A SHORT METHOD FOR DETERMINING THE DISTANCES OF RR LYRAE STARS

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ABSTRACT

Using a previously derived period-luminosity-color relation for RR Lyrae stars, it is shown that to a first approximation a star of given period obeys this relation at each point in its cycle If the period of the star is already known, therefore, a single photometric observation in B and V suffices to determine the distance modulus of the star Practical tests show the method to give distance moduli on the average within 0 1 mag of distance moduli determined from the full light-curve

It is also shown that due to the particular numerical value of the coefficient of the color term in the period-luminosity-color relation, the true distance modulus may be determined photometrically without knowledge of the interstellar absorption.

INTRODUCTION

In a previous paper (Fernie 1964*a*), a period-luminosity-color relation of the form

$$M_V = -0.83 - 2.50 \log P + 2.96(B - V)_0 \tag{1}$$

was derived for RR Lyrae stars. In this, M_V and $(B - V)_0$ were taken to be averages $\langle M_V \rangle$ and $\langle (B - V)_0 \rangle_{\text{mag}}$ as defined by Kraft (1961). This relation was tested on a number of RR Lyrae stars, both field and cluster, for which absolute magnitudes were available by other methods and shown to give good results. In a re-examination of the subject (Fernie 1965), based on the RR Lyrae variables in M3, it was found that while the above relation gave good results for Bailey type a and b variables, the absolute magnitudes predicted for Bailey type c variables were about 0.2 mag. too faint. The distance moduli given in Tables 1 and 2 have been adjusted for this effect. The occurrence of type a and b variables with $P \sim 0^d.3$ among field stars but not among cluster stars leaves it uncertain as to whether or not their absolute magnitudes should also be brightened by 0.2 mag. No such adjustment has been made for these stars in Table 1, but their moduli are marked as uncertain.

USE OF A SINGLE OBSERVATION

Unlike a classical Cepheid, which traces out a fairly well-defined and repeating loop on the HR diagram in the course of the cyclic variation (Fernie 1964*b*), an RR Lyrae star traces out a loop which is generally erratic, poorly defined, and which, partly due to the Blazhko effect, does not repeat well from cycle to cycle. Thus, in general, one may speak not of a loop on the HR diagram but of a band within which a given star is located at any point in its cycle. This band may be approximated to by a straight line in the HR diagram.

It is seen that for a given period the period-luminosity-color relation reduces to the equation of a straight line in the HR diagram. In the case of classical Cepheids it was shown (Fernie 1964*b*) that this straight line is very nearly also the straight line approximating the band or loop in the HR diagram for a star of the appropriate period. The same is now found to be the case for RR Lyrae stars. Figure 1 illustrates the case for RR Lyrae itself. The points are from photoelectric observations by Hardie (1955), while the line is not drawn to fit the points, but is given by the period-luminosity-color relation with the period set equal to the period of RR Lyrae, and M_V transformed to V by

TABLE 1

OBSERVATIONS AND DISTANCE MODULI OF FIELD STARS

Star	Type	log P	$\langle V\rangle_{int}$	$\langle B-V\rangle_{mag}$	$(m-M)_0$*	$(m-M)_0$†	E_{B-V}‡
RR Lyr	ab	−0 246	7 71	0 35	6 88	6 82	0 07
SU Dra	ab	— 181	9 80	.33	9 20	9 27	08
TU UMa	ab	— 253	9 84	33	9 06	9 03	06
BS Aqr	c	— .703	9 37	32	7 69	7 64	10
RS Gru	ab	— 833	8 24	27	6 19:	6 18:	11
YZ Cap	c	— 564	11 23	30	9 98	9 97	11
SW Cru	ab	— 484	12 33	66	10 00:	9 83:	31
V494 Sco	ab	— 370	11 41	43	10 05	9 78	33
S Ara	ab	— 345	10 74	34	9 71	9 75	27
BI Cen	ab	— 344	11 91	48	10 46	10 27	30
DN Pav	ab	— 330	12 45	31	11 53	11 70	09
V499 Cen	ab	— 460	11 09	.37	9 68:	9 69:	04
RY Col	ab	— 320	10 92	.34	9 95	9 97	12
UV Oct	ab	— .265	9 49	40	8 48	8 57	12
WY Ant	ab	— 241	10 85	36	10 02	9 91	.11
RX Eri	ab	— 231	9 67	40	8 74	8 64	06
RV Phe	ab	— .225	11 84	.37	11 05	11 04	.10
YY Tuc	ab	— 197	11 97	32	11 36	11 34	.11
FY Hya	ab	— 196	12 51	34	11 85	11 66	04
W Tuc	ab	— 192	11 40	32	10 80	10 72	11
TY Pav	?	— 149	12 57	41	11 82:	11 83:	24
V675 Sgr	ab	— 192	10 37	.37	9 62	9 43	20
SW For	ab	— 095	12 33	36	11 86	11 70	10
IU Car	ab	— 526	11 92	.44	10 13:	10 02:	20
YZ Boo	?	— 983	10 57	.25	8 20:	8 21:	07
DY Her	ab	— 827	10 43	.34	8 19:	8 15:	08
BS Aqr	c	— .703	9 43	.33	7 72	7 65	10
DH Peg	C	— 592	9 49	.29	8 18	8 10	05
SX UMa	c	— 513	10 84	.25	9 85	9 88	07
RZ Cep	?	— 510	9 39	.45	7 61:	7 68:	27
RS Boo	ab	— 424	10 34	33	9 13:	9 17:	06
SW And	ab	−0 355	9 70	0 40	8.46	8 39	0 20

* Determined from the full light- and color-curves.
† Determined from a single observation
‡ E_{B-V} from Parenago's method, revised by Sharov (1964).

TABLE 2

OBSERVATIONS AND DISTANCE MODULI OF M3 STARS

Star	Type	log P	$\langle V\rangle_{int}$	$\langle B-V\rangle_{mag}$	$(m-M)_0$*	$(m-M)_0$†
1	ab	−0 283	15 53	0 35	14 61	14 47
6	ab	— 289	15 64	35	14 70	14 65
9	ab	— 266	15 70	38	14 73	14 75
12	c	— 498	15 55	23	14 77	14 74
16	ab	— 291	15 73	33	14 85	14 86
18	ab	— 287	15 72	36	14 77	14 98:
19	ab	— 199	15 71	46	14 68	14 64
24	ab	— 178	15 49	42	14 63	14 80
25	ab	— 319	15 59	32	14 67	14 72
37	c	— 486	15 67	26	14 71	14 75
56	c	— 482	15 65	27	14 67	14 61
60	ab	— 150	15 57	40	14 83	15 12:
72	ab	— 341	15 70	31	14 71	14 91
105	c	−0 541	15 56	0 21	14 61	14 60

* Determined from full light- and color-curves
† Determined from a single observation.

the use of a distance modulus of 6.9 (Eggen and Sandage 1959). The line is seen to fit the band of points rather well. Thus the period-luminosity-color relation is not only applicable to a series of stars by the use of each star's average light and color, but it is also applicable to an individual star throughout its cyclic variation. This suggests that the period-luminosity-color relation is an integral of the equation of pulsation, with the period appearing as an eigenvalue determined by the mass and average radius of the star (Fernie 1965). However, this picture is undoubtedly complicated by second-order (phase-lag) effects. The point to which we wish to draw attention here is that a single observation at any point in the cycle suffices to determine the distance modulus of the star to within the accuracy set by the finite width of the band in V.

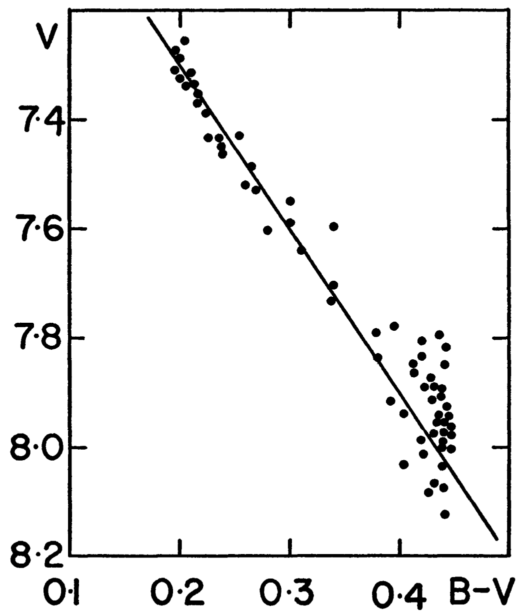

FIG. 1.—Observations of RR Lyrae by Hardie. The line is not intended necessarily to fit the points, but is derived by substituting the period of RR Lyrae into the period-luminosity-color relation and taking the distance modulus of the star to be 6 9.

Not all RR Lyrae stars have the line fitted to the points as well as the case in Figure 1. For the thirty-two stars listed in Table 1 photoelectric data are available from the following sources: Hardie (1955), Spinrad (1959, 1961), Kinman (1961), and Preston, Spinrad, and Varsavsky (1961). A diagram similar to Figure 1 has been constructed for each of these stars. It is found that the slope of the band ranges from 2.1 to 5.1, although there is a strong clustering of values around 2.96, and the over-all average is 2.96. The cause of this range is unknown. The slope does not correlate with the star's period, color, or chemical composition (Preston's ΔS). From the practical standpoint of determining distance moduli from single observations we assume that the existence of this range in slopes is unimportant.

In Tables 1 and 2 two distance moduli are listed. The first is derived by applying the period-luminosity-color relation to the averaged values of $\langle V \rangle_{\text{int}}$ and $\langle B - V \rangle_{\text{mag}}$. The second is derived by applying the period-luminosity-color relation to any single pair of observed V and $(B - V)$ taken at random from among the published lists of observations. The two moduli differ on the average by only 0.08 mag. for both tables, and rarely by more than 0.2 mag. These errors could easily arise from the typical errors of single photometric observations, and our disregarding of the range in band slopes as unimportant is therefore justifiable.

EFFECTS OF INTERSTELLAR ABSORPTION

The period-luminosity-color relation is shown in equation (1). Normally, in using this to determine a true distance modulus from a pair of observations V and $(B - V)$, one would first correct V for the interstellar absorption A_V, and $(B - V)$ for the color excess E_{B-V}. If, however, we write out this procedure explicitly,

$$V_0 - M_V = V - M_V - A_V = V - M_V - RE_{B-V} , \tag{2}$$

$$(B - V)_0 = (B - V) - E_{B-V} , \tag{3}$$

and substitute these relations into equation (1), we obtain

$$V_0 - M_V = V + 0.83 + 2.50 \log P - 2.96(B - V) - (R - 2.96)E_{B-V} . \tag{4}$$

But $R = A_V/E_{B-V} = 3.0$, and therefore the final term in equation (4) vanishes. Hence, the true distance modulus is obtainable from V, $(B - V)$, and P without knowledge of the interstellar absorption. Thus the moduli given in Tables 1 and 2 are true moduli.

A major application of this result would be to redetermine the distance to the galactic center by Baade's method, since the major difficulty associated with this method has always been to make accurate allowance for interstellar absorption. A major program of BV photometry in these fields will first have to be undertaken, however.

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