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THE PROBLEM OF BETA LYRAE. I. SIX-COLOR PHOTOMETRY

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ABSTRACT

The 12 9-day period eclipsing binary Beta Lyrae has been observed on 12 nights with a six-color photometer. The colors of the individual stars have been computed. The influence of various uncertainties (whether the eclipse is partial or total, the distorting effect of gas streams, and the accuracy of the rectification of the light-curve) has been crudely assessed. It is concluded that the spectral type of the secondary is A7 ± 3. The variation of color with phase and the variation of equivalent widths of Fe II absorption lines measured by Boyarchuk are consistent in predicting the same temperature variation around the ellipsoidal primary. The spectral type of the primary at maximum light is B8.5 and at secondary minimum is B9 5.

I. INTRODUCTION

Although the eclipsing binary Beta Lyrae has been studied many times both spectroscopically and photometrically, no agreement about the nature of the two stars and their probable masses has yet been reached. The present status of the problem may be seen in papers by Struve (1958), Sahade, Huang, Struve, and Zebergs (1959), Boyarchuk (1959), and Abt, Jeffers, Gibson, and Sandage (1962).

In most interpretations, the depths of the two minima of the light-curve are used to determine the spectral type of the secondary. If the primary is of spectral type B9, with effective temperature about 11000° K, then the secondary has a temperature of about 8000° K (Kopal 1941). However, in a system with severely distorted stars and with optically thick gas streams, this procedure is not obviously safe. The interpretation could be confirmed by a study of the color changes of the system. Unfortunately previous studies, e.g., Wood and Walker (1960), have been made with too short a color base line to be very useful in this context. We have, therefore, observed Beta Lyrae in the six-color system of Stebbins and Whitford (1943).

II. OBSERVATIONS

The photometer employed a refrigerated RCA 7102 photomultiplier with an S1 color response. It was used with the 22-inch Tauchmann reflector at Lick Observatory. A description of the color filters can be found elsewhere (Stebbins and Kron 1957).

The comparison star chosen for this study was 9 Lyrae. Its light was found to be constant by Wood and Walker; however, one observation of 8 Lyrae was made each night as a check on this assumption.

Observations were made on 12 nights over a period of 2 months in the summer of 1961. The observations were made at phases which give a good representation of the light-curve throughout the cycle, although the observations do not refer to a single cycle.

The phases of Beta Lyrae were calculated from a new ephemeris, kindly provided by Dr. D. B. Wood, that is based on observations of 465 minima. The ephemeris is

$$\text{Minimum} = \text{JD } 2433289.47 + 12.928481E + 0.3556 \times 10^{-5} E^2 - 0.648 \times 10^{-10} E^3 \pm 0.18 \text{ days} .$$

The colors and phases are given in full in Table 1. The instrumental green magnitudes of 9 Lyrae-Beta are given in Table 2. The nightly means of the green magnitudes are

TABLE 1

COLORS OF BETA LYRAE

Hel JD 2437000+	Phase	$U-G$	$V-G$	$B-G$	$R-G$	$I-G$
481 7607	0 239	−1 39	−0 84	−0 34	0 43	0 91
7971	242	−1 42	− 86	− 36	40	89
8521	245	−1 41	− 86	− 36	43	93
8964	249	−1 41	− 84	− 36	42	94
486 7201	622	−1 39	− 86	− 36	38	86
7278	623	−1 38	− 85	− 36	41	88
7514	625	−1 37	− 84	− 35	38	84
7841	627	−1 38	− 83	− 35	41	85
8542	633	−1 41	− 88	− 37	42	84
8741	634	−1 41	− 86	− 36	41	85
8908	636	−1 41	− .85	− 36	41	.84
487 6921	697	−1 39	− 86	− 34	43	84
7261	700	−1 40	− 87	− .36	39	83
7528	702	−1 39	− 86	− 31	41	86
7808	704	−1 38	− 84	− 34	41	87
8248	708	−1 39	− 85	− 35	39	85
8514	710	−1 39	− 85	− 35	40	83
8761	712	−1 38	− 83	− 35	41	82
8957	713	−1 40	− 85	− 36	40	81
9268	716	−1 40	− .86	− .36	40	84
9457	717	−1 37	− 83	− 36	40	85
491 6978	008	−1 25	− 76	− 29	36	71
7148	009	−1 31	− 80	− 34	36	69
7428	011	−1 27	− 79	− 30	40	78
7571	013	−1 30	− 83	− 31	38	75
7778	014	−1 28	− 76	− 31	41	78
7964	016	−1 28	− 75	− 31	39	76
8114	017	−1 30	− 77	− 32	40	75
8571	020	−1 28	− 77	− 32	37	74
8835	022	−1 29	− .77	− .29	39	76
9064	024	−1 29	− 77	− 31	38	75
9242	025	−1 29	− 75	− .30	.38	75
9507	028	−1 29	− 79	− 33	35	69
497 6841	471	−1 33	− .82	− 33	45	92
7071	473	−1 33	− 84	− 35	44	90
7285	474	−1 33	− 84	− .33	44	90
7535	476	−1 36	− 84	− .36	42	87
7792	478	−1 33	− 85	− 37	41	87
8007	480	−1 33	− 83	− 35	42	89
8571	484	−1 39	− 86	− 36	42	87
8714	485	−1 31	− 81	− 34	44	91
8888	487	−1 36	− 87	− 39	43	87
9057	488	−1 35	− 87	− 35	43	.87
9181	489	−1 35	− 86	− 36	42	90
9449	491	−1 36	− 86	− .36	43	85
9592	492	−1 34	− 84	− 34	42	90
498 6978	549	−1 38	− 85	− 36	40	86
7221	551	−1 39	− 85	− 35	38	86
7500	553	−1 39	− 89	− .35	.41	88
7756	555	−1 42	− 87	− .38	41	86
7992	557	−1 36	− 82	− 34	43	90
8578	562	−1 38	− 84	− 36	42	87
8892	564	−1 37	− 84	− 34	.39	84
9156	566	−1 39	− 85	− 34	41	83
9414	568	−1 43	− 90	− .35	40	85
9685	570	−1 40	− 86	− 35	38	85
9892	572	−1 39	− 84	− 32	45	87
503 6500	932	−1 27	− 79	− 32	44	81
6735	0 934	−1 27	−0 80	−0 33	0 38	0 81

TABLE 1—*Continued*

Hel JD 2437000+	Phase	$U-G$	$V-G$	$B-G$	$R-G$	$I-G$
503 7014	0 936	−1 28	−0 84	−0 33	0 41	0 83
7535	940	−1 28	− 78	− 31	40	81
7728	942	−1 27	− 79	− 33	39	78
7964	944	−1 29	− 78	− 33	37	78
8156	945	−1 29	− 81	− 34	39	79
8342	946	−1 25	− 79	− 33	38	77
8571	948	−1 29	− 81	− 32	37	78
8885	951	−1 27	− 78	− 31	43	78
508 6121	316	−1 39	− 85	− 33	43	87
6249	317	−1 42	− 85	− 35	41	86
6407	318	−1 41	− 84	− 35	41	86
6727	320	−1 41	− 84	− 36	40	86
7021	323	−1 40	− 83	− 34	42	87
7771	328	−1 39	− 83	− 34	41	85
7949	330	−1 40	− 82	− 35	41	86
8307	333	−1 38	− 82	− 34	41	86
8500	334	−1 40	− 82	− 36	40	83
509 7355	403	−1 39	− 80	− 32	42	86
7563	404	−1 39	− 83	− 35	40	84
7770	406	−1 36	− 82	− 33	40	83
7941	407	−1 37	− 84	− 35	40	84
8391	411	−1 36	− 83	− 34	40	83
8791	414	−1 37	− 82	− 34	41	85
9020	415	−1 42	− 82	− 34	41	85
9291	417	−1 36	− 83	− 34	41	85
9670	420	−1 33	− 73	− 32	40	86
519 6569	.169	−1 43	− 86	− 37	42	87
6747	171	−1 42	− 84	− 35	43	84
6905	172	−1 33	− 84	− 34	40	86
7362	176	−1 41	− 85	− 35	38	81
7526	177	−1 40	− 84	− 35	40	83
7783	179	−1 37	− 82	− 33	42	84
8062	181	−1 38	− 78	− 32	41	83
8269	183	−1 35	− 79	− 34	42	86
540 6877	796	−1 42	− 86	− 36	42	87
7480	801	−1 41	− 86	− 39	41	84
8020	805	−1 40	− 86	− 37	43	87
8656	810	−1 40	− 84	− 35	44	89
9220	815	−1 42	− 86	− 36	45	88
554 6529	876	−1 37	− 83	− 35	45	89
6729	878	−1 37	− 80	− 34	43	89
7136	881	−1 38	− 84	− 34	48	87
7415	883	−1 37	− 83	− 36	43	86
7643	0 885	−1 40	−0 82	−0 34	0 42	0 84

given in Table 3, but with an arbitrary zero adjustment to make them international visual magnitudes. They are compared with the visual observations by Danjon (1928). Table 3 also gives the nightly mean colors of Beta Lyrae.

III. THE STANDARD SIX-COLOR SYSTEM

On the night of August 28/29, 9 Lyrae was observed six times at air masses well distributed between 1.0 and 2.7. Extinction coefficients for each color were determined from these observations. The following standard stars were also observed: α Ser, α Oph, β Oph, α Aql, β Aql, γ Aql, α Peg, α Ari, α Per, θ Tau, and γ Ori. Using these observations, the colors for 9 Lyrae were transformed to the standard system. In the standard system, these colors are as shown in the accompanying table.

U	*V*	*B*	*G*	*R*	*I*
−0 60	−0 76	−0 36	−0 04	0 40	0 78

TABLE 2

INSTRUMENTAL GREEN MAGNITUDES, 9 LYRAE — BETA LYRAE

Phase	Mag.	Phase	Mag.	Phase	Mag.	Phase	Mag
0 008	1 094	0 318	1 827	0 489	1 435	0 710	1 824
009	1 097	.320	1 827	491	1 440	.712	1 839
.011..	1 128	323	1 832	492	1.439	713	1 828
.013 .	1.087	.328	1 823	.549	1 498	716	1 820
.014.	1 106	330	1 818	.551 .	1.545	.717	1.838
.016. .	1 086	.333	1.817	553	1.553	.796 . .	1 880
.017.	1 075	.334	1 808	.555 .	1 539	.801	1 819
.020 .	1 067	.403	1 737	557 .	1.550	.805	1 861
.022.	1 060	.404	1 694	562 .	1 559	.810	1 933
024	1 065	.406	1 689	562	1 571	.815	1 831
025	1 083	407	1 689	.566	1 576	876	1 756
028	1 050	411	1 689	568..	1 548	878	1 761
.169	1 734	.414	1 695	570	1 591	881	1 748
.171	1 771	415	1 695	572	1 625	883	1 733
172	1.804	.417	1 695	622	1 719	885	1 707
.176	1 779	.420	1 672	623	1.727	932	1 307
.177	1 784	.471	1 529	625	1 723	934	1 336
179	1 805	.473	1 519	627	1 734	936	1 322
.181	1 813	474	1 513	633	1 744	940	1 308
183	1 838	.476	1 484	.634	1 740	942	1 259
.239	1 866	.478	1 475	636	1 748	944	1 220
.242	1.836	480	1 486	697	1 824	945	1 211
245	1 854	.484	1.468	.700	1 811	946	1 196
249. .	1 860	.485	1 479	.702	1 824	.948	1 171
316..	1 813	.487.	1.445	704	1 841	0 951	1 166
0 317	1 828	0 488	1 445	0 708	1.832		

TABLE 3

NIGHTLY MEAN COLORS AND GREEN MAGNITUDES FOR BETA LYRAE

Phase	$U-G$	$V-G$	$B-G$	$R-G$	$I-G$	Green Mag.	Visual Mag (Danjon)
0.017 .	−1 286	−0 776	−0 311	0 381	0 743	4 12	4 11
176 .	−1 386	− 826	− 344	410	.843	3 42	3 41
244	−1 408	− 850	− 355	042	.918	3 35	3 35
324	−1 400	− 833	− 347	411	857	3 39	3 39
411	−1 372	− 813	− 337	406	846	3 51	3 58
481	−1 344	− .845	− 353	428	886	3 73	3 77
.561	−1 391	− 855	− 349	407	.861	3 65	3 62
.629 .	−1.393	− 853	− 359	403	.851	3 48	3 46
708	−1 389	− 850	− .348	.404	840	3 38	3 36
806	−1 410	− 856	− 366	430	870	3 35	3 38
880	−1 378	− 824	− .346	.442	870	3 47	3 52
0 941	−1.276	−0 797	−0 327	0 393	0 794	3 96	3 89

These may be compared with the mean of two previous determinations by Dr. R. L. Sears (private communication).

U	*V*	*B*	*G*	*R*	*I*
−0 60	−0 78	−0 36	−0 04	0 40	0 80

The agreement is satisfactory.

IV. THE COLOR VARIATION OF BETA LYRAE

The color variation of the system in $U - V$ and $V - I$ and the light-curve are shown together in Figure 1. The points are nightly means. The mean internal error of a point is 0.006^{m}. The deviations from a symmetrical curve are much greater than this; thus the changes in color may not be completely repetitive. All color-curves not including ultraviolet light are similar to the $V - I$ curve.

The system is coolest (in color gradient) at primary minimum. This has been long known (e.g., Elvey [1935]). It has not been previously demonstrated that the system shows color-temperature maxima associated with light maxima, but the present results appear to establish this. The $V - I$ curve also shows a color-temperature maximum associated with the secondary light minimum. This will be interpreted as a result of the eclipse of the cooler star. In the $U - V$ curve, the ultraviolet is reduced with respect to the violet at both minima. This variation is similar to the variation in $U - B$ found by Wood and Walker. Unfortunately, the variation in $B - V$ that they found was too small compared with its uncertainties to confirm the present results from the $V - I$ curve.

V. THE WAVELENGTH DEPENDENCE OF THE RECTIFICATION COEFFICIENT

We shall only use the simplest model in this discussion. Assume that the stars can be represented by similar ellipsoids. We thus neglect the difference between the pointed and blunt ends of the primary. Russell and Merrill (1952) have shown that the combined effects of limb and gravity darkening upon the light outside eclipses may be represented by increasing the ellipticities of the components by a factor N, given approximately by

$$N = \frac{(15 + x)(1 + y)}{15 - 15x}, \tag{1}$$

where x is the limb-darkening coefficient and y is the coefficient representing the gravity effect. These quantities are fully described by Russell and Merrill. The observed light between eclipses is then to first order in z,

$$\mathfrak{I} = (1 - \tfrac{1}{2} N z \cos^2 \theta)\, I^m . \tag{2}$$

Here z is the geometrical rectification coefficient.

The observed values for Nz were computed for each color by a least-squares solution. The values were computed for each maximum separately, and for both together. In Table 4 are given solutions for Nz (*a*) using points from the maximum following primary minimum alone; (*b*) using points from the maximum before primary minimum alone; (*c*) using points from both maxima.

In these solutions, points corresponding to θ outside the ranges $30° < \theta < 150°$ and $210° < \theta < 330°$ were omitted. We assume that within these ranges there are no effects introduced by the eclipses and that gas-stream effects are negligible.

In Figure 2 we compare the wavelength dependence which has been observed for Nz with theoretical curves obtained from equation (1), and various values of z. To compute N from equation (1), we have assumed a limb-darkening coefficient of 0.6 which is supposed independent of wavelength. For computing y, a gray atmosphere with effective temperature 11000° K was assumed.

It is seen that the observed rectification coefficients agree that the geometrical coefficient, z, is close to 0.19. Since Danjon's visual light-curve was obtained with a similar

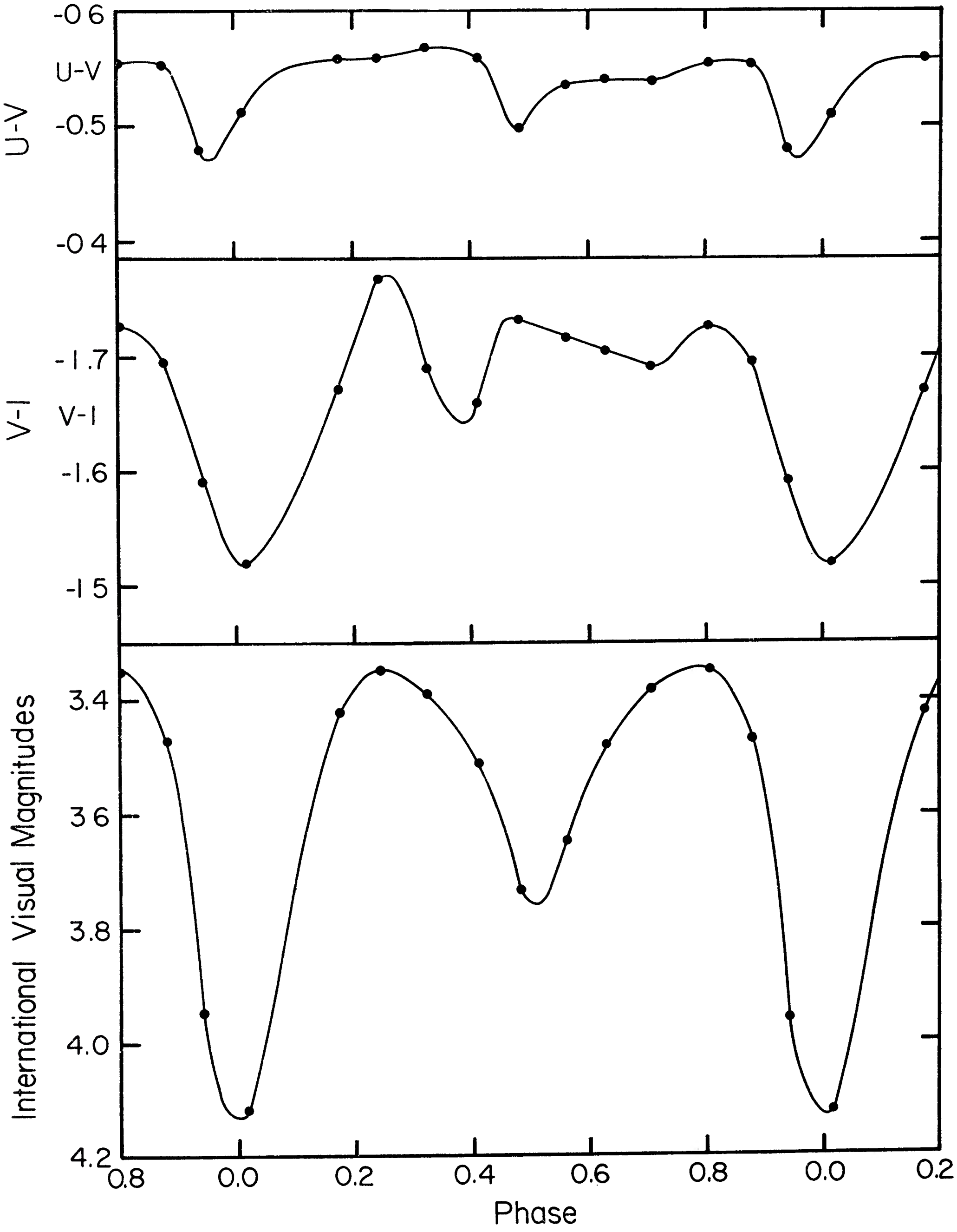

FIG. 1.—The $U - V$, $V - I$ color and green light-curves of Beta Lyrae Each point represents the mean of one night's observations.

effective wavelength to our green color, we should expect him to obtain a similar coefficient. For $z = 0.19$ he should have found a coefficient of 0.406. He found 0.415. Results by Sandig (1934) are in poorer agreement. He found Nz to be 0.47 and 0.48 in yellow and blue light, respectively. However, he commented upon the uncertainty of his values.

The exact value of Nz is of great importance, since the depth of secondary minimum after rectification depends critically on this. Falsely large values of Nz give falsely shallow secondary eclipses. Then the temperature of the secondary appears anomalously low. The variation of Nz with wavelength shows reasonable agreement with theoretical predictions. To the best of our knowledge this variation has not previously been observed for any system. If, as seems likely, the primary fills its lobe of the critical equipotential, the ellipsoidal model and neglect of higher-order terms than $\cos^2 \theta$ in rectification may not be adequate. If this is so, the depth of secondary minimum that we determine may be incorrect, and this could seriously affect the remainder of our analysis.

TABLE 4

THE WAVELENGTH DEPENDENCE OF THE RECTIFICATION COEFFICIENT

COLOR	EFFECTIVE WAVELENGTH	Nz OBSERVED METHOD OF COMPUTATION*			THEORETICAL VALUE IF z=0 19	O−C
		(a)	(b)	(c)		
U	3530	0 461	0 453	0 455	0 481	−0 026
V	4220	464	432	466	447	+ 019
B	4880	425	413	427	425	+ 002
G	5700	386	449	401	406	− 005
R	7190	365	407	374	381	− 007
I	10300	0 335	0 525	0 379	0 356	+0 023

* See text, Sec V, for descriptions of methods (a), (b), and (c)

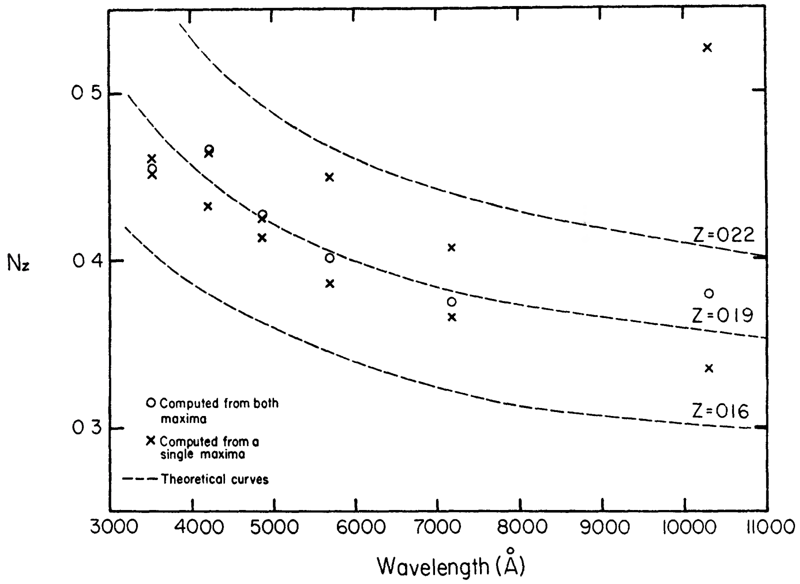

FIG 2 —The wavelength dependence of the rectification coefficient Nz The dotted curves are based on the first-order theory of Russell and Merrill. The points correspond to the results in Table 4

VI. THE COLORS OF THE COMPONENTS

To determine the colors of the individual stars we do not search for solutions consistent with the light-curve, since such solutions do not lead to a unique model (Kuiper 1941). Instead, we begin with the spectroscopic observation that the B9 star, behind at primary eclipse, always dominates the light of the system. Even at primary eclipse, the light received from the secondary must be at least 1 mag. less than the light received from the primary. With this observation, we use the rectified light-curve of Danjon, and the six-color observations.

After rectification, the depth of primary minimum was 36 per cent and of secondary minimum was 13 per cent. We shall assume that at primary minimum the effects of limb and gravity darkening of the primary are approximately compensatory. Then the relative surface brightnesses of the two stars J_1 and J_2 are

$$J_2/J_1 = 13/36 = 0.36 \pm 0.03 .$$

We have assumed the depth of each eclipse is uncertain by 1 per cent. If the temperature of the B9.5 end of the primary is 10500° K, and assuming that both stars radiate like black bodies, the temperature of the secondary is 7700° ± 400° K. The corresponding spectral type is A7 ± 3.

TABLE 5

THE COLORS OF THE COMPONENTS (TOTALLY ECLIPSING MODEL)

Star	Redden-ing	Spectral Type	*U*	*V*	*B*	*G*	*R*	*I*
67 Oph	0 09	B5 Ib	−1 44	−0 88	−0 34	−0.05	0 40	0 89
β Lyrae A (end-on)	065	B9 5 II	(−1 39)	− 87	− 37	− 02	40	.85
13 Mon	09	A0 Ib	−0.97	− 89	− 38	− .03	41	.74
α Lep	09	F0 Ib	−0 18	− 58	− 26	− 01	28	48
β Lyrae B	065	?	(−0 90)	− 54	− 22	− 03	25	42
ν Per	0 05	F5 II	−0 06	−0 37	−0 12	−0 03	0 15	0.26

With these surface brightnesses and eclipse depths, the secondary eclipse must be almost total if the spectrum of the secondary is not to show in the combined spectrum at primary minimum. Assume that the secondary eclipse is total. The color observed during this eclipse is the color of the primary. During primary eclipse, we receive the full light of the secondary (13 per cent of the green light out of eclipse) and the remainder comes from the primary. We know the color of the system at this time, and the color of the primary; thus we can calculate the color of the secondary. This estimate is independent of the shape of the secondary.

The calculation has been made, and the colors of the two components are compared with the colors of stars suffering similar reddening (Kron 1958) in Table 5. The reddening of Beta Lyrae is taken from Abt *et al.* (1962). From Table 6 it appears that the color of the secondary is perhaps more like that of an F0 star than of an A7, but since the effective temperatures for stars of this luminosity are simply assumed to be the same as main-sequence ones (themselves doubtful), we regard the agreement as good. Since the $U - V$ color-curve shows minima associated with both primary and secondary minimum, we cannot interpret this variation in terms of the mutual eclipses of the stars. This variation could be the result of the gas streams between the stars emitting copious ultraviolet radiation, the emission being eclipsed by the two stars in turn. Thus we regard the U color as an unreliable guide to the properties of the stars.

Using the colors of both stars, we have computed the color that the system should show out of eclipse. These calculated colors are compared with the colors at times nearest light maxima in Table 6. It is seen that the residuals are systematic. The system is hotter at light maximum than this model predicts. If the color discrepancy in $V - I$ corresponds to a change in spectral type, the change should be 1.1 subclasses. This figure was found by plotting $V - I$ as a function of spectral type in data by Stebbins and Kron (1956). This discrepancy is the surface gravity effect (Von Zeipel 1924). These results should be compared with line spectrophotometry by Boyarchuk. He found that the equivalent widths of six Fe II lines changed by a factor of 1.7 between light maximum and secondary minimum. These same lines change their equivalent widths by a factor of 4.4 between Alpha Cygni A2 Ia, and 2H Persei, B9 Ia, or a factor of 1.65 per subclass. The agreement between the two methods is excellent. The mean spectral type of Beta Lyrae is B9 (Struve 1934), and at secondary minimum it is B9.5 (Boyarchuk 1959). Thus the spectral type of Beta Lyrae is best given as B8.5 II broadside-on and B9.5 II end-on.

In this totally eclipsing model, the ratio of mean end-on projected areas of primary to secondary is 1:0.40. Since there is no clear indication that secondary eclipse is total, we have also computed a model in which 25 per cent of the secondary is still visible during secondary eclipse. At primary minimum with this model, the secondary gives 1.4 and 0.9 mag. less light than the primary, in green and infrared light, respectively. In view of the strength of the B9 lines at this phase, despite the "washing out" by the lines of the gas streams, it appears that the secondary cannot contribute more light than this. In the partially eclipsing model, the ratio of end-on areas is 1:0.58. The $V - I$ colors of primary and secondary are changed from −1.72 to −1.76 and from −0.96 to −0.88, an unimportant change.

TABLE 6

COMPARISON OF OBSERVED AND COMPUTED COLORS OUT OF ECLIPSE

	U	*V*	*B*	*G*	*R*	*I*
Colors at light max	−1 422	−0 867	−0 374	−0 026	0 396	0 851
Computed colors	−1 338	− 834	− 354	− 022	377	787
O−C	−0 084	−0 033	−0 020	−0 004	0 019	0 064

We have also considered using a model in which the pointed end of the primary is noticeably cooler than the blunt end. However, we realized that such a model should have shown a pronounced anti-reflection effect. The reflection effect is about +1 per cent (Danjon 1928), and this is inconsistent with such a model. We have also considered that the depth of primary minimum should really either be deeper or shallower because of the effect of gas streams. For no reasonable change did the change of color or size of the secondary become appreciable.

VII. CONCLUSIONS

The colors proposed for the two components of Beta Lyrae are by no means definitive. However, much of their uncertainty is caused by doubt about the validity of first-order rectification theory for highly distorted systems. Nevertheless, the results do appear to establish changes in the colors due to the secondary component of Beta Lyrae which are in agreement with models in which a B9 star and a fainter star of late A or early F type are involved. The spectroscopic observations eliminate models in which more than $\frac{1}{4}$ of the secondary is visible at secondary minimum.

During this work it has been possible to demonstrate the variation of the coefficient of ellipticity with wavelength. This variation is consistent with present theory and is a partial confirmation of that theory.

We have observed the color variation of the system due to gravity darkening. The results are compatible with existing spectrophotometric studies.

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