THE PROBLEM OF BETA LYRAE. II. THE MASSES AND THE SHAPES

N. J. Woolf
Princeton University Observatory
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ABSTRACT

By consideration of the present rate of mass transfer, the primary of Beta Lyrae is shown to have its photosphere in contact with its critical equipotential. The primary is slightly less distorted than the Roche model because its rotation is slower than synchronism demands. These results are used in a new determination of the mass ratio of the system. The mass ratio is smaller than previous estimates by Huang and by the author. The mass ratio is now in agreement with one of the two possible spectroscopic estimates of this quantity.

Arguments from the observed spectrum and from the six-color photometric study of the system are given to support Huang's suggestion that the secondary is a star imbedded in a disk. From the luminosity of the star in the disk, it is inferred that at least half the mass of the secondary must be in the disk. It is suggested that the disk has been produced as a result of the mass transfer from the primary conveying considerable angular momentum with the mass. The time scale for the disk to lose angular momentum is probably longer than the time scale for mass transfer, while the expected future life of the primary is longer than either of these. Thus it is reasonable that we see the system in its present state. It is pointed out that Beta Lyrae should soon evolve into a system with resemblances to V356 Sgr.

I. THE SHAPE OF THE PRIMARY AND THE MASS RATIO

In this paper, the results of Belton and Woolf (1965; hereinafter called "Paper I") are used to discuss the masses, luminosities, and shapes of the components of Beta Lyrae. There have been some recent advances in our understanding of this system. Huang (1962*a*, 1963*a*) has proposed that the secondary is a disk-shaped object. He has also estimated the rate of mass transfer from the primary to the secondary (Huang 1963*b*), using a suggestion that this transfer is responsible for the observed period change of the system. He and the author have independently fitted the primary to the Roche model (Huang 1962*b;* Woolf 1962) and thus shown that the primary is probably the less massive component of the system.

In fitting the primary to the Roche model, Huang has questioned how closely the star fits its contact lobe. This fit is critical in determining the mass ratio. Plavec (1958) has shown that the finite density of a stellar envelope is not important in this fit, but that the rotation of the star is crucial. The primary rotates more slowly than synchronism demands (Sahade, Huang, Struve, and Zebergs 1959). Thus according to Plavec, the star will be unstable only if its dimensions are somewhat larger than the critical equipotential normally calculated. The star will also be more spherical than the Roche model. Using this argument, the same luminosity primary demands an even more extreme mass ratio than the values previously estimated.

It should be possible to confirm such estimates from the observed geometrical ellipticity of the primary, 0.19 (Paper I). Unfortunately, the theoretical structure for such estimates is not yet adequate. When an ellipsoid with $z = 0.19$ is drawn, it appears significantly less elongated than a Roche critical equipotential. While this deviation is in the sense predicted, it is not clear whether the comparison is meaningful.

It is possible to use the rate of mass transfer computed by Huang to show that the photosphere of the primary is very close to the surface of the contact lobe. The rate of mass transfer is about 10^{21} gm sec^{-1}. The surface area of the photosphere inferred from the luminosity and temperature is 10^{25} cm^2. The photospheric density is estimated to be about 10^{-10} gm cm^{-3}. At this density, the rate of mass transfer requires a velocity of 10 km sec^{-1}, the speed of sound. To cause the matter to flow at such a speed, most of the pressure that would be exerted from above, if the photosphere were normal, must be missing. This is interpreted as showing that the photosphere is at most a few scale heights below the critical equipotential. The dimensions of the star are about 10^{12} cm, while the atmospheric scale height is about 10^9 cm; thus the photosphere is effectively at the critical equipotential.

It is instructive to compare the rate that mass leaves the surface of Beta Lyrae A with the rate that it leaves the surface of Betelgeuse (Weymann 1962). In Betelgeuse the mass loss proceeds without the aid of a critical equipotential. The total rate of mass flow from Beta Lyrae A is four times greater: the rate per unit area of surface is 10^4 times greater. At a radius of 0.95 of the critical equipotential, the potential barrier for mass exchange of Beta Lyrae A is about equal to the potential barrier for mass loss of Betelgeuse; however, the barrier decreases linearly to zero, as the surface of Beta Lyrae A approaches the critical equipotential.

This evidence suggests that a more detailed calculation than the previous ones for the mass ratio is now justified. In this new calculation, the deviation from the Roche model is computed using the observed equatorial velocity of the primary of 45 km sec^{-1} (Böhm-Vitense 1954; Mitchell 1954). In the fit to the critical equipotential, we no longer use the "effective radius" (Kopal 1954) but rather the individual axes of the distorted ellipsoid. The deviation of this from the Roche model has been computed using the results of Plavec. The fit has been made at light maximum. Instead of the indirect method of effective temperatures and bolometric corrections used previously, we now use brightness temperatures. For the primary B8.5 II side-on, we use a brightness temperature of 12000° K. The solution is shown in the accompanying tabulation below.

$L^*/L_\odot$ (UBV, V band) = $-8^{m}62 \pm 0{:}10$	2806	± 269
$J^*/J_\odot$	12 6	
Projected area of star/projected area of Sun	222 9	$\pm$ 21 4
Projected area of Sun	1 523	$\times 10^{22}$ cm^2
Projected area of star	3 394	$\times 10^{24}$ cm^2
(Projected area of star)$^{1/2}$	1 842	$\times 10^{12}$ cm
$a_1 \sin i$ (from spectroscopic data)	3 29	$\pm 10^{12}$ cm
(Projected area of star)$^{1/2}/a_1 \sin i$	0 5599	$\pm$ 0.0280
Mass ratio corresponding to above ratio, using the Roche model and $\sin i = 1$	0.195	
Mass ratio corrected for the rotation	0 175	$\pm$ 0.020
Mass of primary	2 05	$\pm$ 0.30 M$\odot$
Mass of secondary	11 74	$\pm$ 0.40 M$\odot$

Huang has shown that such results are rather insensitive to the value of $\sin i$ if $90° > i > 70°$. From Sahade *et al.* (1959), we see that the "mass-ratio derived from the velocity amplitude of the emission 'peaks' yields a mass of about 2.0 $\odot$ for the primary and 11.5 $\odot$ for the secondary." The agreement of the two totally different methods of deriving the masses of the components should be considered a coincidence in view of the large uncertainties of both methods, but the agreement is probably not entirely fortuitous. Larger mass ratios have been calculated from the spectroscopic data when attempts have been made to measure the entire emission lines (Belopolsky 1895), rather than the sharp peaks. The broad emission lines are very difficult to measure because they are cut into by broad, deep absorptions. There is also probably some emission not associated with the secondary. Thus the mass ratios derived this way are not regarded as a serious objection to the true mass ratio being close to 0.175. It is noteworthy that the elements given here fit the empirical mass-ratio–luminosity-deviation law for subgiant secondaries (Smak 1962).

II. THE SHAPE OF THE SECONDARY

Because of the recent distance determination, the light-curve of Beta Lyrae presents an acute problem to analysis. The secondary cannot be an opaque ellipsoid of approximately similar shape to the primary. The distance and absolute magnitude of the primary give the dimensions of that star. From this, we know that $a_1 \sin i$ is considerably larger than the radius of the primary. From the light-curve, the two stars are almost in contact. Thus it can be shown that if the secondary is a normal ellipsoid, it has a surface area at least four times greater than the primary. From rectification of the light-curve, $J_2/J_1 = 0.36$, thus the secondary should be more luminous than the primary.

Both the spectroscopic and colorimetric evidence disagrees with this. The lines of the primary are clearly visible, even at primary minimum, while the very blue color of the system (Paper I) shows that the cooler secondary cannot contribute any large fraction of the light of the system. This paradox can be resolved either by rejecting the distance determination or rejecting the supposition that the shapes of the stars are similar, or by assuming that the secondary is semitransparent.

The distance of Beta Lyrae has been inferred from the distance of its common proper motion main-sequence companions (Abt, Jeffers, Gibson, and Sandage 1962). The true distance cannot be much greater than the 260 pc inferred by Abt *et al.* There is only a single sharp interstellar absorption line component in the spectrum, while the color of the system is so blue that there can be little interstellar absorption between the star and us. The galactic latitude of the star is 14°. Furthermore the spectroscopic luminosity criteria for the primary suggest a small distance (Boyarchuk 1959), although this point should carry little weight. The star deviates from a normal mass-luminosity relationship, and its chemical composition is abnormal. In summing up, it seems that the distance determination is not at fault, and that the secondary is abnormal.

Huang (1963*a*) has ingeniously uncovered the probable solution. He proposes that the secondary is a disk with a star at its center. He has also considered the possibility that the secondary is semitransparent but does not consider it likely. The color variation of the system appears also to show that this possibility is unlikely. The system appears blue at secondary minimum and yellow at primary minimum. If the secondary had a hot transparent envelope, it would absorb light by electron scattering, which is wavelength-independent. If it were somewhat cooler, H^- absorption would dominate. Over the wavelength range we are considering, the absorption coefficient of H^- increases with wavelength. The optical depth of the supposed envelope would be only about $\frac{1}{3}$ so that multiple scattering could be neglected. It is seen that a scattering envelope is unable to explain the color changes associated with the eclipses. On the contrary, it tends to predict a change in the wrong sense. Huang's disk is the only explanation left. If the disk has a temperature of about 7700° K, it will explain the color changes of the system. We therefore accept Huang's model and attempt to determine its nature, origin, and immediate evolutionary future.

III. THE DISK AND THE CENTRAL STAR

In Huang's model, the disk is about double the radius of the primary. In terms of Paper I, we must use a partially eclipsing model, since neither star is ever totally obscured. It was shown in Paper I that to be consistent with the spectroscopic observations, no more than 25 per cent of the light of the secondary is seen during secondary eclipse. A flat, thick disk seen edge-on would show 50 per cent. A tilted, thin disk would show 39 per cent. We must therefore postulate a central condensation in the disk to supply the extra area and luminosity. This condensation is also required to uncover the area of the primary on which the red and violet satellite absorption lines are projected. Thus the model requires a central bulge or star, and once again Huang's model is able to explain the observations.

The bolometric magnitude of the secondary should be about -4.0 if it fulfils a normal mass-luminosity relationship. The disk and star together have a "temperature" of about 8000° K. There is no evidence of a very luminous source at the center of the disk, and there should be some sign of it since it should be at least as luminous as the primary. From the light-curve and from spectroscopic evidence discussed, it appears that the secondary is about 2 mag. underluminous for its mass.

It might seem that this underluminosity is merely the effect of the disk being between the star and us. The light coming toward us from the star would be redirected away by the disk. If this were so, we should expect to see the secondary in other Beta Lyrae systems with orbital planes less inclined to our line of sight.

Two Beta Lyrae systems with planes sufficiently inclined are HD 30353 (Bidelman 1950; Heard 1962) and υ Sgr[1] (Greenstein 1940; Gaposchkin 1945). These binaries are, like Beta Lyrae, hydrogen-deficient. They have a large mass function, they show hydrogen emission, they rotate slowly, but in neither case is the secondary seen. We therefore prefer other explanations for the low luminosities of the secondaries.

Besides the above-mentioned systems, there are some eclipsing stars that present similar problems in their analysis to Beta Lyrae. These include ϵ Aurigae (Struve 1956), VV Cephei (Frederick 1960), and W Crucis (Woolf 1962). Because these stars are fairly late in spectral type, it will be difficult to determine whether they too are hydrogen-deficient. There are arguments from the distances of VV Cephei and W Crucis that we see the less massive components. In ϵ Aurigae, there is evidence that the secondary is disk-shaped. In all of these systems, there is a large mass function, hydrogen emission, and a slowly rotating primary. W Crucis and VV Cephei show signs of a considerable elliptical distortion of the primary. The secondary in all of these systems is either invisible or visible merely through the presence of emission lines.

Before this digression, it was shown that the secondary of Beta Lyrae is probably subluminous. The primary is hydrogen deficient (Boyarchuk 1959). The author interpreted this as evidence that the primary is merely the core of the original star, most of its initial mass being either transferred to the secondary or lost to the system. Since the final layers transferred should be helium rich, the secondary should be overluminous, increasing the luminosity discrepancy.

Sahade (1962) has suggested that the secondary is on the way to becoming a white dwarf, and the author previously supported this view, supposing that the stars had exchanged mass twice. However, this interpretation fails. The failure of a white dwarf to radiate is due to the degeneracy of the interior. But for this, the energy output would be fairly normal, although the surface temperature would be unusually high. Stars with masses greater than 5.75 μ^{-2} cannot become degenerate (Chandrasekhar 1935), so this is completely impossible for a star of 11.7 $M_{\odot}$. Beta Lyrae systems excepted, to the author's best knowledge there is no convincing evidence for any star's being both subluminous and more massive than 1.5 $M_{\odot}$.

Huang has suggested that the secondary of Beta Lyrae may be contracting and that its condensation is slowed up by its excess angular momentum. Underluminosity does not normally occur when a star is contracting (Henyey, LeLevier, and Levée 1954; Hayashi 1961). Huang's suggestion about excess angular momentum can solve this problem. If the mass in the disk is equal to or greater than the mass in the star, the system will behave as we see it. The central star will radiate with a normal mass luminosity relationship. The disk will radiate little, because it can only generate a small central pressure by collapse in one dimension.

From the small contribution of the secondary to the total light, the maximum thickness of the disk can be shown to be less than $\frac{1}{6}$ of its diameter. This is far flatter than any stable Maclaurin or Roche spheroid (Jeans 1928). Thus the disk must be broken into rings or fragments. As the disk rids itself of angular momentum, the material will slowly go to forming the central star.

L. B. Lucy has pointed out to the author that the more massive stars of semidetached systems would be expected to acquire an excessively large angular momentum if most of their mass has recently been transferred from their companions. Beta Lyrae would be an extreme case of this. Struve (1949) has shown that the few adequately observed Algol-like systems have primaries rotating faster than synchronism, in agreement with Lucy's

[1] Gaposchkin's evidence shows that υ Sgr does in fact eclipse. The amplitude is very small, and the major contribution is undoubtedly the ellipsoidal shape of the primary. From the small amplitudes, it seems that the inclination of the system should be less than 70° so that our argument is appropriate.

suggestion. While the mechanism for these stars to lose this acquired angular momentum is not obvious, it presumably does operate, since many Algol-like systems are found with the more massive star well below the limit of rotational instability.

In the initial collapse of a star, the disposal of surplus angular momentum probably occurs with the aid of magnetic fields (Burbidge 1960). These fields would decay somewhat during the life of a star, and the process now might be rather ineffective. On the other hand, there is a moderately large field exposed at the surface of VV Cephei (Babcock and Cowling 1953); thus there may be enough magnetic coupling for the mechanism still to work in these systems. The time scale for such processes in the initial contraction of a star is about 10^6 years according to Lüst and Schlüter (Burbidge 1960). This is shorter than the future maximum life of the primary, 5×10^6 years if it is now burning helium. To date, no sufficiently efficient mechanism has been proposed for transferring angular momenta of individual stars into orbital angular momentum while the stars remain at constant mass. However, mechanisms involving tidal friction have not yet been fully explored.

The time for mass transfer to occur is about 3×10^4 years. We are assuming that mass transfer occurs by the rapid process proposed by Reddish (1957). Morton (1960) has shown that such a mass transfer occurs on a Kelvin time scale. Further considerations relating to this process have been discussed by Smak (1962). The inequality that the future life of the primary should be greater than the contraction time of the disk, which in turn should be greater than the time to transfer mass, is necessary for us to be able to see the system in its present state.

If the disk is distended by angular momentum, the rotational velocity at the edge should be about 260 km sec^{-1}. The velocity will doubtless change toward the central star, but a priori it is difficult to predict whether the inner velocities will be higher or lower since the velocities depend upon the mass distribution in the disk. Thus if emission lines are produced at the surface and edge of the disk, they will have a Doppler width of at least 520 km sec^{-1}, possibly greater. The width of the emission lines has been estimated by Struve (1941) to be 685 km sec^{-1}. The two edges of the disk should be eclipsed in turn before and after secondary minimum. In the spectra of Sahade *et al.* (1959) there is a suggestion that the violet edge of these lines is missing before minimum, and the red edge is missing after minimum. This should be confirmed on the original spectra. The necessary rotation to give such an effect would be direct, in the same sense as the satellite gas streams.

We have previously discussed the disk and its central star as though they emit radiation at the same temperature. If they emitted at different temperatures, this should have shown itself in the photometric results of Paper I. A composite source can be recognized by there being an excess of energy at the ends of the spectrum compared with a black body. However, in Table 5 of Paper I the color deduced for the secondary does not show such an effect. Presumably, either the energy carried by the gas streams equalizes the temperature over the disk and star, or else the major contribution to the light of the secondary comes from "reflection" of the light of the primary. The gas streams flowing from the primary only carry about 10^{33} erg sec^{-1}, so that they cannot be responsible for the radiation of the disk. On the other hand, "reflection" effect does generate a luminosity for the star and disk that is correct in order of magnitude.

As Beta Lyrae evolves, and the disk collapses, the central star will become hotter and more luminous. There will be a time when the luminosities of the two stars are rather similar. At that time, Beta Lyrae will bear some resemblance to V 356 Sgr (Popper 1955). The present resemblance is shown in Table 1.

IV. CONCLUSIONS

The primary of Beta Lyrae has its photosphere at its critical potential. It is slightly less elongated than a Roche critical equipotential. The masses of the components are 2.0 $M_\odot$ and 11.7 $M_\odot$. The secondary, in agreement with Huang's model, is a disk with a central condensation. The underluminosity of the secondary is explicable by the disk containing at least half of the mass. The model is consistent with the spectroscopic and photometric evidence, and the qualitative predictions of the Reddish-Morton evolutionary scheme.

TABLE 1

A COMPARISON OF BETA LYRAE AND V356 SGR

	Beta Lyrae	V356 Sgr
Period . .	12 9^d	8 9^d
Mass of primary	2 0	4 7
Mass of secondary	11 7	12.1
Type of primary	B9 II	A2 II
M_{bol} of primary	−4 5	−3 2
Type of secondary	A7:	B3:
M_{bol} of secondary	−2 2	−3 9
Volume of primary	Fills lobe	Fills lobe

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