THE PERIOD-RADIUS RELATION FOR PULSATING VARIABLE STARS

II. A MODIFICATION

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ABSTRACT

The period-luminosity-color relation for classical Cepheids based on the period-radius relation of Paper I (Fernie 1964) is found to differ systematically for very short-period Cepheids from the period-luminosity-color relation of Kraft. A test on the Cepheids of the Small Magellanic Cloud shows both period-luminosity-color relations to be incorrect. The observations are used to show that the period-radius relation of Paper I should be modified to include a mass term, and that the full relation is $P = kR^2/\mathfrak{M}^{1/2}$. Revised period-luminosity-color relations are deduced for the classical Cepheids and β Cep stars and shown to be satisfactory.

While the same quantitative period-radius-mass relation applies to both classical Cepheids and β Cep stars, it is found that application of this relation to RR Lyr stars requires the masses of the latter to be 0.1 $\mathfrak{M}\odot$. This is considered unlikely, and instead it is tentatively suggested that the constant of proportionality, k, in the period-radius-mass relation is different for these (and probably other post–red-giant) stars.

Other revised conclusions of Paper I are (*a*) the pulsation "constant" $Q = P\rho^{1/2}$ is given by $Q = kR^{1/2}$; (*b*) A_V/E_{B-V} for classical Cepheids is more nearly 3.0 than 3.4; (*c*) Bailey type c RR Lyr variables have a zero point to their period-luminosity-color relation 0 2 mag. different from the type ab variables.

I. INTRODUCTION

In an earlier paper (Fernie 1964; hereinafter referred to as "Paper I") it was shown that pulsating variable stars tend to fall into two groups, each having the period of pulsation proportional to the square of the radius of the star, but with differing constants of proportionality. Substitution of these relations into the $L = 4\pi R^2\sigma T_e^4$ relation allowed the prediction of detailed period-luminosity-color (P-L-C) laws for all classes of variables for which the relations between bolometric correction, effective temperature, and color were known. These P-L-C laws were tested against observation wherever possible, and rather good agreement was obtained in all cases. In particular, it was shown that the P-L-C law obtained for classical Cepheids gave results that were not systematically different from those based on an earlier P-L-C law obtained by Kraft (1961*b*). This conclusion was based on a comparison of Cepheids with periods between about 5 and 45 days.

Subsequent applications of the P-L-C law, however, showed that for Cepheids of period about 2 days there was a systematic difference of between 0.5 and 1 mag. in the predicted M_V between Kraft's P-L-C law and that of Paper I. Sufficiently accurate tests to decide which (if either) of the two P-L-C laws is correct are difficult to devise. There exist no Cepheids of sufficiently short period in galactic clusters. There are only two short-period Cepheids, DT Cyg and SU Cas, which have radial velocity-curves of the precision required for applying Wesselink's method, and for these the published photometry is too uncertain. In any case, short-period Cepheids often have such sinusoidal light- and color-curves that phases of equal color are also phases of equal light, and Wesselink's method then breaks down. It was hoped that a dynamical test based on galactic rotation might serve. If the short-period Cepheids tabulated by Kraft and Schmidt (1963) have had systematically incorrect absolute magnitudes assigned to them, they should yield a value of the Oort parameter A different from that obtained from the longer-period Cepheids. It is found, however, that there are too few short-period Cepheids too inhomogeneously distributed in longitude to make the test sufficiently accurate. Dynamical tests between short- and long-period Cepheids are further confused by the fact that the latter have ages one to two orders of magnitude less than the former, and therefore constitute practically a different stellar population.

Finally, a test has been devised based on the classical Cepheids in the Small Magellanic Cloud (SMC) for which accurate photometry is available from Arp (1960). If the distance modulus of each SMC Cepheid is computed from a P-L-C law and Arp's observations, and the results plotted as a function of period, it is, of course, required that

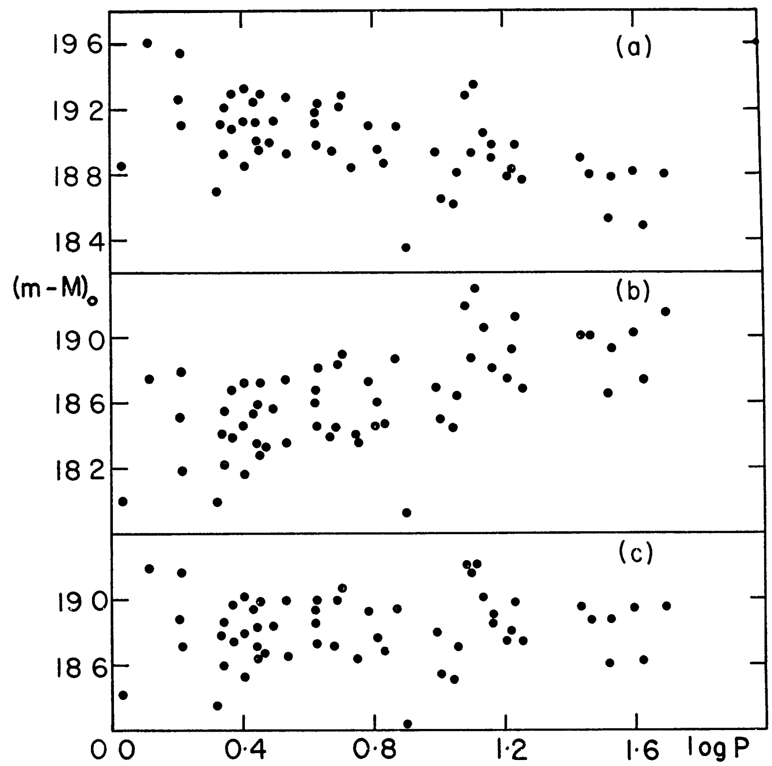

FIG. 1 —Distance moduli of individual Small Magellanic Cloud Cepheids determined by (*a*) the P-L-C relation of Paper I; (*b*) Kraft's P-L-C relation; and (*c*) the revised P-L-C relation given in this paper. The trend with period in (*a*) and (*b*) indicates that these P-L-C relations are unsatisfactory.

the moduli should not correlate with period. In applying the observations in Arp's Table VII, the following relations have been adopted:

$$E_{B-V} = 0.09 \text{ mag.}, \qquad A_V = 0.3 \text{ mag. (Fernie 1963)} .$$

Both the P-L-C law of Paper I and Kraft's P-L-C law have been applied and the results are shown in Figure 1. Clearly, both sets of results show a trend with period. The figure also indicates why only at the shortest periods did the discrepancy become readily detectable. Before concluding that both P-L-C laws are incorrect we briefly examine other possible explanations for Figure 1: (*a*) The SMC Cepheids are different from galactic Cepheids and the P-L-C laws for the latter do not apply. There is no real evidence that this is so. The only well-established difference between the two sets of Cepheids is in the relative distribution of their numbers and amplitudes with period. The Cepheids in various galaxies (except possibly the LMC) have the same slope to their mean period-luminosity laws (Sandage 1962), which speaks for their similarity. (*b*) There exists a scale error in Arp's photometry. This seems quite unlikely at the apparent magnitudes concerned. Also, the two latter arguments under (*a*) indicate that there is no significant scale error.

We conclude that both the P-L-C law of Paper I and Kraft's P-L-C law are incorrect. Furthermore, since the color terms in the two laws both stem from common data, yet the trends in Figure 1 are opposite, we conclude that it is not the color terms that are at fault.

In fairness it must be remarked that the derivation of Kraft's P-L-C law requires the use of a mean period-color relation, and that there is some evidence (Arp and Kraft 1961) that these may be different in the Small Magellanic Cloud and the Galaxy. In this case the conclusion regarding Kraft's law is merely that it is inapplicable to the Cepheids of the Small Magellanic Cloud, not that it is totally incorrect.

II. MODIFICATION OF THE PERIOD-RADIUS RELATION

The implication of the above conclusions for Paper I is that there exists some modifying factor to the $P \propto R^2$ law which is a function of the period of the Cepheid concerned. Two possibly significant factors correlated with the period are age and mass. Age might be important if chemical composition depended on it, but there is no evidence that the

TABLE 1

RADII OF SELECTED SMC CEPHEIDS

Star	$\langle M_V \rangle$	$\langle (B-V)^{\circ} \rangle$	$\log T_e$	$\langle M_{\rm bol} \rangle$	$\log R/R_{\odot}$	$\log \mathfrak{M}/\mathfrak{M}_{\odot}$	$\log P$
HV 2064	−5 43	0 87	3 736	−5 56	2 11	1 00	1 528
1954 ..	−5 28	52	3 797	−5 28	1 93	0 96	1 223
1768	−4 26	69	3 767	−4 30	1 80	0 85	0 991
1987	−3 15	53	3 795	−3 15	1 51	0 72	0 496
1850	−3 23	43	3 813	−3 23	1 49	0 73	0 441
848	−2 79	52	3 797	−2 79	1 44	0 68	0 338
11198 .	−2 54	0 41	3 816	−2 54	1 35	0 66	0 209

composition of short-period classical Cepheids is significantly different from that of long-period classical Cepheids. On the other hand, considering the simple theory of pulsating stars, the mass is very likely to be important. We therefore inquire if it is the mass which is the modifying factor, i.e., that the period-radius relation of Paper I is in reality a period-radius-mass relation. At this point one comes to realize that there is an inadvertent selection effect in Paper I: the stars of Group I (except Mira, which is not well determined anyway) all have a similar mass of order 10 $\mathfrak{M}_{\odot}$, while those of Group II also all have a similar mass of order 1 $\mathfrak{M}_{\odot}$. The significant difference between them, therefore, may have nothing to do with the fact that one group is in the pre–red-giant phase of evolution and in the other in the post–red-giant phase, as was suggested in Paper I; the two groups may just reflect two groups of mass. This point is returned to in § VI.

In order to determine the nature of the mass term in the period-radius-mass relation we proceed as follows. The relative positions of stars of different mass in the log R–log P plane are of critical importance. Accordingly these relative positions will be best determined by using a selection of stars from among the SMC Cepheids rather than galactic Cepheids with radii determined by Wesselink's method, since if we adopt an arbitrary distance modulus for the SMC we obtain an arbitrary *but constant* error in the calculated log R for those Cepheids. Their *relative* values of log R are therefore better determined than for the galactic Cepheids, which have comparatively large *random* errors in their determined values of log R. We select seven SMC Cepheids (Table 1) that lie close to the center of the scatter in Figure 1, *a*, and that have periods ranging from 1.6 to 34 days. The zero point in log R has been set to give radii for the longer-period Cepheids in general accordance with radii by Wesselink's method for similar galactic Cepheids. From Paper I we take it that for stars of equal mass, $P \propto R^2$. For each of the seven Cepheids we then use this to calculate what the period would have been had the radius been 1 $R_\odot$, i.e., we determine $(\log P)_{\log R=0}$. A plot of the latter against $\log \mathfrak{M}_\odot$ then gives the form of the mass term in the period-radius-mass relation.

The masses of the Cepheids can only be determined by adopting a form for their evolutionary tracks from their main-sequence progenitors. It has been assumed that in the range of interest evolution proceeds horizontally across the M_{bol}–$\log T_e$ plane after the initial rise to the Schönberg-Chandrasekhar (S-C) limit. Kraft (1961*a*) has given evidence for the plausibility of this assumption. A mass-luminosity relation for fairly massive stars at the S-C limit may be derived from the work of Henyey, LeLevier, and Levée (1959). This then is also the mass-luminosity law applicable to Cepheids. Thus we obtain the masses of the seven SMC Cepheids under consideration.

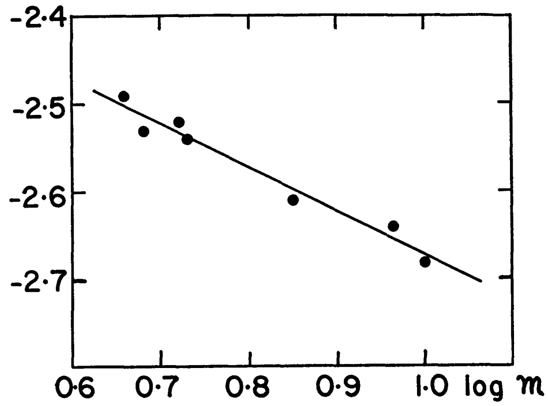

FIG. 2 —A plot of the quantity $(\log P)_{\log R=0}$ against log (mass) for seven SMC Cepheids. The line has slope $= -0.5$ and indicates that at constant radius the period varies inversely as the square root of the mass.

Figure 2 shows a plot of $(\log P)_{\log R=0}$ versus $\log \mathfrak{M}$ for the seven SMC Cepheids. The line drawn through the points has the equation

$$(\log P)_{\log R=0} = -2.171 - 0.50 \log \mathfrak{M} .$$

The period is therefore also inversely proportional to the square root of the mass. The full period-radius-mass relation therefore becomes

$$\log P = -2.171 + 2.00 \log \frac{R}{R_\odot} - 0.50 \log \frac{\mathfrak{M}}{\mathfrak{M}_\odot}$$

or

$$P = 0^{\mathrm{d}}.00675 \left(\frac{R}{R_\odot}\right)^2 \left(\frac{\mathfrak{M}}{\mathfrak{M}_\odot}\right)^{-1/2} .$$

In arriving at this expression we have assumed that it is the square of the radius that enters the expression. If we now write in general

$$P \propto R^n \mathfrak{M}^m ,$$

we may investigate the consequences for m of adopting a different value for n. In particular, if $m = -0.5$ were also consistent to within the errors of the observations with

$n = +1.5$ then we would recover the standard $P\sqrt{\rho} = Q$ relation. However, when the procedure outlined above is repeated with $n = 1.5$, it is found that $m = +0.57 \pm 0.06$. There is therefore a very close interdependence between m and n, and the $P\sqrt{\rho} = Q$ relation is quite definitely inconsistent with observation. Of course, combinations such as $n = 1.5$, $m = +0.57$ cannot be ruled out from the Cepheids alone. However, we find that if this were so and we apply the relation to other classes of variables, we predict a mass of about 1 $\mathfrak{M}_\odot$ for β Cep and about 17 $\mathfrak{M}_\odot$ for RR Lyr, which is entirely unreasonable. On the other hand, the combination $n = 2$, $m = -0.5$ leads to a mass of about 8 $\mathfrak{M}_\odot$ for β Cep and about 0.4 $\mathfrak{M}_\odot$ for RR Lyr (using the radii given in Paper I), which is in much better accord with expectation. We therefore take $n = 2$, $m = -0.5$ to be the correct combination. However, in § VI we consider the possibility that, while $P \propto R^2 \mathfrak{M}^{-1/2}$ is universal among pulsating variables, the constant of proportionality is not.

III. THE $P\sqrt{\rho}$ RELATION

If the relation

$$P = kR^2 \mathfrak{M}^{-1/2}$$

is substituted into the period-density relation,

$$P \mathfrak{M}^{1/2} R^{-3/2} = Q \, ,$$

one obtains

$$Q = kR^{1/2} \, .$$

Thus Q increases with increasing radius and, therefore, in general with increasing period, which is in qualitative agreement with the discussion in Paper I. We apply the above equation to calculate Q for a typical 10-day Cepheid of radius 64 $R_\odot$ and a typical β Cep star of radius 10 $R_\odot$. With $k = 0.00675$ from above, we obtain $Q = 0.054$ and 0.022 for the Cepheid and β Cep star, respectively. These may be compared with the directly determined values of 0.054 and 0.028 given in Paper I. Again, however, in § VI we will find that k may not be a universal number.

In Paper I it seemed possible to explain why Cepheids and long-period variables (LPV) appear to have approximately equal values of Q. This is no longer possible, since the relation $Q = kR^{1/2}$ calls for $Q_{\mathrm{LPV}} \sim 2\,Q_{\mathrm{cep}}$. At least three possible explanations present themselves: (*a*) The masses of long-period variables are uncertain by at least 100 to 200 per cent and their radii by perhaps 50 per cent (Fernie and Brooker 1961; Feast 1963). Thus the directly calculated Q-values are uncertain by about 100 per cent. Hence, although a rather extreme combination of errors is called for, the observations do not entirely rule out the value of Q predicted from $Q = kR^{1/2}$. (*b*) The value of k may be different for long-period variables. (*c*) The concept of radius may be poorly defined for stars whose average density is only 10^{-7}gm cm^{-3}. It is not inconceivable that the "effective" radius for pulsation is something different from the radius determined by Pease (1925) with an interferometer for Mira, on which the discussion largely hinges.

IV. P-L-C RELATION FOR CLASSICAL CEPHEIDS

With the period-radius-mass relation known we may now proceed as in Paper I to derive period-luminosity-color relations for various classes of pulsating variables, except that now an explicit expression involving the mass appears in the relation. This may be removed by substituting either an average mass-period relation or a mass-luminosity relation. Which of these is adopted is purely a matter of procedure, since the origin of the data is the same. Trial and error show that substitution of a mass-period relation leads to slightly better agreement with observation. Consideration of the size and position of the Cepheid instability strip in the H-R diagram shows that the intrinsic scatter in the mass-period relation should not cause an error of more than a few hundredths of a magnitude in the predicted absolute magnitude. From the discussion given in § II the mass-period relation for classical Cepheids is found to be

$$\log \mathfrak{M} = 0.280 \log P + 0.595 \ .$$

Using this to remove the mass term from the period-radius-mass relation and then proceeding as in Paper I, the P-L-C relation for classical Cepheids is found to be

$$\langle M_V \rangle_{\text{int}} = -2.87 - 2.850 \log P + 2.06 \langle B - V \rangle_{\text{mag}} \ ,$$

where the cyclic averages are as defined by Kraft (1961*a*).

TABLE 2

COMPUTED AND OBSERVED ABSOLUTE MAGNITUDES OF CLASSICAL CEPHEIDS

Star	log P	$\langle (B-V)^\circ \rangle$	$\langle M_V \rangle_{\text{obs}}$	$\langle M_V \rangle_{\text{calc}}$
SV Vul	1 655	0 94	−5 45	−5 65
T Mon	1 432	90	−5 21	−5 21
δ Cep	0 730	63	−3 65	−3 66
η Aql	0 856	67	−4 27	−3 94
l Car	1 551	93	−5 43	−5 38
CF Cas	0 687	70	{−3 39*, −3 61†}	−3 38
U Sgr	0 828	65	{−3 87*, −4 07†}	−3 90
DL Cas	0 908	77	{−3 79*, −3 98†}	−3 88
S Nor	0 989	80	{−4 04*, −4 12†}	−4 04
HV 2064	1 528	87	−5 43	−5 44
1954	1 223	52	−5 28	−5 28
1768	0 991	69	−4 26	−4 27
1987	0 496	53	−3 15	−3 17
1850	0 441	43	−3 23	−3 23
848	0 338	52	−2 79	−2 77
11198	0 209	0 41	−2 54	−2 62

* Adopting $A_V/E_{B-V} = 3\ 0$ † Adopting $A_V/E_{B-V} = 3\ 4$.

This relation is tested on the same galactic Cepheids as were used in Paper I and also the seven SMC Cepheids of Table 1. Results are shown in Table 2. For the four Cepheids of Table 2 that are in galactic clusters two values of $(M_V)_{\text{obs}}$ are listed. These correspond to the adoption of $A_V/E_{B-V} = 3.0$ and 3.4 for the upper and lower values, respectively. In Paper I it was concluded that $A_V/E_{B-V} = 3.4$ gave better agreement with the predicted absolute magnitudes. Table 2 shows clearly that this is no longer so and that the previous result was due to the fault in the earlier P-L-C law. We therefore return to $A_V/E_{B-V} = 3.0$ as being the most likely value applicable to Cepheids.

The relation is also applied to determine the distance modulus of all the SMC Cepheids, as in § I. The results are shown in Figure 1, *c*, where the trend in Figure 1, *a*, is shown to have been satisfactorily removed. The average true distance modulus of the SMC Cepheids is found to be 18.81 mag. (apparent distance modulus 19.1). Since the same P-L-C law applied to the galactic Cepheids in Table 2 gave an average error in M_V of only ± 0.08 mag., this is estimated to be about the uncertainty in the modulus of the SMC given above.

V. P-L-C RELATION FOR β CEPHEI STARS

We proceed as for the Cepheids to derive a P-L-C relation for the β Cep stars. First we check that the period-radius-mass relation for the latter is the same as that for the former. Using the observational absolute magnitudes and colors for the known β Cep stars as given by Schmalberger (1960) and the bolometric-correction–effective-temperature–color relations given in Paper I, the radii may be calculated. The β Cep stars are stars just at the S-C limit, so the mass-luminosity law contained in Henyey *et al.* (1959) is applicable (as it was to the Cepheids in § II). When we write

$$\log P = k + 2 \log R - 0.5 \log \mathfrak{M} ,$$

the individual radii, masses, and periods are applied to determine k. The average value of k is found to be

$$k = -2.14 \pm 0.03 \text{ (s.e.)} ,$$

which is in good agreement with the value found from the classical Cepheids in § II. The Cepheids and β Cep stars appear to obey the same period-radius-mass relation.

TABLE 3

COMPUTED AND OBSERVED ABSOLUTE MAGNITUDES OF β CEPHEI STARS

Star	log P	$\langle (B-V)^\circ \rangle$	$\langle M_V \rangle_{obs}$	$\langle M_V \rangle_{calc}$
β CMa	−0 602	−0 280	−4 9	−4 7
β Cru .	− 602:	− 28	−4 5	−4 7:
σ Sco. .	− 609	− 26	−4 3	−4 5
ξ^1 CMa	− 660	− .280	−4 2	−4 5
BW Vul..	− .697	− .270	−4 5	−4 3
HD 21803	− 699	− 24	−4 0	−4 0
12 Lac .	− 705	− 265	−4 3	−4 2
β Cep. .	− 719	− .275	−4 5	−4 2
15 CMa	− .735	− 25	−3 5	−4 0
ν Eri . .	− 750	− .255	−4 1	−3 9
τ^1 Lup	− 752	− 24	−2 9:	−3 7
16 Lac	− 767	− 260	−4 0	−3 9
δ Cet .	− 793	− 245	−3 1	−3 7
γ Peg	− 821	− 240	−3 8	−3 6
θ Oph...	−0 851	−0 22	−3 0	−3 3

Since this relation is better determined by the Cepheids, we apply the result of § II to determine the P-L-C relation for β Cep stars. This, together with a mass-period relation of the form

$$\log \mathfrak{M} = 1.17 \log P + 1.96$$

leads to a P-L-C relation

$$M_V = -5.15 - 3.96 \log P + 6.9 (B - V) .$$

This is applied to the stars listed by Schmalberger (1960). Results are given in Table 3. The average residual between the observed and predicted absolute magnitude is 0.28 mag. The predictions of Paper I gave an average residual of 0.32 mag. The improvement is therefore only very slight.

VI. P-L-C RELATION FOR RR LYRAE STARS

A difficulty is encountered in applying the period-radius-mass relation to RR Lyr stars in that the masses and the trend of mass with period are not well known. We may first investigate whether there is any evidence that the mass varies with the period of the stars. As in other cases, the P-L-C law will be of the form

$$M_V = a + b \log P + c(B - V) .$$

If there is no trend of mass with period, then b should have the value -2.5 as was found in Paper I. If there is a trend, then b will have some different value. We find which is the case by using the observations of RR Lyr stars in M3 by Roberts and Sandage (1955) and Preston (1961). The color term in the above equation remains unchanged from Paper I. Therefore, if one plots $[m_V - c(B - V)]$ versus $\log P$, the points should lie on a line of slope b and zero point involving a and the distance modulus of M3. This is shown in Figure 3. The line through the points representing the Bailey type ab stars has slope -2.5 and seems to fit reasonably well. We therefore conclude that there is no significant

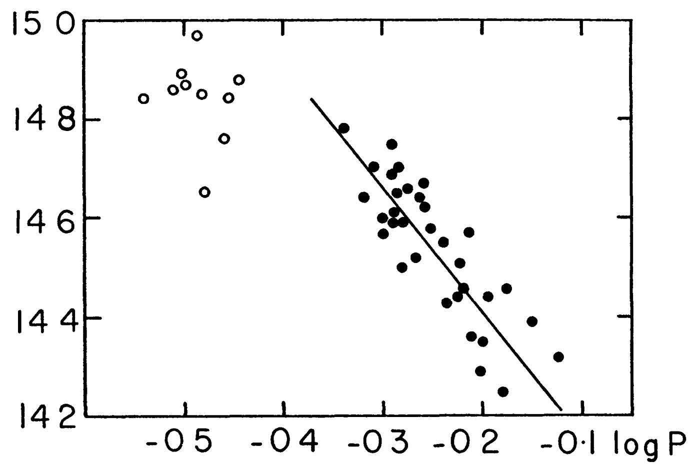

FIG. 3.—A plot of the quantity $[\langle m_V \rangle - 2.96 \langle B - V \rangle]$ against log (period) for the RR Lyrae stars in M3. Open circles denote Bailey type c variables, closed circles Bailey type ab. The line has slope = -2.5 and indicates that there is no significant variation in mass among the ab variables. The fact that the type c variables do not fit the same relation as the ab variables indicates that either they are pulsating in a different overtone or have a different mass from the ab variables.

trend of mass with period for these stars. The Bailey type c stars may either be stars of different mass or may be pulsating in a different overtone, although if the latter is the case the ratio of fundamental to overtone period is only about 1.25. In either case we revise our conclusion of Paper I that the same P-L-C relation fits all Bailey types. This came about because of the use of too few stars in the test. We now find that if the P-L-C law of Bailey type ab stars is applied to the type c stars the predicted distance moduli will be about 0.2 mag. less on the average.

We may now choose an average mass for an RR Lyr star (restricting ourselves to Bailey type ab). We do this by noting that the mean period-radius relation used for the RR Lyr stars in Paper I lead to absolute magnitudes in good agreement with observation. This relation was

$$\log R = 0.5 \log P + 0.83 .$$

From § II we find

$$\log R = 0.5 \log P + 0.25 \log \mathfrak{M} + 1.086 .$$

Equating the two expressions leads to an average mass

$$\mathfrak{M} = 0.1\, \mathfrak{M}_\odot .$$

It has been customary to take the masses of these stars as about 1.2 $\mathfrak{M}_\odot$, these being the masses of their probable main-sequence progenitors. This, however, neglects the possibility, if not probability, of their having lost mass in the red-giant stage of evolution. Hayashi, Hōshi, and Sugimoto (1962) have constructed models of horizontal-branch globular-cluster stars and obtained reasonable agreement with observation using a mass of 0.7 $\mathfrak{M}_\odot$. Osaki (1963) has done the same for 0.8 $\mathfrak{M}_\odot$, while Demarque and Hartwick (in preparation) have found that a mass of 0.6 $\mathfrak{M}_\odot$ is required. Oke, Giver, and Searle (1962) in their study of SU Dra found a mass of 0.3 $\mathfrak{M}_\odot$ to be possible. From these findings an average mass of 0.6 $\mathfrak{M}_\odot$ may be reasonable, and at first sight this may not seem grossly different from the 0.1 $\mathfrak{M}_\odot$ arrived at above. However, if we retain the mass explicitly in the P-L-C relation we find

$$M_V \sim -1.25 \log \mathfrak{M} ,$$

in which case, changing the mass from 0.1 to 0.6 $\mathfrak{M}_\odot$ will brighten M_V by 1.0 mag. This is definitely ruled out by the observations listed in Paper I. On the other hand, it is very doubtful that these stars could have masses of only 0.1 $\mathfrak{M}_\odot$. Apart from the severe

TABLE 4

OBSERVATIONAL DATA FOR THREE STARS

Star	P	$R_\odot$	$\mathfrak{M}_\odot$
β Cep	0ᵈ190	9	17 2
HV 1768	9 80	63	7 1
RR Lyr .	0 567	7	0 6

theoretical difficulties of explaining how a star of such low mass could be so luminous, a mass of only 0.1 $\mathfrak{M}_\odot$ would imply that horizontal-branch stars in globular clusters had lost about 92 per cent of their masses in the course of their evolution, and that there must therefore be very considerable quantities of interstellar matter in globular clusters.

It is interesting to see whether a slight adjustment of n and m in the $P = kR^n \mathfrak{M}^m$ relation might not lead to a universally applicable relation without the difficulties of a low mass for the RR Lyrae variables. Three stars, a classical Cepheid, β Cep, and RR Lyr were selected, with the values of P, R, and $\mathfrak{M}$ given in Table 4. These data were used to solve for k, n, and m in the above equation. The result was $k = 0.013$, $n = +1.81$, $m = -0.45$. This was then used to derive a new P-L-C relation for classical Cepheids following the procedure of § IV. The absolute magnitudes calculated from this, however, when compared with observation gave residuals which were not only fairly large (0.2 mag.) but which showed a pronounced trend with period.

It would seem, therefore, that unless the masses of the RR Lyrae stars are indeed as small as 0.1 $\mathfrak{M}_\odot$, which is unlikely, the period-radius-mass relation cannot be universal among pulsating variables. It may be that the values $n = 2$, $m = -0.5$ are universal, and that only k depends on the type of star, or it may be that all three depend on the type of star. Such observational data as are available at present are too crude to allow a decision to be reached.

It is a pleasure to acknowledge numerous discussions on the theoretical aspects of this subject with Dr. Pierre Demarque.

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