A SHORT METHOD FOR DETERMINING THE APPARENT DISTANCE MODULI OF CLASSICAL CEPHEIDS

J. D. FERNIE
David Dunlap Observatory, University of Toronto
Received March 16, 1964; revised April 22, 1964

ABSTRACT

A method is presented for determining the apparent distance moduli of classical cepheids without knowledge of the full light-curve. The method is tested on forty-four cepheids, utilizing only a single observation for each star, and the average error in distance modulus, when compared to that obtained by the utilization of the full light-curve, is found to be less than 0.1 mag.

Current techniques for determining the distance moduli of classical cepheids by the period-luminosity-color relation (Kraft 1963) are rather tedious and time-consuming in practice. They require detailed photoelectric observing of the full light-curve (typically twenty or more observations per star), and thereafter a laborious reduction of the light-curve to intensity units, numerical integration by planimetry to determine the average intensity, reconversion to a magnitude, and a similar planimetry of the color-curve. The present note indicates a method whereby the apparent modulus may be obtained to within about 0.1 mag. of that given by the detailed method, utilizing only a single observation in V and $B - V$.

It has been shown elsewhere (Fernie, in press) that the period-luminosity-color (PLC) relation for classical cepheids may be represented to an accuracy of a few hundredths of a magnitude by the equation

$$M_V = -3.33 - 2.500 \log P + 2.06(B - V) .$$

In the detailed method for determining distance moduli, M_V and $B - V$ will be averaged values, as indicated above. However, for a given period, the above equation describes a straight line sloping downward to the right in the H-R diagram. At the same time, an individual cepheid in the course of its cycle describes an elongated, roughly elliptical loop in the H-R diagram, with the axis of the loop also sloping downward to the right. In fact, detailed examination in a number of cases shows that the line of the PLC relation and the axis of the loop are often essentially coincident. This suggests that were it not for phase-lag phenomena introducing second-order effects, the period-luminosity-color relation would apply at every point in the cycle and not merely to the average magnitude and color alone. In any case, since the amplitudes of the loops are only of the order of a few tenths of a magnitude, and all cepheids describe the loops in a clockwise manner, it should be possible to derive empirical corrections which, in effect, collapse the loop into a line and allow one to apply the PLC relation at any single point in the cycle. This has been attempted using the data for forty-five cepheids given by Bahner, Hiltner, and Kraft (1962).

The procedure was as follows. A color excess for each star was adopted, based on the excess given in Table 2 of Bahner *et al.*, revised according to the precepts given by Fernie (1963). This excess was applied to $B - V$ of the first observation of a star appearing in the lists of individual observations given in Table 1 of Bahner *et al.* These first observations occur at random phases among the forty-five stars, and are arbitrarily selected as the single observations to which the present method is applied. With $B - V$ corrected for the excess, and $\log P$ for each star, the PLC relation given above was entered and M_V calculated. This was subtracted from the listed V of the first observation to form the apparent distance modulus. Table 2 of Bahner *et al.* gives the properly averaged values of V and $B - V$ from the full light-curves. Applying the same color excesses as before, these were used with the PLC relation to determine the most accurate possible apparent distance moduli. The distance moduli obtained by the two methods are compared in Table 1. The corrections for the non-zero amplitudes in V of the loops were derived by plotting the residuals of Table 1 as a function of the phase of the first observation of a star. The corrections are also functions of the period and amplitude of variation, since the loops generally grow in size with increasing period. However, for periods less than about 6 days it was found that the corrections were generally smaller than the errors introduced by the errors of a single photometric observation. Accordingly, no correction for the size of the loop has been applied to stars with periods less than 6 days. For periods greater than 6 days the empirical corrections became significant. The average curve is given numerically in Table 2. The

TABLE 1

COMPARISON OF DISTANCE MODULI BY SHORT AND DETAILED METHODS

Star	$(m-M)_{DM}$*	$(m-M)_{SM}$†	Difference	Star	$(m-M)_{DM}$*	$(m-M)_{SM}$†	Difference	$(m-M)_{SM}$‡	Difference
SU Cas	9 08	9 12	−0 04	AB Cam	15 85	15 82	+0 03	. .	. .
AY Cas	14 81	14 85	− 04	FM Cas.	12 86	12 70	+ 16	. .	. . .
BY Cas	14 05	13 97	+ 08	VW Cas .	14 67	14 47	+ .20	14 53	+0 14
DW Per	15 08	15 15	− .07	VV Cas .	14 74	14 87	− 13	14 70	+ 04
DF Cas	14 32	14 18	+ 14	CR Cep	13 36	13 27	+ 09	13 31	+ 05
SY Cas	13 66	13 67	− 01	BP Cas	14 85	14 65	+ 20	14 77	+ 08
UZ Cas	15 02	14 83	+ .19	RS Cas	13 65	13 52	+ 13	13 69	− 04
Y Lac	12 75	12 82	− .07	RR Lac	12 90	12 76	+ 14	12 92	− 02
CG Cas	15 10	14 93	+ 17	AP Cas	15 77	15 63	+ 14	15 77	00
DF Lac	15 42	15 53	− 11	AK Cep	15 45	15 44	+ 01	15 47	− 02
XY Cas	13 39	13 35	+ 04	CD Cas	14 87	14 71	+ .16	14 75	+ 12
UX Per	15 26	15 32	− 06	RX Cam .	11 85	11 93	− 08	12 07	− 22
V Lac.	12 78	12 74	+ 04	IX Cam	16 20	16 16	+ 04	16 32	− 12
DW Cas	14 75	14 79	− 04	DD Cas	14 20	14 19	+ 01	14 08	+ 12
TV Cam	15 72	15 58	+ 14	Z Lac	12 71	12 92	− 21	12 92	− 21
BG Lac	12 63	12 54	+ 09	RY Cas	14 51	14 24	+ .27	14 40	+ 11
UY Per	15 06	15 08	− 02	SZ Cas	14 49	14 47	+ 02	14 58	− 09
δ Cep	7 81	7 97	− 16	CY Cas	16 25	16 18	+ 07	16 31	− 06
SW Cas	13 59	13 40	+ 19	RW Cas	13 80	13 59	+ 21	13 76	+ 04
X Lac	12 34	12 29	+ 05	CH Cas .	15 94	15 79	+ 15	15 85	+ 09
VY Per	15 03	14 98	+ 05	RW Cam	13 43	13 67	− 24	13 54	− 11
CZ Cas	15 49	15 44	+0 05	CP Cep	15 61	15 27	+0 34	15 39	+0 22

* Apparent distance modulus by detailed method (DM) (full light-curve)
† Apparent distance modulus by short method (SM) (single observation)
‡ Apparent distance modulus by short method (SM) with correction applied.

TABLE 2

CORRECTION-CURVE FOR PERIODS OVER 6 DAYS

Phase*	Correction Mag	Phase*	Correction Mag	Phase*	Correction Mag
0 0	0 00	0 4	+0 17	0 8 .	−0 16
1	+ 07	5	+ 16	0 9	− 10
.2	+ 12	6	+ 11	1 0	0 00
0 3 .	+0 16	0 7	−0 01		

* Measured from maximum visual light

corrections derived from it have been applied to the individual moduli determined by the short method as indicated in Table 1.

The average residual in distance modulus for all stars is 0.09 mag. This, of course, does not reflect the total uncertainty in the true distance modulus, which must include errors in the PLC relation itself and errors arising from absorption and reddening corrections. The total uncertainty may be as much as 0.2 or 0.3 mag., so the average 0.09-mag. error of the short method shows it to be quite satisfactory where the highest accuracy is not demanded, although its application to cepheids of periods over about 20 days may involve somewhat greater errors, since the loops then become much larger and more complex in shape.

The short method requires previous knowledge of the period and reddening of a star, and for periods over 6 days the ephemeris also. The period and ephemeris, however, are usually already known from earlier photographic observations, although the ephemeris often only poorly so. Nevertheless, as Table 1 shows, omission of the phase correction entirely often makes very little difference to the result. When no account at all of the phase is taken, the average residual in Table 1 rises from 0.09 mag. to only 0.11 mag. The reddening problem could be overcome in a systematic survey by including with the V and $B - V$ measures a measure of Γ (Kraft 1963), although possibly more than one observation would be required to obtain sufficient accuracy.

A major application of this short method might be to an extended survey of very faint cepheids for galactic structure studies. By its use the demands on observing time would be very much reduced.

REFERENCES

Bahner, K., Hiltner, W. A , and Kraft, R. P. 1962, *Ap. J. Suppl.*, **6**, 319.
Fernie, J. D 1963, *A J* , **68**, 780
Kraft, R. P. 1963, *Basic Astronomical Data*, ed. K Aa. Strand (Chicago: University of Chicago Press), chap. xxi.