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A SEARCH FOR INTERSTELLAR ICE ABSORPTION IN THE INFRARED SPECTRUM OF MU CEPHEI

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ABSTRACT

The infrared spectra of μ Cephei and α Orionis obtained in the second flight of Stratoscope II are compared in an attempt to detect an absorption band at 3.1 μ due to interstellar ice particles. The absence of the band suggests that no more than one-quarter of the interstellar reddening is due to ice.

The water-vapor bands in the spectra of these two supergiants are surprisingly strong, and no satisfactory explanation for this phenomenon is known.

I. INTRODUCTION

The particles responsible for the interstellar absorption have often been assumed to consist of the ices of water, methane, and ammonia with an admixture of heavy-element oxides. These ices have many infrared absorption bands corresponding to the absorption bands in the gaseous phase. The second flight of Stratoscope II offered the opportunity to look for these absorption bands. The range of the infrared spectrometer used in this flight was 0.8–3.1 μ (see Woolf, Schwarzschild, and Rose [1964]; hereafter called "WSR," for a description of the second flight). As shown in Section IV, the fundamental of solid H_2O at 3250 cm^{-1} (3.1 μ) is by far the strongest band in the wavelength range of Stratoscope II and the only one in which one might expect to see an absorption due to interstellar ice. Thus the most promising band is at the long-wavelength limit of the spectrometer. The band does, however, extend from 2.8 to 3.4 μ and thus one might reasonably expect to see it between 2.8 and 3.1 μ if it existed.

A promising type of object in which to search for this band is a late-type supergiant which is reddened. Mu Cephei (M2 Ia) is such a star. Its color has been measured as $B - V = 2.41$ (Johnson and Morgan 1953). Alpha Orionis is a good comparison object. It is usually listed as spectral type M2 (but is sometimes listed as M2–M3), and it is somewhat less luminous than μ Cephei, being of luminosity class Iab. Johnson and Mitchell (1963) give $B - V = 1.87$ for α Orionis, which suggests that μ Cephei is reddened. Blanco (1954) points out that "μ Cephei lies in the area of the I Cepheus aggregate which shows the effect of obviously near obscuration," and hot stars near μ Cephei in the Cepheus I association show a typical reddening of $\Delta\, B - V = 0.5$. Blanco concludes on the basis of color measurements on early M-type supergiants that the intrinsic color of μ Cephei is approximately the same as the observed color of α Orionis, confirming that μ Cephei is reddened due to interstellar absorption. Both μ Cephei and α Orionis were scanned several times during the second flight. The data obtained are presented in the next section.

II. THE OBSERVATIONS

The spectrum of μ Cephei was observed during the second flight of Stratoscope II. There were four complete scans of its spectrum. The details of the observations, the method of reduction of the data, and most other relevant facts are described by WSR. The spectra of μ Cephei obtained with both detectors are tabulated in Table 1. The

TABLE 1

THE SPECTRUM OF μ CEPHEI

$\lambda(\mu)$	$1/\lambda$ (cm^{-1})	$F_A(\lambda)$	$F_B(\lambda)$	$\lambda(\mu)$	$1/\lambda$ (cm^{-1})	$F_A(\lambda)$	$F_B(\lambda)$
0 82	12195	286		1 78	5618	175	153
0 83	12048	286		1 80	5556	159	146
0 84	11905	258		1 82	5495	167	140
0 85	11765	269		1 83	5464	163	139
0 86	11628	277		1 85	5405	162	144
0 87	11494	273		1 87	5348	155	130
0 89	11236	289		1 88	5319	146	119
0 90	11111	264		1 90	5263	145	96
0 91	10989	287		1 92	5208	136	109
0 93	10753	213		1 94	5155	128	112
0 94	10638	252		1 96	5102	129	112
0 96	10417	266	167	1 98	5051	129	109
0 97	10309	259	215	2 00	5000	125	102
0 99	10101	251	203	2 02	4950	124	107
1 00	10000	260	263	2 03	4926	124	115
1 01	9901	276	286	2 05	4878	123	123
1 03	9709	267	269	2 07	4831	125	105
1 04	9615	274	202	2 08	4808	125	98
1 06	9434	257	228	2 10	4762	124	108
1 07	9346	250	232	2 12	4717	117	111
1 09	9174	252	200	2 13	4695	120	103
1 10	9091	232	197	2 15	4651	116	98
1 12	8929	232	183	2 17	4608	112	98
1 13	8850	244	189	2 18	4587	108	83
1 15	8696	236	201	2 20	4545	102	75
1 16	8621	251	227	2 22	4505	107	72
1 18	8475	241	205	2 24	4464	95	67
1 19	8403	269	217	2 26	4425	87	73
1 20	8333	271	239	2 28	4386	71	65
1 22	8197	279	256	2.30	4348	74	65
1 23	8130	269	243	2 31	4329	67	56
1 25	8000	258	221	2 33	4292	68	50
1 27	7874	264	223	2 34	4274	67	49
1 28	7813	277	220	2 36	4237	60	60
1 30	7692	266	169	2 37	4219	61	48
1 32	7576	258	165	2 39	4184	57	45
1 34	7463	241	176	2 40	4167	59	49
1 35	7407	237	158	2 41	4149	61	51
1 37	7299	223	159	2 43	4115	61	60
1 38	7246	205	161	2 44	4098	59	46
1 40	7143	200	151	2 46	4065	51	47
1 42	7042	198	144	2 47	4049	46	39
1 44	6944	198	131	2 49	4016	45	46
1 45	6897	201	155	2 50	4000	47	43
1 47	6803	195	158	2 51	3984	46	42
1 48	6757	212	225	2 53	3953	52	43
1 50	6667	207	195	2 54	3937	53	49
1 52	6579	224	190	2 56	3906	51	53
1 54	6494	228	199	2 57	3891	52	45
1 56	6410	236	184	2 59	3861	58	40
1 58	6329	242	208	2 60	3846	56	45
1 60	6250	242	191	2 61	3831	54	46
1 62	6173	249	198	2 63	3802	50	35
1 64	6098	250	221	2 64	3788	44	36
1 66	6024	238	204	2 66	3759	41	35
1 68	5952	224	188	2 67	3745	42	35
1 70	5882	208	158	2 69	3717	43	36
1 72	5814	201	160	2 70	3704	38	29
1 73	5780	190	168	2.71	3690	33	38
1 75	5714	172	170	2 73	3663	40	39
1 77	5650	169	152	2 74	3650	40	41

TABLE 1—*Continued*

$\lambda(\mu)$	$1/\lambda$ (cm^{-1})	$F_A(\lambda)$	$F_B(\lambda)$	$\lambda(\mu)$	$1/\lambda$ (cm^{-1})	$F_A(\lambda)$	$F_B(\lambda)$
2 76	3623	38	39	2 94	3401	41	39
2 77	3610	35	40	2 95	3390	41	38
2 79	3584	42	34	2 96	3378	42	36
2 80	3571	39	31	2 97	3367	40	36
2 81	3559	44	30	2 98	3356	39	36
2 83	3534	38	41	2 99	3344	39	39
2 84	3521	41	39	3 00	3333	36	38
2 86	3497	43	38	3 01	3322	37	36
2 87	3474	43	36	3 02	3311	33	36
2 89	3460	42	35	3 03	3300	30	34
2 90	3448	42	38	3 04	3289	30	28
2 91	3436	42	33	3 05	3279	30	26
2 93	3413	41	35				

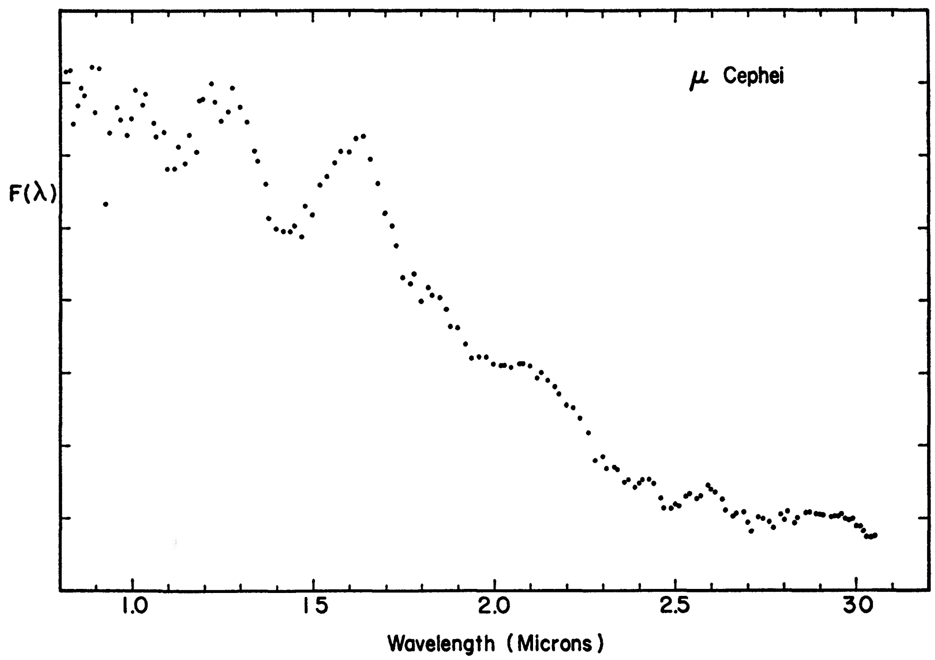

FIG. 1.—The relative energy per unit wavelength interval from μ Cephei (detector A)

spectrum from detector A is shown in Figure 1. The spectrum of μ Cephei should be compared with the spectrum of α Orionis (WSR, Fig. 4), since this star is similar in type to, but slightly less luminous than, μ Cephei. A closer comparison can be made when both spectra are divided by a black-body spectrum for 3000° K (the approximate effective temperature of these stars). This comparison is between Figure 2 of this paper and Figure 12 of WSR. It is apparent that the absorption bands are more prominent in μ Cephei.

The equivalent widths of the bands at 1.4 and 1.9 μ have been estimated, and are compared with corresponding values of α Orionis in Table 2. If the band strengths in μ Cephei are interpolated in the data for normal giants (WSR), the spectral type would be

M6, whereas by the criteria of the MK classification system (Johnson and Morgan 1953), μ Cephei has a type of M2 Ia.

The two stars of WSR most comparable in band strength to μ Cephei are α Orionis and ρ Persei. The spectrum of μ Cephei (detector A) has been divided by the corresponding spectra of these stars, and the results are shown in Figures 3 and 4. In Section V, these results will be used in the discussion of the interstellar ice band. In these figures, any differential error in wavelength calibration between the two stars is serious for the spectral region from 2.9 to 3.05 μ. The sensitivity function for the apparatus peaks sharply in this region of the spectrum as shown in Figure 5. In the inset for Figure 3 we have recomputed the division of the spectra putting in both positive and negative differential wavelength corrections. The amplitude of the corrections we have used is 1 wavelength marker or approximately 0.01 μ in this spectral region. The two stars were

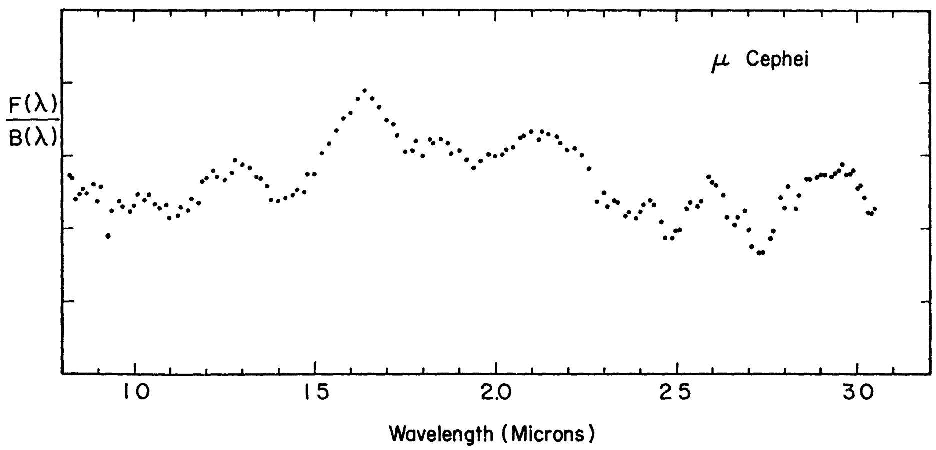

FIG. 2.—The relative energy per unit wavelength interval from μ Cephei divided by the Planck function for $T = 3000°$ K (detector A).

TABLE 2

EQUIVALENT WIDTHS OF ABSORPTION BANDS

	1.4 μ	1.9 μ
μ Cephei	220 cm^{-1}	140 cm^{-1}
α Orionis	120 cm^{-1}	35+23 cm^{-1}

observed at times separated by 75 minutes in the flight, and as the wavelength zero point only changed by 3 markers during the last 8 hours of flight, it seems improbable that the differential correction should be any larger.

III. THE ABSORPTION BANDS IN μ CEPHEI

As shown in Section II, μ Cephei exhibits a band structure which is intermediate between ρ Persei (M4 III) and R Leonis at mean light (M8 III). Similarly, α Orionis exhibits a band structure which is far stronger than in μ Geminorum (M3 III) and more nearly like that in ρ Persei. Thus the two observed supergiants have deeper bands than the less luminous giants of the same spectral type. Furthermore, the more luminous of the two supergiants (μ Cephei) has deeper bands than the other supergiant (α Orionis). Since μ Cephei appears to have the same bands as the other stars discussed in WSR, we

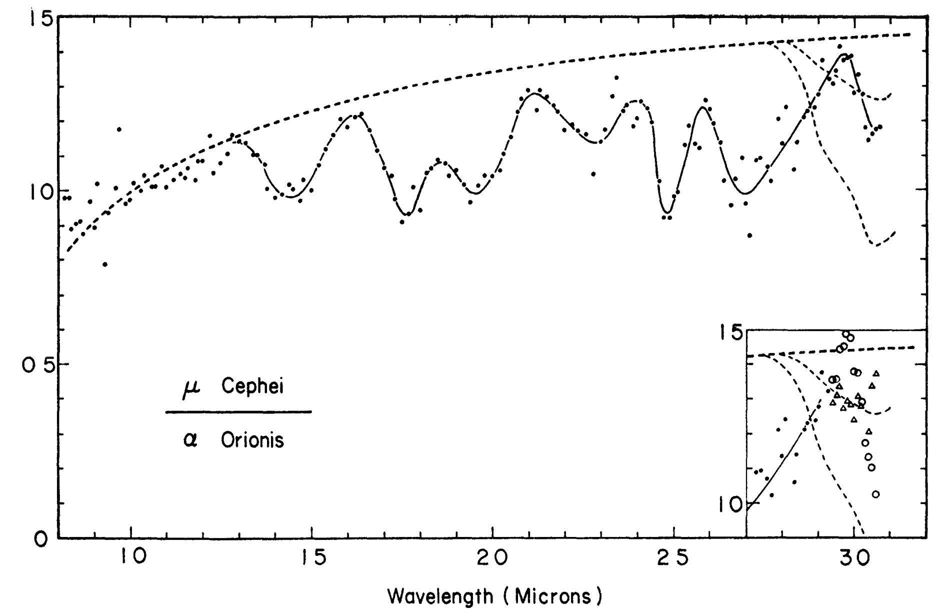

FIG. 3.—The infrared signal of μ Cephei divided by the infrared signal of α Orionis normalized to unity at 1 μ (detector A). The continuum curve (*large dashes*) is the expected continuum due to interstellar reddening (see text). The predicted ice band at 3 05 μ is the small dashed curve.

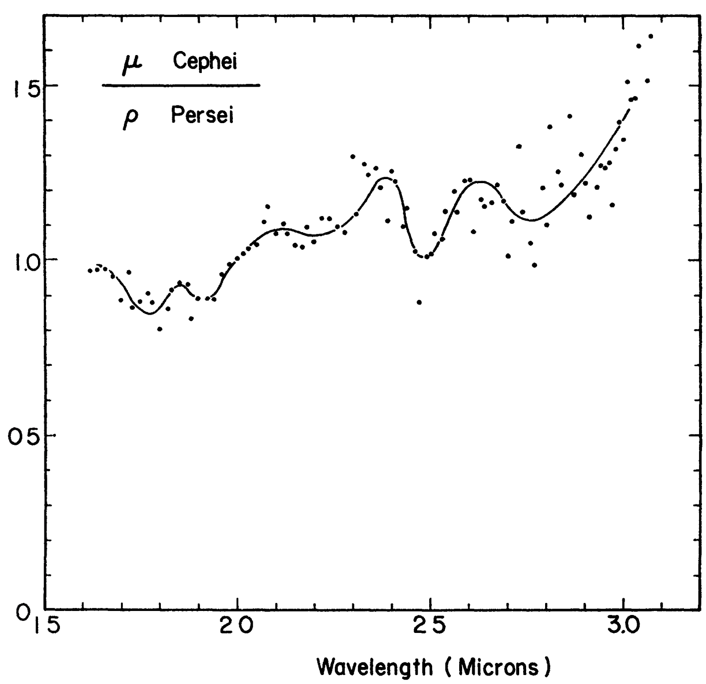

FIG. 4.—The infrared signal of μ Cephei divided by the infrared signal of ρ Persei normalized to unity at 2 μ (detector A).

conclude that the bands in μ Cephei are mainly due to water vapor. This is corroborated by Figure 3 in which the comparison with α Orionis reveals bands at 1.4, 1.9, and 2.7 μ as well as an indication of a band near 1.13 μ.

The reason that the water-vapor bands in the supergiants are deeper than in the giants of the same spectral type is not clear. A provisional scale of effective temperatures (Keenan 1963) suggests that for the same spectral type, higher luminosities have lower effective temperatures. Lower temperatures would increase the amount of water vapor above the photosphere. However, the lower photospheric pressures corresponding to the higher luminosities will reduce the amount of water vapor above the photosphere and will probably more than compensate for any likely reduced temperature. In this connection, see graphs of molecular abundances above the photosphere due to Stanger (Aller 1963).

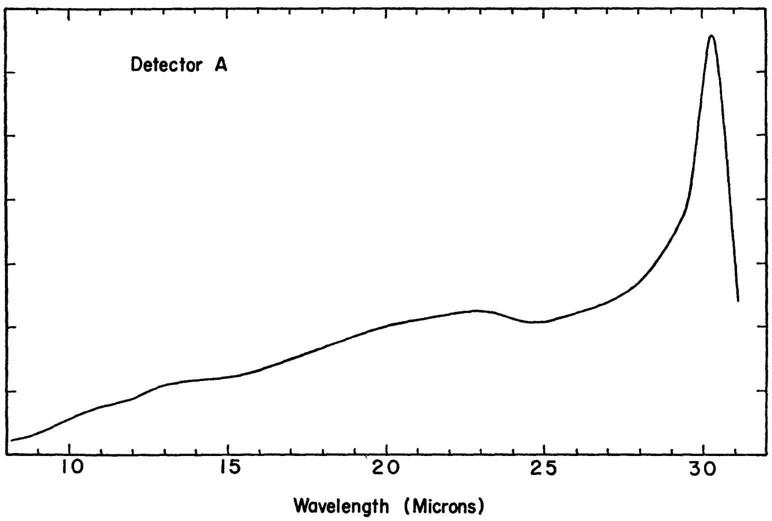

FIG. 5.—The sensitivity function of the spectrograph (detector A)

One possible explanation for the deeper bands is that the more luminous stars have higher turbulent velocities than the less luminous stars. Spitzer (1939) concluded that α Orionis had turbulent velocities of about 8 km/sec while α Herculis (an M5 II giant) had velocities about half as large. These velocities should be compared with the velocity of sound which is 6.5 km/sec for a mean molecular weight $M = 1$ and a temperature $T = 3000°$ K. The bands in μ Cephei have equivalent widths which are about a factor of 2 larger than in α Orionis. If these are to be attributed to increased turbulent velocities, one requires velocities in μ Cephei of the order of 20 km/sec. Velocities of this magnitude have been observed in μ Cephei (Deutsch 1964; see also Weyman [1963]). It is not certain, however, that these velocities are such as to produce a Doppler-like broadening in the individual lines making up the water-vapor bands.

Another possibility is that water-vapor bands occur in the supergiants' circumstellar envelopes. Weymann (1960) has studied the circumstellar envelope of α Orionis and has estimated that the total number of hydrogen atoms in the line of sight is $2 \times 10^{22}/\text{cm}^2$. He has also estimated that the temperature of the circumstellar envelope is roughly 1000° K. If one assumes an O/H ratio of 10^{-3}, one obtains 6×10^{-4} gm cm^2 of H_2O in the circumstellar envelope of α Orionis if all the oxygen is in water vapor. It is estimated in WSR that of the order of 1 gm/cm^2 of water vapor is required to produce the large absorptions (equivalent widths approximately 500 cm^{-1}) in o Ceti.

To produce a 1.9-μ water-vapor band in α Orionis (with equivalent width about 50

cm^{-1}) one requires about 10^{-3} gm cm^2 in the star's atmosphere, but considerably more, about 10^{-2} gm cm^2, in a circumstellar envelope. However, in this estimate we assume the same Doppler broadening as in *o* Ceti. The calculation also assumes that, as the line strength required changes by a factor of 10, the number of lines of that strength or greater changes by a factor of 2. Some allowance is made for the variation of the number of lines with temperature. In reality less water will be needed because the Doppler broadening of the lines of α Orionis is greater than in *o* Ceti. Thus this crude estimate cannot eliminate the circumstellar envelope of α Orionis as a source of its strong H_2O bands. In the case of μ Cephei, the 1.9-μ band has an equivalent width of 140 cm^{-1}. No estimate of the amount of matter in the envelope is known to us. To produce a band this strong, either the amount of water must be 30 times as great as in α Orionis, or velocity broadening must be 2–3 times as great. Again, we cannot eliminate these possibilities.

Figure 3 gives the ratio of the intensities of μ Cephei and α Orionis normalized to unity at 1 μ. The continuum was computed from van de Hulst's (1949) model No. 15 for the interstellar particles. The continuum is also normalized to unity at 1 μ. Since model No. 15 was obtained by fitting the six-color measurements of Stebbins and Whitford (1945), the six-color measurements of μ Cephei and α Orionis were used in conjunction with model No. 15 to determine the differential reddening and extinction between μ Cephei and α Orionis. Translated into the *UBV* system, the differential reddening is $\Delta\, B - V = 0.42$. The calculation of the continuum further assumes that μ Cephei has the same temperature as α Orionis. If μ Cephei were cooler than α Orionis, the continuum ratio would increase toward long wavelengths. However, a temperature difference of 350° K is needed to produce the continuum shown in Figure 3. It seems unlikely that μ Cephei is 350° cooler than α Orionis. It is therefore concluded that the continuum increase indicated in Figure 4 is primarily due to the interstellar reddening.

IV. LABORATORY SPECTRA OF SOLIDS

Van de Hulst (1949) suggests that the interstellar particles are composed primarily of solid H_2O, CH_4, NH_3, and H_2. Of these substances, that for which the most complete laboratory data are available is ice. The work up to 1957 has been summarized by Ockman (1958). The strongest band observed is the ν_3 fundamental at 3250 cm^{-1} (3.1 μ), which has an integrated absorption of 5×10^6 cm^{-1} (gm $cm^2)^{-1}$. The librational band at 12 μ is about one-third as strong, the ν_2 fundamental at 4.4 μ one-tenth as strong, whereas the overtone and combination bands at 2.0, 1.7, 1.5, 1.3, and 1.0 μ are more than one hundred times weaker. These estimates are obtained from the data given by Ockman (1958), Fox and Martin (1940), and Kislovskii (1959). The strength of the ν_3 band is about a factor 40 greater in the solid phase than in the gas phase, due to the formation of strong hydrogen bonds between neighboring molecules.

The methane molecule does not interact strongly with its neighbors in the solid phase, and it is therefore expected that the absorption-band strengths will be approximately the same as in the gas phase. Very few absolute-intensity measurements have been made of the spectrum of solid CH_4, but the work of Ewing (1964) on the ν_3 fundamental and that of Glasel (1961) on two overtone bands indicate that the difference between the solid- and gas-phase band strengths is not more than a factor of 2. The intensity of the ν_3 band at 3010 cm^{-1} (3.3 μ) is 4×10^5 cm^{-1} (gm $cm^2)^{-1}$ and that of the ν_4 band at 1303 cm^{-1} (7.7 μ) is half as great (Armstrong and Welsh 1960). The overtone and combination bands are one hundred times weaker (Dennison 1925).

It is expected that CH_4 is less abundant in the interstellar particles than H_2O both because of the lower cosmic abundance of carbon and the high vapor pressure of CH_4. Since the band strengths of the CH_4 fundamentals are also weaker than those of ice, it is likely that the absorption bands produced by methane in the interstellar particles are very weak compared to the 3.1-μ ice band.

No absolute measurements of the band intensities of solid ammonia have yet been made, and since NH_3 does form strong hydrogen bonds with neighboring molecules upon solidification, it is likely that the solid-phase intensities will be somewhat different from the gas phase. Indeed, Reding and Hornig (1951) found that the ratio of band strengths ν_3/ν_1 was at least 20 times greater in the solid phase than in the gas phase. If we assume that, as with water, the perpendicular stretching mode ν_3 is enhanced, rather than that the ν_1 mode is depressed, then we can estimate from the gas phase intensity measurements of McKean and Schatz (1956) and the ν_3/ν_1 ratio measured by Reding and Hornig (1951) that the ν_3 band of solid NH_3 at 3380 cm^{-1} (3.0 μ) has an intensity of 3×10^5 cm^{-1} (gm $cm^2)^{-1}$. The ν_2 band at 9.4 μ has this same order of strength, whereas the ν_4 fundamental at 6.1 μ is ten times weaker and the overtone and combination bands are one hundred times weaker. These band strengths are much less than those for ice, and since nitrogen has a much lower cosmic abundance than oxygen, we also expect that bands produced by solid ammonia in the interstellar particles will be much weaker than the 3.1-μ ice band.

Solid hydrogen has a fundamental absorption band at 4484 cm^{-1} (2.2 μ), by the measurements of Gush, Hare, Allin, and Welsh (1960) indicate that the intensity is only 2×10^3 cm^{-1} (gm $cm^2)^{-1}$. Therefore, this band will also be much weaker than the ice band.

In summary, if the interstellar particles are composed of solid H_2O, CH_4, NH_3, and H_2, then the strongest absorption band is expected to be the ν_3 fundamental of ice at 3.1 μ.

V. THE PREDICTED ICE BAND IN μ CEPHEI

For the purpose of computing the predicted strength of the ice band, we adopt van de Hulst's (1949) model No. 15 for the interstellar particles, since this model agrees well with modern extinction measurements extending to 2.2 μ (Johnson and Borgman 1963). In this model, over half the mass is in particles less than 0.3 μ in radius. Therefore for the 3.1-μ band the particles are essentially all small compared to the wavelength, and only the first term, proportional to a^3, in the expansion for the absorption cross-section (van de Hulst 1957, p. 270), need be taken into account in calculating the absorption. Further, since the refractive indices, n-1 and n', are both small for ice, the absorption by a collection of small particles is approximately the same as that by the same mass of solid material. Therefore to an accuracy better than 20 per cent we may use the laboratory measurements directly in computing the expected absorption of ice particles.

We have shown in Section III that the differential reddening between μ Cephei and α Orionis is $\Delta\ B - V = 0.42$. For model No. 15 this corresponds to a visual absorption $A_V = 1.3$ magnitudes and a density of particles along the line of sight of 4×10^{-5} gm cm^{-2}. We assume that this entire mass is ice and use the experimental absorption data of Fox and Martin (1940) to compute the expected absorption profile shown in Figure 3. For this amount of ice, the optical depth at the band center (3.05 μ) is 0.54. The entire profile is clearly much deeper than is consistent with the observed spectrum. Also shown in the figure is the expected profile for one-fourth the expected amount of ice. This is the maximum strength of the absorption band that can be fitted to the observed curve.

The same expected profiles are shown in the inset to Figure 3, which shows that the above conclusions are not affected by the uncertainty in the wavelength calibration near 3 μ (Sec. II).

Since the water-vapor bands in the spectrum of μ Cephei are deeper than those in α Orionis, it might be thought that a difference in the intrinsic spectra of the two stars at 3 μ could mask the presence of the ice band. However, the comparison shown in Figure 4 of μ Cephei with ρ Persei, which has water-vapor bands of perhaps more similar strength, also shows no indication of the ice band.

Although a particular model of the particle size distribution has been adopted here, it is not critical, for as long as only the a^3 term is important in the expansion for the absorption cross-section, the absorption depends only on the total mass of the particles. The higher-order terms contribute less than 50 per cent to the absorption cross-section as long as the particle radius is less than 0.7 μ. It is unlikely that there exist many particles of as large a radius as this.

The laboratory measurements of Fox and Martin were made on a sample at 263° K, whereas the temperature of the interstellar particles is thought to be of the order of 20° K. However, the spectra obtained by Hornig, White, and Reding (1958) at 195° and 83° K show the bands with the same half-width and the maximum absorption at the same wavelength as at 263° K. These measurements suggest that the absorption does not depend sensitively on temperature.

The presence of other substances than water in the grains may affect these results in two ways. First, the strength and the wavelength of the ice band depend on the presence of strong hydrogen bonds between neighboring molecules. Other molecules in the grain may modify this bonding somewhat, but it is not likely that the characteristic ice spectrum would be removed except under extreme dilution. A laboratory study of this problem would be very valuable. Second, there may be a contribution to the extinction by a metallic or graphite core, in which case a smaller amount of ice will be associated with a given color excess.

VI. CONCLUSIONS

The ice absorption band does not appear at its predicted strength in the spectrum of μ Cephei. This prediction is based on the grains' being entirely composed of H_2O and on the assumptions that the absorption band position, shape, and strength are the same as determined for ice in the laboratory, and that the particle size distribution is given by the van de Hulst model No. 15. Laboratory results show no change in the ice band between 263° and 83° K, and the van de Hulst size distribution is not critical. Other particle distributions that reproduce the wavelength dependence of interstellar absorption will produce absorption bands of similar strength. From an attempt to fit the strongest possible ice band consistent with the data, we find that no more than one-quarter of the interstellar reddening is produced by ice. This result is not consistent with suggestions that three-quarters of the particles constituting interstellar grains are ice crystals. Studies of the absorption of ice mixed with other molecules and molecular fragments at low temperatures are desirable to strengthen this conclusion. The absence of detectable ice absorption may perhaps be an argument in favor of interstellar extinction's being caused by material other than dielectric grains (Platt 1956; Hoyle and Wickramasinghe 1962).

The water-vapor bands in the spectrum of μ Cephei are surprisingly strong. So far no satisfactory explanation has been found for this phenomenon, but it may be partly due to the large turbulent velocities in the atmosphere of this star. The difference in band strength between μ Cephei and α Orionis is surprising since the spectral types of these objects are so similar. It points a warning against using late-type stars to determine interstellar extinction at long wavelengths.

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