ON RELATIVISTIC ASTROPHYSICS

F. Hoyle
St. John's College, Cambridge, England, and
California Institute of Technology, Pasadena, California

William A. Fowler
California Institute of Technology, Pasadena, California

G. R. Burbidge and E. Margaret Burbidge
University of California, San Diego, La Jolla, California
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ABSTRACT

In this paper we have attempted to discuss the relation of massive highly condensed objects to astrophysics in general, rather than only to the radio-source problem. Because situations in which general relativistic effects play a dominant role have not received much attention in astrophysics, a brief review of the relativistic properties of collapsed objects is given in Section I of the paper.

In Section II we have given especial attention to two problems: (1) Galaxies may contain considerable quantities of inert or "hidden" mass, not simply in the form of white dwarfs or neutron stars. (2) Highly collapsed objects may act as energy sources, not just explosively, but over extended time intervals.

The first of these problems has implications both for nucleosynthesis and for the mass-to-light properties of stellar systems. The second appears applicable to a wide range of phenomena, ranging from supernovae in our Galaxy to whole radio galaxies. A continuous source of optical synchrotron electrons appears necessary both in the Crab Nebula and in the jet of M87. We suggest their origin lies in processes that are physically similar and in which considerations of general relativity play a dominant role.

I. THE FATE OF MASSIVE BODIES

a) *Introduction*

Gravitation tends to pull aggregates of matter together. A logical question evidently arises as to the ultimate outcome of this process. In astronomy it has usually been supposed that astrophysical evolution proceeds in such a way that, given sufficient time, a cold body in hydrostatic equilibrium remains a body that can maintain its equilibrium over all further time. Yet it is known that no such equilibrium state can be found for a mass appreciably greater than $M_\odot$. The Chandrasekhar limit gives 5.76 $M_\odot/\mu_e^2$ for the maximum mass that can be supported by degenerate electron pressure. A similar result applicable at higher densities for neutron degeneracy was obtained by Oppenheimer and Volkoff (1939)—in fact, the maximum mass was about 0.7 $M_\odot$. Even the doubtful assumption of a hard-core neutron potential, allowing the nuclear fluid at high densities to behave like an incompressible liquid, only extends the maximum mass to about 3 $M_\odot$, a result obtained immediately from the Schwarzschild interior solution, using nuclear densities. Evidently, however, it is possible for gravitation to pull together much larger masses than these. The conventional view is that in all such cases a process of mass ejection takes place, conveniently reducing the final mass below one or the other of the limits just mentioned. We wish to emphasize that this view is no more than a superstition. The existence of white dwarfs has sometimes been taken as supporting evidence, but the white dwarfs can be explained in terms of the evolution of stars whose masses have never appreciably exceeded the limits mentioned above. For massive stars (and very massive objects) there is no evidence, either from theoretical studies of stellar evolution or from observation, that mass loss plays any such role. Indeed, it would be a curious situation if astrophysical processes occurring long before the onset of the final implosion crisis were to operate always to prevent the crisis from arising. This would imply an unlikely "foreknowledge" on the part of natural processes.

In this paper we propose to accept the situation that stars with masses greater than the critical mass can reach a stage of catastrophic implosion in which general relativity becomes dominant. In the following section we give a brief review of what might be called the classical implosion problem (Datt 1938; Oppenheimer and Snyder 1939). In later sections we shall consider modifications demanded by (i) non-zero internal temperature, (ii) rotation, (iii) the non-classical discussion of Hoyle and Narlikar (1963). This will conclude Section I of the paper.

b) The Classical Implosion Problem

The object is assumed uniform and spherically symmetric and the internal pressure is zero. Outside is empty space, and the line element is Galilean at infinity. The object is taken to implode from rest without rotation.

The interior solution, obtained from Einstein's equation, is found to be identical to the simple elliptic cosmology with line element

$$ds^2 = dt^2 - S^2(t)\left(\frac{dr^2}{1-r^2} + r^2 d\Omega^2\right), \tag{1}$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2\,, \qquad c = 1\,. \tag{2}$$

The coordinates r, θ, φ are intrinsic—that is to say, they always have the same values for any particular particle. It is possible to choose such coordinates so that $r = 0$ for any assigned particle. These conditions arise out of the special conditions of homogeneity and isotropy treated in cosmology. The same conditions arise in the case of a finite object from uniformity and spherical symmetry and from considering implosion from rest without rotation.

Einstein's equations give

$$\frac{\dot{S}^2}{S^2} = -\frac{1}{S^2} + \frac{8\pi G\rho}{3}, \tag{3}$$

where the proper density ρ satisfies the condition $\rho S^3 =$ constant. The right-hand side of equation (3) always has a zero. In the cosmological case the zero gives the value of the scale factor S at maximum expansion of the universe, while in the case of a finite object the zero refers to the initial state of rest. If quantities at the zero are referred to by the subscript zero then

$$S_0 = \left(\frac{8\pi G\rho_0}{3}\right)^{-1/2}. \tag{4}$$

It is evidently possible to redefine the r coordinates so that $S = 1$ at maximum phase. The line element then takes the form

$$ds^2 = dt^2 - S^2(t)\left(\frac{dr^2}{1-\alpha r^2} + r^2 d\Omega^2\right), \tag{5}$$

where

$$\alpha = \frac{8\pi G\rho_0}{3}. \tag{6}$$

Equation (3) becomes

$$\frac{\dot{S}^2}{S^2} = \alpha\,\frac{(1-S)}{S^3}. \tag{7}$$

It is usual to work with equations (1) and (3) in cosmology, but equations (5) and (7) are more convenient in the implosion problem. It should be noted that, whereas in cosmology, $r = 0$ can be any assigned particle, in the implosion problem $r = 0$ refers uniquely to the center of the body.

A further difference is that an exterior solution must also be found for the implosion problem. At first sight it might be thought most convenient to use Schwarzschild coordinates for the exterior,

$$ds^2 = e^\nu dt^2 - e^\lambda dR^2 - R^2 d\Omega^2 . \tag{8}$$

(The radial coordinate is characterized by the property that the sphere through R, T has area $4\pi R^2$, independent of T.) The advantage of these coordinates is that for a non-radiating object the T coordinate can be chosen such that

$$e^\nu = e^\lambda = 1 - \frac{\text{constant}}{R} \tag{9}$$

and the line element is static. An awkward problem arises, however, in matching the interior and exterior solutions at the boundary of the object. The boundary has a constant r coordinate, r_b say, but the R coordinate is not constant. For this reason it is convenient to complete the exterior solution in terms of the r, t coordinates. With the solution obtained a transformation to R, T can be found.

The exterior solution has the remarkable property that Einstein's equations, instead of being partial in r, t, become ordinary equations with $r^{-3/2}\, t$ as variable. The line element is

$$d s^2 = dt^2 - S^2 \left(\frac{t r_b^{3/2}}{r^{3/2}}\right) \left(\frac{K^2 d r^2}{1 - a r_b^3 / r} + r^2 d\Omega^2\right), \tag{10}$$

where $K = (1/S)\partial/\partial r(rS)$, and S satisfies equation (7) but with differentiation taken with respect to $tr_b^{3/2}/r^{3/2}$. The exterior solution is, therefore, closely similar to the interior solution.

Turning now to the transformation to Schwarzschild coordinates, our aim is to determine the constant in equation (9). This is achieved most easily by noticing that Schwarzschild coordinates can also be used for the interior—subject to a proviso to be mentioned later—and that e^ν, e^λ are continuous at the boundary. In fact

$$R = r S(t) \qquad \text{and} \qquad T = \Phi \left(\int \frac{r\, d r}{1 - a r^2} + \int \frac{dt}{S \dot{S}}\right) \tag{11}$$

transform the line element from equation (5) to equation (8) with

$$e^\lambda = \frac{1}{1 - a r^2 - r^2 \dot{S}^2}, \qquad e^\nu = \frac{S^2 \dot{S}^2 (1 - a r^2)}{(1 - a r^2 - r^2 \dot{S}^2)\Phi_1^2} . \tag{12}$$

Here Φ is any differentiable function and Φ_1 is the derivative with respect to the argument. For the purpose of determining the constant in equation (9) nothing more need be known about Φ.

Inserting for $\dot{S}^2$ from equation (7) we have

$$e^{-\lambda} = 1 - \frac{a r^2}{S} = 1 - \frac{a r^3}{R} . \tag{13}$$

Applying equation (13) at the boundary $r = r_b$ we see that the constant in equation (9) is $a r_b^3$. Remembering the definition of a we can write e^λ in the conventional form

$$e^{-\lambda} = 1 - \frac{2GM}{R} \tag{14}$$

by defining

$$M = \frac{4\pi}{3} \rho_0 r_b^3 . \tag{15}$$

For an observer in the exterior, M is the gravitational mass of the object. It remains constant throughout the implosion for a non-radiating object.

An event horizon develops at $R = 2GM$. Once the object has contracted inside the sphere with this R coordinate no further signals can cross the sphere. Writing $R = r_b S(t)$ for the boundary, we see that the boundary coincides with the event horizon when

$$S = \frac{2GM}{r_b} = \alpha r_b^2 , \qquad \alpha = \frac{8\pi G \rho_0}{3} . \tag{16}$$

The proper time required for the object to fall inside the event horizon is

$$\int_{S=1}^{S=\alpha r_b^2} dt = \int_1^{\alpha r_b^2} \frac{dS}{\dot{S}} = \frac{1}{\alpha^{1/2}} \left[\frac{\pi}{2} - \sin^{-1} \alpha^{1/2} r_b + \alpha^{1/2} r_b \sqrt{(1 - \alpha r_b^2)} \right] , \tag{17}$$

and implosion into a singularity occurs in a proper time

$$\int_0^1 \frac{dS}{\dot{S}} = \frac{\pi}{2 \alpha^{1/2}} . \tag{18}$$

The square root of equation (7), choosing the negative sign, is used in the evaluation of these integrals.

Initially, we have $R = r_b$. At the event horizon $R = 2GM = \alpha r_b^3$. Hence, if implosion occurs from a dispersed state, αr_b^2 must be $\ll 1$, since by a dispersed state we mean that the initial area of the boundary is large compared with the area of the event horizon.

The proper time required for implosion from the event horizon into the singularity is

$$-r_b (1 - \alpha r_b^2)^{1/2} + \alpha^{-1/2} \sin^{-1} \alpha^{1/2} r_b \simeq \tfrac{2}{3} \alpha r_b^3 , \; \alpha r_b^2 \ll 1 , \tag{19}$$

i.e., for implosion from a dispersed state. In terms of M, $2\alpha r_b^3/3 = 4GM/3$. So far units with $c = 1$ have been used. In conventional units the proper time for implosion from the event horizon into the singularity is

$$\frac{4GM}{3c^3} \simeq 6.6 \times 10^{-6} \frac{M}{M_\odot} \sec \backsim 660 \sec \text{ for } M = 10^8 M_\odot . \tag{20}$$

It was mentioned above that the transformation (11) is subject to a proviso, namely, that e^λ, e^ν must not become negative. In fact the transformation is singular for $S = \alpha r^2$, and e^λ, e^ν are negative for $S < \alpha r^2$. Since $S \to 0$ it is clear that the transformation to Schwarzschild coordinates fails sooner or later at all points of the interior. The transformation also fails at all exterior points within the event horizon.

The transformation can, of course, be made even when $S < \alpha r^2$, but R, T are not the usual Schwarzschild coordinates. Because of the negative values of e^ν, e^λ the space- and timelike roles of R, T are inverted—R becomes the time coordinate and T a space coordinate. These transformation difficulties explain why Schwarzschild coordinates are unsuitable for the solution of the whole implosion problem.

c) The Effects of Internal Pressure

Even a uniform pressure is capable of modifying equation (7) for S. The equivalent equation (3) can be written as

$$\frac{\dot{S}^2}{S^2} = -\frac{1}{S^2} + \frac{8\pi G E}{3} , \tag{21}$$

since the energy density E is just ρ in the absence of internal pressure. When there is a non-zero pressure, E is not equal to ρ, however. Equation (21) holds for uniform pressure, but not when there is a pressure gradient.

Now at very high densities, $>\sim 10^{16}$ gm cm^{-3}, quantum statistics prevent $E = \rho$. Thus even for zero temperature we have $E \propto \rho^{4/3}$ at very high densities. For $\rho \propto S^{-3}$, $\dot{S}^2$ is determined by a term of order S^{-2} for small S, not by a term of order S^{-1}. This evidently only accentuates the onset of the singularity.

Pressure is also important in a discrete object because a pressure gradient is always present, on account of the necessity for the pressure to be zero at the boundary. The effect of a pressure gradient is to change the mathematical structure of Einstein's equations from ordinary equations to partial equations in the coordinates r, t. The problem is then harder to handle and less is known about its solution, except in the special case of equilibrium when the t coordinate drops out.

One might be tempted to speculate that pressure gradients will ultimately halt an imploding object, since this is the case in the Newtonian approximation whenever the ratio of specific heats is greater than 4/3. But the same result is not true if relativity effects are sufficiently important. While a formal solution, applicable in detail to the most general case, has not yet been given, a disproof follows from a simple *reductio ad absurdum* argument. For example, consider a cold object with degeneracy pressure—that is, $P \simeq \frac{1}{3}E \propto \rho^{4/3}$. If such a pressure were able to halt an implosion the resulting motion would be oscillatory between radii R_1, R_2, say—where R is once again the Schwarzschild coordinate. An equilibrium solution for the object must then exist with a radius between R_1, R_2. But this is not possible if the total mass exceeds the limiting values mentioned in the Introduction. Hence implosion cannot be halted for sufficiently large mass, $>\sim M_\odot$.

A similar argument can be used for a non-zero temperature. Although an equilibrium solution with a radius between R_1 and R_2 may be possible, the thermal energy necessary to maintain the equilibrium state becomes impossibly large as M is increased, a result recently demonstrated by Iben (1963). The mass in question depends on the value of R at which the equilibrium solution is sought, failure occurring when $2GM/R$ becomes of order unity.

A hot object evolves through the escape of energy. This leads to R being ultimately reduced to the stage where $2\,GM/R$ is of order unity. Hence a pressure gradient must finally fail to prevent implosion; the only exception to this statement in the spherically symmetric case being when the mass falls below one or the other of the limiting values, $\sim M_\odot$.

d) Neutrino Losses

Because of the development of the event horizon one might take the view that singularities are of no consequence to an external observer, since the observer loses contact with an imploding object after it has fallen inside the event horizon. But while this is true for electromagnetic communication the external observer still has contact with the object in the sense that he can detect a gravitational field—unless there is some way in which the object can radiate its gravitational mass. This question, with particular reference to neutrinos as the radiating agent, has recently been discussed by Michel (1963). In the present section we shall re-examine this question.

To understand the issue more clearly it is convenient to use the Schwarzschild form (8) for the line element outside the object, and to consider a situation in which the energy-momentum tensor is small at all points between the object and the observer. Over a small time interval the line element can be considered static, with e^ν and e^λ given by equation (9). However, over a long time the constant in equation (9) may change secularly. Thus the 41-component of Einstein's equation is

$$\frac{d\lambda}{dt}\frac{e^{-\lambda}}{R} = -\,8\pi G T_4{}^1 . \tag{22}$$

If we define M by

$$e^{-\lambda} = 1 - \frac{2GM}{R}, \tag{23}$$

equation (22) gives

$$\frac{dM}{dt} = -4\pi R^2 T_4^{\ 1}. \tag{24}$$

The right-hand side of equation (24) is just minus the energy flow through the sphere of coordinate R—for example, in the electromagnetic case $T_4^{\ 1}$ is the radial component of the Poynting vector.

We continue by estimating the energy flow in the form of neutrinos, on the basis that an object of initial mass M_0 implodes from a comparatively dispersed state in which the density ρ and the temperature $T_9 = T/10^9\,^\circ$ K are related by

$$\begin{aligned}\rho &= 2.0 \left(\frac{a}{G}\right)^{3/4} \frac{T^3}{M_0^{1/2}} \\ &= 2.8 \times 10^5 \left(\frac{M_\odot}{M_0}\right)^{1/2} T_9^{\ 3} \text{ gm cm}^{-3}.\end{aligned} \tag{25}$$

This equation was obtained by Hoyle and Fowler (1963*a*, *b*) for polytropes of index 3. Iben (1963) has investigated the *matter* density-temperature relation for the equilibrium state using the adiabatic approximation and has shown that equation (25) holds quite accurately up to $M \simeq 10^5 M_\odot$ and is only a factor of 2 too high at $M \simeq 10^9 M_\odot$. Evidently in an imploding situation the temperature corresponding to a given density is less than it is in an equilibrium state. Hence the adoption of equation (25) must tend to exaggerate the neutrino emission, as it also does for masses above $10^5 M_\odot$. Since we shall show that neutrino emission does not reduce M significantly, our conclusion applies *a fortiori*.

A sample of material moves in the ρ, T_9-plane along a track determined by the usual thermodynamic relation,

$$\frac{P}{\rho^2} d\rho - dU = \frac{dU_\nu}{dt} \cdot dt, \tag{26}$$

in which P is the pressure, U the internal energy per mass, and dU_ν/dt the neutrino loss per unit mass per unit time. Equation (26) holds for an observer who is co-moving with the material and who uses locally flat coordinates. According to Fowler and Hoyle (1964) the left-hand side of equation (26) takes the simple form

$$\frac{P}{\rho^2} d\rho - dU = \frac{11}{3} \frac{aT^4}{\rho} \left(\frac{d\rho}{\rho} - 3\frac{dT}{T}\right) \tag{27}$$

at T_9 appreciably in excess of unity—the case with which we shall be concerned. The expression (27) includes contributions to pressure and internal energy both from radiation and from electron pairs. The pairs contribute 7/4 of the contributions of the radiation in both cases. We also have from Fowler and Hoyle (1964) for $T_9 > 2$

$$\frac{dU_\nu}{dt} = \frac{4.3 \times 10^{15}}{\rho} T_9^{\ 9} \text{ erg gm}^{-1} \text{ sec}^{-1} (T_9 > 2) \tag{28}$$

for ρ in gm cm^{-3} provided all neutrinos and antineutrinos generated by $e^- + e^+ \to \nu + \bar{\nu}$ escape, and provided the neutrinos are not seriously redshifted by the gravitational field. Below $T_9 \sim 2$ neutrino losses are much smaller than given by equation (28).

We shall find that all significant neutrino loss occurs before nuclear densities are reached. Implosion can be considered to take place in accordance with

$$\frac{\dot{S}^2}{S^2} \simeq \frac{8\pi G \rho_0}{3S^3}, \tag{29}$$

where we neglect the $-S^{-2}$ term in equation (7) since we are concerned with $S \ll 1$, and where we also neglect the slowing of the implosion due to the pressure gradient, and the slight acceleration due to the contribution of pairs and of radiation to E. This last contribution is $11aT^4/4$ and becomes equal to ρc^2 when $\rho = 23T_9{}^4$ in our units. For stars with $M \leq 10^9 M_\odot$ which follow equation (25) the condition $\rho = 23T_9{}^4$ is never reached before the event horizon.

From $\rho S^3 =$ constant, we have

$$3\frac{dS}{S} + \frac{d\rho}{\rho} = 0. \tag{30}$$

Combining equations (29) and (30), and remembering that the negative sign must be taken in the square root of equation (29),

$$dt = -\left(\frac{3}{8\pi G\rho}\right)^{1/2} \frac{dS}{S} = \frac{1}{(24\pi G\rho)^{1/2}} \frac{d\rho}{\rho}. \tag{31}$$

Inserting in equation (26) leads immediately to

$$\frac{d\rho}{\rho}\left[1 - \frac{1}{(24\pi G\rho)^{1/2}} \frac{dU_\nu/dt}{11aT^4/3\rho}\right] = 3\frac{dT}{T}, \tag{32}$$

and using equation (28) this can be expressed numerically in the form

$$\frac{d\rho}{\rho}\left(1 - 7\cdot 10^{-5}\frac{T_9{}^5}{\rho^{1/2}}\right) = 3\frac{dT_9}{T_9}. \tag{32'}$$

This can be integrated straightforwardly and the constant of integration determined with the aid of equation (25) to yield

$$\rho \simeq 2.8\times 10^5 \left(\frac{M_\odot}{M_0}\right)^{1/2} T_9{}^3 \left(1 - 10^{-4}\frac{T_9{}^5}{\rho^{1/2}}\right)^{-3/5}. \tag{33}$$

This result shows that the material continues to follow a track $\rho \propto T_9{}^3$ so long as the second term in the second parentheses is small compared with unity. However, with increasing temperature the second term increases in importance. In the limit for high temperature the term in the second parentheses yields

$$\rho \simeq 10^{-8}\, T_9{}^{10}. \tag{34}$$

The track (33) of the material in the ρ, T_9-plane is a curve with equations (25) and (34) as asymptotes. The asymptotes intersect at the point

$$\rho \simeq 1.6\times 10^{11}\left(\frac{M_\odot}{M_0}\right)^{5/7} \text{gm cm}^{-3}, \qquad T_9 = 83\left(\frac{M_\odot}{M_0}\right)^{1/14}. \tag{35}$$

We note at this point that pressure gradients can be effectively treated as decreasing the value of G used in equation (32). However, we see that a pressure gradient which reduces G to 10 per cent of its full value will only increase the time of fall, $(24\pi G\rho)^{-1/2}$, by a factor of $10^{1/2}$. Thus the calculations made below for neutrino losses using the free-fall time cannot be seriously low in value for this reason, and in fact other approximations more than compensate for the free-fall assumption.

Subject to the assumption that all neutrinos escape without redshift effects arising, we are now in a position to estimate the total neutrino loss along the path (33). This proviso means that the integration must not be carried beyond the event horizon where the neutrinos are redshifted to zero energy. According to the work of Section II the event horizon has Schwarzschild coordinate $R = \alpha r_b^3$, while $M_0 = 4\pi\rho_0 r_b^3/3$, $S = \alpha r_b^2$. The proper density at the event horizon is obtained from $\rho = \rho_0 S^{-3}$, which gives $\rho = \rho_0\alpha^{-3}r_b^{-6}$ at the event horizon. Using the definition of α, $\alpha = 8\pi G\rho_0/3$,

$$\rho_{\max} = \left(\frac{3}{8\pi G}\right)^3 \frac{1}{\rho_0^2 r_b^6} = \frac{3}{32\pi G^3}\cdot\frac{1}{M_0^2}\,. \tag{36}$$

It will be recalled that units with $c = 1$ were used in the former work. To express ρ in conventional units, a factor c^6 must be introduced on the right-hand side of equation (36). Numerically, we obtain

$$\rho_{\max} = 1.85\times10^{16}\left(\frac{M_\odot}{M_0}\right)^2 \text{gm cm}^{-3}\,. \tag{37}$$

Equations (25) and (37) yield a maximum temperature

$$(T_9)_{\max} = 4.1\times10^3\,(M_\odot/M_0)^{1/2}\,. \tag{37'}$$

Thus stars with $M > 10^7 M_\odot$ do not reach $T_9 \sim 2$ and neutrino losses are much smaller than calculated in what follows.

The expression for the energy loss per gram of material along the path (33) from $\rho = 0$ to $\rho_{\max}$ or from $t = 0$ to $t(\rho_{\max})$ is

$$\begin{aligned} E_\nu &= \int_0^\infty \frac{dU_\nu}{dt'}\,dt' = \int_0^{t(\rho_{\max})} \frac{dU_\nu}{dt}\,dt \\ &= \int_0^{\rho_{\max}} \frac{dU_\nu/dt}{(24\pi G\rho)^{1/2}}\,\frac{d\rho}{\rho} \\ &= 1.92\times10^{18}\int_0^{\rho_{\max}} \frac{T_9^9}{\rho^{5/2}}\,d\rho \text{ erg gm}^{-1}, \end{aligned} \tag{38}$$

where dt' is the time interval and dU_ν/dt' is the energy loss measured by an external observer. It will be clear that the redshift decrease in the energy loss is just canceled by the time dilation for such an observer. It is simpler to use the calculations of the local co-moving observer and to cut off the loss which he calculates at the time he measures, $t(\rho_{\max})$, corresponding to infinite time for the external observer. Eliminating the temperature by means of equation (33) yields

$$E_\nu = 5.0\times10^{18}\left(\frac{M_0}{M_\odot}\right)^{3/7}\int_0^{Z_{\max}} \frac{Z^{2/7}dZ}{(1+Z)^{9/5}}, \tag{39}$$

where

$$Z = 8.5\times10^{14}\left(\frac{M_0}{M_\odot}\right)^{5/6}\rho^{7/6}$$

and

$$Z_{\max} = 8.1\times10^5\left(\frac{M_\odot}{M_0}\right)^{3/2}.$$

The integral in equation (39) is $\Gamma(9/7)\,\Gamma(18/35)/\Gamma(9/5) \sim 1.67$ for $Z_{\max} = \infty$. For our purposes a sufficiently accurate integration of (39) for $Z_{\max} > 1$ or $M_0 < 10^4\, M_\odot$ is

$$E_\nu = 8.3 \times 10^{18} \left(\frac{M_0}{M_\odot}\right)^{3/7} \left[1 - 10^{-3} \left(\frac{M_0}{M_\odot}\right)\right]^{27/35} \text{erg gm}^{-1}, \qquad \frac{M_0}{M_\odot} < 10^4. \quad (40)$$

This expression has a maximum value at $(M_0/M_\odot) \simeq 2 \times 10^3$ at which value $E_\nu(\max) \simeq 1.37 \times 10^{20}$ erg gm^{-1} = $0.15\, c^2$. Thus only 15 per cent of the rest mass is lost in the maximum case. As might be expected it can be shown that $E_\nu(\max)$ is independent of the numerical coefficient, 4.3×10^{15}, in equation (28). Furthermore, it can be shown that $E_\nu(\max)$ does not depend critically on the power of T_9 in equation (28). For very large exponent $E_\nu(\max)$ approaches $\sim$22 per cent. The point is that any mechanism of great energy loss is quenched by the large redshift which arises when collapse to the event horizon occurs.

These arguments apply to nuclear processes which customarily depend on a high power of the temperature. High temperatures are reached only near the termination of collapse when redshifts are large. On the other hand the second expression in equations (38) indicates for any dU/dt = constant, for example, that the energy release diverges on evaluation at the lower limit of integration. Thus it is not possible to rule out large losses for processes such as gravitational radiation which do not depend critically on temperatures.

For $Z_{\max} < 1$ which corresponds to stars with $M_0 > 10^4 M_\odot$ one finds

$$E_\nu \simeq 1.5 \times 10^{26} \left(\frac{M_\odot}{M_0}\right)^{3/2} = 1.7 \times 10^5 \left(\frac{M_\odot}{M_0}\right)^{3/2} c^2 \,\text{erg gm}^{-1}, \qquad 10^4 < \frac{M_0}{M_\odot} < 10^7. \quad (41)$$

The upper limit, $10^7\, M_\odot$, occurs because equation (28) grossly overestimates neutrino losses along the evolutionary path of such stars. Thus for $M_0 = 10^6 M_\odot$, $E_\nu = 1.5 \times 10^{17}$ erg gm^{-1} = $1.7 \times 10^{-4}\, c^2$ and the fractional energy loss is very small indeed. Michel (1963) has suggested that the energy gained by the envelope if the imploding core loses *all* of its rest-mass energy and thus no longer acts gravitationally on the envelope is $2.9 \times 10^{49}\, (T_9)_{\text{coll}}\, (M/M_\odot)^{3/2}$ erg, where M is the total mass of the star and $(T_9)_{\text{coll}}$ is the temperature at which collapse of the core begins. This expression must be multiplied by E_ν/c^2 on the basis of our analysis and the mass of the collapsing core (M_0 in our notation) must be estimated. Michel (1963) used $M_0 \simeq 0.37\, M$. The explosion energy of the envelope thus becomes

$$E_{\text{env}} \simeq 2 \times 10^{55}\, (T_9)_{\text{coll}} \text{ erg},$$

or

$$\frac{E_{\text{env}}}{M c^2} \sim 10 \left(\frac{M_\odot}{M}\right) (T_9)_{\text{coll}}, \qquad 10^4 M_\odot < M < 10^7 M_\odot, \quad (42)$$

which is $\sim 10^{-3}$ at $M \sim 10^4 M_\odot$ and only 10^{-6} at $M \sim 10^7 M_\odot$ when the reasonable estimate $(T_9)_{\text{coll}} \sim 1$ is employed. Thus neutrino losses from collapsing cores cannot lead to envelope explosions with appreciable fractions of the rest-mass energy of a star.

It is worth noticing, by way of concluding the present section, that the track (33) applied to ordinary stellar masses determines the situation under which neutrino emission could produce a free-fall implosion. This track lies far below the track actually followed by evolving stellar material (Hoyle and Fowler 1963*a*, *b*). This shows that free-fall implosion is never produced in ordinary stars by neutrino emission.

e) Rotation

The implosion of a discrete body including rotational effects has not been solved in general relativity. Rotation effects have been studied in cosmology, and we are encouraged to think that what is known about the cosmological case may be adequate for a general discussion. As emphasized in Section II, the implosion of a finite body without rotation is closely similar to the cosmological case.

We have so far regarded $S(t)$ as a scale factor applicable to all three space coordinates. Now we regard S as applicable only to a coordinate taken parallel to the axis of rotation. A different scale factor $R(t)$ will be used for two coordinates taken perpendicular to the rotation axis. In Newtonian mechanics the differential equations for R, S are of the form

$$\frac{\dot{R}^2}{R^2} \simeq \frac{a}{R^3} - \frac{\Omega^2}{R^4}, \qquad a = \frac{8\pi G \rho_0}{3}, \tag{43}$$

$$\frac{\dot{S}^2}{S^2} \simeq \frac{2a}{R^2 S}, \tag{44}$$

where Ω is the angular velocity at $R = 1$, and terms which become small as the implosion proceeds have been omitted.

The right-hand side of equation (43) has a zero for sufficiently small R, implying that implosion is halted and reversed to explosion so far as the coordinates perpendicular to the axis of rotation are concerned. But no such effect occurs for the coordinate parallel to the rotation axis. In the cosmological case implosion continues indefinitely for this coordinate (Narlikar 1963). However, for a finite body a different situation can arise due to a pressure gradient, as will now be seen.

Suppose that at the onset of implosion from a dispersed state the body is essentially spherical in shape, with $R = S = 1$. Rotation eventually causes S/R to decrease below unity. Now the pressure gradient necessary to maintain equilibrium in a direction parallel to the rotation axis can be considerably less than would be necessary for the whole mass in the absence of rotation. In fact the problem of support is reduced to that for a mass of only $M(S/R)^2$. For sufficiently small S/R this can be reduced below the limiting value of order $M_\odot$, and equilibrium is possible either through electron degeneracy or nucleon degeneracy. Hence it is in principle possible for a rotating object to maintain itself whatever the total mass M, as was pointed out by Hoyle (1947). For $M \gg M_\odot$ the shape must be that of a thin disk. Although in the imploding case oscillations must occur, we expect they will eventually become damped away, perhaps by repeated bursts of neutrino emission.

At first sight one might suppose that, because of this effect, implosion to a singularity never occurs in any actual case, since presumably there is never a complete absence of rotation. This view is incorrect, however. If ρ attains nuclear densities, the energy density E cannot be taken as ρ and the Newtonian equations (43) and (44) require modification. We must have $E \propto \rho^{4/3}$, and the terms in a are changed to the proportionality R^{-4} for the $\dot{R}^2$ equation and to $R^{-8/3} S^{-4/3}$ for the $\dot{S}^2$ equation. Hence both terms on the right-hand side of the $\dot{R}^2$ equation behave in the same way as $R \to 0$, and there is no root if Ω is small enough. The gravitational field can be strong enough even to crush rotation, a result that never occurs in the Newtonian case.

The situation is further complicated, even in the case $E \simeq \rho$, by the circumstance that a rotating disk can be locally unstable. Contraction parallel to the rotation axis increases the density by R/S. A local aggregation is then unstable against contraction *perpendicular* to the rotation axis. Indeed, a spherical aggregation of mass $\sim(S/R)^2 M$ can shrink as a whole by a factor R/S before rotation again prevents contraction perpendicular to the axis. Hence the density can rise by a total of $(R/S)^4$. We may state this in the following way: If a rotating disk of mass M and radius a breaks up into more or less spherical fragments of mass $x^{-2} M$ $(x \gg 1)$, the density in each fragment can increase to $\sim x^4 M/a^3$.

The important issue is whether nuclear densities are reached before or after the fragment masses become of order $M_\odot$. If before, no equilibrium is possible, since rotation becomes ineffective before the fragment masses are small enough for pressure gradients to maintain equilibrium. If after, we expect the implosion to be halted, and a final state reached in which the object has divided into stable fragments with masses $\sim M_\odot$ that move in nearly coplanar orbits about a common center.

Set $x^{-2} M \simeq M_\odot$. The density then increases by $(M/M_\odot)^2$ above the value at which rotation first impeded shrinkage perpendicular to the axis. Write ρ_1 for this density, so that the fragments have density $\rho_1 (M/M_\odot)^2$. Our two cases depend on whether this value is $>$ or $< \sim 10^{16}$ gm cm^{-3}. Hence our cases are given by

$$\rho_1 \gtrless \sim 10^{16} \left(\frac{M_\odot}{M}\right)^2 \text{gm cm}^{-3}. \tag{45}$$

This density is close to that which occurs at the event horizon. Hence we arrive at the conclusion that, if rotation does not become dynamically important until after an imploding object has retreated within the event horizon, singularities develop.

When the "less than" sign applies in equation (45) two cases appear to arise. If ρ_1 is sufficiently small compared with the right-hand side of equation (45) the relevant pressure can arise from electron degeneracy and the object fragments into white dwarfs. At larger values of ρ_1 nucleon degeneracy occurs, however, and the object fragments into neutron stars.

f) The Prevention of Singularities in C-Field Cosmology

Hoyle and Narlikar (1963) have shown that equation (21) is modified to

$$\frac{\dot{S}^2}{S^2} = -\frac{1}{S^2} + \frac{8\pi G E}{3} - \frac{A^2}{S^6} \tag{46}$$

by the presence of the C-field of steady-state cosmology where A is a small constant. Since E is proportional only to S^{-4} at very high densities, the right-hand side of equation (46) can have two zeros. In fact, for any physical motion there must be two roots, a larger root with the $-S^{-2}$ term essentially compensating the E term—this is just the dispersed state—and a smaller root with the $-A^2 S^{-6}$ term essentially compensating the E term. This result arises from the negative-energy density of the C-field. Although the concept of a negative-energy density is strange, several arguments can be advanced in its support:

1. A negative-energy field is gravitationally repulsive. The expansion of the universe can be taken as evidence of the existence of such an effect—otherwise we must take the unsatisfactory step of imposing special initial boundary conditions.
2. Matter must be "created" either continuously or at the "origin" of the universe. A negative-energy field appears necessary to obtain a mathematical description of the creation process.
3. A negative-energy field now seems to be necessary if singularities are to be avoided. The singularities are prevented through the gravitational repulsion of the field.

The two roots of the right-hand side of equations (46) imply that the object oscillates between a maximum S_1 and a minimum S_2 instead of imploding into a singularity. It is to be expected (Hoyle and Narlikar 1963) that the oscillations are gradually damped in the sense that S_1 decreases. Neutrino emission is probably the main damping agent, although charged particles may also be emitted. The object never attains a static state —that is, S_1 can never be reduced to S_2 because the damping ceases when the redshift cuts off the emission. The maximum S_1 then takes its value, αr_b^2, at the event horizon, whereas $S_2 \ll \alpha r_b^2$, that is, at minimum the object lies far inside the event horizon. This is because the coefficient A^2 is very small.

According to Hoyle and Narlikar it seems unlikely that the maximum density can be less than 10^{30} gm cm^{-3}, corresponding to an interparticle separation of 10^{-18} cm, or less.

Nuclear physics gives no real guidance as to what might happen under such conditions. Boson fields are to be expected with energy densities proportional to S^{-4}, individual bosons have energies proportional to S^{-1}. At $\rho \simeq 10^{16}$ gm cm^{-3} the energies are of order 1 BeV, while at $\rho \simeq 10^{30}$ gm cm^{-3} energies of order 10^5 BeV would be expected. The possibility exists that such bosons emerge from the surface of the object. At first sight it might seem as if nothing of this kind could be detected by an outside observer, since the object lies far inside the event horizon at stages where the density is very high. However, particles can be pushed outside the event horizon by the expansion of the object itself. Any massless boson that emerges radially from the surface always stays ahead of the surface. So long as the surface crosses the event horizon the particle is ultimately disgorged into the outside world. Decay could provide electrons and protons or neutrinos and neutrons if appropriate weak coupling interactions exist.

It is clear that at just this point a serious gap exists in our knowledge, a situation that has been strongly emphasized by Wheeler (cf. Wheeler, Wakano, and Harrison 1958). A filling of this gap would provide an interesting connection between high-energy physics and astronomy.

II. ASTROPHYSICAL CONSEQUENCES

In the previous sections we have described the various situations which have been envisaged as the final stages of evolution of a star which reaches the end point of thermonuclear evolution with a mass greater than the mass which can be supported either by degenerate electron pressure or degenerate neutron pressure. While it is not clear which of these theoretical possibilities is the correct one, in each case the star eventually reaches a situation in which it ceases to communicate with the outside world except through the action of its gravitational field. In the theory of Hoyle and Narlikar there is the possibility that such an object will be able to radiate some fraction of its mass energy as it pulsates, but such pulsations will eventually damp out and the remnant will disappear.

Thus we conclude that the following facts should be taken into account.

1. Invisible mass is likely to be present wherever the condensation and evolution of stars have occurred. How much mass is present will depend on the masses of the stars which condensed, and on what fraction of this mass can be ejected in the stages of evolution prior to catastrophic implosion.

2. In these circumstances star formation is a one-way process which removes uncondensed material in the universe and transforms it into a form which makes it immune to further evolution.

In modern theories concerning the evolution of stars and galaxies it has been supposed that much of the material which is condensed into a first generation of stars will be ejected and condensed into a second generation, etc. While such processes do occur, it is necessary to take into account that fraction of the mass which goes into an invisible form and no longer plays a direct role in the evolution.

3. In the theory of Hoyle and Narlikar mass energy can be radiated by the object after it has reached the collapsed state. This energy is bound to be radiated in the form of high-energy quanta. It thus provides a continuous energy input for non-thermal radio sources.

We now discuss the implication of these results for astrophysics in more detail.

a) The Presence of Invisible Mass

We first discuss the stellar mass function and the mass-to-light ratios for aggregates of stars and for galaxies.

In our Galaxy the number of stars born with masses between M and $M + dM$ is proportional to $\sim M^{-1.4}\, d \log M$ at any rate for masses up to 5 $M_\odot$ (for a review of the work on clusters cf. Burbidge and Burbidge 1958). This means that, whereas most stars lie at the bottom end of the mass range, the total mass of a group of stars comes mainly from the upper end of the range. Except for a few stars in very young clusters, stars near the upper end have already evolved. Hence most of the mass originally condensed into stars has evolved, in the sense that the stars into which it was originally condensed are no longer visible. The question evidently arises as to what has happened to this material. There seem to be only two possibilities: (i) The bulk of the material was ejected from the stars in the course of their evolution and is now condensed into further stars. (ii) The stars evolved into imploding objects and the mass is now invisible. The possibility of the material being stored permanently in the interstellar medium as gas appears to be excluded, since not more than 10 per cent of the mass of the Galaxy seems to be in the form of gas.

The first of these possibilities can be tested by considering what would be expected for the mass-to-light ratio in our own Galaxy. Consider first the simple case of main-sequence stars with the mass distribution $M^{-2.4}\,dM$ extending from $M_0 \approx 0.1\ M_\odot$ up to M of the order of, or greater than, $M_\odot$. Write M^* for the upper limit of mass and L^* for the upper limit of luminosity. The mass-to-light ratio is

$$\sim \int_{M_0}^{M^*} M^{-1.4} dM \div \int_{M_0}^{M^*} L M^{-2.4} dM . \tag{47}$$

With $L = L^*(M/M^*)^4$ on the main sequence, equation (47) is approximately evaluated to give $6.5(M^*/L^*)\ [(M^*/M_0)^{0.4} - 1]$. Next, we require values for M^*, L^*. In the Galaxy $M^* \simeq M_\odot$ for the old star distribution, while $L^* \simeq 2L_\odot$ at the so-called break point of the main sequence. Hence in solar units we have a mass-to-light ratio of $\sim 3[(M^*/M_0)^{0.4} - 1]$. Similar data do not exist for other galaxies. However, the theory of stellar evolution would not permit a significantly larger value to be taken for M^*/L^*, unless the galaxies in question were substantially older than our own.

The mass-to-light ratio calculated in the previous paragraph must be reduced to allow for young clusters and for stars that have evolved off the main sequence. Giants, in particular, contribute appreciably to the light but not to the mass. Hence the expected mass-to-light ratio cannot be much greater than ~ 3.

The mass-to-light ratio for the solar neighborhood is about 4 (Schmidt 1963*a*), while Burbidge, Burbidge, and Prendergast, and Page (cf. Burbidge 1961) have found values of order 3 or less for a number of spiral galaxies, mainly of type Sc. These values are consistent with a choice of $0.1\ M_\odot$ for M_0. On the other hand, the mass-to-light ratios for some spirals are $\sim$10–15 (M31, NGC 253, M81, and others; cf. Burbidge 1961) while the average value for elliptical galaxies (Page 1962) is about 30. Such values point to the existence of hidden mass, unless the mass function for stars is grossly different in other galaxies from what it is in our own. Obscuration of light might perhaps falsify the observed values in dusty spirals, but obscuration cannot be important for the ellipticals.

Hidden mass in the above sense could come from (i) faint white dwarfs, (ii) very faint main-sequence dwarfs, corresponding to a very small value for M_0, (iii) imploded evolved stars that have not lost appreciable fractions of their original masses, (iv) imploded objects of very large mass. As far as our Galaxy is concerned the number of white dwarfs in the solar neighborhood is not sufficient for (i) to make more than a modest contribution to the total mass. Nor is there any direct evidence from the solar neighborhood to support (ii). However, it seems probable that considerable numbers of red dwarfs are present in some galaxies (Spinrad 1962) and that the mass function is not the same as it is in our own Galaxy. We have pointed out elsewhere the difficulties associated with the assumption of a universal mass function (Burbidge, Burbidge, and Hoyle 1963). On the other hand, either a very low value for M_0 or a very gross difference in the mass function for main-sequence stars has to be assumed to obtain the mass-to-light ratios for ellipticals, and the observations do not indicate that such large differences as these, in fact, exist.

An upper limit for the importance of (iii) can readily be calculated. Suppose the

$M^{-2.4}\,dM$ distribution extends *at birth* to an upper limit M^{**}, M^* being the upper limit to which it applies *at present*. Then the total mass born over the whole range from M_0 to M^{**} exceeds the range from M_0 to M^*—the latter being the present-day observed range—by the factor

$$[1-(M_0/M^{**})^{0.4}]/[1-(M_0/M^*)^{0.4}] \approx 1/[1-(M_0/M^*)^{0.4}]$$

for sufficiently large M^{**}. Taking $M^* \simeq M_\odot$, $M_0 \simeq 0.1\ M_\odot$, this factor is 5/3. The mass-to-light ratios, including hidden mass in the form (iii), then becomes $\sim$5. Such a value is clearly compatible with the mass-to-light ratios of galaxies such as our own, and perhaps with M31, M81, NGC 253, etc. However, the much larger ratios which are found for many elliptical galaxies might indicate that very massive objects have imploded and are present in the form of invisible mass, that is, possibility (iv) may be required to explain the observational results.

It is self-evident that direct *observational* evidence for the presence of invisible mass can never be forthcoming. The arguments given above concerning the mass-to-light ratios are indirect but give some indication of phenomena which may be explained by these developments in the theory. However, we do not believe that the discussion in the first part of the paper is strongly dependent on the indirect observational arguments. We do assert that masses which arrive at the end point of evolution and implode must follow one of the paths described in that section. Another phenomenon which might suggest that some mass is present in the form of invisible matter is the presence of invisible (often called "infrared") members of binary star systems.

Another problem of some significance is that concerning the stability of clusters of galaxies (cf. the papers published in *A.J.*, **66**, 533–636, 1961). It is found by applying the virial theorem to clusters and groups of galaxies that in nearly all cases it must be supposed either that the systems are expanding, or else that a very large amount of invisible matter must be present. Probably systems of both types occur. In the case of clusters and groups which are stable by virtue of their containing a large fraction of mass in invisible form, it is quite reasonable to suppose that much of this is made up of large masses $\geq 10^5\ M_\odot$ which have imploded.

b) The Effect of the Concept of Imploded Mass on Stellar Evolution and Nucleosynthesis

The rate at which the material in a galaxy is enriched in the heavy elements is dependent, first, on the efficiency of the processes of nucleosynthesis in the interiors of stars and, second, on the processes which will redistribute this material into the interstellar medium. The discussion given in Section I of this paper strongly suggests that the latter process is much less efficient than has been thought up to the present, since in some proportion of the stars with masses above the critical mass we may expect that the implosion will occur and the mass will vanish. Thus some modification of the calculations of the rate of enrichment of heavy elements as a function of star formation as they have been made by Schmidt (1963*b*) is required, since he made the assumption that all of the mass in a star in excess of the white dwarf mass would be returned to the interstellar medium. In the case of massive supernovae (Burbidge, Burbidge, Fowler, and Hoyle 1957; Hoyle and Fowler 1960; Fowler and Hoyle 1964) the possibility that mass vanishes when these stars have evolved is of particular interest. It is usually supposed that there have not been more than $\sim 10^8$ supernovae in the lifetime of the Galaxy, whereas the number of stars that have been born with $M > 2M_\odot$ is probably of order 10^{10}. It therefore seems necessary to argue that only a small fraction of imploding stars, about 1 per cent, become supernovae. A similar result is obtained if we simply take the known number of star deaths among A, B, and O stars in young clusters, together with an assumed present-day supernova rate of $\sim$1 per 10^2 years. Rotation may supply the restraining factor. Imploding stars with too much rotation may attain stable disklike structures in accordance with the considerations of Section I(*e*) and these also have the property of hiding mass. It may well be necessary for a star to satisfy the "greater than" condition of equation (45) in order that it become a supernova. The physical reason could be that superdense conditions are needed in order that relativistic particles be generated.

It has been stressed previously (Fowler and Hoyle 1960) that, if we suppose that the *r*-process elements are made in supernovae in sufficient amount to explain the light-curves on the californium hypothesis, then taking an average rate of supernovae as 1 per 300 years far too much *r*-process material would be produced. This difficulty can be avoided if Type I supernovae are taken to have such small mass and such long evolution times that none evolved in the Galaxy before the solar system formed. However, an alternative explanation which involves the rejection of the californium hypothesis will be explored in the next section. A similar difficulty is encountered when we consider the production of iron peak elements in Type II supernovae, that is, too much iron is produced. However, it now appears possible that much of the elements synthesized in these massive stars are never ejected but are contained in the imploded mass, so that this difficulty can be surmounted.

Finally, in this section we turn briefly to the evolution of a globular cluster. In a recent analysis of the dynamics of M3, Oort and van Herk (1959) came to the conclusion that more than half the mass was originally in the form of stars with average masses of 4.4 $M_\odot$. Since these stars must have evolved long ago they argued that the excess mass above the white dwarf mass, i.e., 3.8 $M_\odot$ per star, must have been ejected and lost from the cluster. This assumed mass loss through stellar evolution is far greater than the mass loss due to the evaporation of stars. However, according to our earlier considerations, this mass may not have been lost to the cluster but may still be present in invisible form. While the observable dynamical effects would be small (a change in mass by a factor of 2 corresponds to a change in velocity dispersion by $\sqrt{2}$) the possible presence of such mass rather than its assumed ejection should be considered; although it could still be the case that the dynamical processes involved produced a sufficient recoil on the remnants to cause escape from the cluster.

c) The Injection of Energy into the Interstellar and Intergalactic Medium from Collapsed Stars

The imploding bodies discussed in Section I have the property that gravitation is in principle capable of yielding $\sim 9 \times 10^{20}$ erg gm^{-1}, much greater than the energy yield from any nuclear reaction (Hoyle and Fowler 1963*a*, *b;* Burbidge 1962*a*). But classical implosion into a singularity does not seem to provide an atomic or nuclear mechanism whereby the full dynamical energy stored in a collapsing body can be returned in an observable form to the outside world.

As pointed out by Hoyle and Fowler (1963*a*) it is probable that hydrogen burning can retard the implosion at a central temperature near 8×10^{7} ° K for a limited time ($\sim 10^6$ years) and supply the positive internal energy ($\sim 10^{59}$ ergs for $M \sim 10^8 M_\odot$) necessary for hydrostatic equilibrium under general relativistic conditions (Iben 1963) as well as that required to match the radiated energy ($\sim 10^{60}$ ergs for $M \sim 10^8 M_\odot$). In fact, radio stars may well represent the early stage of the implosion of massive objects where the luminosities predicted by Hoyle and Fowler, $L = 2 \times 10^{38} (M/M_\odot) \rightarrow 2 \times 10^{46}$ ergs sec^{-1} for $M = 10^8 M_\odot$, are of the order of magnitude observed for 3C48 for example (Greenstein and Matthews 1963). The radius and effective surface temperature depend critically on the structure of the star during hydrogen burning. Hoyle and Fowler (1963*a*) give $R \sim 10^{11} (M/M_\odot)^{1/2}$ cm for polytropic index $n = 3$, but this could be low by as much as a factor of 10^2 to 10^4, since a larger index is indicated if general relativistic considerations are taken into account. Thus $R \sim 10^{14} (M/M_\odot)^{1/2}$ cm with considerable uncertainty. Similarly the surface temperature $T_e \sim 7 \times 10^4$ ° K given by Hoyle and Fowler (1963*a*) may be too high and a value $T_e \sim 10^4$ ° K is probably to be preferred with an uncertainty of a factor of 3 either way.

During the hydrogen-burning period the central density is $\sim 10^{-1}$ to 10^{-2} gm cm^{-3} an the ratio of gas pressure to total pressure is $\sim 10^{-2}$ to 10^{-3}, these values holding fo $M \sim 10^6 M_\odot$ to $10^8 M_\odot$ using equations given by Hoyle and Fowler (1963*a*). Settin $\gamma\alpha = 3\gamma - 4 \sim \beta/2 \ll 1$ in equation (130.3), p. 192, of Eddington (1930), these value lead to periods of pulsation $\Pi \sim (80\pi/3G\beta\rho)^{1/2} \sim 1.2\,(G\beta\langle\rho\rangle)^{1/2} \sim 10^6$ to 10^7 sec. Smit and Hoffleit (1963) have found evidence in 3C273 for light variations with periods i this range. In view of this relatively satisfactory model for radio-star behavior we mus emphasize once again that somewhat less than 1 per cent of the rest-mass energies o these objects is required to account for the energy requirements discussed above. Hydro gen burning can supply this energy. However, if the light output of such an object i to be attributed to the comparatively steady luminosity of a massive star during it "main-sequence" evolution, at least one previous outburst involving the evolution an collapse of another massive star must be invoked to account for the radio flux from th source.

It is also possible that rotation prevents the development of a singularity, at any rat in some cases, and that collapse to disklike aggregations of white dwarfs or neutron star can give rise to large energy sources. The mechanism whereby such aggregations act a sources is far from clear, however.

Another possibility is the release of energy in the form of gravitational radiation. Be cause of the critical dependence of this release on rotation and on the model of collaps as discussed in Hoyle and Fowler (1963*b*) it is difficult to make realistic estimates o the energy release and we have not attempted to do so. Our remarks in Section I(*d*) concerning the difficulty in abstracting energy from collapsing systems are subject to the qualification that gravitational radiation rates have not yet been determined.

The non-classical oscillations discussed in Section I(*f*) seem to provide a more convenient energy source. Energy can be derived through a damping of the oscillations until the redshift cutoff is reached. The yield is then about 5×10^{20} erg gm^{-1} (Hoyle and Narlikar 1963). If the fraction of all material involved in this process is f, the energy density for the whole universe is $\sim 5 \times 10^{20} f\rho_c$, where ρ_c is the cosmological mass density, usually taken as $\sim 3 \times 10^{-29}$ gm cm^{-3}. This gives $\sim 10^{-8} f$ erg cm^{-3}. As stated in Section I, the main damping agent is probably neutrinos, and the present estimate can be taken as referring to the energy density of ν, $\bar{\nu}$.

The speculations of Hoyle and Narlikar on the oscillatory problem lead to extraordinarily great densities at maximum compression, which has the advantage that very high-energy particles would surely be generated. These speculations also have the advantage of relating high-energy physics with astrophysics and cosmology. If k is the fraction of the energy released as relativistic electrons and protons, the cosmic-ray energy density for the whole universe would be $\sim 10^{-8} fk$ erg cm^{-3}.

Burbidge and Hoyle (1964) have suggested that massive objects may be the source of cosmic rays and that the energy density may be ~ 1 eV cm^{-3} everywhere in space. This would require $fk \simeq 10^{-4}$. It seems unlikely that k could be more than a few per cent, in which case Burbidge and Hoyle require f to be not much less than 1 per cent. Since matter condensed into galaxies has mean density $\sim 3 \times 10^{-31}$ gm cm^{-3}—i.e., only ~ 1 per cent of the value taken above for the total cosmological density, we evidently require an appreciable fraction of the mass of the galaxies to have evolved into imploded objects. This question has been discussed in the previous sections.

To account for the energy contained in non-thermal radio sources, if we take the point of view that this is energy released in collapse to very high-density configurations, it is therefore necessary to suppose that the model of Hoyle and Narlikar, and not the classical implosion solution, is correct. If we make this assumption, then the supernova remnants and the objects which give rise to the powerful extragalactic radio sources continuously inject high-energy particles and quanta into the surrounding medium, and the bulk of the energy injected must be in the form of neutrinos. This hypothesis leads to quite a different picture of the evolution of a supernova remnant than that previously developed.

First of all, it is possible to suppose that the interacting particles which are ejected at the earliest phase are the source of visible radiation of the supernova, either through direct synchrotron radiation or through the penetration of the particles into an expanding envelope, ejected perhaps as a consequence of a nuclear explosion in the outer parts of the star.

We now turn to the situation in the Crab Nebula, the remnant of the supernova of AD 1054 which shows evidence of its continuing activity through the moving wisps observed by Baade and the presence of high-energy electrons with synchrotron lifetimes much less than the age of the remnant.

It is remarkable that no star brighter than about 18^m appears at the present day to be associated with the Crab Nebula. This, taken together with the estimates of Osterbrock (1957) and O'Dell (1962) of no more than 0.3 $M_\odot$ for the mass of the nebula, leads to the challenging question of what has happened to the original star. Since it has not been disintegrated, the major part of the mass must still exist as a compact object. It cannot be a white dwarf, since cooling could not have reduced its luminosity sufficiently in only 10^3 years. It can scarcely be a single neutron star since it is unlikely that the original mass was less than $M_\odot$. The interesting possibility is that the star is now in the oscillatory state discussed in Section I(f) and that the continuing activity of the Crab Nebula is due to a steady output of high-energy particles.

The present picture allows a different point of view to be taken about the origin of the filaments of the nebula. The volume of the nebula is $\sim 10^{56}$ cm^3. The mass of interstellar gas normally present within such a volume, situated as it is quite near the galactic plane, would be $\sim 3 \times 10^{32}$ gm. If the filaments only have mass $\lesssim 0.3\ M_\odot$ it follows that they must be largely interstellar gas, unless the region of the Crab is extraordinarily void of gas. We take the view that the whole filamentary structure is interstellar in origin, that no significant expanding envelope was emitted, and the light of the supernova was produced by particles of high energy, perhaps by the synchrotron process. A similar supposition for all Type I supernovae explains the absence of detectable lines in the spectra of these stars.

On this point of view, the approximately exponential decay of the light-curves of Type I supernovae can then no longer be attributed to Cf^{254}, or to other very heavy nuclei subject to spontaneous fission. Damping of the oscillations of the central object may be expected to proceed quite rapidly to the stage where redshift effects become important, and exponential decay of the light-curve could arise from this damping. We urge that observational astronomers turn their attention to the question of which of the two alternatives we have suggested for the light-curves of Type I supernovae is the correct one. Is the source of the energy radioactivity or gravitation?

The present outward speed of the filaments is ~ 1000 km sec^{-1}, the total kinetic energy being $\sim 10^{48}$ erg. It has been argued in the past that the energy of the relativistic particles associated with the Crab cannot be greater than the kinetic energy of the filaments; otherwise the nebula would expand faster than it is observed to do. This argument is only correct, however, if the relativistic particles are trapped within the filaments. This has always been assumed because in the past it was felt to be difficult to explain the origin of even 10^{47} erg in the form of high-energy electrons. The same difficulty does not arise in the present picture. The rest energy of a star of mass 1–10 $M_\odot$ is 10^{54}–10^{55} erg. Even though most of this energy emerges as neutrinos, the energy available for charged particles could readily exceed 10^{50} erg. This leads us to the quite different point of view that the filaments have derived their outward motion from the *momentum* of the relativistic particles, not from their energy. We suppose that the particles are emitted by the central object and that they experience deflection by the filaments—the interstellar magnetic field within the filaments forces the particles to fan out in much the shape of a comet's tail in order to get around a filament on which they are incident. The filamentary structure itself may arise from the ability of the high-energy particles to punch holes in the gas. Indeed, the ravaged appearance of the nebula could be a consequence of such violent fluting.

If the filaments take up a fraction θ of the outward momentum of the relativistic particles the filamentary kinetic energy E_f will be related to the particle energy E_p by

$$E_f \simeq \theta \frac{V}{c} E_p , \tag{48}$$

where V is the filamentary velocity. With $E_f \simeq 10^{48}$ erg, $V \simeq 10^8$ cm sec^{-1}, $E_p \simeq 3 \times 10^{50}$ θ^{-1} erg. Individual relativistic particles spend perhaps 10 years within the nebulosity—the radius is about 3 light years while allowance for deflections and spiraling motions reasonably increases the time to 10 years. Hence in the 10^3 years of existence of the Crab we expect there to have been about 10^2 generations of relativistic particles. The energy content at any moment is then $\sim 10^{-2} E_p \simeq 3 \times 10^{48} \theta^{-1}$ erg. Oort and Walraven (1956) and Woltjer (1957) estimate the energy content, excluding protons and excluding electrons that do not emit synchrotron radiation either in the radio band or the visible spectrum, as 10^{47} to 3×10^{48} erg depending on the intensity of the magnetic field. The energy content could be as high as 10^{49} erg.

Deflection around a knot of gas arises from compression of the magnetic field. Deflection becomes appreciable when the magnetic pressure $H^2/8\pi$ increases to a value comparable with the energy density of the incident beam of particles. Taking the total resident energy as $\sim 10^{49}$ erg and the volume as 10^{56} cm^3, the latter is $\sim 10^{-7}$ erg cm^{-3}. Hence we expect compression to raise H to a value of order 10^{-3} gauss, a value that has been used in past investigations. Partly because of the compression of the field, and partly because the electrons spend a comparatively long time in the region of compression, we expect synchrotron emission to occur mainly at the inside faces of knots and filaments of gas—i.e., the faces turned toward the central star.

If the central star emits particles at a steady rate, one might at first sight expect the outward motion of the gas to be decreasing, because of the increasing quantity of interstellar material swept up by the system. However, this need not be the case since θ also depends on the amount of gas in the system. In fact, so long as $\theta < 1$ we expect θ to be proportional to the total mass. Then it can easily be shown that the system accelerates. Deceleration sets in when θ increases toward unity.

It is also easy to see that interesting polarization effects can arise. For the simple case of a coplanar fluting around a knot of gas, an observer with a face-on view finds the polarization structure shown in Figure 1. This detail is mentioned because the polarization structure of the Crab contains an example of this special case (Woltjer 1957). A general discussion of polarization lies far outside the present work. The problem is complicated by the unknown three-dimensional structure of the system, by the lack of regularity in the arrangement of the knots and filaments, and by uncertainty in our knowledge of the initial run of the magnetic field lines.

In this picture of a supernova remnant tremendous fluxes of particles will escape into the disk of the Galaxy, and this picture would be inconsistent with the estimates of the relativistic electron energy density in the disk $\leq 5 \times 10^{-14}$ erg/cm^3 if the particles were contained in the Galaxy. Taking 10^{59} erg as the total production in the lifetime T of the Galaxy and taking 3×10^{66} cm^3 as the volume of the disk the requirement is that

$$10^{59} \frac{t}{T} \leq 5 \times 10^{-14} \times 3 \times 10^{66} ,$$

where t is the escape time. Thus $t \leq \sim 10^{-6}\, T \sim 10^4$ years. Thus the Galaxy must be entirely open as is indicated on other grounds (cf. Ginsburg and Syrovatsky 1961).

The extragalactic radio sources probably have short lifetimes $\sim 10^6$ years (Burbidge 1962*b;* Burbidge, Burbidge, and Sandage 1963), but in many cases much shorter time scales within these are indicated. Examples are M87 which contains optical synchrotron electrons with half-lives $\sim 10^2$–10^3 years, the nuclei of Seyfert galaxies, and the starlike extragalactic objects (Matthews and Sandage 1963; Schmidt 1963*c;* Hazard, Mackey, and Shimmins 1963; Oke 1963; Greenstein and Matthews 1963) which show light variations over time scales of months and years (Matthews and Sandage 1963; Sandage 1963; Smith and Hoffleit 1963; Shklovsky 1963). These characteristics all indicate that continuous activity takes place in such objects. Since the tremendous energies required to explain these objects most likely come from gravitational sources, the Hoyle-Narlikar hypothesis of continuous energy injection appears to be indicated.

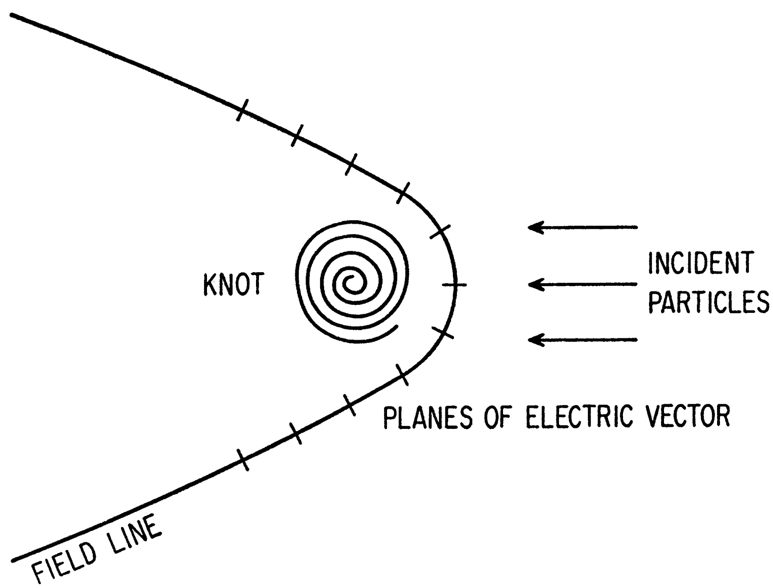

FIG. 1.—Magnetic field lines bent around a knot of gas by a beam of incident relativistic particles produce the polarization vectors shown above for an observer with line of sight perpendicular to the field lines.

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