BINARY STARS AMONG CATACLYSMIC VARIABLES
VI. ON THE MEAN ABSOLUTE MAGNITUDE OF U GEMINORUM VARIABLES

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ABSTRACT

From twenty-five proper motions and eleven radial velocities, the solar motion and mean parallax of U Geminorum stars have been estimated. The solar apex is $A_{\odot} = 263°$, $D_{\odot} = +19°$, and $s_{\odot} \cong 30$ km/sec. The mean absolute magnitude of U Gem stars at minimum light is estimated to be $M_V = +7.5 \pm 0.7$. If there is a spread in the absolute magnitudes depending on the period of the orbit, it seems possible to explain why the spectroscopic absolute magnitudes of SS Cyg, AE Aqr, RU Peg, and EY Cyg are approximately 2 mag. brighter than the mean value.

I. INTRODUCTION

It is by now fairly well established (cf. Kraft 1962; hereinafter cited as "Paper I") that all U Geminorum stars are close binaries with periods of the order of a few hours. In most cases, the spectrum consists of a hot subdwarf with superimposed emission lines of H, He I, and Ca II; in a few objects, viz., AE Aqr, SS Cyg, RU Peg, Z Cam, and EY Cyg, one finds also the spectral lines of a cool companion with spectral characteristics similar to those of the Sun. There is a tendency for the late-type companion to be found in the spectrum only when the period is longer than about 6 hours; the presumption is strong that the stars with periods shorter than 6 hours have companions that are still fainter and later in type. In all cases it is believed that the late-type component overflows its lobe of the inner Lagrangian surface, and that the material flowing out through the inner Lagrangian point takes up an orbit around the hot component, forming a ring.

Less well understood, however, is the absolute magnitude of these stars, the evidence from different types of investigations being contradictory. From the spectral classification of four of the five red stars mentioned above (Z Cam has lines so evanescent that classification is not possible), one finds a mean $M_V = +5.4$ for the red components, or about $M_V = +4.8$ for the total light of the systems. The red stars are, therefore, either on or slightly to the right of the main sequence, as judged from the spectral types. Trigonometric parallax measurements, however, yield much fainter absolute magnitudes. Strand's (1948) value of $p = +0\overset{''}{.}032 \pm 0\overset{''}{.}010$ for SS Cyg leads to $M_V = +9.5 \pm 0.8$; van Maanen's (1938) parallax, however, was $-0\overset{''}{.}012 \pm 0\overset{''}{.}012$. If we average Van Maanen's parallax with Strand's, we still obtain the very low absolute magnitude of $+7.5$. Though not much weight can be given to a parallax of $+0\overset{''}{.}010 \pm 0\overset{''}{.}012$ for U Gem (Van Maanen 1938), the corresponding absolute magnitude of $M_V = +9.1$ agrees with the low value obtained for SS Cyg, and further widens the gulf between the trigonometric and spectroscopic absolute magnitudes.

Kraft (1962) has emphasized that, if the red component fills one lobe of the inner Lagragian surface, the gravity is lowered relative to a non-rotator of the same mass; thus there may be a tendency to assign too high a luminosity class to the late-type components. This is confirmed by the change in apparent luminosity class of RU Peg from V to III (!) during the cycle, the highest luminosity being achieved when the inner Lagrangian point is on the side toward the observer. Furthermore, the radiation field of the red star may be affected somewhat by its hot companion (Joy 1954; Crawford and Kraft 1956). These effects indicate that the spectroscopic absolute magnitudes are unreliable, and might certainly account for the classification of some red components as lying above the main sequence. However, unless the temperatures are seriously disturbed, absolute magnitudes as faint as +8 or +9, which seem to be required by the trigonometric parallaxes, would place the red, as well as the blue, components well below the main sequence. The precise location of the red components in the H-R diagram takes on added significance with Krzeminski's (1965) discovery that the late-type member of U Gem is very likely the seat of the outbursts.

In this paper we shall derive a statistical parallax based on proper motions and radial velocities. Our view is that, in light of the paucity as well as the great inaccuracy of the trigonometric parallax determinations and the physical uncertainty surrounding the spectroscopic absolute magnitudes, a parallax based on motions is probably the most reliable. The sizes of the early proper motions of U Gem and SS Cyg by Kukarkin and Parenago (1934) were several times their probable errors, suggesting that a program of proper motions was indeed feasible. Subsequent motions measured by Mannino and Rosino (1950) and by Miczaika and Becker (1948) were, however, not very accurate (see § III). In this paper we give new proper motions, with an average mean error of $\pm 0\overset{''}{.}005$, for twenty-five U Gem stars.

The radial velocity situation is, however, less satisfactory, because the stars are not only faint ($\langle m_v \rangle_{\min} = +14.6$ for the twenty-five stars with proper motions), but each is presumably a velocity variable with a period of a few hours. We quote here eleven radial velocities from various sources; some of these are new. A departure in this work, compared with earlier studies of the motions, is a direct determination of solar motion from the radial velocities themselves.

A summary of earlier absolute magnitude determinations by a variety of techniques has been given by Schmidt-Kaler (1962).

II. THE RADIAL VELOCITIES

Systemic velocities for eleven stars are listed in Table 1. The radial velocities were determined principally with the nebular spectrograph of the 200-inch telescope at dispersions of 90 and 180 Å/mm in the blue; observational details have been given in Paper I. The orbits for SS Cyg and AE Aqr are taken from the work of Joy (1954, 1956), and the systemic velocity for WZ Sge has been given by Krzeminski and Kraft (1964; hereinafter cited as "Paper V").

New velocities are given for Z Cam, HV 8002 (Paczynski 1963), and T Leo, and an improved value is quoted for SS Aur. Z Cam will be discussed in detail elsewhere because of its great intrinsic interest as a double-lined binary that probably eclipses (Kraft, Krzeminski, and Mumford 1965). Velocity-curves for the single-lined binaries (emission lines of H, He I, and Ca II) HV 8002 and SS Aur are shown in Figures 1 and 2; the individual velocities are listed in Tables 2 and 3. The number of velocities for the former is small, and the systemic velocity is therefore a little uncertain. A number of spectrograms taken over short nightly runs as well as over an interval of some years indicate only small and rather erratic changes in velocity for T Leo; our dispersion does not permit the resolution of an orbit, in all probability either because the system is seen nearly pole-on and/or because the masses are small. We take the γ-velocity as a mean of all the available material. It is estimated that the systemic velocity of an individual star is correct to about ± 10 km/sec (estimated outside error) on the average. Since the systemic velocities derived from the absorption and emission lines in the double-lined

TABLE 1

PROPER MOTIONS AND RADIAL VELOCITIES FOR U GEM VARIABLES

Star	α(1950)	δ(1950)	m_V (min)	μ_α (″/yr)	(m.e.)	μ_δ (″/yr)	(m.e.)	μ_τ (″/yr)	μ_υ (″/yr)	$\mu_\upsilon \sin\Delta$ (″/yr)	V (km/sec)	V_p Case I (km/sec)	V_p Case II (km/sec)	V_p Case III (km/sec)	V_p Case IV (km/sec)	P hrs, min	Ref.
HV 8002.	00^h08^m8	−11°45′	14.5	+0.010	±0.004	+0.012	±0.006	+0.015	+0.007	+0.007	+30:	+19:	+22:	+24:	+26:	3 50	(1), (2)
RX And	01 01 8	+41 02	13.6	+ 005	003	− 009	003	− 006	− 008	+ .008	−18	−21	−20	−19.5	−19	5 05	(3), (4)
TZ Per	02 10 3	+58 09	15.2	− .011	004	+ 006	005	− 001	− 012	− .012							
VW Hyi	. 04 09 5	−71 25	13.4	+ .019	010	+ 044	.006	+ 006	+ 048	+ 037							
CN Ori.	05 49 1	− 5 26	14.7	+ 031	007	+ 005	.004	− 025	+ 018	− .004							
SS Aur .	06 09 5	+47 45	14.8	+ 013	004	− 025	.002	− 016	− 023	+ .021	+33	− 6	+ 3	+12	+21	4 20	(2)
CZ Ori.	06 13 8	+15 25	16.2	− 011	006	− 010	006	+ 005	− 015	+ .009							
U Gem .	07 52 1	+22 08	14.1	− 021	002	− 039	.003	− 007	− 044	+ 035	+42	+ 4	+14	+20	+25	4 15	(4), (5), (6)
SU UMa . .	08 08 1	+62 46	14.5	+ 022	007	− 009	004	− 024	+ 002	− 002							
Z Cha	. 08 08 7	−76 29	14.8:	− .020	006	+ 073	.008	+ 012	− 074	+ .067							
Z Cam .	08 19 7	+73 17	13.5	− .018	003	− 002	.002	+ 010	− 015	+ 015	−33	−28	−29	−31	−33	7 00	(7)
SY Cnc.	08 58 3	+18 06	13.7	− 023	005	− 006	003	+ 004	− 023	+ .020							
X Leo.	09 48 3	+12 07	15.1	− 049	007	− 015	.007	+ 008	− 053	+ 050							
T Leo.	. . 11 35 .9	+ 3 39	15.4	− 068	.005	− 061	.004	− .038	− .083	+ .083	+11	+11	+11	+11	+11	.. ....	(2)
TW Vir	. 11 42 8	− 3 36	16 :	+ 019	010	− 038	.010	− 039	+ .017	− .017							
BV Cen .	13 28 1	−54 43	14.0	− .025	008	− 011	004	+ 004	− 026	+ .026							
AY Lyr .	18 42 7	+37 55	15.2:	− .011	.004	− 008	.004	000	− 014	− 007							
CY Lyr	18 50 6	+26 41	17.0:	+ 018	010	+ 024	012	+ .015	+ .026	+ .005							
FO Aql.	. 19 14 1	+00 02	17.5:	+ 006	004	− 009	004	− .006	+ 009	+ .003							
AB Dra .	19 51 1	+77 37	15.6	− 054	007	− .016	003	+ .002	− .059	− 038							
EY Cyg .	19 52 7	+32 14	15.0:	+ 011	006	− 005	.006	− .008	+ .010	+ .005	−10:	+41:	+29:	+18:	+ 8:	. . ...	(4)
WZ Sge	20 05 4	+17 33	15.5	+ 077	004	− 014	004	− 010	+ 071	+ 041	−25	+25	+13	+ 2	− 8	1 21	(3), (8)
AE Aqr	20 37 6	−01 03	12.0	+ 065	003	+ 018	004	+ 047	+ .050	+ 038	−53	−12	−22	−31	−40	10 ?	(3), (9), (10), (11)
SS Cyg.	21 40 7	+43 21	12.1	+ 115	002	+ 036	002	+ 017	+ .121	+ 102	− 9	+25	+17	+ 9	+ 1	6 38	(3), (12)
RU Peg	22 11 6	+12 27	12.7	− 007	008	+ 034	007	+ .035	+ 001	+ 001	− 7	+18	+12	+ 6	0	8 54	(3), (4)

References

(1) Paczynski 1963 (2) This paper (3) Walker 1957 (4) Kraft 1962 (5) Mumford 1963 (6) Krzeminski 1965 (7) Kraft, Krzeminski, and Mumford 1965 (8) Krzeminski and Kraft 1964 (9) Joy 1954 (10) Crawford and Kraft 1956 (11) Walker 1965 (12) Joy 1956

systems are the same, within the errors, it does not seem likely that either star is ejecting material secularly from the system, and therefore no K-term has been included in the analysis of the velocities.

III. THE PROPER MOTIONS

We list in Table 1 the proper motions of twenty-five U Gem variables, together with their mean errors. The determination of the proper motions, the time length of the base line, and a criticism of earlier proper motion determinations for U Gem stars is given by Luyten and Hughes (1965), to which the reader is referred for details. The spatial symmetry of the sample is enhanced by the inclusion of a few southern objects, e.g., VW Hyi, BV Cen, and Z Cha, even though the accuracy of these motions is somewhat poorer than the bulk of the material.

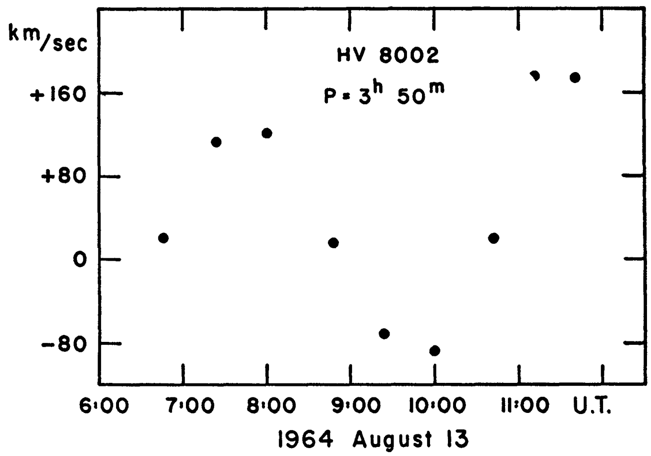

FIG. 1.—Radial-velocity-curve of HV 8002

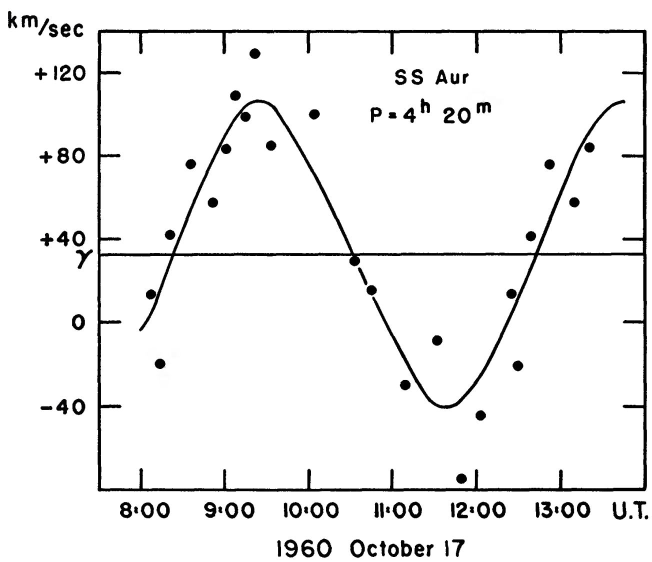

FIG. 2—Radial-velocity-curve of SS Aur. The velocities for 1965 are assembled on the same curve with those of 1960.

TABLE 2

RADIAL VELOCITIES FOR HV 8002

Plate	Date, 1964 (U T) (Mid-exp)	V (km/sec)
N 2297:		
a	Aug 13 282	+ 20
b	13 310	+114
c	13 334	+121
N 2298:		
a	13 367	+ 15
b	13 392	− 72
c	13 419	− 86
N 2299:		
a	13 446	+ 20
b	13 468	+178
c	13 488	+176

TABLE 3

RADIAL VELOCITIES FOR SS AUR

Plate	Date (U T) (Mid-exp)	V (km/sec)
N 1164	1960 Aug. 30 494	+108
N 1165:		
a	Aug. 31 381	+ 77
b . .	31 409	+ 99
N 1217:*		
a	Oct. 17 389	+130
b	17 420	+101
c .	17 448	+ 15
N 1218:*		
a	Oct. 17 479	− 9
b	17 502	− 45
c	17 520	− 21
N 2376:		
a	1965 Jan. 2 269	+ 11
b	2 300	+ 56
c	2 330	+ 84
N 2377:		
a	Jan. 2 370	+ 27
b	2 395	− 32 5
c .	2 423	− 77
N 2378:		
a	Jan. 2 455	+ 40
b	2 486	+ 82

* In Table 6 of Paper I the velocities from this plate are given (incorrectly) in inverted order

IV. DISCUSSION

We include in our discussion two stars not usually found in the list of conventional U Gem variables; this needs a little justification. WZ Sge is usually regarded as a nova, but attention has been drawn (Kraft 1964*b*) to its greater similarity to a U Gem star. Briefly, it fails to satisfy the life-luminosity relation for novae by several magnitudes, and its spectroscopic characteristics are similar to a U Gem star in degree of excitation. Furthermore, its small outburst range is only a little larger than that of a large-amplitude U Gem variable. AE Aqr does not have very large outbursts, and its orbital period is somewhat longer than the longest period known for a bona fide U Gem star, but it is spectroscopically so similar to RU Peg or SS Cyg that we include it in the group. The star EX Hya, while spectroscopically and photometrically similar to WZ Sge, has been omitted because of its unusually large radial velocity, viz., -124 km/sec (Kraft 1964*b*).

Solution for the solar motion based on radial velocities has been carried out in the usual way by least squares and yields the following results:

$$u_{\odot} = -0.3 \text{ km/sec}, \quad v_{\odot} = -41.3, \quad w_{\odot} = +24.2;$$

or

$$s_{\odot} = +47.8 \pm 14 \text{ (p.e.) km/sec}, \quad A_{\odot} = 269^{\circ}40' = 17^{\text{h}}59^{\text{m}}, \quad D_{\odot} = +30^{\circ}20' .$$

On the other hand, if we ignore the radial velocities altogether, we can obtain the direction of the solar apex using Airy's (1859) equations (cf. Trumpler and Weaver 1953, p. 319) from the twenty-five proper motions alone. These yield

$$A_{\odot} = 257^{\circ}10' = 17^{\text{h}}09^{\text{m}}, \quad D_{\odot} = +6^{\circ}\,2' .$$

The differences, especially in $D_{\odot}$, between these two apices are somewhat surprising, but it must be remembered that the amount of available observational material is quite small. The star of largest absolute radial velocity is AE Aqr, and since its inclusion might be regarded as questionable for other reasons, we have solved for the solar motion (from radial velocities) omitting it. The motion, however, is not significantly changed, and is taken in a direction slightly toward the "standard" apex and away from the apex suggested by the proper motions. If we take $s_{\odot}$ as given above, we can derived the components $u_{\odot}$, $v_{\odot}$, $w_{\odot}$ of the solar motion based on proper motions alone; if these are then averaged with the components obtained from the radial velocities, we obtain our best direct estimate of the solar motion for U Gem variables:

$$u_{\odot} = -5.3 \text{ km/sec}, \quad v_{\odot} = -43.4, \quad w_{\odot} = +14.7;$$

or

$$s_{\odot} = +46.1 \pm 14 \text{ (p.e.) km/sec}, \quad A_{\odot} = 263^{\circ}02' = 17^{\text{h}}32^{\text{m}}, \quad D_{\odot} = +18^{\circ}36'.$$

While the direction of the solar motion, especially that obtained from the radial velocities, is more-or-less normal (cf. Vyssotsky 1957), the speed is rather large, and indicates that we are dealing with a disk population of moderately, but not very, old stars. The probable error in the determination of the speed is so large that we are certainly justified in assuming that the motions of the U Gem stars are consistent with those of the G- and K-type dwarfs. Additional evidence in favor of this identification is found in the dispersion of the peculiar radial velocities which, as we shall see below, lies near 20 km/sec, and is virtually independent of the solar motion. This dispersion is quite consistent with that of the G- and K-type dwarfs (Vyssotsky 1957). Further evidence that a solar motion smaller than 46 km/sec is correct for U Gem stars will be provided later in the discussion when the absolute magnitudes derived from the total τ-, and υ-components of proper motion are compared.

Because of the large uncertainty in the speed of the Sun, we have considered four values of the solar motion, and have computed the dispersion σ_V for each case. The deviations V_p (cf. Trumpler and Weaver 1953, p. 290), in km/sec, for the four cases (Case I, $s_\odot$ = 60 km/sec; Case II, $s_\odot$ = 46.1 km/sec; Case III, $s_\odot$ = 32 km/sec; and Case IV, $s_\odot$ = 20 km/sec) are listed in Table 1. Cases I–III represent the derived solar motion plus or minus its probable error, and Case IV was included to see the effect produced by a "standard" value. The values of σ_V for the four cases are 20.3, 18.2, 18.8, and 20.9 km/sec, respectively, indicating that the dispersion in the peculiar radial velocities is quite insensitive to the adopted value of the solar motion.

The τ- and υ-components of the proper motion are also listed in Table 1. Because the τ-components are insensitive to the speed of the solar motion, we consider first the mean parallax derived from them. For the twenty-five stars, after applying a correction for the average probable error of the proper motions (cf. Trumpler and Weaver 1953, p. 346), we obtain $\langle|\mu_\tau|\rangle = +0\overset{''}{.}0138 \pm 0\overset{''}{.}002$ (p.e). If, then, we can write $\mathcal{E}(|t_\tau|) = \sqrt{(2/\pi)}\sigma_V$ for the expected value of the τ-component of the tangential velocity, the mean parallax $\langle p \rangle$ can be obtained from

$$\langle p \rangle = \kappa \frac{\langle |\mu_\tau| \rangle}{\mathcal{E}(|t_\tau|)},$$

where $\kappa = 4.74$ for each of the four cases. These are listed in Table 4. Since, however, $\langle \log p \rangle$ is not in general equal to $\log \langle p \rangle$, we must evaluate the former by returning to the original data of Table 1; otherwise bias will be introduced into our estimate of $\langle M_V \rangle$.

TABLE 4

$\langle M_V \rangle$, $\langle p \rangle$, AND $\langle \log p \rangle$ FROM τ-COMPONENTS OF PROPER MOTION

Solar Motion $s_\odot$	Case I (60 km/sec)	Case II (46 km/sec)	Case III (32 km/sec)	Case IV (20 km/sec)
$\langle p \rangle$	0\".00403	0\".00451	0\"00436	0\".00394
$\langle \log p \rangle$	−2 491	−2 479	−2 419	−2 337
$\langle M_V \rangle$*	+7 0	+7.1	+7.4	+7 8

* Includes 0 10-mag visual absorption corresponding to $\langle r \rangle$ = 160 pc

The estimates of $\langle \log p \rangle$ listed in Table 4 were obtained from $\langle \log p \rangle = \langle \log |\mu_\tau| \rangle - \langle \log |V_p| \rangle + \log \kappa$. The corresponding values of $\langle M_V \rangle$, listed in Table 4, are corrected for an amount of interstellar absorption of 0.10 mag., which is appropriate to the mean distance $[\langle r \rangle = (1/\kappa)\langle 1/\mu \rangle/\langle 1/t \rangle]$ of 160 pc.

The mean absolute magnitudes of Table 4 are quite stable, reflecting the insensitivity of the τ-components to the adopted solar motion. As a check on these estimates, and as a means of gaining insight into the correct choice of the solar motion, we consider the mean parallax derivable from the components μ_α and μ_δ, and also from the υ-components of the proper motion, both of which are dependent on the Sun's speed. Returning to Airy's equations, we derive from μ_α and μ_δ a value of the secular parallax $q = 0\overset{''}{.}0333$. In Table 5 we list the mean parallax for each of the four cases of solar motion, obtained from the equation $\langle p \rangle = q(\kappa/s_\odot)$. If we now demand equality between this mean parallax and that derived from the τ-components of proper motion (the latter averages about $0\overset{''}{.}0042$, independent of $s_\odot$), we obtain a solar motion of 30 km/sec. This lies comfortably within the mean error of the solar motion derived from radial velocities, and is consistent with expectation based on the small dispersion of the peculiar radial velocities.

We may similarly consider the υ-components of the proper motion, deriving the mean parallax from

$$\langle p \rangle = \frac{\kappa}{s_\odot} \frac{\Sigma \mu_\upsilon \sin \Delta}{\Sigma \sin^2 \Delta} = \frac{+0.1415}{s_\odot},$$

where Δ is the angular distance between the star and the solar antapex. In this case, equality between the mean parallaxes derived from the τ- and υ-components of proper motion demands again a solar speed of 30 km/sec, if $\langle p_\tau \rangle = 0\overset{''}{.}0048$. It therefore appears we would be justified in adopting the following best estimates of the motion parameters of U Gem stars:

$$s_\odot \simeq 30 \text{ km/sec toward } A_\odot = 263^\circ, \quad D_\odot = +19^\circ; \quad \langle p \rangle = 0\overset{''}{.}0042,$$

corresponding to $\langle M_V \rangle = +7.5$ with 0.1 mag. absorption. The error in the mean absolute magnitude is difficult to estimate, but from all sources together, can probably be taken as ± 0.7 mag. (estimated outside error).

V. A SUGGESTION FOR RESOLVING THE SPECTROSCOPIC ANOMALY

A mean absolute magnitude of $+7.5$ is in striking disagreement with the much brighter spectroscopic absolute magnitudes cited earlier. If the result based on motions is to be believed, then the red components would indeed seem to lie significantly below the main sequence. Both the motions and the appearance of the spectra militate against their identification with the extreme subdwarfs. We suggest, therefore, an *ad hoc* hypothesis by which this apparent anomaly may be removed:

TABLE 5

$\langle p \rangle$ FROM THE SECULAR PARALLAX

Solar Motion $s_\odot$	Case I (60 km/sec)	Case II (46 km/sec)	Case III (32 km/sec)	Case IV (20 km/sec)
$\langle p \rangle$	$0\overset{''}{.}00264$	$0\overset{''}{.}00345$	$0\overset{''}{.}00495$	$0\overset{''}{.}00792$

1. There is a range of 4–5 magnitudes in the absolute magnitudes of U Gem stars at minimum light, in the sense that the stars of longest orbital period are the brightest.
2. Since only the stars of longest period show the spectra of both components and those of shorter period show the spectrum of only the blue component, supposition 1 requires that, as we go to shorter and shorter periods, the magnitude of the red component declines faster than the blue.
3. The red components actually lie on the main sequence, with radii and temperatures in equilibrium with their masses, and fill one lobe of the inner Lagrangian surface defined by the restricted problem of these bodies.

The preceding suggestions imply, for example, that stars such as SS Cyg and RU Peg, having two spectra visible and comparatively long periods, are in fact intrinsically brighter than stars such as SS Aur and U Gem. The determination of the mean absolute magnitude by a statistical parallax is heavily weighted by stars showing the spectrum of only the blue component; i.e., of the twenty-five stars used in this discussion, seventeen have been observed spectroscopically at minimum light, and of these, only six show the spectrum of the red star. It is therefore not surprising, if our suppositions are correct, that the mean absolute magnitude so derived is fainter than that suggested by the spectral types of the red stars in the longer-period systems. The view that there might be an orbital period-luminosity law among the red components is supported by an apparently similar relation for the broader group of cataclysmic binaries, including novae (Kraft 1964*a*).

We can compute roughly the period-luminosity relation to be expected for the red components of U Gem systems. For, if the red stars are on the main sequence and fill a lobe of the inner Lagrangian surface, then for some assumed mass ratio we have log $(R_{red}/a) = 0.318 \log [\mathfrak{M}_{red}/(\mathfrak{M}_{red} + \mathfrak{M}_{bl})] - 0.327$ (Kuiper and Johnson 1956), and the separation a of the components can be estimated. Applying Kepler's third law, we obtain the period in the orbit. As typical of the longer-period U Gem stars, we adopt a mean of the masses, mass ratios, and absolute magnitudes for SS Cyg and RU Peg (Paper I), assuming the mass-luminosity law to be satisfied (Harris, Strand, and Worley 1963); viz., $\langle M_V \rangle = +5.3$, $\langle \mathfrak{M}_{red} \rangle = 0.89\, \mathfrak{M}_\odot$, $\langle \mathfrak{M}_{bl} \rangle = 0.79\, \mathfrak{M}_\odot$, $\langle \mathfrak{M}_{bl}/\mathfrak{M}_{red} \rangle = 0.88$. At the other end of the scale, we find our faintest object, WZ Sge, for which $M_V(\mathrm{bl}) = +11.2$, $M_V(\mathrm{red}) > +16$, $\mathfrak{M}(\mathrm{bl}) = 0.59\, \mathfrak{M}_\odot$, $\mathfrak{M}(\mathrm{red}) \sim 0.03\, \mathfrak{M}_\odot$ (Paper V). Over this extreme range, the mass of the blue component varies only slightly; in the elementary discussion to follow, we adopt a constant mass of $0.7\, \mathfrak{M}_\odot$ for the blue star. Since the periods go as the square root of the masses, the conclusions reached here would not be greatly modified by some other choice of mass in the vicinity of $1\, \mathfrak{M}_\odot$.

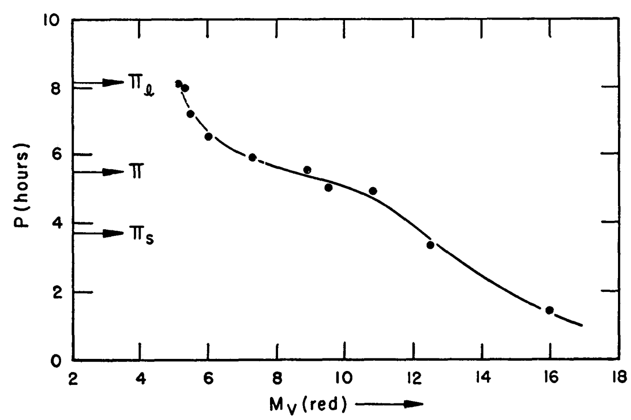

FIG 3 —Relation between orbital period and absolute magnitude for the red star It is assumed that the red star fills its lobe of the zero-velocity surface and satisfies the mass-luminosity relation Dots correspond to computed points. Π_l, Π, and Π_s refer, respectively, to the mean period of the three groups: (1) all double-line binaries, (2) all stars, (3) all single-line binaries

The period (in hours) as a function of M_V for the red star is plotted in Figure 3. We denote by Π, Π_s, and Π_l the mean period of all stars for which an orbital period is known (nine stars), those for which only one spectrum is seen (five stars), and those for which both spectra are seen (four stars), respectively. Thus the red star in a system of average period (5^h32^m) should have $M_V = +8.5$, $M_{pg} = +10.0$. If this system has, by our earlier discussion, $M_V(\mathrm{bl}) = +7.5$ and M_{pg} (bl) $= +7.3$, it is not surprising that the red star, an object of type M0 V, is not seen in the photographic region of the spectrum.

Though the preceding arguments are perhaps plausible, they do not explain why the brightness ratio between the two stars should be a function of the orbital period. This might be understandable, however, if the direction toward decreasing period were also an evolutionary sequence in which the blue star gravitationally contracts to the white-dwarf stage while the red star loses mass through the inner Lagrangian point or through explosions and slides down the main sequence, always maintaining an instantaneous equilibrium between mass and luminosity. If we begin with the stars about equal in brightness, we would require the rate of loss of luminosity with time to be slower for the blue star than the red. This is perhaps not unreasonable since the blue star eventually goes to the white-dwarf stage where the time scale is enormously lengthened by simple cooling; there is presumably no such floor under the red component. However, further consideration of these matters is far beyond the scope of this paper.

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