1964ApJ...140..544D

THE AGE OF GALACTIC CLUSTER NGC 188

P. R. Demarque and R. B. Larson[*]
David Dunlap Observatory, University of Toronto

Received November 13, 1963; revised March 16, 1964

ABSTRACT

The early evolution of lower main-sequence stars of Population I (composition parameters $X = 0.67$, $Z = 0.03$) is calculated on an IBM 7090 computer using the entirely automatic method of Larson and Demarque (1964). The agreement between theory and Sandage's color-magnitude diagram of NGC 188 is good and yields an age of $9 - 10 \times 10^9$ years for that cluster. From another evolutionary track for $X = 0\;75$ and $Z = 0.01$, the age of NGC 188 is estimated at 12×10^9 years, but it is found in this case that agreement with observation is not as satisfactory as for composition ($X = 0\;67$, $Z = 0\;03$).

I. INTRODUCTION

NGC 188 is believed to be the oldest known galactic cluster (Sandage 1962*a*). A determination of the age of NGC 188 is clearly very important to our understanding of the age and evolution of the galaxy. Using the calculations of Hoyle (1959), Sandage (1962*b*) recently obtained an age of $14\text{–}16 \times 10^9$ years for NGC 188 under the assumptions that the hydrogen and metal relative abundances by mass of the cluster stars are, respectively, $X = 0.75$ and $Z = 0.01$. The purpose of this paper is to present the results of new calculations of evolutionary models made with the IBM 7090 computer at the University of Toronto, following the method described by Larson and Demarque (1964).

II. THE CHOICE OF CHEMICAL COMPOSITION

The chemical composition of the cluster stars must first be estimated. Spectroscopic evidence points to a metal abundance for NGC 188 very nearly the same as that of the Sun. We shall adopt $Z = 0.03$. For the hydrogen content, which cannot be determined by direct observation, our only guides are the hydrogen abundance of the Sun and the empirical mass-luminosity law for Population I stars. The last uncertain parameter, the ratio l/H of mixing length to pressure-scale height in the outer convection zone will also be obtained from the Sun and the position of the observed zero-age main sequence.

a) The Sun

The results of a study of the evolution of the Sun from its homogeneous main-sequence state to the present (Demarque and Percy 1964) indicate that, assuming an age of 4.5×10^9 years and $Z = 0.03$, one finds the present solar luminosity approximately for $X = 0.67$, and the present radius for $l/H = 1.6$.

b) The Empirical Mass-Luminosity Law

The empirical mass-luminosity law, as given by Sandage (1962*b*) may be compared with three main sequences for $Z = 0.03$, corresponding to $X = 0.57$, 0.67, and 0.77, respectively, which are listed in Table 1 and plotted in Figure 1. Clearly $X = 0.67$ agrees best with the Population I empirical mass-luminosity law. Comparing further three main sequences for $X = 0.67$, $Z = 0.03$, and $l/H = 1.0$, 1.6, and 2.0, respectively, with Sandage's (1962*b*) zero-age main sequence, we find that $l/H = 1.6$ fits best the observations (see Fig. 2 and Table 2).

---

[*] Now at the California Institute of Technology.

TABLE 1

MAIN SEQUENCES FOR $Z = 0.03$, $l = H$

$M/M_\odot$	$L/L_\odot$	$R/R_\odot$	$\log T_e$	M_{bol}	10 K
	$X = 0$ 77				
1 0	0 357	0 923	3 668	+5 88	0 107
1 1	0 584	0 994	3 705	+5 34	0611
1 2	0 915	1 071	3 737	+4 86	0286
1 3	1 376	1 154	3 766	+4 41	0102
1 4	1 992	1 229	3 792	+4 01	0 00208
	$X = 0$ 67				
0 9	0 407	0 882	3 692	+5 74	0 0710
1 0	0 698	0 960	3 732	+5 15	0318
1 1	1 130	1 045	3 766	+4 63	0104
1 2	1 735	1 124	3 796	+4 16	0 00171
1 3	2 511	1 196	3 823	+3 76	$<5\times10^{-4}$
1 4	3 501	1.230	3 853	+3 40	$<5\times10^{-4}$
	$X = 0$ 57				
0 8	0 459	0 837	3 716	+5 61	0 0420
0 9	0 830	0 923	3 759	+4 96	0131
1 0	1 387	1 009	3 796	+4 41	0 00202
1 1	2 138	1 093	3 825	+3 95	$<5\times10^{-4}$
1 2	3 143	1 133	3 859	+3 52	$<5\times10^{-4}$

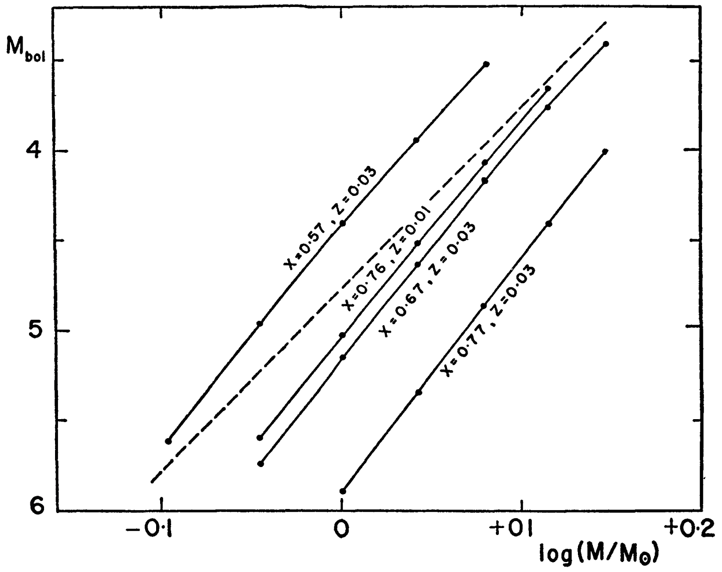

FIG. 1.—Mass-luminosity laws for the four compositions considered. The broken line shows the position of the empirical mass-luminosity law for Population I stars as given by Sandage (1962*b*).

It seems then that, for the present, it is reasonable to accept as typical for old Population I stars, the following parameters:

$$X = 0.67,\ Z = 0.03, \text{ and } l/H = 1.6\,.$$

The uncertainty in X still remains, nevertheless, for several reasons: (1) the ages of the stars used to determine the empirical mass-luminosity law are unknown, and it is

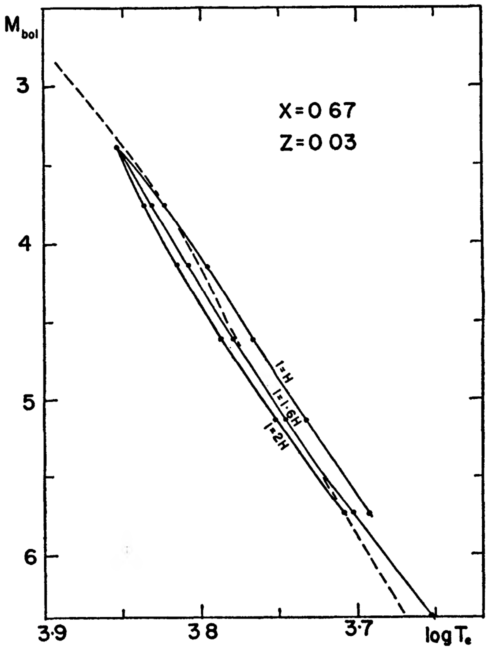

FIG. 2.—Main sequences for $X = 0.67$ and $Z = 0.03$ and three values of the mixing length

TABLE 2

MAIN SEQUENCES FOR $X = 0.67$, $Z = 0\ 03$

$M/M_\odot$	$L/L_\odot$	$R/R_\odot$	log T_e	$M_{\rm bol}$	10 K
			$l = 1\ 6H$		
0 8	0 223	0 781	3 653	+6 39	0 166
0 9	0 409	0 837	3 704	+5 73	110
1 0	0 702	0 898	3 747	+5 14	0675
1 1	1 134	1 022:	3 780:	+4 62	0344:
1 2	1 739	1 065	3 808	+4 16	0102
1 3	2 512	1 154	3 831	+3 76	0 00168
			$l = 2H$		
0 9	0 410	0 819	3 709	+5 73	0 129
1 0	0 703	0 877	3 752	+5 14	0830
1 1	1 135	0 948	3 787	+4 62	0452
1 2	1 740	1 035	3 815	+4 16	0171
1 3	2 512	1 131	3 835	+3 76	0 00408

then impossible to estimate accurately their original main-sequence luminosity; (2) we have no way of telling what the original mass-luminosity law of NGC 188 was; (3) we do not know exactly the position of NGC 188's main sequence. The work of Eggen and Sandage (1963) shows the possibility of a departure of a few tenths of a magnitude from the Hyades main sequence for other main-sequence stars. As pointed out by Iben (1963), it is very risky to try to estimate relative helium contents from such an assumption as the uniqueness of the main sequence on the theoretical H-R diagram.

For the sake of comparison, models with $X = 0.76$ and $Z = 0.01$ were also constructed.

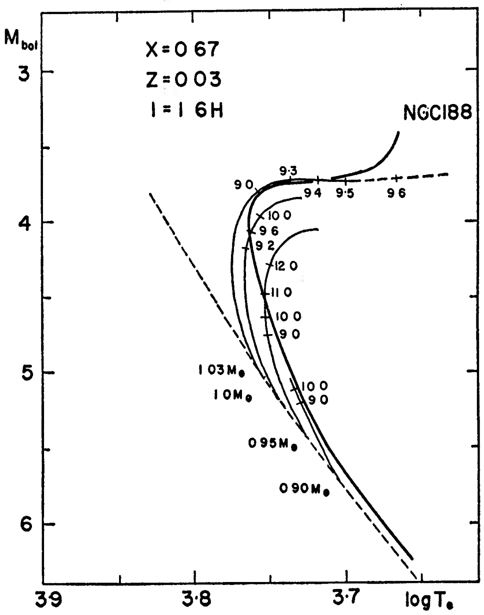

FIG. 3.—Evolutionary tracks for stars with $X = 0.67$, $Z = 0.03$, and $l = 1.6H$ together with Sandage's locus for the stars in NGC 188. Ages in 10^9 years are indicated along the tracks.

III. EVOLUTIONARY TRACKS

The evolutionary tracks were constructed with the computer program described by Larson and Demarque (1964). In that program, the opacities are those of Keller and Meyerott (1955) and the energy generation rates, which include the three branches of the proton-proton chain and the carbon-nitrogen cycle, are taken from Reeves (1964).

Stars with the following masses were evolved: 1.03, 1.00, 0.95, 0.90, and 0.80 $M_{\odot}$. The results are reported in Table 3. On such evolutionary paths one can draw isochrone curves, one of which should correspond to the color-magnitude array of NGC 188. It is, of course, an advantage to be able to evolve several stars of different masses, rather than having to rely on a single evolutionary track and appropriate scaling. As shown in Figure 3, the agreement between the theory and Sandage's (1962*a*) locus for the stars in NGC 188 is remarkably good, and the differences are well within the limits of observational uncertainties. The age of NGC 188 thus found is 9–10 × 10^9 years.

It seems that this satisfactory agreement with observation indicates that our choice of a "typical" composition was reasonable. This belief is strengthened by another evolutionary track for the composition $X = 0.76$, $Z = 0.01$, and $l/H = 1.0$ (Tables 4 and 5). As seen in Figures 1 and 4, the main sequence for this composition and mixing length also agrees well with the empirical mass-luminosity law and the observed main sequence. Nevertheless, the evolutionary track for a star of 0.95 $M_{\odot}$ which, as shown in Figure 5, yields an age around 12 × 10^9 years for NGC 188, does not fit the observations as well.

1964ApJ...140..544D

TABLE 3

EVOLUTIONARY TRACKS FOR $X = 0.67$, $Z = 0.03$, $l = 1.6H$

$t(10^9$ years)	$L/L_{\odot}$	$R/R_{\odot}$	log T_e	$M_{\rm bol}$
		$M = 0\ 8M_{\odot}$		
0 .	0 223	0 781	3 653	+6 39
2 5	239	791	3 658	+6 32
5 0	258	799	3 664	+6 23
7 5.	280	811	3 669	+6 14
10 0	307	825	3 676	+6 04
12 5	339	843	3 682	+5 93
15 0	0 379	0 868	3 688	+5 81
		$M = 0.9M_{\odot}$		
0 .	0 409	0 836	3 704	+5 73
3 0	.469	859	3 713	+5 58
6 0 .	549	894	3 721	+5 41
7 5	602	.915	3 726	+5 31
9 0	668	944	3 731	+5 20
10 5 .	0 752	0 979	3 736	+5 07
		$M = 0.95M_{\odot}$		
0..	0 540	0 866	3 726	+5 43
2 0..	0 602	0 888	3 733	+5 31
4 0	0 680	0 915	3 739	+5 18
6 0 .	0 786	0 955	3 746	+5 02
8.0..	0 926	1 014	3 751	+4 84
9 10..	1 017	1 056	3 752	+4 74
9 81 .	1 096	1 094	3 753	+4 66
10 53	1 201	1 141	3 753	+4 56
11 24	1 341	1 205	3.753	+4 44
11 70	1 456	1 264	3 752	+4 35
12 24	1 619	1 353	3 749	+4 24
12 54	1 732	1 441	3 743	+4 16
12.83	1 851	1 560	3 733	+4 09
13.00	1.903	1 669	3 721	+4 05
		$M = 1M_{\odot}$		
0 .	0 702	0 900	3 747	+5 14
2 0	0 801	0.933	3 753	+5 00
4 0	0 935	0 974	3 760	+4 83
5 2	1 038	1 014	3 763	+4 72
6 4.	1 151	1 057	3 765	+4 61
7 09	1 237	1 095	3 765	+4 53
7 87	1 369	1 147	3 766	+4 42
8 39	1.479	1 191	3.767	+4 34
8 85	1 601	1 242	3 766	+4 25
9 32	1 756	1 309	3 765	+4 15
9 66	1 896	1 374	3.763	+4 07
9 94	2 028	1 452	3 758	+3 99
10 15	2 140	1.526	3 753	+3 93
10 35	2 249	1 633	3 744	+3 88
10 48	2.318	1 735	3.734	+3 85

TABLE 3—*Continued*

$t(10^9$ years)	$L/L_{\odot}$	$R/R_{\odot}$	$\log T_e$	M_{bol}
	$M = 1\ 03M_{\odot}$			
0 .	0 815	0 921	3 758	+4 98
2 0	0 945	0 963	3 764	+4.82
4 0	1 126	1 018	3 771	+4 63
5 3	1 265	1 072	3 773	+4 51
6 21	1 398	1 124	3 773	+4 39
6 92	1 548	1 178	3 774	+4 28
7 51	1 709	1 237	3 774	+4 18
7 97	1 871	1 302	3 773	+4 08
8 28	1 998	1 356	3 771	+4 01
8 58	2 152	1 429	3 768	+3 93
8 83	2 299	1 508	3 763	+3 86
9 02	2 431	1 595	3 757	+3 79
9 15	2 512	1 673	3 750	+3 76
9 28	2 589	1 792	3 739	+3 73
9 32	2 601	1 847	3 733	+3 72
9 37	2 577	1 917	3 724	+3 73
9 43	2 523:	2 022:	3 710:	+3 76:
9 48	2 527:	2 082:	3 703:	+3 75:
9 556	2 684:	2 222:	3 696:	+3 69:
9 571	2 642:	2 326:	3 684:	+3 71:
9 586	2 618:	2 452:	3 672:	+3 72:
9 613	2 636:	2 578:	3 661:	+3 72:
9 632	2 632:	2 698:	3 652:	+3 71:
9 651	2 652:	2 807:	3 644:	+3 70:

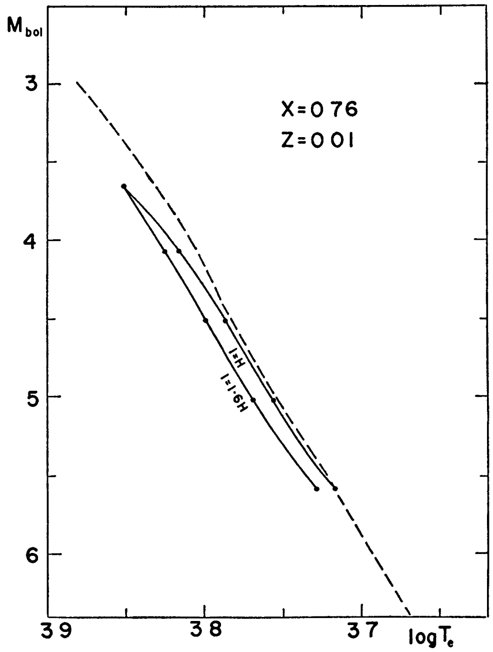

FIG. 4.—Main sequences for $X = 0.76$ and $Z = 0\ 01$ and two values of the mixing length

1964ApJ...140..544D

TABLE 4

MAIN SEQUENCES FOR $X = 0.76$, $Z = 0.01$

$M/M_{\odot}$	$L/L_{\odot}$	$R/R_{\odot}$	log T_e	M_{bol}	10 K
			$l = H$		
0 9	0 466	0 835	3 718	+5 59	0 0874
1 0 .	0 788	0 910	3 757	+5 02	.0364
1 1 .	1 258	1 001	3 787	+4 51	0 00738
1 2 .	1 913	1 079	3 816	+4 06	$<5\times10^{-4}$
1 3	2 785	1 107	3 851	+3 65	$<5\times10^{-4}$
			$l = 1\ 6H$		
0 9 . .	0 467	0 796	3 729	+5 59	0 135
1 0	0 789	0 857	3 770	+5 02	0758
1 1	1 259	0 946	3 799	+4 51	0247
1 2	1 913	1 034	3 825	+4 06	0 00413
1 3	2 780:	1 092:	3 856:	+3 65:	$<5\times10^{-4}$

TABLE 5

EVOLUTIONARY TRACK FOR $X = 0.76$, $Z = 0\ 01$, $l = H$, and $M = 0\ 96M_{\odot}$

t(10^9 years)	$L/L_{\odot}$	$R/R_{\odot}$	log T_e	M_{bol}
0 .	0 636	0 878	3 741	+5 25
1 3	0 685	0 894	3 745	+5 17
2 6 .	0 740	0 914	3 749	+5 09
3 9	0 807	0 940	3 752	+4 99
5 2	0 886	0 969	3 756	+4 89
6 5 .	0 987	1 007	3 759	+4 77
7 8	1 121	1 057	3 762	+4 64
9 1.	1 292	1 123	3 765	+4 48
10 01	1 448	1 187	3 765	+4 36
10 70	1 615	1 256	3 765	+4 24
11 17	1 761	1 318	3 764	+4 15
11 55	1 909	1 384	3 762	+4 06
11 85	2 054	1 458	3 758	+3 98
12 10	2 193	1 535	3 754	+3 91
12 30	2 323	1 615	3 750	+3 84
12 52 .	2 491	1 754	3 739	+3 77
12 67	2 598	1 887	3 728	+3 72
12 75.	2 662	2 001	3 718	+3 70
12 805	2 658	2 108	3 706	+3 70
12 859	2 633	2 218	3 696	+3 71
12 940	2 699	2 320	3 687	+3 68
12 978 .	2 800	2 408	3 678	+3 65
13 011	2 757	2 581	3 666	+3 66
13 041 .	2 735	2 764	3 650	+3 67

Furthermore, it is unlikely that the heavy-element content is as low as $Z = 0.01$ if the solar value is $Z = 0.03$ or 0.04.

We may then say with fair confidence that the age of NGC 188 lies between 9 and 12×10^9 years, and likely closer to the range $9\text{–}10 \times 10^9$ years. This age may be regarded as the age of the galaxy in its present form.

The diagram of Figure 3 shows that the most advanced stages of the evolutionary track for $M = 1.03\ M_\odot$ do not resemble the observations and continue toward the cool end of the effective-temperature scale instead of turning upward to the giant region. This turning point was found and discussed by Hoyle (1959) in his calculations. There are two reasons for the breakdown of our method: (1) numerical accuracy decreases substantially, particularly near the center of the models and in the energy producing shell. Clearly more points should be taken for the integration at this stage; (2) also, the neglect of conductive opacity becomes important in this region. A preliminary calculation indicates that models would indeed move up along the giant branch at this particular point in their evolution.

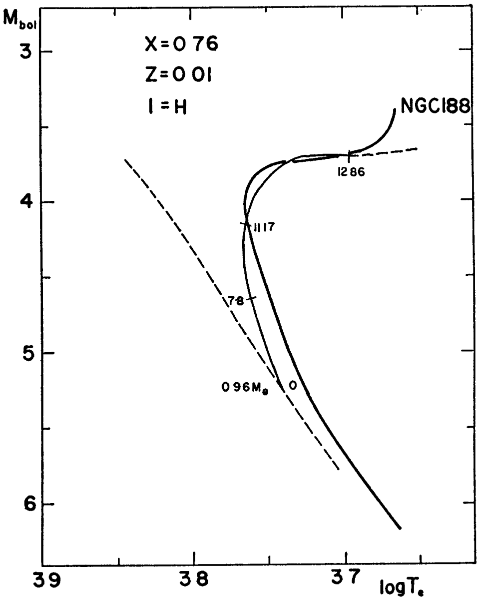

FIG. 5.—Evolutionary track for a star with $X = 0.76$, $Z = 0.01$, and $l = H$ together with Sandage's locus for the stars in NGC 188. Ages in 10^9 years are indicated along the track.

It is a pleasure to thank the Institute of Computer Science of the University of Toronto for a generous time allotment on the IBM 7090. We are also indebted to Mr. J. R. Percy for the evolutionary runs of 0.8 and 0.9 $M_\odot$. One of us (R. B. L.) was supported by a postgraduate bursary from the National Research Council of Canada.

REFERENCES

Demarque, P. R., and Percy, J. R. 1964, *Ap. J.*, **140**, 541.
Eggen, O. J., and Sandage, A. R. 1963, *Ap. J.*, **136**, 735.
Hoyle, F. 1959, *M.N.*, **119**, 124.
Iben, I., Jr. 1963, *Ap. J.*, **138**, 452.
Keller, G., and Meyerott, R. E. 1955, *Ap. J.*, **122**, 32.
Larson, R. B., and Demarque, P. R. 1964, *Ap. J*, **140**, 524.
Reeves, H. 1964, "Stellar Energy Sources," in *Stellar Structure*, Vol. VIII of *Stars and Stellar Systems*, ed L. H. Aller and D McLaughlin (Chicago: University of Chicago Press).
Sandage, A R. 1962*a*, *Ap. J.*, **135**, 333.
———. 1962*b*, *ibid.*, p. 349.