NOTES

ON THE DENSITY OF NEUTRAL HYDROGEN IN INTERGALACTIC SPACE

Recent spectroscopic observations by Schmidt (1965) of the quasi-stellar source 3C 9, which is reported by him to have a redshift of 2.01, and for which Lyman-α is in the visible spectrum, make possible the determination of a new very low value for the density of neutral hydrogen in intergalactic space. It is observed that the continuum of the source continues (though perhaps somewhat weakened) to the blue of Ly-α; the line as seen on the plates has some structure but no obvious asymmetry. Consider, however, the fate of photons emitted to the blue of Ly-α. As we move away from the source along the line of sight, the source becomes redshifted to observers locally at rest in the expansion, and for one such observer, the frequency of any such photon coincides with the rest frequency of Ly-α in his frame and can be scattered by neutral hydrogen in his vicinity. The calculation of the size of the effect is very easily performed as follows:

Let us consider a cosmological model with the metric

$$ds^2 = dt^2 - R^2(t)\,(du^2 + \sigma^2(u)d\gamma^2)\,,$$

where

$$\sigma(u) = \begin{cases}\sin u\,, \\ u \\ \sinh u\end{cases}$$

depending on the curvature, and $d\gamma$ is the increment in angle.

The probability of scattering of a photon in a *proper* length interval $dl = R(t)du$ at cosmic time t is clearly

$$dp = n(t)\sigma(\nu_s)dl\,,$$

where $n(t)$ is the number of neutral hydrogen atoms per unit volume at time t (assumed uniform for the present) and $\sigma(\nu)$ is the radiative excitation cross-section for the Ly-α transition. This has the form

$$\sigma(\nu) = \frac{\pi e^2}{m c} f g(\nu - \nu_a)\,,$$

where f is the oscillator strength (here $f = 0.416$), and g the profile function, which is strongly peaked at $\nu = \nu_a$, the Ly-α frequency (2.46×10^{15} sec^{-1}), and has unit integral:

$$\int_{-\infty}^{\infty} g(x)\,dx = 1\,.$$

Let the redshift of the object observed be z_0, and suppose the redshift of the resonance scattering layer as seen from here is z; $z < z_0$, and if the observed frequency is ν,

$$\nu_s = \nu(1 + z)$$

is the frequency seen by an observer stationary with respect to the scattering layer. Thus the total optical depth at ν is

$$p = \int_0^{z_0} dp = \int_0^{z_0} n[t(z)]\,\sigma[\nu(1+z)]\,\frac{dl}{dz}\,dz\,.$$

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Now dl/dz is clearly something like cH^{-1}. It can in fact be shown easily using the development angle formalism (Sandage 1961*b*) that

$$\frac{dl}{dz}=R(t)\frac{du}{dz}=\frac{cH_0^{-1}}{(1+z)^2(1+2q_0z)^{1/2}}$$

for the relativistic models with vanishing pressure and cosmological constant, and with deceleration parameter q_0 (see, e.g., Sandage 1961*a* for a discussion of these models). For the steady-state model, $u = z$, and so

$$\frac{dl}{dz}=\frac{cH^{-1}}{1+z}.$$

We then have, for the relativistic models,

$$p=\int_1^{Hz_0}\left\{\frac{n[t(z)]\pi e^2 f}{mH_0\nu(1+z)^2(1+2q_0z)^{1/2}}\right\}g[\nu(1+z)-\nu_a]\nu d(1+z).$$

The function g is strongly peaked at zero; its width depends on the intergalactic temperature, but even at 10^6 ° K its width expressed in velocity units is only 2×10^{-4} c (compared to the redshift, which is of the order of 2). Thus we can take the factor in braces out of the integral, evaluated at $(1 + z) = \nu_a/\nu$; the integral that is left is unity, and we obtain (for $H_0 = 10^{-10}$ yr^{-1})

$$p=\frac{n_s}{(1+z)(1+2q_0z)^{1/2}}\left(\frac{\pi e^2 f}{m\nu_a H_0}\right)\simeq(5\times 10^{10}\text{ cm}^3)\frac{n_s}{(1+z)(1+2q_0z)^{1/2}}.$$

Here n_s is the number density of neutral hydrogen in the scattering region. The flux will be reduced, of course, by a factor e^{-p}, but it is difficult to say just how large the effect is on the plates of 3C 9. Intensity tracings were made of two plates, and tracings of the neighboring night-sky spectra allowed approximate subtraction of the night-sky contribution. The blueward component and the wing of the line are noticeably depressed (they are enhanced on the plates by a strong, broad night-sky feature at 3640 Å that gives the line an almost symmetric profile before the sky is removed), but the exact amount is difficult to measure; best estimates place the depression at about 40 per cent, which corresponds to an optical depth of about $\frac{1}{2}$. This yields, for $q_0 = \frac{1}{2}$, a number density $n_s = 6\times 10^{-11}$ cm^{-3}, or a mass density $\rho_s = 1\times 10^{-34}$ gm cm^{-3}—a figure five orders of magnitude below the limit (for the *present* density, which should be 27 times smaller because of the expansion) obtained from 21-cm observations by Field (1962).

For the $q_0 = \frac{1}{2}$ model, the total density at $z = 2$ is 5×10^{-28} gm cm^{-3}; thus only about one part in 5×10^6 of the total mass at that time could have been in the form of intergalactic neutral hydrogen. For the steady-state model $\rho_s = 2\times 10^{-35}$ and the (constant) total density is 4×10^{-29}; the factor here is somewhat less, about 2×10^6.

We are thus led to the conclusion that either the present cosmological ideas about the density are grossly incorrect, and that space is very nearly empty, or that the matter exists in some other form. Oort has shown that only about 1 per cent of the $q_0 = \frac{1}{2}$ density is accounted for by galaxies, and it has been generally assumed that the remainder exists as an intergalactic gas which is presumably mostly or entirely hydrogen. It is possible that this interpretation is still valid but that essentially all of the hydrogen is ionized; this conclusion can be defended if we are allowed to make the intergalactic electron temperature high enough. Field and Henry (1964) have shown that the temperature cannot exceed 2×10^6 ° K for the steady state and about 4×10^5 ° K for the evolving models in the vicinity of $q_0 = \frac{1}{2}$. Higher temperatures than these would result in more free-free (bremsstrahlung) emission from the intergalactic medium than the

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total flux observed in the X-ray region. One finds (Allen 1963) that the mean recombination time for ionized hydrogen is about

$$t_r = 1.2 \times 10^{11}\, T^{1/2} n^{-1}\, [\ln (1+1.58 \times 10^6/T)]^{-1}\,,$$

where T is the electron temperature and n the total number density. Consider first the $q_0 = \frac{1}{2}$ relativistic case. At $n = 3 \times 10^{-4}$ (the value at $z = 2$, assuming essentially all the density in the form of hydrogen) and $T = 2 \times 10^5\,°$ K (corresponding to a present value of 2×10^4 and adiabatic expansion), $t_r = 7 \times 10^{16}$ sec, or 2×10^9 years. At $z = 2$, the age of the universe was 2×10^9 years; we must thus find a mechanism by which the ionization level may be *maintained* at 5×10^6. This requires a process with a mean ionization time per hydrogen atom of $t_{\text{ion}} = 1.6\; 10^{10}$ sec, or 500 years. Electron collisions are inadequate; the mean lifetime for collisional ionization is about (Allen 1963)

$$t_{\text{coll}} = 1.75 \times 10^{10}\, T^{-1/2} n^{-1} (1 + T/1.6 \times 10^6) \exp (1.58 \times 10^5/T)\,,$$

or about 10^4 years. Consider now radiative ionization: we find that we need a mean intensity at the Lyman limit of 1.2×10^{-20} erg cm^{-2} (c/s)$^{-1}$ sec^{-1} ster^{-1}, if the spectrum does not fall off too rapidly to the blue.

The flux can come from three sources; normal galaxies, radiogalaxies, and QSS's, and the intergalactic medium itself. The contribution from the first two sources can be estimated roughly, and almost certainly does not exceed 3×10^{-24} units at $z = 2$, of which about 10 per cent is from quasi-stellar sources (assuming that one can extrapolate the visual radiation into the UV with a spectral index of -0.7, and assuming a present space density of [600 Mpc]$^{-3}$).

The intergalactic medium is more promising. For $q_0 = \frac{1}{2}$, the computations of Field and Henry give, for $T = 2 \times 10^5\,°$ K at $z = 2$, an intensity 6×10^{-21} units at the Lyman limit. This is a lower bound, and correction of the Gaunt factor (Karzas and Latter 1961) for this temperature and frequency range gives an estimate of about 1.1×10^{-20} units, which is very nearly equal to the required value; the treatment of the singularity at $t = 0$ is quite uncertain, and the excellent agreement is probably fortuitous. Going to larger q_0 increases the free-free intensity, but this is more than compensated by the increased level of ionization required and the decreased recombination time. If we decrease the temperature slightly, the recombination time will drop but will be exactly compensated by an increase in the free-free intensity down to a present temperature of about $3 \times 10^3\,°$ K, below which the intensity drops precipitously. We have computed this equilibrium assuming that all the mass is in the gas; we find, however, that the number density of neutrals is independent of the gas density, since the recombination time is proportional to n^{-1} and the ionization time to n^{-2}. Lower densities do, of course, make collisional ionization relatively more important. Because of this independence of total number, the dependence on q_0 enters only through the factor $(1 + 2q_0 z)^{1/2}$ in dl/dz, and differences between reasonable models are smaller by far than the uncertainty in the treatment of the free-free intensity integrals near $t = 0$. It seems therefore inappropriate at this time to discuss differences between relativistic models with small q_0. As we go to temperatures (at $z = 2$) higher than about 5×10^5, the recombination time increases more rapidly than the radiative ionization time; collisions become more important, and the ionization ratio decreases.

For the steady state, the observed ionization ratio is obtained *collisionally* at a temperature of about $7 \times 10^5\,°$ K; t_{coll} is about 1.5×10^{12} sec, and the radiative ionization time is very much longer, about 10^{13} sec. If the older density of 2×10^{-29} g/cm^3 is taken, the equilibrium temperature drops to about 4×10^5. D. W. Sciama (private communication) has pointed out that recombinations which result in reionization should be deleted from the equilibrium considerations, but it is easily shown that the medium is optically

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thin in the ionization continuum, so that high-energy photons are redshifted past the limit before they can ionize.

One interesting consequence of an essentially completely ionized medium is that there appears to be an appreciable optical depth in Thompson scattering for very distant objects. Since the cross-section for this process is independent of frequency, its only effect is to attenuate the over-all level of radiation. The optical depth to redshift z is

$$p = \int \sigma n dl ,$$

which for the relativistic models is

$$p = \frac{n_0 \sigma}{H_0} \left[\frac{3 q_0 + q_0 z - 1}{3 q_0^2} (1 + 2 q_0 z)^{1/2} - \frac{3 q_0 - 1}{3 q_0^2} \right]$$

and for the steady state is

$$p = \frac{n_0 \sigma}{H} \ln (1 + z) .$$

At $z = 2$, these quantities are 0.38, 0.23, and 0.18 for $q_0 = 1$, $q_0 = \frac{1}{2}$, and the steady state, respectively, and are in such a direction as to *decrease* the sensitivity of the m–log z relation to the model. In addition, these figures assume that all the material is in the ionized intergalactic medium, a fact that is certainly not known a priori; thus one does not know quite how much to correct for the effect, and the interpretation of data from very distant sources is no longer a straightforward application of the usual simple cosmological tests that involve only the parameters of the metric tensor plus knowledge of intrinsic source properties.

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