ON THE FISSION THEORY OF THE ORIGIN OF BINARY STARS

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ABSTRACT

The pre–main-sequence evolution of a rotating non-magnetic star is investigated. Stars forming out of interstellar gas clouds are shown to have sufficient angular momentum to cause centrifugal force to balance gravity before reaching the stable Hayashi phase, so that during subsequent evolution matter must be left behind from the equatorial regions. During contraction through the fully convective Hayashi phase the coupling of central and surface regions by convection determines a definite rotation law which we take to be uniform rotation. With continued contraction the star develops a radiative core and the "viscosity" effect of the turbulence is no longer operative; each element of the growing core therefore conserves its angular momentum causing an inward increase in angular velocity. It is shown that the ratio of centrifugal force to gravity increases in the central regions and that for stars with mass $> 0.8\ M_\odot$ rotational instability is likely to occur. This is imagined to cause the splitting of the original star into two components and so form a binary system. Assuming conservation of angular momentum on fission it is shown that stars with mass $< 4\ M_\odot$ can form a contact binary system whereas more massive stars will produce separated binaries. The theoretical limits of $0.8\ M_\odot$ and $4\ M_\odot$ for the total mass of contact binaries are in good agreement with observations of W Ursae Majoris systems, as is the distribution of total angular momentum with mass.

I. INTRODUCTION

This paper is concerned with one of the oldest hypotheses in stellar-evolution theory, namely, that a binary system originates from the splitting of a single star into two components due to rotational instability during the contraction of the star. In spite of many attempts, no satisfactory theory of this process has been developed (see Struve 1950 for an account of previous work), and for reasons of mathematical simplicity the case that has received most attention is that of an incompressible liquid with gradually increasing angular velocity, this being formally equivalent to constant angular momentum and increasing density. An account of these investigations has been given by Lyttleton (1953), where he criticizes Jeans's (1929) conclusion that a rotating liquid will split to form a binary system. However, although Jeans's analysis includes many errors, this does not necessarily mean that his conclusions were wrong. Lyttleton argues that, as the Jacobi ellipsoid becomes both dynamically and secularly unstable at the same point, then the subsequent development of the system can be examined without considering the effect of friction. This means that the equations must be time-reversible. But reversing the arrow of time in a binary system just makes the two stars go around each other in the opposite direction and not coalesce to form a single rapidly rotating star. Therefore Lyttleton argues that a single rotating star cannot form a binary system due to rotational instability.

We may criticize Lyttleton's conclusion since, even though the system becomes dynamically unstable, its subsequent development will depend on friction and the equations are not time-reversible. This is of particular importance to the theory developed here, as the rotational instability occurs in the central regions which are surrounded by a large part of the star. Again, if some material is expelled to infinity during the rotational breakup, then on reversing the arrow of time we must bring back this material, and this could be sufficient to make the whole configuration coalesce to form a single rotating object.

However, even if Lyttleton's objections can be overcome there is a much more serious

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objection to the fission theory of the origin of close binary stars. Whereas a liquid becomes unstable and breaks up either into two stars or a collection of smaller objects, a compressible object does not become unstable, provided that it is uniformly rotating and has an effective polytropic index greater than 0.8 (Jeans 1929; James 1964), and, with increasing angular velocity, centrifugal force balances gravity at the equator and during contraction the star will leave behind a disk in the equatorial plane.

The two different forms of behavior, rotational instability and equatorial mass loss, are correlated with the ratio of centrifugal force to gravity over the bulk of the star. With only a low degree of central condensation the major part of the star is influenced by rotation, whereas for a highly centrally condensed star, even though centrifugal force may balance gravity at the equator, its effect is very slight over the bulk of the star. One way of increasing the effect of rotation over the bulk of the star is to have a higher angular velocity in the central regions than in the surface layers, thus increasing the effect of rotation over the major part of the star while still keeping centrifugal force less or equal to gravity at the equator. We shall show that during the pre–main-sequence of a rotating star, this does indeed occur.

At the present state of the theory it is difficult to give any reliable criterion for fission rather than equatorial mass loss. It is known that a rotating liquid reaches a point of bifurcation when

$$\alpha = \frac{\Omega^2}{2\pi G\rho} = 0.187\,, \tag{1.1}$$

where Ω is the angular velocity and ρ the density, and then with increasing Ω the liquid evolves along the Jacobi series until $\alpha = 0.142$, at which stage the liquid becomes unstable and fission may occur; α decreases along the Jacobi series so that its maximum value is 0.187 where the Maclaurin series becomes unstable. If we are to take α as a stability parameter we must conclude that instability will occur when α exceeds the value 0.187. For a uniformly rotating polytrope of index 1.5, centrifugal force balances gravity at the equator before instability can set in, and then (Monaghan and Roxburgh 1965)

$$\alpha = \frac{\Omega^2}{2\pi G\rho_c} = 0.04\,, \tag{1.2}$$

where ρ_c is the density at the center. Now in the central regions of a star the density is approximately constant (the density gradient is zero at the center), so we may suppose that such regions behave like a liquid. Consequently in the subsequent work we shall suppose that a star becomes rotationally unstable and splits into two, when

$$\alpha = \frac{\Omega^2}{2\pi G\rho_c} = 0.187\,. \tag{1.3}$$

For a polytrope of index 1.5 this corresponds to an inward increase in angular velocity by a factor of 2. This is in fair agreement with the results of Stoeckly (1965). An upper bound on α is given by the condition that centrifugal force balances gravity in the central regions, and this gives

$$\alpha = \frac{\Omega^2}{2\pi G\rho_c} < 0.667\,. \tag{1.4}$$

Fortunately the results of the theory given here are not that sensitive to the value of α, and the other uncertainties in the theory are greater than an error of 2 in the value of α. Consequently we shall use condition (1.3) as our condition for rotation to cause fission of the central regions.

Of the close binary systems that we may hope to form by fission by far the most numerous are contact systems of the W Ursae Majoris type. These systems are observed to form a very compact group of stars, with spectral types lying between F0 and G9. Of the systems that have been well determined (Kopal and Shapley 1956) it is found that the total mass of the systems lies between 0.74 and 3.8 $M_{\odot}$ and the mass ratio is of order of 0.5. The mass ratio and detailed structure of such configurations will be considered in a separate communication as this is a problem of the structure of main-sequence contact binaries rather than one of formation of such systems, but a theory of the origin of close binary stars should aim at explaining the mass range and the angular momentum of such systems, and this will be the testing ground for our

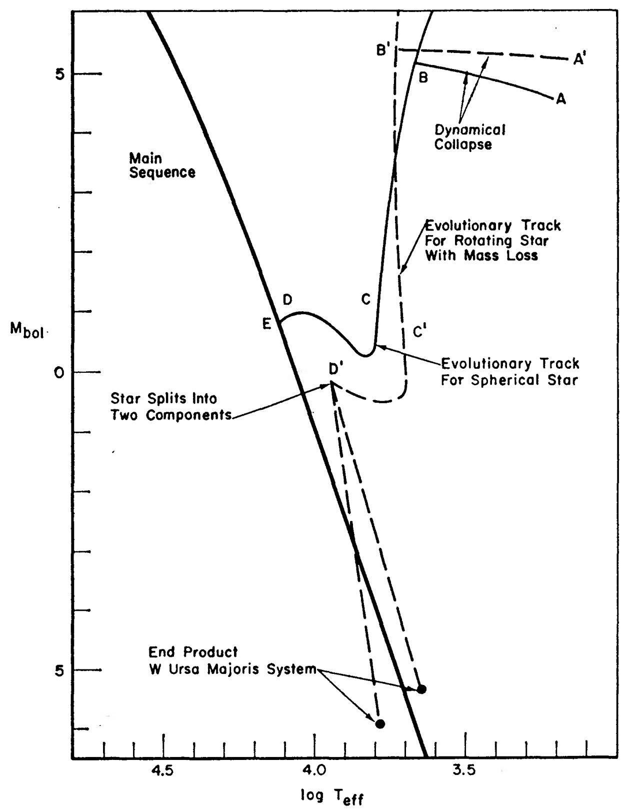

FIG. 1.—Pre-main-sequence evolution of non-rotating and rotating stars

theory. The predominance of W Ursae Majoris systems over other types of binary systems is due to the preferential formation of stars with masses from 0.5 to 4 $M_{\odot}$, and is a property of the original dynamical collapse of the gas cloud from which the stars were formed.

II. PRE-MAIN-SEQUENCE CONTRACTION OF SPHERICAL STARS

The dynamical collapse of a gas cloud ceases when the internal pressure has increased sufficiently to halt the motion and the star will then oscillate about a position of hydrostatic equilibrium; at this stage the star has a radius of order 50 times its main-sequence value, point B on the track in Figure 1. Subsequent contraction is then only necessary to release the energy that is radiated away from the surface. When the star has such large dimensions, the surface conditions force the star to be fully convective and have a high luminosity (Hayashi 1961). Contraction continues releasing energy to be radiated away, and the star moves almost vertically in the H-R diagram (section BC in Fig. 1) during which period it is fully convective.

With continued contraction and decreasing luminosity it will eventually be possible to transport the energy with a subadiabatic temperature gradient, at which stage the convection begins to die away. This first starts at the center of the star. Point C on the track in Figure 1 is where the radiative core starts to develop.

The star continues to contract with a growing radiative core until almost all the star is in radiative equilibrium. As the star approaches the main sequence, the more massive stars will develop a convective core. Section CD of the track in Figure 1 is the stage during which the radiative core is growing, and section DE is the stage of adjustment to the main-sequence configuration during which the convective core begins to grow.

Results of computations on this stage of evolution are given in Table 1, where the numbers are taken from Iben (1965). The central density at point C in Figure 1 is ρ_{cH}, ρ_{cM} is the central density in the main-sequence configuration, R_H the radius at point C, and R_{MS} the main-sequence radius of the star.

TABLE 1

SUMMARY OF RESULTS OF PRE-MAIN-SEQUENCE MODELS

$M/M\odot$	$\log \rho_{cH}$	$\log \rho_{cM}$	$\log R_H/R\odot$	$\log R_{MS}/R\odot$
0.5.......	+0.95	1.95	−0.11	−0.28
1.0.......	−0.27	1.95	+0.40	− .06
1.5.......	−1.09	1.87	+0.73	+ .07
3.0.......	−2.53	1.67	+1.31	+ .24
5.0.......	−3.5	1.29	+1.75	+0.38

III. PRE-MAIN-SEQUENCE CONTRACTION OF ROTATING STARS

If the star is rotating its evolutionary pattern will be different from that of a spherical star. If we try to form stars out of interstellar gas clouds of a density of 10^{-20} gm cm^3 rotating with the galactic angular velocity $\Omega \simeq 10^{-15}$ sec^{-1}, then the ratio of centrifugal force to gravity before collapse is

$$\frac{\Omega^2 R^3}{GM} \simeq 4.10^{-4} \tag{3.1}$$

with dynamical contraction (the track $A'B'$ in Fig. 1) and conservation of angular momentum; then this ratio increases like R^{-1}. By the time the star reaches point B', where the collapse is stopped by the pressure force, the radius has decreased by a factor of 10^5 and the ratio of centrifugal force to gravity exceeds unity. This is clearly not possible and the star must leave behind some material, but on reaching the point of dynamical stability we may expect the star to be rapidly rotating, with centrifugal force balancing gravity at the equator.

As in the spherical case, once the star becomes dynamically stable then the surface conditions are such as to make the star completely convective and to have a very high luminosity. The effect of the turbulence is not only to transport energy outward, but also to completely mix the star and create a very large "turbulent viscosity." This coupling of the internal and external regions by convection will maintain the star in uniform rotation if the turbulence is approximately isotropic. The subsequent contraction can then be investigated assuming uniform rotation (Roxburgh 1965). The star then contracts loosing matter from the equator, thus changing its angular momentum. If the star has equatorial radius R_e, angular velocity Ω, mass M, then the balance of gravity and centrifugal force at the equator gives

$$\frac{\Omega^2 R_e^3}{GM} = 1 . \tag{3.2}$$

The angular momentum of the star is

$$H = kMR_e^2\Omega = kG^{1/2}M^{3/2}R_e^{1/2} , \tag{3.3}$$

where $k^{1/2}R_e$ is the radius of gyration of the star. For a completely convective fully rotating star, the parameter k is 0.132 (Roxburgh 1965) and is even smaller for a radiative star, being of order 0.04. The rate of change of angular momentum is equal to the angular momentum carried away by the mass loss, so

$$\frac{d}{dt}\left(kG^{1/2}M^{3/2}R_e^{1/2} \right) = \frac{dM}{dt} R_e^{1/2}G^{1/2}M^{1/2} , \tag{3.4}$$

which integrates to give

$$M \propto R^{k/(2-3k)} , \tag{3.5}$$

so that the angular momentum

$$H \propto R^{1/(2-3k)} . \tag{3.6}$$

With such a small value of k, the mass does not change very much and $H \propto R^{1/2}$.

The star contracts down the track $B'C'$ in Figure 1, losing mass and angular momentum until it reaches point C', where the energy can be transported by radiation and a radiative core begins to develop, just as in the spherical case. Since the centrifugal force is only a small perturbation over most of the star, we can take the results of the investigations of spherical stars to indicate when point C' is reached. The development of the radiative core now produces a significant change in the star's evolution. As long as the star was completely convective we could assume that the turbulence distributed the angular momentum throughout the star, so that uniform rotation was a valid approximation. With the growth of the radiative core the inner regions are no longer coupled to the outer regions and the star no longer rotates even approximately uniformly.[1] Each element of the radiative core contracts conserving its angular momentum, as the star continues to lose mass from the convective envelope. The star moves along the track $C'D'$ in Figure 1.

As long as the star was in uniform rotation then centrifugal force balanced gravity at the equator causing matter to be left behind in a disk, and α was too small to cause rotational instability. However with the uncoupling of the central regions from the surface by the development of the radiative core, the effect of rotation can be felt in the central regions, a prerequisite for fission of a fluid form. It is in the very central regions where the rotational effect first becomes significant.

At the end of the fully convective phase we find

$$\alpha = \frac{\Omega^2}{2\pi G\rho_c} = 0.04 \tag{3.7}$$

(Monaghan and Roxburgh 1965). If we consider a small volume surrounding the center, then this contracts conserving its angular momentum so that $\Omega \propto R^{-2}$. The density $\rho \propto R^{-3}$ so that

$$\alpha = \frac{\Omega^2}{2\pi G\rho_c} = 0.04 \left(\frac{\rho_c}{\rho_{cH}}\right)^{1/3} , \tag{3.8}$$

[1] Assuming there is no other coupling mechanism such as a magnetic field.

where ρ_c is the central density at any stage and ρ_{cH} is the value at the end of the completely convective phase. The parameter α can then be calculated at any stage during the subsequent contraction, and if it becomes larger than the limit for stability 0.187, the central region will become unstable and split into two.

In Table 2 we give the values of α when the star reaches its point of maximum central density before the growth of any convective core. This exceeds the critical value 0.187 for stars with $M > 0.8\ M_\odot$. Also in Table 2 we give the radius of the star at the point where $\alpha = 0.187$, R_I, using Iben's (1965) results for the variation of central density with radius. We deduce that stars of mass greater than 0.8 $M_\odot$ will become unstable in the central regions and will possibly split into two stars. The agreement between the lower limit and the observed lower limit of W Ursae Majoris stars $\simeq$ 0.74 $M_\odot$ (Kopal and Shapley 1956) is remarkable.

TABLE 2

CRITICAL VALUES DURING CONTRACTION

$M/M_\odot$	$(\Omega^2/2\pi G\rho_c)_{MS}$	$\log (R_I/R_\odot)$
0.5........	0.093	....
1.0........	0.24	0.05
1.5........	0.40	0.33
3.0........	0.93	0.86
5.0........	1.65	1.30

TABLE 3

ANGULAR MOMENTUM AT DIFFERENT STAGES OF COLLAPSE

$M/M_\odot$	$\log H_H$	$\log H_L$	$\log H_U$	$\log H_I$	$\log H_d$
1.0.......	51.10	50.92	51.03	50.98	51.28
1.5.......	51.52	51.32	51.43	51.38	51.60
3.0.......	52.27	52.04	52.16	52.10	52.17
5.0.......	52.82	52.59	52.71	52.65	52.56

IV. ANGULAR MOMENTUM OF THE BINARY SYSTEM

A question of considerable interest is how much angular momentum does the system have at the onset of instability? With the radius at the end of the Hayashi phase given in Table 1 we can compute the angular momentum of the star, H_H, using equation (3.3) with $k = 0.132$; this is given in Table 3. However, this is not equal to the angular momentum at the onset of instability H_I since the star continues to contract, losing matter and angular momentum from a decreasing outer convective zone until it reaches the unstable state. A lower limit on the angular momentum at the onset of instability, H_L, is given by assuming the star to remain completely convective and hence uniformly rotating since this would correspond to complete convection, and then by equation (3.6) we have

$$\log \left(\frac{H_L}{H_H}\right) \simeq \tfrac{1}{2} \log \left(\frac{R_I}{R_H}\right). \tag{4.1}$$

The values of H_L computed using this formula are given in Table 3.

An upper limit on the angular momentum at the onset of instability, H_U, can be computed by assuming conservation of angular momentum by each element during the contraction from the end of the Hayashi phase to the onset of fissional instability, since this would be the correct assumption if the star were completely radiative. With this assumption there will be a distance R_c from the center of the fully convective star, such that on contraction with angular momentum conservation the material originally at R_c is at the surface of the star at the onset of instability. Material originally outside the distance R_c would have too large an angular velocity to be part of the star and is therefore lost from the system. R_c is given by the condition

$$\Omega_H R_c^2 = \Omega_I R_I^2 , \tag{4.2}$$

where Ω_H and Ω_I are the angular velocity at the end of the fully convective stage and at the surface of the star at the onset of fissional instability, respectively. With centrifugal force balancing gravity at the equator, this gives

$$\frac{G^{1/2}M^{1/2}}{R_H^{3/2}} R_c^2 = G^{1/2}M^{1/2}R_I^{1/2} , \tag{4.3}$$

provided the mass loss is small. Hence

$$R_c = (R_H^{3/2}R_I^{1/2})^{1/2} . \tag{4.4}$$

TABLE 4

VALUES OF RADII DURING COLLAPSE

$M/M_\odot$	$R_H/R_\odot$	$R_I/R_\odot$	R_c/R_H	ξ_c	$R_{M/2}/R_\odot$
1.	2.51	1.12	0.818	2.987	0.525
1.5.	5.37	2.14	.794	2.901	0.676
3.0.	20.42	7.24	.772	2.820	1.17
5.0.	56.23	19.95	0.772	2.820	1.58

With R_I and R_H given in Tables 1 and 2, the value of R_c/R_H is readily calculated and the results are given in Table 4. With R_c known we can now calculate the angular momentum loss by assuming all the material exterior to R_c is lost to the system. This gives

$$\Delta H = \Omega_H \int_{R_c}^{R_H} \frac{8\pi}{3} \rho r^4 dr . \tag{4.5}$$

Since the convective star is approximately polytrope of index 1.5 we can express this as

$$\frac{\Delta H}{H_H} = \left(\int_{\xi_c}^{\xi_0} \theta^{1.5} \xi^4 d\xi\right) \Big/ \left(\int_0^{\xi_0} \theta^{1.5} \xi^4 d\xi\right), \tag{4.6}$$

where θ and ξ are the ordinary polytropic variables, $\xi_0 = 3.6538$ the value of ξ at which $\theta = 0$, and $\xi_c = R_c\xi_0/R_H$. The values of ξ_c are given in Table 4. Since θ is the Emden function of index 1.5, these integrals are readily evaluated and the upper limit on the angular momentum at the onset of instability, $H_U = H_H - \Delta H$, is readily calculated. The results are shown in Table 3 (see above).

The actual value of the angular momentum must be somewhere between the two values H_I and H_U and for simplicity we shall take

$$H_I = \frac{H_U + H_L}{2} , \tag{4.7}$$

and these values are given in Table 3 and illustrated in Figure 2. As the two determinations are so close, we may expect H_I to give an adequate estimate of the angular momentum at the onset of instability.

The evolution of the system after the onset of instability is, of course, difficult to discuss, and all we can do is to look for the final configuration. If we assume that after instability the star splits into two stars which continue to contract but do not eject matter and hence conserve total angular momentum, a tentative discussion is possible.

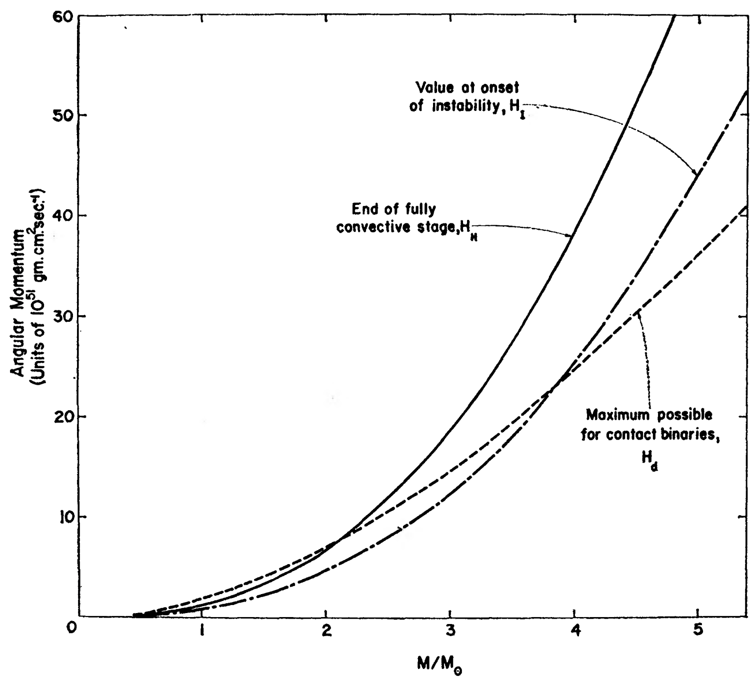

FIG. 2.—Theoretical angular momentum for contracting and binary stars

If the system is to form a pair of main-sequence stars then, assuming components of roughly equal mass, the total orbital angular momentum is

$$H_d = \tfrac{1}{4} G^{1/2} M^{3/2} d^{1/2} , \tag{4.8}$$

where d is the distance apart of the two stars. If the stars are to form a contact configuration then the diameter of each lobe will be approximately $\frac{5}{2}$ $R_{M/2}$ where $R_{M/2}$ is the main-sequence radius of a star of mass $M/2$ (see Fig. 3). The distance apart is therefore 3 $R_{M/2}$, and the angular momentum is

$$H_d = \frac{\sqrt{3}}{4} G^{1/2} M^{3/2} R_{M/2}{}^{1/2} . \tag{4.9}$$

The values of $R_{M/2}$ are given in Table 4 and the resulting values of H_d are given in the final column of Table 3. This is the maximum amount of angular momentum that can be stored in a contact binary configuration. If the system has more angular momentum, it must consist of two separated stars while if it has less it must have a common envelope. The relation between angular momentum and mass given by equation (4.9) is illustrated in Figure 2.

An examination of Figure 2 brings out the fact that the curves for H_I and H_d intersect at about $M = 4\ M_{\odot}$. Stars more massive than this have too much angular momentum at the onset of instability to form a contact configuration and, therefore, assuming angular momentum conservation, they must form a separated system. Stars with $M < 4\ M_{\odot}$ can become contact configurations with a common envelope. The agreement between this limit and the observed maximum total mass of W Ursae Majoris stars of $3.8\ M_{\odot}$ (Kopal and Shapley 1956) is satisfactory.

The resulting variation of angular momentum with mass is given in Figure 4 where it is compared with the observational values of W Ursae Majoris stars. (Observational results from Kopal and Shapley 1956.) The agreement is satisfactory.

The main point of this discussion is to emphasize that there exist certain theoretical limits on the total mass of a contact binary system. The numbers $0.8\ M_{\odot}$ and $4\ M_{\odot}$ found here are at best only a crude estimate of these limits since the theory used is only approximate, but the fact that they are the right order of magnitude is at least encouraging. The upper mass limit is the most difficult to determine since it depends on the assumption of conservation of angular momentum after the onset of instability.

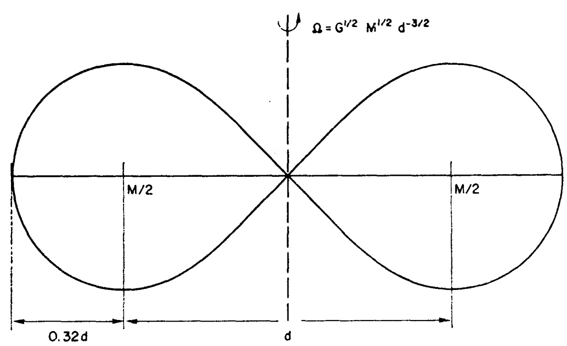

FIG. 3.—Contact binary configuration with maximum angular momentum for a given total mass

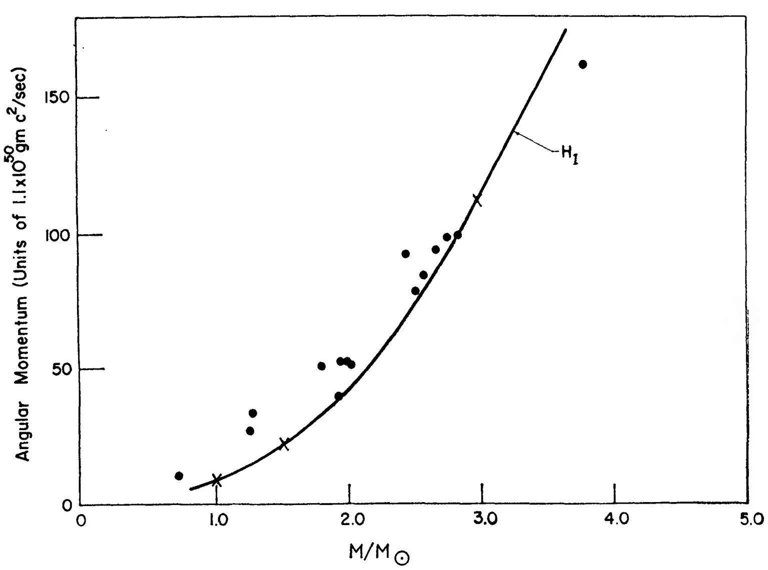

FIG. 4.—Comparison of predicted angular momentum—mass relation with observations of W Ursae Majoris stars.

The next step in the refinement of the theory is to integrate accurate models of this pre–main-sequence stage of contraction of rotating stars, at each stage testing the star for stability. We hope to consider this problem in a subsequent publication.

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REFERENCES

Hayashi, C. 1961, *Pub. Astr. Soc. Japan*, **13**, 450.
Iben, I. 1965, *Ap. J.*, **141**, 993.
James, R. 1964, *Ap. J.*, **140**, 551.
Jeans, Sir J. 1929, *Astronomy and Cosmogony* (Cambridge: Cambridge University Press).
Kopal, Z., and Shapley, L. 1956, *Element of Binary Stars* (Joddrell Bank Annals, Vol. 1).
Lyttleton, R. 1953, *The Stability of Rotating Liquid Masses* (Cambridge: Cambridge University Press).
Monaghan, J., and Roxburgh, I. 1965, *M.N.*, **131**, 13.
Roxburgh, I. 1965, *Zs. f. Ap.* (in press).
Stoeckly, R. 1965, *Ap. J.*, **142**, 208.
Struve, O. 1950, *Stellar Evolution* (Princeton, N.J.: Princeton University Press).