THE EXISTENCE OF A MAJOR NEW CONSTITUENT OF THE UNIVERSE: THE QUASI-STELLAR GALAXIES

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ABSTRACT

Photometric, number count, and spectrographic evidence is presented to show that most of the blue, starlike objects fainter than $m_{pg} = 16^m$ found in color surveys of high-latitude fields are extragalactic and represent an entirely new class of objects Members of the class called here quasi-stellar galaxies (QSG) resemble the quasi-stellar radio sources (QSS) in many optical properties, but they are radio-quiet. The QSG brighter than $m_{pg} = 19^m$ are 10^3 times more numerous per square degree than the QSS that are brighter than 9 flux units. The surface density of QSG is about 4 objects per square degree to $m_{pg} = 19^m$.

The evidence is developed in three parts: (1) Photoelectric photometry shows that a fundamental change occurs in the color distribution of high-latitude blue objects at about $V = 14.^m5$. Brighter than this, the objects fall near the luminosity class V line of the $U - B, B - V$ diagram. Fainter than this, 80 per cent of the objects lie in the peculiar region known to be occupied by the quasi-stellar radio sources. (2) The observed integral-count-curve, log $N(m)$, for objects in the Haro-Luyten catalogue undergoes a profound change of slope between $m_{pg} = 12^m$ and $m_{pg} = 15^m$, steepening and reaching a constant slope for m_{pg} fainter than 16^m. This magnitude interval is the same as that in which the color distribution changes, as discussed above. The slope fainter than 16^m is $d \log N(m)/dm = 0.383$. It is shown that this is the expected value from the theory of cosmological number counts for uniformly distributed objects with large redshifts. (3) Spectra of five of the faint blue objects are similar to spectra of quasi-stellar radio sources Intense, sharp emission lines of forbidden [O III], [O II], and [Ne III], together with very broad (35 Å wide) lines of Hβ, Hγ, Hδ, Hϵ, and [Ne V] are present in two of the five. Two broad emission lines are present in another at λ 3473 and λ 4279, identified as C IV (1550) and C III (1909). The other two objects have featureless spectra with only a blue continuum showing. The redshifts ($\Delta\lambda/\lambda_0$) for the three objects with lines are 0.0877, 0.1307, and 1.2410. The position of the objects in the redshift–apparent-magnitude diagram shows each of the three to be superluminous.

The space density of the quasi-stellar galaxies is estimated to be about 5×10^{-80} QSG/cm^3, which is to be compared with the space density of normal galaxies of about 1×10^{-75} galaxies/cm^3. The ratio, per unit volume, of QSG to QSS is estimated to be 500, which gives a lifetime of the QSG phase as 5×10^8 years if the lifetime of the radio source is 10^6 years.

The objects would seem to be of major importance in the solution of the cosmological problem. They can be found at great distances because of their high luminosity. QSG at $B = 22^m$ are estimated to have a mean redshift of $\Delta\lambda/\lambda_0 \simeq 5$ for a model universe of $q_0 = +1$. At these redshifts, we are sampling the universe in depth to 0.63 of the distance to the horizon (for $q_0 = +1$), and are looking back in time more than 0.9 of the way to the "creation event" in an evolutionary model. Study of the $[m, z]$- and log $N(m)$-curves using the QSG should eventually provide a crucial test of various cosmological models. But even more important, comparative study of the quasi-stellar galaxies and the intimately connected quasi-stellar radio sources is expected to shed light on the evolutionary processes of the violent events that characterize the two classes.

I. INTRODUCTION

A systematic search for quasi-stellar radio sources (hereinafter called "QSS") has been in progress at the Mount Wilson and Palomar Observatories for the past several years. Following the discovery of an excess in the ultraviolet radiation of 3C 48, 3C 196, and 3C 286 (Matthews and Sandage 1963), the search was begun with the 100-inch reflector using the two-color photographic method previously described (Ryle and Sandage 1964).

When the telescope was set at a catalogued radio position, nearly every ultraviolet object that occurred near the plate center proved to be a correctly identified QSS, as shown by later optical position determinations (Veron 1965) and photoelectric measurements. However, a curious circumstance developed as many survey plates accumulated. Objects were found on the plates that imitated the ultraviolet excess of the true QSS but that did not occur at the radio positions. The first four such images were found on plates centered on the Cambridge 3C R positions of 3C 194, 3C 205, 3C 225, and 3C 280. Consultation with Ryle concerning the possibility that the radio positions could be in error showed that the four objects could not possibly be identified with the four catalogued radio sources because they were displaced from the radio positions by amounts far in excess of the radio probable errors. The nature of these so-called interlopers remained obscure and no further work was done on them for several months.

Many more interlopers were found when the two-color search was transferred to the wide-field 48-inch Schmidt telescope. The observed frequency of about 3 per square degree to a limiting magnitude $B \simeq 18.^m5$ was low enough so as not to impair the discovery program for the QSS, because the chance of finding a random interloper within a radio error square of 30″ by 30″ was only one in 5000. But the frequency was high enough to suggest that they might be the same type of objects found by Luyten (1953, 1954, 1955, 1956*a*), Iriarte and Chavira (1957), Chavira (1958), and Haro and Luyten (1962) in high galactic latitudes, which occur with a frequency of 4 objects per square degree to $B \simeq 19.^m0$.

This identification was adopted as a working hypothesis until photometry of the first four interlopers showed that the $U - B$, $B - V$ indices, given in Table 1, were far different from the run of colors tabulated in the Haro-Luyten (HL) catalogue, so the identification with the HL objects remained ambiguous. The colors closely resemble those for U Gem and SS Cyg eruptive variables (Walker 1957; Grant and Abt 1959; Mumford 1964; Eggen and Sandage 1965), and because eruptive variables are known to occur among the blue-halo objects (Luyten and Haro 1959; Haro and Chavira 1960; Haro and Luyten 1960), the objects in Table 1 were tentatively identified as members of this class. However, evidence accumulated in the past several months shows that this cannot be true. The interlopers and the HL objects are one and the same, but most of them are not stars.

TABLE 1

PHOTOELECTRIC COLORS FOR THE FIRST FOUR INTERLOPERS

Object	V	$B-V$	$U-B$
Near 3C 194	17 72	0 45	−0 83
Near 3C 205	18 13	46	−0 77
Near 3C 225	19 25	23	−0 81
Near 3C 280	19 48	0 23	−1 16

The majority of the blue objects in high galactic latitudes appear to be superluminous galaxies with very large redshifts. They appear to be optically similar to quasi-stellar radio sources, but are radio-quiet to a flux level of 10^{-25} W/m^2 Hz at 178 MHz. Their average absolute blue magnitude is of the order of $\langle M_B \rangle \simeq -25^m \pm 2^m$ (with $H = 75$ km/sec 10^6 pc). The average redshift at $B = 22.^m0$ is estimated to be $\Delta\lambda/\lambda_0 \simeq 5$, and may be higher. At this redshift, the metric distance reached by members of the class is about 63 per cent of the way to the horizon of the universe (if $q_0 = +1$). To this magnitude limit we are looking back into an evolutionary universe ($q_0 = +1$) about 90 per cent of the way in time to the "creation event" ($R \approx 0$). Because they are radio-quiet to the limit of existing surveys, the name "QSG," denoting quasi-stellar galaxies, might be appropriate for the class.

The evidence for these conclusions is given in the following sections, presented in the order in which the clues and final confirmation unfolded between February and May of this year. The large redshifts were predicted on the basis of §§ II and III. Confirmation came from spectral observations made with the 200-inch telescope between April 23 and May 6.

II. EVIDENCE FROM THE COLORS

We consider in this section the colors of two classes of objects: (*a*) the true quasi-stellar radio sources identified to April 15, 1965, and (*b*) the blue objects with $9^m < B < 19^m$ found in surveys at high galactic latitudes.

a) *QSS*.—Identification and photoelectric confirmation of forty-four quasi-stellar radio sources are now available. The data, taken from the literature (Matthews and Sandage 1963; Ryle and Sandage 1964; Sandage and Wyndham 1965; Sandage, Veron,

TABLE 2

PHOTOELECTRIC COLORS AND MAGNITUDES FOR QSS AVAILABLE TO APRIL 15, 1965

Object	*b*	*V*	*B−V*	*U−B*	Object	*b*	*V*	*B−V*	*U−B*
3C 9	−47°	18 21	+0 23	−0 74	3C 263 . .	+50	16 32	+0 18	−0 56
47	−41	18 1	+ 05	−0 65	270 1 ..	+81	18 61	+ 19	− 61
48	−29	16 2	+ 42	−0 58	273 . .	+64	12 8	+ 21	− 85
93.	−38	18 1	+ 34	−0 50	275 1	+79	19 00	+ 23	− 43
138*	−11	18 84	+ .53	−0 16	277 1 .	+60	17 93	− 17	− 78
147†	+10	17 8	+ 65	−0 37	279	+57	17 75	+ 26	− 56
181	+15	18 92	+ 43	−1 02	280 1 ...	+77	19 44	− 13	− 70
186	+26	17 60	+ 45	−0 71	281 . .	+69	17 02	+ 13	− 59
190	+22	17 46	− 20	−0 90	286	+81	17 30	+ 22	− 84
191	+21	18 4	+ 25	−0 84	287 .	+81	17 67	+ 63	− 65
196	+33	17 6	+ 60	−0 43	298 . .	+61	16 79	+ 33	− 70
204	+36	18 21	+ 55	−0 99	334 ..	+41	16 41	+ 12	− 59
207 .	+30	18 15	+ 43	−0 42	336 .	+42	17 47	+ 44	− 79
208 .	+33	17 42	+ 34	−1 00	446..	−49	18 39	+ 44	− 90
215	+37	18 27	+ 21	−0 66	CTA 102.	−38	17 32	+ 42	− 79
216	+43	18 3	+ 50	−0 60	MSH 13-011	+55	17 68	+ 14	− 66
217	+43	18 50	+ 25	−0 86	0106+01 .	−61	18 39	+ 15	− 70
245	+56	17 25	+ 45	−0 83	0922+14	+41	17 96	+ 54	− 52
247	+62	18 82	+ 52	−0 25	0957+00 .	+40.	17 57	+ 47	− 71
249 1	+38	15 72	− 02	−0 77	1116+12	+64	19 25	+ 14	− 76
254 .	+66	17 98	+ 15	−0 49	1217+02	+64	16 53	− 02	− 87
261 .	+73	18 24	+0 24	−0 56	1252+11	+75	16 64	+0 35	−0 75

* 3C 138, corrected by $E(B-V) = 0$ 30, $E(U-B) = 0$ 22, gives $V_0 = 17$ 9, $(B-V)_0 = +0$ 23, $(U-B)_0 = -0$ 38

† 3C 147, corrected by $E(B-V) = 0$ 30, $E(U-B) = 0$ 22, gives $V_0 = 16$ 9, $(B-V)_0 = +0$ 35, $(U-B)_0 = -0$ 59.

and Wyndham 1965; Bolton, Clarke, Sandage, and Veron 1965), are collected in Table 2. In all but a few cases, the identifications are confirmed by excellent agreement between the radio positions and the new optical positions measured by Veron.

The distribution of colors is shown in Figure 1. Closed circles are the measured colors with no correction for galactic reddening. The two open circles represent 3C 138 and 3C 147, where reddening corrections of $E(B-V) = 0.^m30$ and $E(U-B) = 0.^m22$ have been applied. The correction for 3C 147 was determined directly by measurement of *UBV* for neighboring galactic stars. The same correction was arbitrarily applied to 3C 138—justified by the equal latitudes of the two QSS on either side of the galactic plane and by the similarity of galactic longitude. The five open triangles are for the N-type galaxies 3C 109, 3C 212, 3C 234, 3C 287.1, and 3C 459, which were measured in the general identification program for radio sources.

The two straight lines represent the black-body relation and the distribution

$$F(\nu)d\nu = Ae^{-C\nu}\,d\nu\,, \tag{1}$$

where A and C are constants. The $U - B$ and $B - V$ colors were computed by relations given elsewhere (Matthews and Sandage 1963).

The distinguishing ultraviolet excess of the QSS relative to normal stars is evident. About half of the QSS fall in the region below the black-body line, known to be occupied by the white dwarfs (Johnson and Morgan 1953; Eggen and Greenstein 1965). The other half fall above the black-body line in a region occupied by old novae, SS Cyg, U Gem, and Z And–type variables, as first described by Walker (1957).

b) High-latitude blue objects.—Many surveys for blue stars in high galactic latitudes have been made in the past 40 years by a number of people. The earliest are the two studies by Malmquist (1927, 1936), which covered an area of about 150 square degrees near the north galactic pole to $m_{\mathrm{pg}} \simeq 15^m$. Eighteen stars with negative colors were found in a sample which contained nearly 8200 stars. Following this pioneer work, surveys were

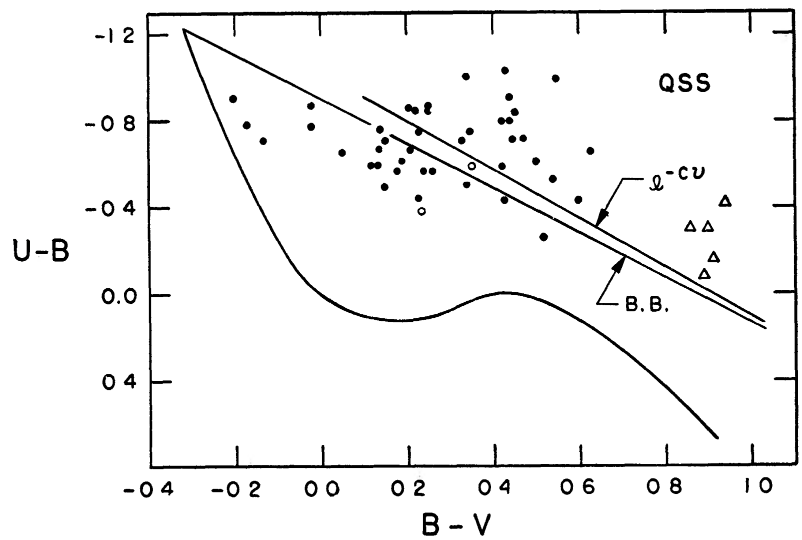

FIG. 1.—The distribution of color indices for 44 quasi-stellar radio sources (*circles*) plus 5 N-type galaxies. The normal luminosity class V line is shown together with lines for black bodies and for objects that radiate with a distribution shown by eq. (1) of the text.

made by Humason and Zwicky (1947), Luyten (many papers from 1952 to the present), Iriarte and Chavira (1957), Chavira (1958), Cowley (1958), Feige (1958), Slettebak and Stock (1959), and Haro and Luyten (1962).

Most of the detailed spectroscopic and photometric work on these objects has been confined to stars brighter than $B = 15^m$. Spectroscopic work by Humason (Humason and Zwicky (1947), Greenstein (1956, 1961), Slettebak (Slettebak, Bahner, and Stock 1961), Klemola (1962), and Berger (1963) had shown that the blue objects brighter than $B \simeq 15$ are stars of a variety of types including white dwarfs, hot subdwarfs, horizontal-branch stars, and composite stars. Photometry by Iriarte (1958, 1959), Slettebak, Bahner, and Stock (1961), Klemola (1962), Harris (unpublished), and Eggen and Sandage (1965) indicated that the spectral classifications are consistent with the colors. Furthermore, statistical parallax determinations giving $\langle M_B \rangle \simeq +3$ for stars brighter than $B \simeq 15^m$ by Luyten (1956*b*, 1959, 1960, 1962*a*, *b*), and Klemola (1962) are consistent with assigning many of the stars to the globular-cluster-like halo population.

But study of the photometric data already in the literature gives a hint that something unexpected occurs for $B > 15^m$. Klemola's discussion (1962, Fig. 2) of the available data, especially those of Iriarte (1959), suggested that a transition in the color distribution occurs at about $B = 14.^m5$, but the data were too scanty for definite conclusions.

In February of this year, following the clue of Table 1, Veron and I made a special color survey on two plates taken with the Palomar 48-inch Schmidt to see if abnormal colors are a universal property of the interlopers. Veron marked thirty-one objects that had large negative $U - B$ indices. Photoelectric photometry was obtained for twenty-one of these objects at the end of February. The unexpected result (Sandage and Veron 1965) was that fifteen of the twenty-one interlopers had colors that imitated the known QSS, five had colors close to the luminosity class V line, and one appears to be an F subdwarf. These data, together with those discussed by Klemola, suggested that the majority of blue objects fainter than $B \approx 15$ are fundamentally different from those brighter than this limit.

Figure 2 shows the results, divided into two groups at apparent visual magnitude $V = 14.^m5$. The data are from Feige (1958), Iriarte (1959), Slettebak, Bahner, and Stock (1961), Klemola (1962), Harris (unpublished observations of the Humason-Zwicky stars made available by Greenstein), Eggen and Sandage (1965), Sandage and Veron (1965), and Table 1 of this paper. The sharp division between Figure 2, *a* and *b*, is clear. Stars

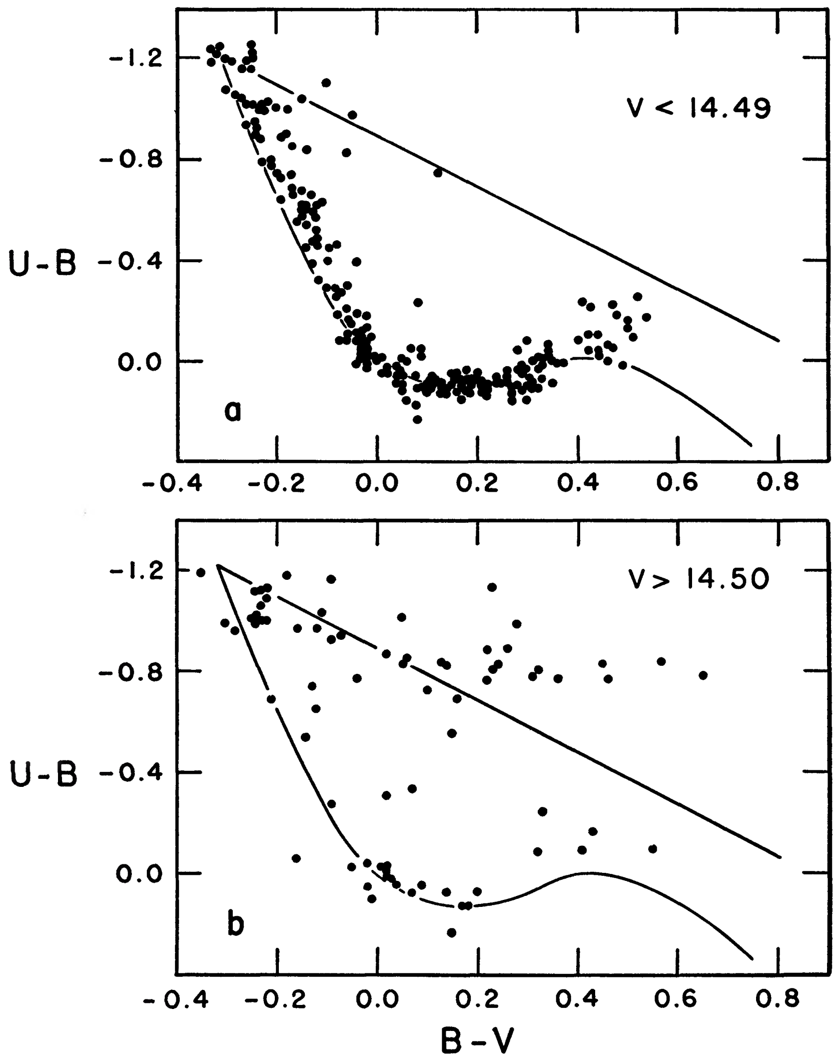

FIG. 2 —(*a*) The two-color diagram for blue objects brighter than $V = 14.^m50$ found in surveys at high galactic latitudes. The data were all photoelectrically determined by authors quoted in the text. (*b*) Same as *a* for objects fainter than $V = 14.^m50$.

in Figure 2, *a*, follow the luminosity class III–V line, while most objects in Figure 2, *b*, imitate the distribution of QSS in Figure 1. The results suggest that we are beginning to run out of galactic halo stars at about $B \simeq 15^m$ and are starting to pick up objects with QSS colors fainter than this limit. But are members of this new class extragalactic? Evidence that this is the case came from analysis of the integral count, $N(m)$, from the catalogue of Haro and Luyten (1962).

III. EVIDENCE FROM THE COUNTS

The HL catalogue lists 8746 blue objects found in 2000 square degrees of the south galactic polar region. The care with which the survey was conducted insures adequate

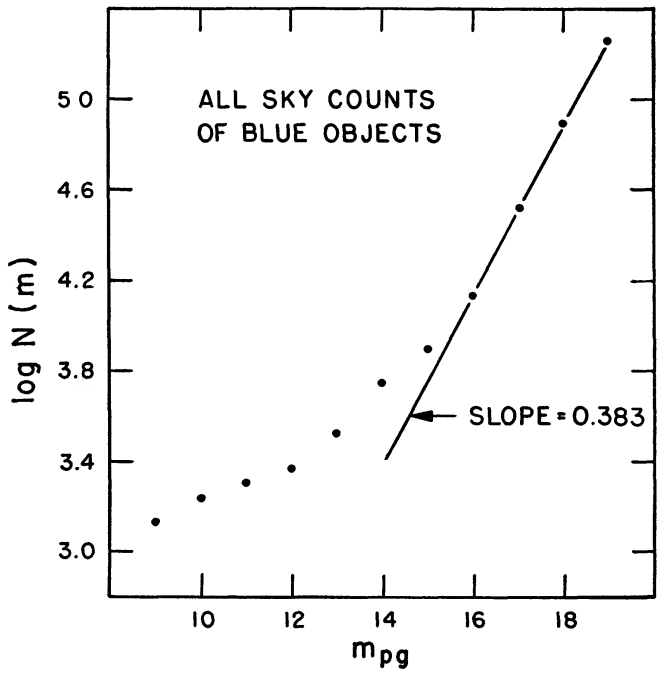

FIG. 3 —The integral-count–apparent-magnitude relation for objects in the Haro-Luyten catalogue with tabulated $U - V$ values smaller than $-0.^m8$. The counts have been normalized to give all-sky values $N(m)$ is the number of objects in 41253 square degrees brighter than apparent magnitude m. Note the sharp change of slope near $m_{pg} = 15^m$.

completeness to $B = 19.^m0$ within the imposed color limits. Figure 3 shows log $N(m)$ versus m_{pg} for objects in the catalogue with tabulated $U - V$ values smaller than $-0.^m8$. The counts are from the HL Table V, but with log $N(m)$ increased by 1.314 to convert the tabulated numbers to all-sky values in 41253 square degrees.

The striking feature of Figure 3 is the sharp change of slope near $m_{pg} = 15$. Brighter than $m_{pg} = 13$, the slope is $d \log N(m)/dm = 0.070$. From $m_{pg} = 16^m$ to the limit of the data, the slope is remarkably constant with a value $d \log N(m)/dm = 0.383$.

Figure 3 *could* be explained by a single class of galactic stars (of constant mean M_B) only if the spatial density $\Delta(r)$ drops precipitously from $m_{pg} = 9$ to $m_{pg} = 12$, undergoing a transition in the gradient $d\Delta(r)/dr$ from magnitude 12 to magnitude 16, and then leveling off to a constant gradient beyond $m_{pg} = 16$. In the distance interval corresponding to apparent magnitudes 9–12, the slope of $d \log N(m)/dm = 0.070$ requires that

$$\frac{d \log \Delta(r)}{dm} = -0.530\,, \tag{2}$$

which means

$$\frac{d \log \Delta(r)}{dr} = -2.65 \tag{3}$$

or

$$\Delta(r) = \text{const.}\, r^{-2\,65} \qquad \text{for } m_{pg} < 12^m\,, \tag{4}$$

where r is the distance from the Sun in the direction of the Haro-Luyten survey. Between $m_{pg} = 12$ and $m_{pg} = 16$, the gradient would have to become less steep, reaching a constant value for distances corresponding to $m_{pg} = 16$, such that

$$\Delta(r) = \text{const.}\, r^{-1\,085}\,, \quad m_{pg} > 16^m\,. \tag{5}$$

Fainter than $m_{pg} = 19$, the density gradient would presumably have to steepen again to a value greater than -2 to prevent the mass of the halo from becoming infinite. A change of density gradient of this type is unknown for any class of object in the Galaxy.

The possibility that all the blue objects from $19 \geq B > 9$ are stars of a single type cannot be excluded on the basis of Figure 3 alone, but, if this were true, the change in color shown in Figure 2, *a* and *b*, at the point of slope transition of the log $N(m)$-curve would not be explained. Of course, two main types of galactic stars could be postulated—the bright class with $\langle M_B \rangle \simeq +3$, as required by the proper-motion calibration (Luyten 1956*b*, 1959, 1960, 1962*a*, *b;* Klemola 1962), and the faint class with a density decline as $r^{-1\,085}$, but this second group would then have to be almost entirely SS Cyg–U Gem-type variables or old novae to explain the colors of Figure 2, *b*. This possibility seemed so remote that a second explanation was sought in terms of QSG.

The principal question now to be answered is: if the objects fainter than $B \simeq 16^m$ are predominantly extragalactic, why does the slope, $d \log N(m)/dm$, have a value of 0.383 rather than 0.60, which applies for normal galaxies at this magnitude level? The answer is: the objects are not of normal absolute luminosity but are brighter. They are so distant for apparent magnitudes fainter than 19^m that the predicted cosmological bending of the log $N(m)$-curve (Sandage 1961*a* [hereinafter called "Paper I"], Fig. 4; 1963, eqs. [26]–[33]) has taken place at these bright apparent magnitudes. The theoretical slope at any desired m can be predicted once the mean absolute luminosity is fixed, and the result is that $d \log N(m)/dm = 0.40$ *if we assign* $\langle M_B \rangle$ *the value that applies to the quasi-stellar radio sources of known redshift*. The argument is as follows.

The important theoretical point is that the log $N(m)$-curve is obtained from theory by a series of transformations from the volume $V(r)$ contained within coordinate distance r. The number count $N(r)$ is proportional to $V(r)$ for uniformly distributed sources. Now $N(r)$ can be uniquely transformed into $N(z, q_0)$ because the metric distance is given once z and q_0 are known (see Sandage 1961*a*, eqs. [50] and [10′]). The fundamental point is that $N(z, q_0)$ is unique—it depends only on the geometry and not upon the objects being counted. However, if we wish to transform $N(z, q_0)$ into the observable $N(m, q_0)$ relation, the shape of the function at any m does depend on a physical parameter of the objects; namely, the mean absolute luminosity, or, more explicitly, on the zero point of Hubble's expansion law, which gives m as a function of z.

These considerations show that the slope of the $N(m)$ relation has a particular value at a particular redshift; but, if we are counting two classes of objects that differ by ΔM in mean absolute magnitude, this given slope will occur at *different* apparent magnitudes, separated by $\Delta m = \Delta M$.

Numerical values of $N(m)$ have been tabulated (Sandage 1961*a*,[1] Tables 3 and 4) for

[1] It has been pointed out by E. L. Schücking and by van Albada (1963) that an error exists in Fig 3 and Table 3 of my paper (1961*a*) due to choice of the incorrect value of the double-valued arc sin P beyond arc sin $P = 1$. The correction procedure on p. 372 (eq [41]) of the paper happens to be valid, but for the wrong reasons. Therefore, Table 4 and Fig. 4 are correct, but the discussion about the antipole is

the special case of average field galaxies, which obey a Hubble relation

$$m_i - K_i = 5 \log z + 22.516 \qquad \text{for } z \ll 1 . \quad (6)$$

Here m_i is the observed heterochromatic magnitude in band pass i, and K_i is the "bolometric" correction due to redshifting the spectrum through the ith filter band (see Humason, Mayall, and Sandage 1956, Appendix B; Sandage 1965, for the theory of the K-correction). For a different class of objects where the constant in equation (6) has a

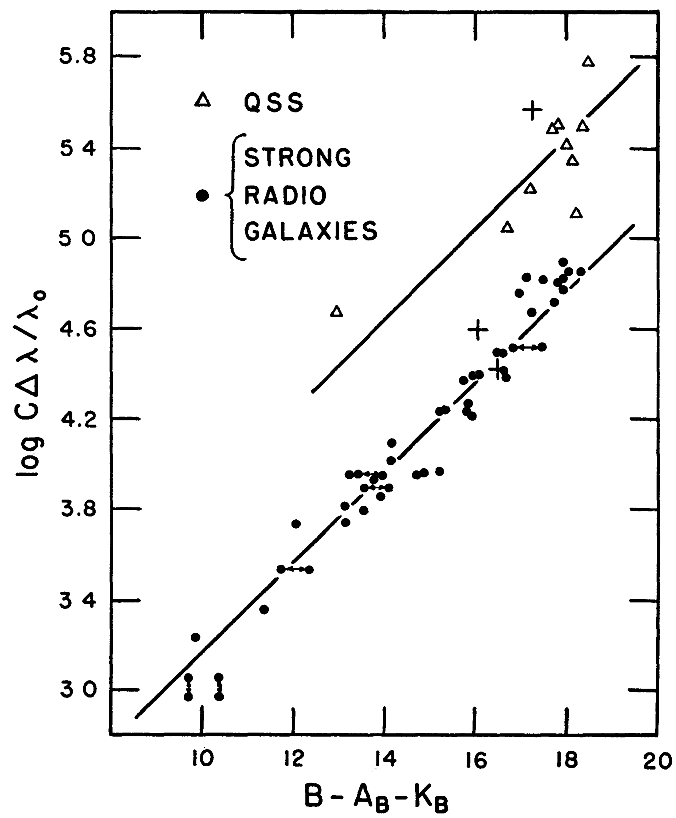

FIG 4.—The redshift-apparent-magnitude relation for strong radio galaxies (*circles*) and for quasi-stellar radio sources (*triangles*). Arrows connect dumbbell galaxies for which the magnitude of the bright component and of the two components combined is given. Correction for K-dimming has been applied to the galaxies. No K-correction has been applied for the quasi-stellar sources. The three crosses are discussed in § IV.

different value C, Tables 3 and 4 of Paper I can still be used to obtain the shape of $N(m)$, providing that the argument of the tables is changed by $22.516 - C$ magnitudes. When this is done, the tables will be entered at the same value of z for the two objects. It is important, therefore, to know the $[m, z]$ relation for the objects under discussion.

Figure 4 shows the Hubble diagram for strong radio galaxies and for quasi-stellar sources as determined from unpublished photoelectric photometry (Sandage 1965), and from redshifts by Schmidt (1965*a*), with a few by Minkowski and by Greenstein. The radio galaxies are shown as closed circles, while the quasi-stellar sources are open tri-

erroneous The caption on Table 5 should read *equator* rather than *antipole*. But again, all numerical values of the critical Table 3, supplemented where necessary by Table 4, can be used without change as they are unaffected by the conceptual mistake.

angles. Plotted[2] is log $c\Delta\lambda/\lambda_0$ versus the photoelectric B magnitude corrected for aperture effect, galactic absorption A_B, and heterochromatic K-dimming discussed elsewhere (Sandage 1965). Redshifts for the quasi-stellar sources are due entirely to Schmidt, except for 3C 48, which was announced by Greenstein and Matthews (1963). The spectroscopic and photometric data on the ten quasi-stellar sources are summarized in Table 3.

The K-correction has been applied in Figure 4 to the galaxies but not to the QSS because it is shown in the Appendix that, if the energy distribution of quasi-stellars is of the form of equation (1), the K-dimming is very small even for large redshifts. This is because the high-energy part of the spectrum is being shifted into the visual region, in contrast to the situation for normal galaxies where the ultraviolet energy is very small. Equation (1) has the further property that the anomalous $U - B$ index will continue to be abnormal for redshifts as high as $z = 3$ (see Table A2 of the Appendix), which explains why 3C 9, with $z = 2.012$, still has the peculiar observed color indices. This situation, ideal from the discovery standpoint, will probably change, however, for redshifts greater than $z = 3$ when the Lyman continuum will be shifted to $\lambda \geq 3600$ Å. If a large Lyman jump

TABLE 3

DATA FOR THE QSS SHOWN IN FIGURE 4

Object	z	log cz	V	$B-V$	$U-B$	Redshift Ref *
3C 9.	2 012	5 781	18 21	+0 23	−0 74	(5)
3C 47	0 425	5 106	18 1	+ 05	− 65	(3)
3C 48.	0 367	5 042	16 2	+ .42	− 58	(2)
3C 147	0 545	5 214	16 9	+ .35	− 59	(3)
3C 245	1 029	5 490	17 25	+ 45	− 83	(5)
3C 254	0 734	5 343	17 98	+ 15	− 49	(5)
3C 273	0 158	4 676	12 9	+ 21	− 85	(1)
3C 286	0 846	5 404	17 30	+ 22	− 84	(4)
3C 287..	1 054	5 500	17 67	+ 63	− 65	(5)
CTA 102	1 038	5 493	17 32	+0 42	−0 79	(5)

* (1) Schmidt 1963; (2) Greenstein and Matthews 1963; (3) Schmidt and Matthews 1964; (4) Schmidt 1962; (5) Schmidt 1965.

is present, the observed U magnitude will be depressed, as explained in the Appendix, and the two-color technique may no longer be useful for survey purposes. This is expected to occur near $B \simeq 21^m$, which is well within the limit of the 200-inch telescope.

Figure 4 shows that the ten QSS show a relatively small scatter about a mean [m, z] line whose equation is

$$m_B = 5 \log z + 18.186 . \quad (7)$$

The data are not yet extensive enough to detect a second-order term, and a straight line ($q_0 = +1$) is adopted, which is accurate enough for the present purpose. (Future work will, of course, aim to find q_0 by using data of this kind.)

The difference in the constant between equations (6) and (7) is $4.^m33$, which is the difference in the mean absolute magnitudes of QSS and the field galaxies in the redshift catalogue (Humason, Mayall, and Sandage 1956). Table 3 of Paper I has been entered with this shift of argument and the predicted log $N(m)$-curves for $q_0 = 0$, and $q_0 = +1$ are shown in Figure 5. Here the $q_0 = 0$ curve has been normalized to fit the observed

[2] It should be noted in passing that multiplication of $\Delta\lambda/\lambda_0$ by the velocity of light is a convention which should probably now be abandoned. The function $c\Delta\lambda/\lambda$ is not the "velocity of expansion" and the relativistic Doppler equation must not be applied to the ordinate in Fig 4. The fundamental parameter in the theory of expanding spaces is $\Delta\lambda/\lambda_0$ itself, which of course becomes larger than 1, approaching ∞ as the horizon is reached.

counts in the HL catalogue, but the normalization is slightly arbitrary because our present knowledge does not yet tell us what percentage of real stars exist at $B = 19^m$. However, the problem at hand, namely, the value of the slope of log $N(m)$, is independent of the normalization. The mean slope between $m_{pg} = 16$ and $m_{pg} = 19$ for the two cases are $d \log N(m)/dm = 0.403$ for $q_0 = 0$, and $d \log N(m)/dm = 0.360$ for $q_0 = +1$, values that nicely bracket the observed slope of 0.383 and that are very different from 0.60 expected for extragalactic objects with negligible redshifts.

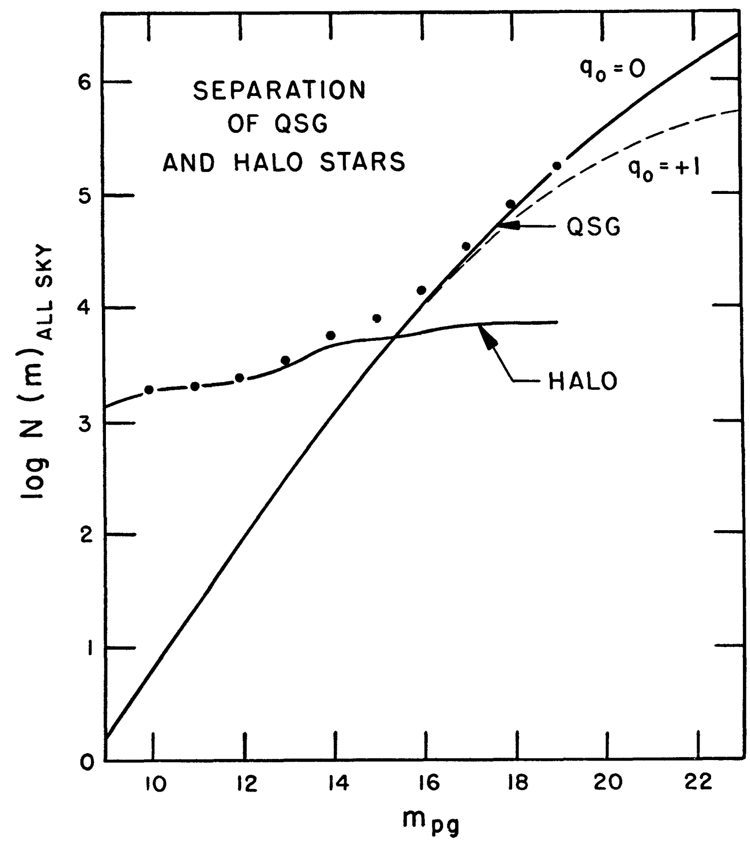

FIG 5.—Predicted integral-count-curves for $q_0 = 0$ and $q_0 = +1$ based on the $[m, z]$ relation for QSS of Fig. 4, fitted to the data of Fig. 3. The normalization has been arbitrarily made for the $q_0 = 0$ curve at $m_{pg} = 19$. The curve marked *halo* is obtained by subtraction of the QSG-curve for $q_0 = 0$ from the observed points. The $N(m)$-values are the number of objects brighter than m_{pg} in 41253 square degrees.

The present analysis should not be carried too far at the moment. For example, no cosmological conclusions on the value of q_0 should be inferred from Figure 5, even though the log $N(m)$-values differ significantly between $q_0 = +1$ and $q_0 = 0$. We must have observations of the QSG at bright magnitudes to properly normalize the log $N(m)$-curves of Figure 5. Equally important, Figure 4 must be better defined because a small change in the constant in equation (7) is equivalent in its effect to the differences between $N(m)$-curves for the various q_0-values in Figure 5.

Figure 5 shows the subtraction of the cosmological $N(m)$-curve (for $q_0 = 0$) from that of the observed HL points. The subtraction separates the true halo stars from the (hypothetical to this point in the discussion) blue galaxies, and shows that the two $N(m)$-curves cross near $m_{pg} = 15$, which is just where the color distribution changes. Table 4 gives the ratio of stars to QSG based on this subtraction, but it should be emphasized that these numbers show only the trend to be expected. The numbers themselves are quite uncertain because the subtraction of two large numbers is involved. All that can safely be generalized is that Table 4 qualitatively explains the color change in Figure 2, *a* and *b*, but the exact ratios must await the results of more detailed surveys.

The results of §§ II and III were established before redshifts were obtained. They constituted a prediction that the radio-quiet blue objects fainter than $B \simeq 15$ that have peculiar colors should have large redshifts. To prove this it was necessary to test for the redshifts, and observations to this end were undertaken April 23–27 and May 2–6, 1965.

IV. EVIDENCE FROM THE REDSHIFTS

Objects for the test were chosen from Iriarte's photometric list (1959) of Tonantzintla blue stars, from Veron's list (Sandage and Veron 1965) for which photometry had been completed in February, and from the interlopers of Table 1. Maarten Schmidt generously offered to observe a few of the candidates during his Palomar run in the last week in April to insure against the possibility of cloudy weather during my scheduled run in the first week of May. The insurance paid off, because the May 2–6 period was abnormally stormy on Palomar Mountain, characterized by mountain fog blown by high winds from the valley, making the seeing images large during the periods of clearing. However,

TABLE 4

PREDICTED RATIO OF HALO STARS TO QSG FROM FIGURE 5

Survey Limit (B Mag)	$N(m)$ Stars/ $N(m)$ QSG	Survey Limit (B Mag)	$N(m)$ Stars/ $N(m)$ QSG
12	26 3	16	0 60
13	9 6	17	24
14	4 1	18	10
15	1 5	19	0 04

spectra were obtained of several of the brighter objects of the candidate list during this time.

A total of ten spectrograms of six objects were taken during the two runs with the 200-inch prime-focus spectrograph, using the F/1 camera at a dispersion of 400 Å/mm. The plates were baked Eastman Kodak IIa-D, covering the wavelength range 3180–6300 Å, or IIa-F, which extends the spectral region to 6700 Å. Schmidt obtained four spectra of three faint objects, one of which proved to be crucial, while I got six spectra of three additional objects, two of which gave positive confirmation. The results are as follows:

BSO 16.—This is one of the blue stellar objects found in the special photographic survey carried out by Veron in February. The photoelectric photometry gave $V = 16.17$, $B - V = -0.25$, and $U - B = -1.01$. One spectrogram by Schmidt shows conclusively that the object is a star of small velocity with Hβ, Hγ, Hδ, and Hϵ in absorption. No other features are clearly visible. The result is expected because the colors are unlike those of QSS, but lie close to the luminosity class III–IV line in Figure 2, *a*, for normal stars.

Near 3C 194.—This was the first of the interlopers, whose photometric properties are given in Table 1. One spectrogram by Schmidt is continuous, with no obvious emission or absorption features. It is similar in this respect to several other QSS, such as 3C 196 (Schmidt 1965*c*). The continuous spectrum extends far into the ultraviolet, as expected from the $U - B$ color, and looks similar in all respects to the continuum of other quasi-stellar radio sources studied to date. However, the absence of lines precludes use of this object as a test of the hypothesis.

BSO 1.—Two spectra by Schmidt show the presence of two very broad emission lines. Visual measurement for wavelength with a microscope gave values of λ 3473 and λ 4279 Å for the two lines. The redward line is exceedingly difficult to see, but analysis of the plates using Westphal's (1965) cross-correlation technique definitely proves that the line is present with a wavelength of λ 4284 Å. The value is well determined by the cross-correlation method if the line is symmetrical. On both plates the blueward line is easy to see and is bisected by a sharp absorption feature, flanked by the broad emission. We interpret this to mean self-absorption. The identification of the lines is not completely certain because the wavelength ratio of 1.233 is given by four possible pairs of coincidences in lists of candidate lines prepared by Schmidt (1965*b*) and by Bowen (private communication). The four cases are (1) Mg II (λ 2798) and [Ne V] (λ 3426), ratio 1.224; (2) O III (λ 3133) and [Ne III] (λ 3869), ratio 1.235; (3) [Ne V] (λ 3346) and Hδ (λ 4102), ratio 1.226; and (4) C IV (λ 1550) and C III (λ 1909), ratio 1.232. Cases (1), (2), and (3) can be excluded because of the absence of other lines that should be present, such as the Balmer series and λ 3727 of [O II]. Furthermore, the wavelength ratios for cases (1) and (3) are outside the measuring errors. Case (2) can be further excluded because the [Ne V] lines (λλ 3346, 3426) should appear, but they do not. Case (4) appears to be the most probable for the following reasons: (1) The λ 1550 line has been previously observed in the quasi-stellar sources 3C 9 and 3C 287, while λ 1909 appears in 3C 254, 3C 245, CTA 102, and 3C 287 (Schmidt 1965*b*). (2) The observed wavelength ratio agrees almost exactly with the laboratory value. (3) The self-absorption of the observed λ 3473 line is consistent with the identification as the ground-state doublet λ 1550, because this is the resonance line of C IV, being the same sort of transition as the D-lines of sodium and the H- and K-lines of singly ionized calcium. The one objection to the identification is the apparent absence of Mg II (2798), which should appear at λ 6269. Schmidt took a special IIa-F plate to attempt to find the line. There may be faint features in that region on both plates, but neither visual inspection nor Westphal's cross-correlation technique has definitely confirmed its existence. The value of the redshift of BSO 1 is, therefore, not beyond all question, but what *is* established is that a radio-quiet object, resembling the QSS in optical properties, exists whose redshift is certainly not zero.

The redshift $\Delta\lambda/\lambda_0 = 1.241$ has been adopted as highly probable for the discussion. The photometry gives $V = 16.98$, $B - V = 0.31$, and $U - B = -0.78$, which places BSO 1 among the brightest QSS in Figure 4. The object is shown as a cross in this diagram at $B = 17.^m29$.

BSO 105.—One spectrogram shows a continuum extending far into the ultraviolet with no obvious emission or absorption features. It closely resembles the object near 3C 194 and has the same bearing on the problem—neither confirming nor denying the hypothesis.

Ton 730.[3]—Three spectra are available. Two were taken in very poor seeing (10″ and 5″ for the seeing disk), and one was obtained under more normal conditions. Photometry by Iriarte (1959) gives $V = 15.91$, $B - V = 0.57$, $U - B = -0.84$—colors that resemble those of QSS. The spectrum shows sharp, strong lines of forbidden [O III] (λλ 5007, 4959), and very broad hydrogen lines of Hβ, Hγ, Hδ, and Hε.

The lines of [O II] (λ 3727) and [Ne V] (λλ 3426, 3346) are also present, but with lower intensity and with smaller contrast with the continuum. The hydrogen lines are about 35 Å wide and resemble in this respect all quasi-stellar radio sources known to date. Ton 730 is, however, radio-quiet to 10 flux units at 178 MHz. It does not appear in the 3C catalogue nor in the summary catalogue of Howard and Maran (1965).

[3] It is of some interest that, after the observing list had been prepared for the May 2–6 Palomar run, a letter, dated April 20, 1965, was received from Iriarte in which he pointed out that the colors of Ton 730 were like those of some of the QSS and that the object was worthy of special attention. A finding chart was inclosed, and I wish to record my indebtedness to him for his letter and to suggest that he had obviously made a tentative connection between certain of the blue objects and the quasi-stellar sources.

The redshift is unambiguous with a value $\Delta\lambda/\lambda_0 = 0.0877 \pm 0.0004$ (m.e.), which shows that the object is extragalactic. The position of Ton 730 in Figure 4, shown by the cross at $B = 15.48$, is fainter than any of the ten QSS with known redshifts. It falls near the line for the strong radio galaxies rather than near the mean QSS line. However, it should be remembered that this fainter line is almost the *upper envelope* of the $[m, z]$ relation for all known normal galaxies, including the brightest optical galaxies in the great clusters (Sandage 1965). Thus, Ton 730 is an extremely bright object as galaxies go ($M_B = -22.^m2$ if $H = 75$ km/sec 10^6 pc). Furthermore, comparison with "normal" QSS is dangerous at the present time because the sample of QSS is much too small to know if the dispersion in absolute magnitude of the quasi-stellars is great enough to contain Ton 730 within the scatter.

Although the results were encouraging, they did not completely verify the hypothesis of stellar-appearing galaxies as the main constituent of the faint blue objects in the poles, for the following reason. The optical image of Ton 730 is slightly fuzzy and elongated on 48-inch Schmidt plates. On very close inspection, this object would not be identified as a star in surveys. The image looks like an extremely compact galaxy on long-exposure plates, although to the eye at the 200-inch telescope it is stellar, and it appears almost stellar on short exposures with the Schmidt. Our present interpretation is that the fuzziness is caused by material blown off from the central core as in the case of 3C 48 and 3C 196, with their attendant small nebulae (Matthews and Sandage 1963). But this circumstance appeared to exclude Ton 730 as a *proof* of the existence of stellar-appearing galaxies. In any case, Ton 730 is exceedingly peculiar and is unlike any non-radio object found to date. Its spectrum is similar to the QSS 3C 249.1 (Schmidt, private communication). Its colors imitate the QSS and it is undoubtedly related to the general class of objects under discussion.

Ton 256.—Iriarte (1959) gives $V = 15.91$, $B - V = 0.57$, $U - B = -0.84$. The object looks completely stellar on the sky survey plates and has the peculiar colors of the QSS. The two spectrograms obtained show intense, sharp lines of [O III] (λλ 5007, 4959), [Ne III] (λ 3869), [O II] (λ 3727), and broad emission lines of Hβ, Hγ, Hδ, Hε, and [Ne V] (λλ 3426, 3346). The spectrum is similar to that of Ton 730 except that the emission lines are more intense relative to the continuum. The best spectrogram is reproduced in Figure 6, where a convenient fiducial wavelength reference is provided by the bright night-sky line of [O I] at λ 5577. All spectral features are shifted by $\Delta\lambda/\lambda_0 = 0.1307 \pm 0.0002$ (m.e.), which is a very large shift for this apparent magnitude for normal galaxies. The object is radio-quiet to the limit of the 3C catalogue.

Ton 256 has been placed in Figure 4 as a cross at $B = 16.^m49$. It is brighter than the radio-galaxy line and has an absolute magnitude similar to the quasi-stellar radio source 3C 47. This object, together with BSO 1, with confirmatory evidence from Ton 730, appears to provide substantial evidence for the hypothesis of the existence of a new class of extragalactic objects—the radio-quiet QSG.

V. THE SURFACE AND SPATIAL DENSITIES OF QSG

Surface density.—If most of the objects fainter than $m_{pg} = 16^m$ in the HL catalogue are QSG, the average surface density at any apparent magnitude can be found from Figure 5. Reading from the curves and reducing to numbers per square degree gives the values in Table 5, where the numbers for $q_0 = +1$ have been renormalized to give the observed surface density at $m_{pg} = 19^m$.

The K-correction will begin to be appreciable for the fainter entries in the table because the redshifts are expected to be about $z \geq 2$ at $m_{pg} - K = 20^m$, and larger than 5 at $m_{pg} - K = 22^m$. At these redshifts the problems involving the Lyman continuum will be paramount, making the observed magnitudes much fainter than would otherwise be the case. Furthermore, the expected disappearance of the ultraviolet excess at redshifts greater than 3, due to the presence of the Lyman jump under the U filter, means

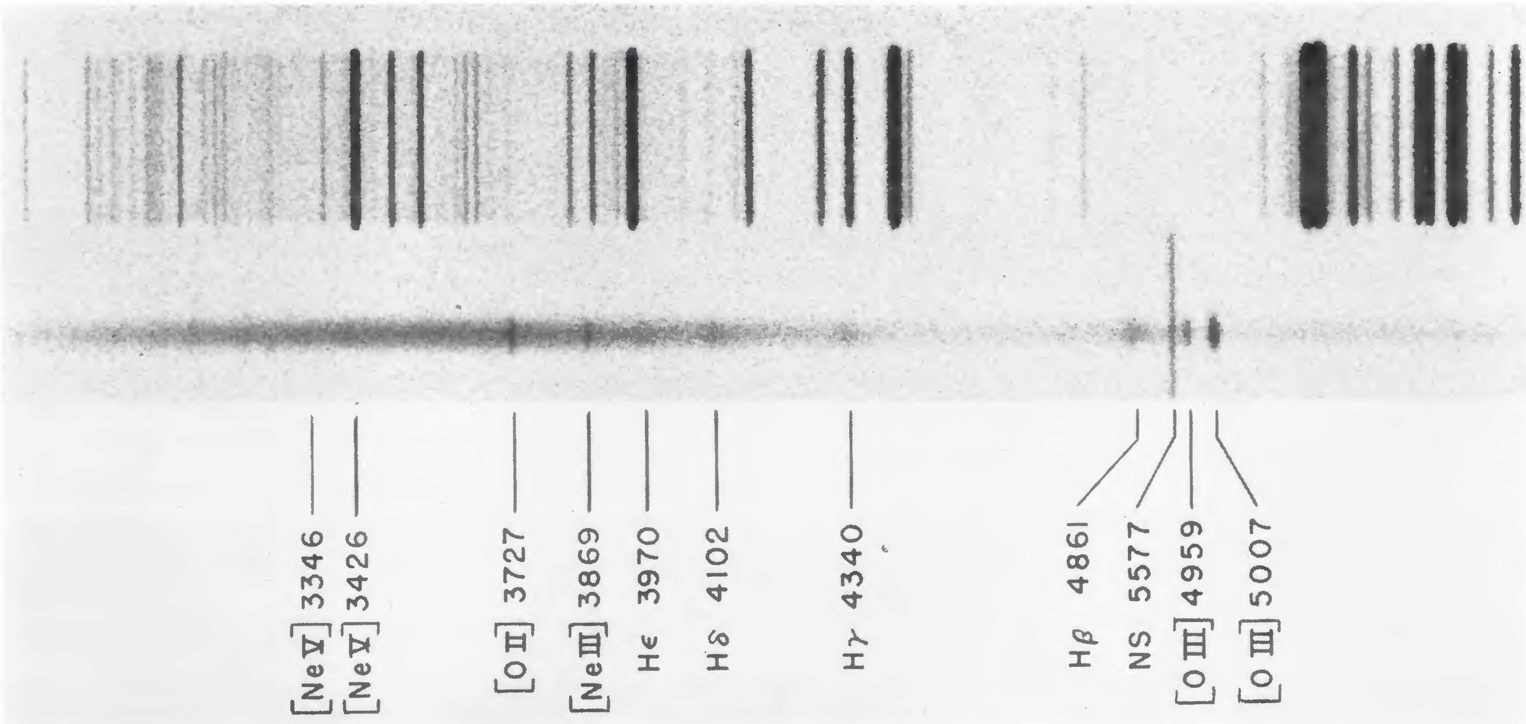

FIG. 6.—Spectrum of Ton 256 showing the narrow forbidden lines of [O III], [O II], and [Ne III], and the broad Balmer series and [Ne V] lines, redshifted from their normal position by a factor of 1.1307.

that new search techniques must be developed before objects of this type can be discovered at the faintest light levels. The most likely bet appears to be long-exposure, two-color plates in the red and near-infrared (to compensate for the redshift), but criteria must be found for picking up the abnormal colors of QSG at these long wavelengths.

Table 5 also shows the necessity of using instruments of wider field than that possessed by the 200-inch prime focus. Even neglecting the K-term at $m_{pg} = 23$, there are only 4.7 expected quasi-stellar galaxies in the 0.077 square-degree field for the $q_0 = 0$ model, and only 1.6 QSG per plate for $q_0 = +1$. Nevertheless, with proper instrumentation and new techniques, the possibility of choosing between model universes on the basis of counts now seems to exist, and every effort toward this goal will be expended.

It is of some interest to note how many bright QSG there should be at various apparent magnitudes. Figure 5 predicts that 6 of these objects should exist brighter than $m_{pg} = 10^m$, 23 brighter than 11^m, 83 brighter than 12^m, and 316 brighter than 13^m over the entire celestial sphere—numbers that can eventually be checked.

Spatial density.—The volume density of QSG can be estimated approximately by combining the data of Figures 4 and 5 at bright magnitudes where the effects of spatial curvature and redshift are negligible. Figure 5 shows that there should be 83 QSG over

TABLE 5

PREDICTED NUMBER OF QSG BRIGHTER THAN m PER SQUARE DEGREE AND PER 200-INCH PRIME-FOCUS PLATE

$m_{pg} - K$	$q_0=0$		$q_0=+1$	
	$N(m)$	$N(m)/12$ 9	$N(m)$	$N(m)/12$ 9
18	1 8	0 14	2 1	0 16
19	4 2	0 32	4 2	0 32
20	8 7	0 67	7 3	0 56
21	17 8	1 4	11 6	0 90
22	33 1	2 6	16 0	1 2
23	60 3	4 7	20 2	1 6

the entire sky brighter than $m_{pg} = 12^m$. The mean redshift at this magnitude, obtained from Figure 4, is $z = 0.058$, which corresponds to a distance of 2.3×10^8 pc, or a volume of 5.2×10^{25} pc^3 (assuming a Hubble constant of 75 km/sec Mpc). There is, then, an average of one QSG every 6.2×10^{23} pc^3, which gives a density of 5.4×10^{-80} QSG/cm^3.

Compare this with the density of galaxies whose mean luminosity is the same as those contained in the redshift catalogue (Humason, Mayall, and Sandage 1956); i.e., galaxies that obey $m_p - K_p = 5 \log z + 23.616$. This equation, together with the integral count relation of Holmberg (1957) given by $\log N(m) = 0.6m_{pg} - 8.87$ for $m_{pg} < 13^m$, gives a spatial density of one galaxy every 1.0×10^{75} cm^3, or 1.0×10^{-75} gal/cm^3. The ratio of the number of average galaxies to QSG is 1.9×10^4. This ratio would be higher by a factor of 8 if the mean absolute magnitude of the galaxies was taken to be $1.^m5$ fainter—justified by noting that the redshift catalogue is biased toward the brighter, more easily observed galaxies. Reasonable limits to the ratio of numbers per unit volume of normal optical galaxies to QSG would then be 2×10^4 to 2×10^5.

Therefore, the QSG contribute negligible mass density to the universe. Even supposing they had a very high mass of $10^{14}\ \mathfrak{M}_\odot$, the average density due to QSG would be only 10^{-32} gm/cm^3, which is about 10 times smaller than that usually assumed for normal galaxies (Hubble 1926; Whitford 1954; Oort 1958).

An important ratio is the number of QSG to QSS per unit volume because *if* a QSS evolves into a QSG, as seems likely, the ratio gives the relative lifetimes. There are 329 non-galactic radio sources in the 3C revised catalogue (Bennett 1962), which covers slightly more than half the sky to 9 flux units. The current optical identification program shows that 30 per cent of the 3C R sources are QSS, giving a surface density of 5×10^{-3} QSS/square degree. This is to be compared with 4 QSG/square degree to $m_{pg} = 19$, assuming that most of the faint Haro-Luyten objects are extragalactic, giving a ratio of surface densities of 800. An estimate of the volume-density ratio can be obtained if we know the radio volume surveyed for QSS to 9 radio flux units and the optical volume corresponding to $m_{pg} = 19^m$. A simple calculation can be made only if the radio and optical absolute luminosities of the QSS and QSG show small dispersion. Schmidt's (1965*b*) data for the nine QSS with known redshifts suggest that this is the case, and we shall adopt it. Reducing his monochromatic radio powers to 178 MHz with a spectral index of -0.7 and adopting a Hubble constant of 75 gives an average absolute power of 2.7×10^{28} W(Hz)$^{-1}$ for the known QSS, a value which permits the distance of a source with flux density 9×10^{-26} W m^2 Hz at 178 MHz to be found from the standard equations (Sandage 1961*a*, eq. [50]). Similarly, adopting the QSS line in Figure 4 to represent the QSG relation, we can find the redshift at $m_{pg} = 19$ and, hence, the coordinate distance surveyed for such sources, using the same value of H. Fortuitously, the distances of the radio and optical surveys to the stated limits turn out to be nearly equal, so the volumes surveyed will be about the same. The actual distance ratio is (optical/radio) = 1.16, which gives a volume ratio of 1.56, with the optical volume being larger. Consequently, the number ratio per unit volume of QSG to QSS is $800/1.56 \simeq 500$, which must be the ratio of the lifetimes if all QSG go through the radio phase.

Evidence has been presented elsewhere (Burbidge, Burbidge, and Sandage 1963) that the lifetime of strong radio sources is of the order of 10^6 years. This then requires that the quasi-stellar galaxies can exist in their observed state for only 5×10^8 years. If this rough estimate is substantially correct, it raises the possibility that we may be witnessing galaxies in the process of formation, because 5×10^8 years is the order of the collapse time of the halo into a disk of a normal galaxy (Eggen, Lynden-Bell, and Sandage 1962). Such a process would agree with the model of Field (1964), arrived at from entirely different grounds. If there is no way out of the short lifetime for QSG, then the conclusion is forced that the QSG phenomenon takes place at times later than the "creation event" of an evolutionary universe, because QSG are present at all distances, spanning nearly 10^{10} years in light-travel time. This fact requires that the objects either periodically flare to their superluminosity from an older, more stable state, or are continuously being born.

VI. COSMOLOGY

Schmidt's discovery (1965*b*) of exceedingly large redshifts for several quasi-stellar objects emphasizes again the importance of the QSS, and now the QSG, in testing different world models. The most sensitive test is still the deviation from linearity of the $[m, z]$ relation (Sandage 1961*a*, Fig. 1). Equations (23), (24), and (25) of Paper I show that very large differences exist in $m - K$ at a given z between the $q_0 = +1$, 0, and -1 cases for redshifts larger than 2, which is now within the observable range. Relative to a straight line ($q_0 = +1$), the $q_0 = 0$ curve should be fainter by $1.^m50$ and the steady-state case of $q_0 = -1$ should be fainter by $2.^m37$ at $z = 2$. For $z = 5$ the deviations become $2.^m25$ and $3.^m89$, respectively.

If the K-corrections can be found empirically by spectrum scanning or multicolor photometry, these theoretical differences should be easy to check by increasing the number of data points in Figure 4. Furthermore, the problem of the evolutionary change in absolute optical luminosity appears to be simpler for QSG than for giant ellipticals (Sandage 1961*b*), because the lifetimes of the QSG are short compared with the interval of cosmic time into which we are gazing; hence averages over a sufficiently large sample of objects should cancel out the evolutionary luminosity decline of a single object. The only remaining problem concerns the cosmic dispersion in luminosity of the sources themselves—a problem which should soon be solved by increasing the sample of both QSS and QSG in Figure 4. As long as the intrinsic spread is smaller than about 2^m, differences between the q_0 models as large as predicted above can be found.

A large fraction of the observable universe can be sampled using the QSG. The equation for the co-moving coordinate distance is given by

$$r=\frac{(2q_0-1)^{1/2}}{q_0^2(1+z)}\{q_0z+(q_0-1)[(1+2q_0z)^{1/2}-1]\} \tag{8}$$

(see eq. [50] and [10′] of Paper I, noting the misprint in eq. [50]). For the simple case of $q_0 = +1$, this reduces to $r = z/(1 + z)$. The horizon in this model occurs at $r = 1$. The volume contained within r is

$$V(r) = 2\,R_0^3\,[\sin^{-1} r - r(1 - r^2)^{1/2}] \tag{9}$$

(Sandage 1963, eq. [26]). Therefore, to a redshift of $z = 5$, $r = 0.833$ and the volume $V(r) = 1.048R_0^3$. The total volume contained from the observer to the horizon at $r = 1$ is πR_0^3. We therefore sample one-third of the total space available using objects with $z = 5$. The metric distance along a geodesic from the observer to the object is $R_0 \sin^{-1}r = 0.986R_0$ for $z = 5$, which should be compared with the maximum distance of $(\pi/2)R_0$ which can be observed in the model, showing that we reach 63 per cent of the way to the horizon with objects of this redshift.

The cosmic time into the past that we are looking, given by equations shown elsewhere (Sandage 1961*b*), is $\tau/t_0 \simeq 0.92$ for $q_0 = +1$, $z = 5$, where t_0 is the proper time between the present and the occurrence of the singularity at $R_0 = 0$.

These numbers show that a significant fraction of the total universe (in the closed case) can be sampled and that any deviation from regularity in the distribution of QSG at $m_{pg} \simeq 19$ would be of cosmic significance. Counts for the surface distribution should therefore be able to give us information about the largest-scale clustering tendencies of objects in space.

It is a pleasure to thank W. C. Miller for specially baking the plates used to obtain the spectrograms, James Westphal for applying his cross-correlation technique to the spectra of BSO 1, and Helen Czaplicki for tying the manuscript under great pressure. Dr. I. S. Bowen was helpful in discussions of expected ultraviolet lines and in comments on the spectrograms. Very special thanks go to Maarten Schmidt, whose help in obtaining the spectroscopic evidence was paramount and whose communication of the results on his spectacular redshifts before publication allowed the prediction of § III to be made at an early date. Finally, Philippe Veron's enthusiastic, hard work on all phases of the QSS identifications and on the BSO survey made parts of this development possible.

APPENDIX

THE *K*-EFFECT FOR QUASI-STELLAR SOURCES

The change of apparent magnitude due to redshifting the continuous spectrum through fixed measuring bands must be known for any class of object before the proper redshift–apparent-magnitude relation can be constructed from observational data. This change in magnitude is called the *K*-correction and has been discussed in recent times by several authors (Humason, Mayall, and Sandage 1956; Sandage 1955). The correction is now determined with moderate accuracy for E and S0 galaxies with known $I(\lambda)$ distribution (Table 1 of Sandage 1965), but the problem for quasi-stellar sources is not yet properly solved because of lack of knowledge of (1) $I(\lambda)$ to $\lambda \simeq 500$ Å, and (2) the variation of $I(\lambda)$ from source to source.

Detailed knowledge of $I(\lambda)$ exists for only 3C 273 (Oke 1965) from $\lambda_0 \simeq 2800$ to $\lambda_0 \simeq 9350$, and 3C 48 from $\lambda_0 \simeq 2640$ to $\lambda_0 \simeq 7220$ due to photometry by Baum. Figure 7 shows log $F(\nu)$ in absolute units of W m^{-2} (Hz)$^{-1}$ versus ν for 3C 48, taken from Table 6 of Matthews and Sandage (1963), where the results of Baum's work are tabulated. For both 3C 48 and 3C 273, the energy distribution per unit frequency interval follows equation (1) of the text with moderate accuracy; 3C 48 follows the equation exactly as Figure 7 shows.

Equation (1) is the form for optically thin bremsstrahlung when $C = h/kT$. The following calculations were made assuming equation (1) is correct, although Figure 1 shows beyond doubt that individual QSS deviate on both sides of the two-color relation.

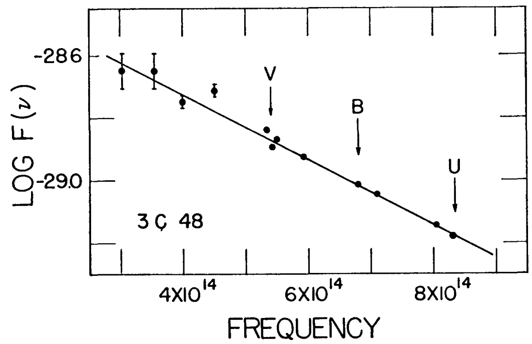

FIG 7.—The continuous spectral distribution for 3C 48 from the measurements by W. A Baum and by the author. The ordinate is the observed flux in W m^{-2} (Hz)$^{-1}$ Effective wavelengths of the U, B, and V filters are shown.

TABLE A1

THEORETICAL COLORS FOR RADIATORS OBEYING $F(\nu)$ = CONST EXP $(-h\nu/kT)$

T (° K)	$U-B$	$B-V$
∞ . . .	−0 909	+0 096
100000	− 832	+ 176
30000	− 648	+ 339
20000	− 518	+ 459
13000	− 303	+ 649
10000	−0 121	+0 810

Expected $U - B$ and $B - V$ values for various parameter temperatures T were computed by the method of Matthews and Sandage (1963, Appendix A) and are given in Table A1, from which the line in Figure 1 was drawn.

The assumed function has the property that, upon redshifting the spectrum by $\lambda/\lambda_0 = 1 + z$, or $\nu/\nu_0 = (1 + z)^{-1}$, the function retains its form, but the distribution then imitates a source of lower-parameter temperature given by $T_z = T_0/(1 + z)$. The color values merely walk down the $\exp(-C\nu)$-curve of Figure 1 but retain the ultraviolet excess relative to luminosity class V stars for all redshifts. This is an important property because the true quasi-stellar sources are known to possess abnormal $U - B$ values even for extreme redshifts, as proved by 3C 9 whose $1 + z = 3\,012$ (Schmidt 1965*b*).

The selective part of the K-correction was computed by quadrature for the cases $T_0 = 100000°$ K and $T_0 = 30000°$ K, using the v sensitivity function previously tabulated (Matthews and Sandage 1963). It was found that, instead of using the complete function, a simpler computation using effective wavelengths gave answers that agreed to within a few per cent with the more exact values. K_B- and K_U-values were calculated by this shorter method. Table A2 gives the results where the tabulated K_i-values represent the total change in magnitude due to redshifting in the band pass i The non-selective term Δ mag. = 2.5 log $(1 + z)$, due to the change in effective band width, has been included in the tabular values. A negative sign means the object becomes brighter due to redshift than it would if the measuring bands were moved along the spectrum by $1 + z$ and were increased in band width by $1 + z$ to keep up with the redshift. The K-values with their tabulated signs reversed should be applied to all observations made at fixed wavelengths so as to place the observed magnitudes on a "bolometric" scale.

The important feature revealed by Table A2 is that the K-corrections are small, even going negative, for redshifts as large as $z = 3$ if $T_0 > 50000°$ K. This is the probable value of the parameter temperatures (for $z = 0$) for many QSS, showing that, to first approximation, no K-

TABLE A2

K-CORRECTIONS FOR VARIOUS PARAMETER TEMPERATURES AND REDSHIFTS FOR RADIATORS OBEYING $F(\lambda)d\lambda$ = CONST $\lambda^{-2}\, d\lambda$ EXP $(-hc/k\lambda T)$*

z	$T_0 = \infty$			$T_0 = 100000°$ K			$T_0 = 50000°$ K			$T_0 = 30000°$ K		
	K_U	K_B	K_V	K_U	K_B	K_V	K_U	K_B	K_V	K_U	K_B	K_V
0 0	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00
0 5	−0 44	−0 44	−0 44	− 22	− 26	− 30	0 00	−0 08	−0 15	+0 28	+0 15	+0 03
1 0	−0 76	−0 76	−0 76	− 31	− 40	− 46	+0 13	−0 04	−0 17	+0 70	+0 44	+0 19
1 5	−1 00	−1 00	−1 00	− 33	− 46	− 57	+0 33	+0 07	−0 13	+1 18	+0 78	+0 42
2 0	−1 19	−1 19	−1 19	− 33	− 47	− 63	+0 55	+0 22	−0 06	+1 71	+1 17	+0 69
2 5	−1 36	−1 36	−1 36	− 27	− 47	− 66	+0 82	+0 40	+0 06	+2 23	+1 60	+1 00
3 0	−1 50	−1 50	−1 50	− 20	− 45	− 66	+1 11	+0 61	+0 20	+2 83	+2 04	+1 32
3 5	−1 63	−1 63	−1 63	−0 11	− 40	− 65	+1 41	+0 84	+0 35	+3 45	+2 49	+1 67
4 0	−1 75	−1 75	−1 75		− 33	− 61		+1 09	+0 53		+2 98	+2 03
4 5	−1 85	−1 85	−1 85		− 26	− 58		+1 33	+0 71		+3 45	+2 40
5 0	−1 94	−1 94	−1 94		− 18	− 54		+1 59	+0 89		+3 98	+2 77
5 5	−2 03	−2 03	−2 03		− 09	− 49		+1 90	+1 09		+4 49	+3 13
6 0	−2 11	−2 11	−2 11	.	+0 01	−0 43		+2 20	+1 30		+5 02	+3 56

* The K-corrections contain the selective term

$$\Delta \text{ mag.} = 2.5 \left[\log \frac{\int_{\lambda_1}^{\lambda_2} S(\lambda) I(\lambda)_0 d\lambda}{\int_{\lambda_1}^{\lambda_2} S(\lambda) I(\lambda)_z d\lambda} \right]$$

plus the non-selective term Δ mag = 2 5 log $(1 + z)$

correction need be applied to the QSS data of Figure 4 in the range of redshifts there encountered The K-corrections will become large near $m_{pg} \simeq 22^m$, where the redshifts are expected to be near $z = 4$. Here special problems in the survey for ultraviolet objects will be encountered because the change in $B - V$ and $U - B$ due to redshift will be large. The objects cannot then be easily distinguished from normal stars A most important factor not considered in Table A2 is the effect of the Lyman continuum at $\lambda = 912$ Å as it is shifted into the observable range. A redshift of $z \simeq 3$ will bring $\lambda = 912$ Å into the center of the U filter at $\langle\lambda\rangle = 3600$ Å and will cause K_U to be much larger than given in Table A2 This is expected to occur near $m_{pg} \simeq 21^m$ and may seriously hamper searches for QSG by color methods. Detailed energy-distribution-curves for QSG, which will be determined in due course, will determine the seriousness of the problem.

Finally, it should be noted that the expected change in color due to redshift is given by $\Delta(U - B) = K_U - K_B$ and $\Delta(B - V) = K_B - K_V$, which can be found for any z from Table A2.

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