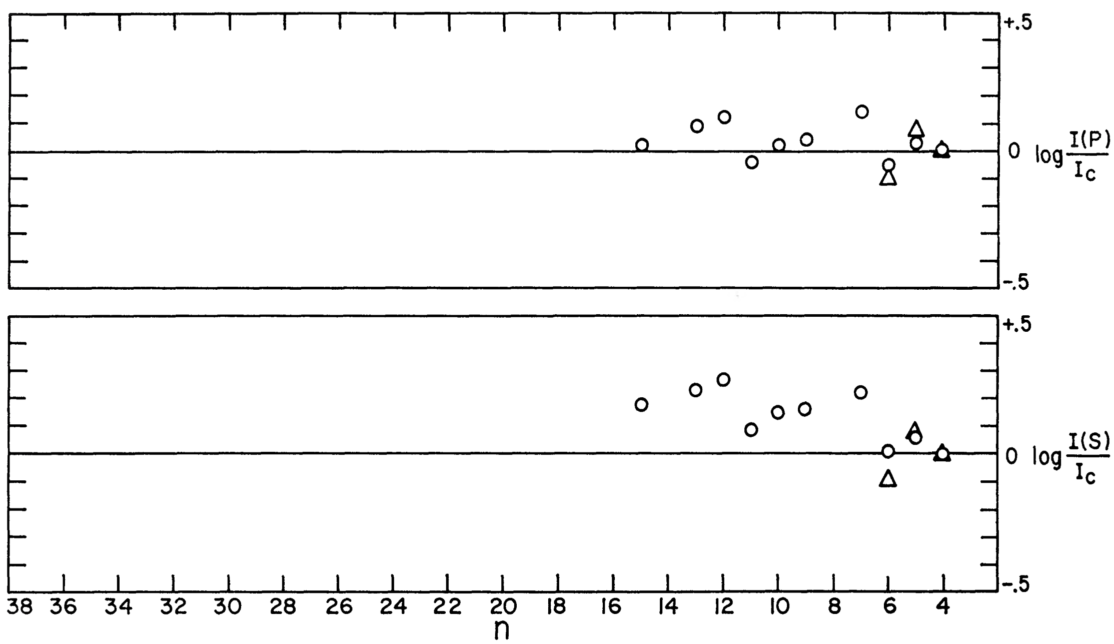

FIG. 12.—NGC 6741

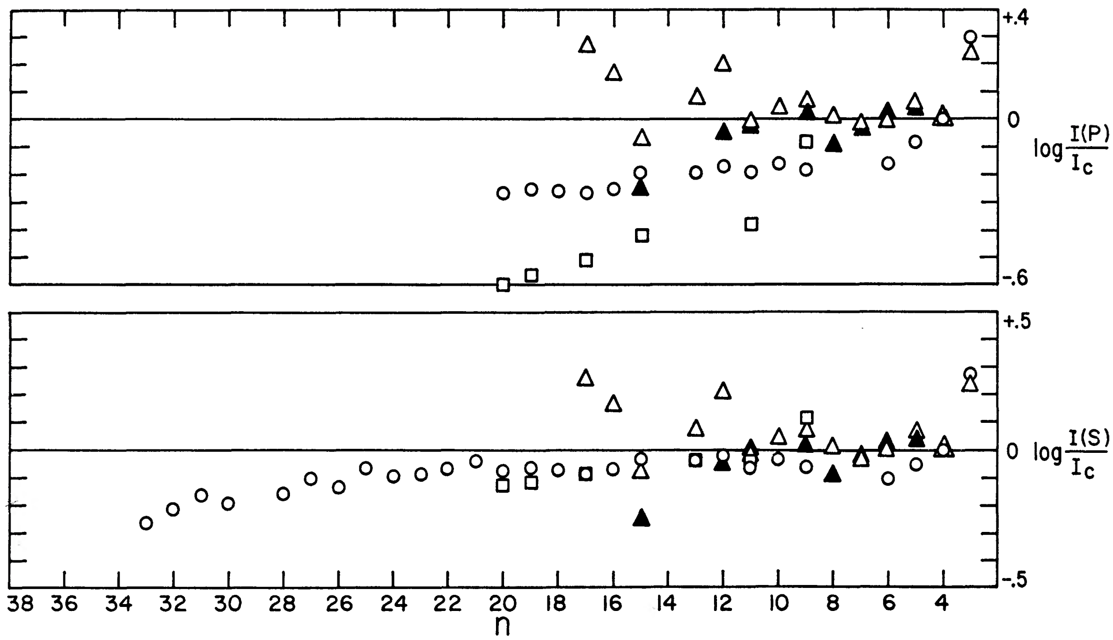

FIG. 13.—NGC 7009

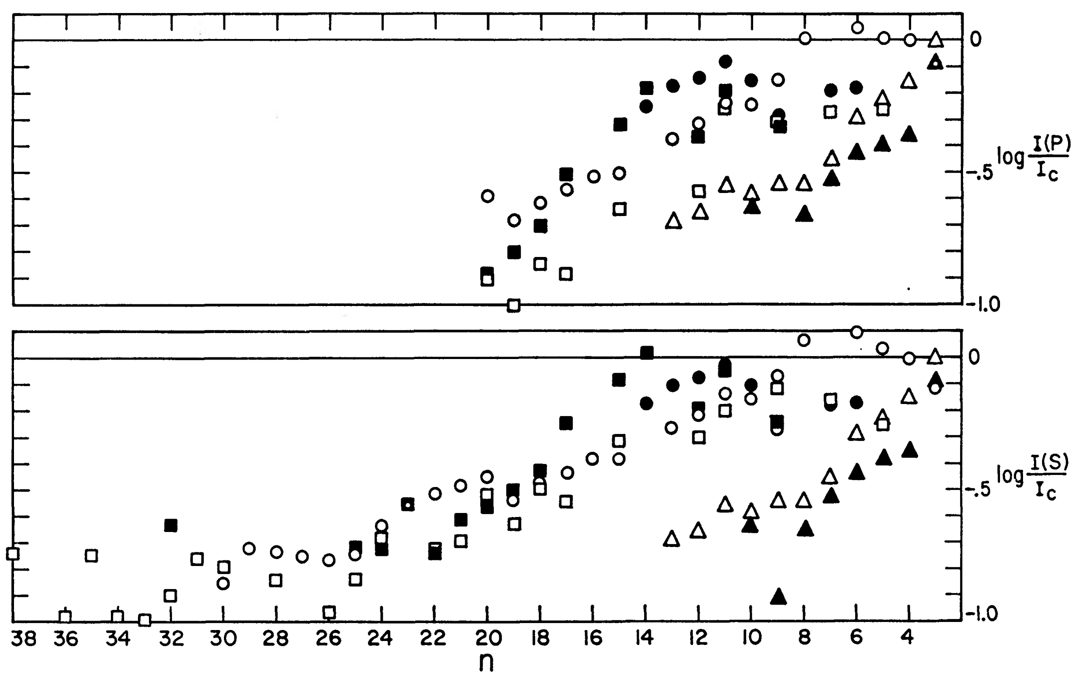

FIG. 14.—NGC 7027

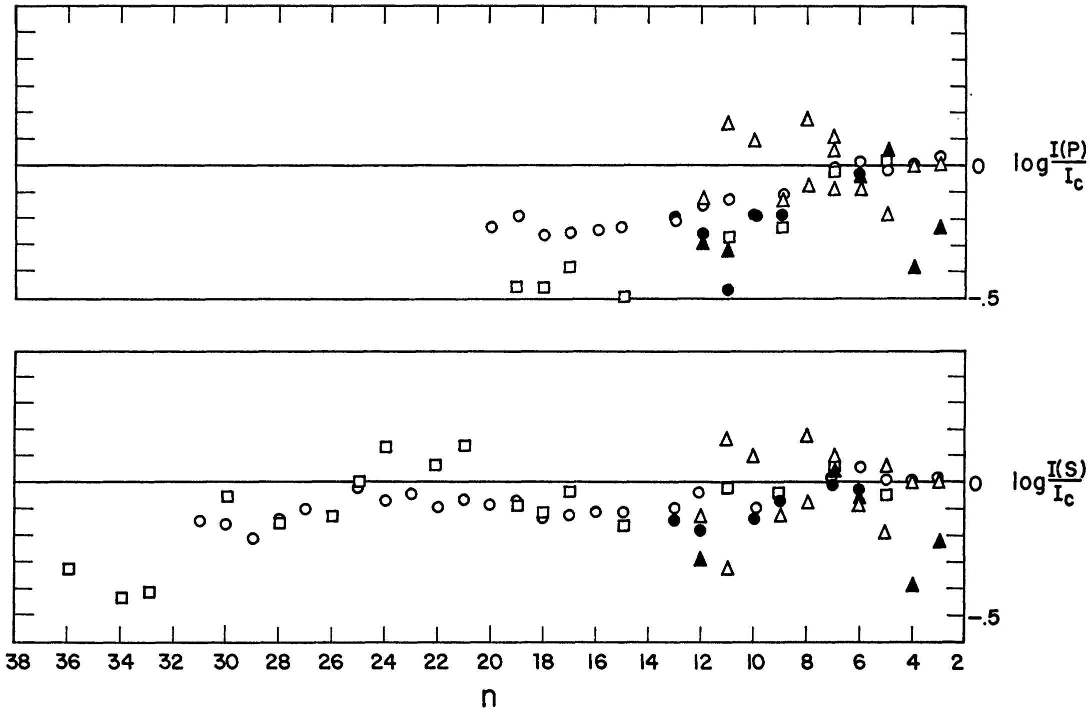

FIG. 15.—NGC 7662

TABLE 7

THE DEVIATIONS OF GROUP I NEBULAE

Nebula	log $d(n)$	n	log $d(30)$	log $d(20)$
Anon 21^h31^m	−0.11	23	−0.20	−0.05
IC 351	.00	16	.00	.00
IC 418	− .29	30	− .29	− .16
IC 2165	− .08	23	− .18	− .05
IC 4997	− .29	30	− .29	− .10
NGC 1535	.00	16	.00	.00
NGC 1976	− .38	30	− .38	− .28
NGC 2392	.00	16	.00	.00
NGC 2440	− .18	23	− .30	− .11
NGC 3242	− .09	24	− .15	− .04
NGC 6572	− .22	30	− .22	− .08
NGC 6741	.00	15	.00	.00
NGC 7009	− .16	30	− .16	− .06
NGC 7027	− .78	30	− .78	− .56
NGC 7662	−0.16	30	−0.16	−0.08

of the Balmer series. The first column of Table 7 gives the nebula, and the second gives log $d = \log I(S)/I_c$ at the highest measured Balmer line, which is itself given in the third column. In order to compare the deviations of different nebulae, the fourth column log d at $n = 30$, extrapolated if necessary, and the fifth, log d at $n = 20$, at which point nearly all the nebulae have been observed. The extrapolated values are rather rough and should be taken with some caution. They probably give lower limits to the deviations. It is hoped that the hydrogen-line intensities will be extended for these nebulae.

There are a number of striking features of the graphs. Before we interpret them, however, we should keep in mind that for the low members of the H and He II series, in the case of agreement with theory, Pengelly's intensities are appropriate, in which case log $I(S)/I_c$ should be positive, and for the high members the degenerate theory holds, and log $I(P)/I_c$ should go negative. This is just what one observes for Anon 21^h31^m (Fig. 1), IC 2165 (Fig. 4), and NGC 3242 (Fig. 10), which apparently agree fairly well with theory. NGC 1535 (Fig. 6) and NGC 2392 (Fig. 8) also appear to agree with theory, although the observations are not carried far enough for the entire pattern to be evident. It seems as if the deviations always appear if the observations are carried far enough.

Perhaps the most outstanding feature of the graphs is that the deviations always are in the sense that the observed lines are too bright as compared to Hβ. The theory appears to predict lower limits to the intensities, or a "steepest decrement."

Note particularly that the He II lines, when they are observed, deviate in the same manner and degree as do the hydrogen lines. The degree of deviation appears to depend only upon the upper level of the transition. One would expect from the collision theory that since the $nl \rightarrow nl \pm 1$ collisions become important at higher n for He^+ than for H, the He II lines would fit Pengelly's intensities to higher n. This effect is observed for NGC 2440 (Fig. 9) and NGC 3242 (Fig. 10), although some nebulae show just the opposite, e.g., Anon 21^h31^m (Fig. 1) and NGC 7662 (Fig. 15). It is clear, however, that all the observed series of H and He II deviate in a similar fashion.

The He I lines also show strong departures from theory. The singlet and triplet series of the nd–$2p$ transition arrays both deviate by the same degree. There is a great deal of scatter in the high members because of the faintness of the lines. In general, the He I points show the same slope as do the H and He II points, with the He I deviation at n the same as the H–He II deviations (either on Pengelly's or Seaton's scheme) at $2n$; i.e., the He I deviations set in at much lower n than do those of H or He II. Since Pengelly's decrements are steeper than Seaton's, naturally the slope of log $I(P)/I_c(n)$ will be greater than log $I(S)/I_c(n)$. It is seen that in some cases (NGC 6572, Fig. 11, and IC 418, Fig. 3) the slope of log $I(T)/I_c(n)$ matches that of the former, and in the others it matches that of the latter. This phenomenon is probably a reflection of the fact that Pengelly's intensities will hold good to higher n in some nebulae than in others, i.e., the $nl \rightarrow nl \pm 1$ collisions take over at higher n.

Now the number of J-states in the upper level of a transition is $2n - 1$ for H and He II, and $4n - 2$ for He I, where for the latter we take the singlet and triplet terms together. Were we to plot the deviation versus the number of J-states in the upper level rather than n, all the points would fall on roughly the same line generally when one considers the Baker-Menzel intensities of Seaton. This fact is seen simply by shifting the He I points to the left by $2n(\text{He I}) - \frac{1}{2}$. NGC 7027, which shows the most spectacular deviation (Fig. 14), gives the best example.

The nebulae of Group II have been treated somewhat differently. Because the results for any one nebula are probably not significant because of low accuracy, and because only the Balmer lines are observed, they are not graphically illustrated, but are simply tabulated in Table 8. The observations of this group are carried to only H12 or so, in the region where the $nl \rightarrow nl \pm 1$–type collisions are competing with radiative transitions, and we thus do not know which set of theoretical intensities is appropriate. The deviation given for a nebula in Table 8 represents a rough mean of the deviations computed according to the two schemes, i.e., the true theoretical intensities at H12–H15 will probably lie somewhere between the two extremes. The first column lists the nebula, the second the deviation at the highest observed n, itself given in the third column. The last column gives log d at $n = 12$. The results of this group corroborate the earlier finding: that the deviations occur in the sense of $I(\mathrm{H}n)/I(\mathrm{H}\beta)$ being too large. For some nebulae, log $I(S)/I_c$ is positive, but in these cases, log $I(P)/I_c$ agrees with theory. The $nl \rightarrow nl \pm 1$ collisions are probably not as effective for these nebulae (e.g., IC 5217, IC 5117, NGC 6309, and NGC 6790).

TABLE 8

THE DEVIATIONS OF GROUP II NEBULAE

Nebula	log $d(n)$	n	log $d(12)$
Anon 18h15m	−0.07	15	−0.06
Anon 18h47m	− .02	15	.00
Anon 22h29m	− .25	14	− .21
Anon 23h24m	− .55	13	− .41
IC 2149	.00	12	.00
IC 4846	− .14	12	− .14
IC 5117	.00	12	.00
IC 5217	.00	12	.00
J900	− .20	12	− .20
NGC 40	− .27	12	− .27
NGC 4361	− .09	13	− .08
NGC 6309	− .06	14	− .02
NGC 6720	− .26	12	− .26
NGC 6790	.00	12	.00
NGC 6803	− .20	12	− .20
NGC 6807	− .03	12	− .03
NGC 6833	− .08	12	− .08
NGC 6884	− .28	12	− .28
NGC 6886	0.00	12	0.00

VI. DISCUSSION

We must bear in mind that none of the intensities is absolute; that the hydrogen intensities are normalized to Hβ, the He II intensities to λ 4686, and the He I intensities to λ 5876. We therefore cannot say whether the high members of a series are too bright, or whether the low members are too weak. We can interpret the deviations as either an overpopulation of the high-energy levels or as an underpopulation of the low levels. It will require further work on the Balmer continuum and the region of the confluence of the Balmer lines to decide between the alternate hypotheses.

In order to find a clue for the physical cause of the deviations, the deviations have been plotted against the following parameters: reddening constant, electron temperature, and electron density derived from both surface brightness (Aller 1965) and [O II] ratios; the difference between the last two; excitation level (Aller 1965) (see Aller 1956, p. 65, for definition of excitation level), surface brightness in Hβ (Aller 1965), expansion velocity (Wilson 1950), distance and radius (O'Dell 1963), central star temperature (Aller 1965; O'Dell 1962; Vorontsov-Velyaminov 1953), absolute photographic and bolometric magnitude of the central star (Aller 1965) (these are due essentially to O'Dell 1962), and galactic latitude and longitude (Abell, private communication). No correlations can be found.

It at first comes to mind that the deviations might be caused by collisions at the high levels, but the non-existence of a correlation with density appears to rule this out. In particular, one would expect the nebula with by far the highest density to show the greatest deviation (IC 4997), which it does not. The work of Seaton (1964) would also appear to rule out this hypothesis.

If one were to assume a very low electron temperature, a fit with theory might be made, but these temperatures would be totally incompatible with the observed line ratios of other ions.

The assumption of Baker-Menzel case A does not help either, as the decrements predicted by both cases are very nearly the same.

The possibility has often been mentioned of the existence of self-absorption in the Balmer lines (Osterbrock 1962; Capriotti 1964), which has the effect of making the decrement flatter. The existence of deviations in the Pickering series and Pfund series of He II rules out this possibility as an explanation, as it is difficult to see how we could obtain significant propulations of the $n = 4$ and $n = 5$ levels of He^+.

A theory sufficient to describe the departures from the standard recombination theory must first of all explain the relation between the various series, as noted in the last section. At the present time, no such theory exists.

In conclusion, we may summarize the work as follows:

1. We have a wide range of agreement between theory and observation, from quite good to very poor.
2. The theoretical intensities appear to be lower limits to the observations.
3. Lines produced from the upper level n in H and He II show the same percentage deviation.
4. Lines produced from levels of H, He I, and He II which have the same number of J-states show the same percentage deviation.
5. No correlations between deviation and nebular parameters can at present be found.
6. There is no known cause of the deviations.

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