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ON THE EXISTENCE OF SUBDWARFS IN THE (M_{bol}, LOG T_e)-PLANE. II

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ABSTRACT

The position of the main sequence as a function of chemical composition is studied in the observed (M_V, $B - V$) diagram and in the corresponding diagram where the effects of differential line blanketing are removed. A sample of stars was chosen for which (*a*) spectroscopic luminosity criteria indicate that they are dwarfs, (*b*) trigonometric parallaxes at least as large as $0''.035$ have been determined at Allegheny, Cape, or Yale Observatory, and (*c*) photometry on the (U, B, V) or (U_C, B, V) systems is available. Our earlier result—that the displacement of a star below the Hyades main sequence in the (M_V, $B - V$) diagram is dependent on the ultraviolet excess—is confirmed. It is shown that the differential blanketing corrections move all stars, independent of the size of their ultraviolet excess, onto the Hyades main sequence with good accuracy. In particular, 16 stars with extreme values of the ultraviolet excess are observed to be $1^{\mathrm{m}}.05 \pm 0^{\mathrm{m}}.04$ fainter than the Hyades before blanketing corrections are applied but move onto the Hyades main sequence to within $+0^{\mathrm{m}}.03 \pm 0^{\mathrm{m}}.05$ after the corrections are made. We conclude that subdwarfs do not exist as a separate sequence when the effects of line blanketing are removed.

The large scatter in the (M_V, $B - V$) diagram for field stars is caused in part by a mixture of stars of different chemical composition and in part by the effects of evolution of the bluer objects. Use of a "standard" subdwarf sequence (Table 1) for stars where $\delta(U - B) > 0^{\mathrm{m}}.16$ is discussed and applied to the globular cluster M13.

I. INTRODUCTION

It has been a common practice in recent years to determine cluster distances by force-fitting their main sequences to that of the Hyades cluster. However, it is now generally recognized that various clusters show an ultraviolet excess compared with the observed colors of the Hyades stars and that this ultraviolet excess is an indication of the chemical composition of the stars. Therefore, before photometric distances of the clusters can be determined with accuracy, it is necessary to know how a change in chemical composition affects the positions of their main sequences in the (M_V, $B - V$) diagram. Theoretical predictions of the effect in the (M_{bol}, log T_e) diagram can be made (e.g., Strömgren 1952; Sandage and Eggen 1959; Demarque 1960) for any assumed abundance of hydrogen, X, helium, Y, and metals, Z; but so little is presently known about the actual variations of the chemical composition among the stars that an empirical approach to the problem is necessary.

In 1959 Sandage and Eggen (1959) showed that a selection of stars whose well-determined trigonometric parallaxes placed them more than $0^{\mathrm{m}}.25$ below the Hyades main sequence in the (M_V, $B - V$) diagram all had ultraviolet excesses relative to the Hyades in the ($U - B$, $B - V$) diagram when the observed $B - V$ was bluer than $0^{\mathrm{m}}.8$. The excesses were, in general, small, with a mean $\delta(U - B)$ of only $+0^{\mathrm{m}}.11$ for the sample of 18 stars including three members of the Groombridge 1830 group for which the group parallaxes were used. Following a suggestion by Strömgren, the ultraviolet excess was attributed to a differential blanketing of the weak Fraunhofer lines, and a theory of this blanketing effect was given. Similar theories were also published by Melbourne (1960) and by Smak (1960). When the predicted corrections for line blanketing were applied to the 18 program stars, they were found to fall, in the mean, on the Hyades main sequence. However, a lack of extreme subdwarfs in the sample of stars discussed constituted a weakness in the argument and left some doubt as to whether the subdwarfs, such as those that occur in globular clusters with values of $\delta(U - B)$ near $+0^{\mathrm{m}}.25$, are completely corrected to the Hyades main sequence by the blanketing corrections.

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From a discussion of eight extreme subdwarfs with generally small parallaxes, Code (1959) concluded that these stars do not correct to the Hyades main sequence in the $G - I$ color system but fall $1\overset{\mathrm{m}}{.}05 \pm 0\overset{\mathrm{m}}{.}4$, in the mean, below the Hyades stars in the $(M_V, G - I)$ diagram. Because the line-blanketing effects are small in the $(G - I)$ color system, Code's result seemed to indicate that subdwarfs do indeed exist even when the effects of line blanketing are removed. If this were the case, the Hyades main sequence could not be used to obtain the distances of globular clusters, at least in the $(G - I)$ color system.

We have investigated the problem anew, using all the available photometric data, on the (U, B, V) system, for stars with trigonometric parallaxes at least as large as $0\overset{''}{.}035$, together with stars in two new moving groups of subdwarfs (Eggen 1962). Several of the possible objections to our first study have been removed, in that (1) stars in the new sample are not restricted to those that fall below the Hyades main sequence in the $(M_V, B - V)$ diagram and (2) the sample includes stars with extreme values of the ultraviolet excess, which then gives some assurance that the results can be directly applied to the members of globular clusters.

II. THE DATA

To insure homogeneity, we have selected only those northern stars whose parallaxes are at least as great as $0\overset{''}{.}035$ as determined at the Allegheny Observatory. The justification for the restriction to this one observatory is that (1) the Allegheny Observatory carries such great weight in the parallax catalogue (Jenkins 1952) that only a small increase in the total weight is achieved by averaging all the available data and (2) any systematic errors in the reduction of one observatory's results to another is avoided. The precepts used in the parallax catalogue were adopted in reducing the Allegheny results to absolute values.

The analysis was first completed by using only the northern stars, but, since it seemed desirable to check the results with data from the south, a second sample was selected from those stars with parallax values at least as large as $0\overset{''}{.}035$ as determined independently by both Yale and Cape Observatories. A further restriction applied to both the northern and the southern sample of stars is that only those which had been spectroscopically classified as dwarfs and for which there was either photometry on the (U, B, V) system or the equivalent Cape (U_C, B, V) system were used.

Figure 1 shows the $(M_V, B - V)$ diagram for the northern stars which meet the above requirements. The continuous curve in each panel of the figure is the Hyades main sequence taken from Table III of Sandage and Eggen (1959). The stars are divided into four groups according to their ultraviolet excesses, a division which separates them into different chemical-composition groups. These groups are $0\overset{\mathrm{m}}{.}05 \geq \delta(U - B) \geq -0\overset{\mathrm{m}}{.}05$ for data *A;* $0\overset{\mathrm{m}}{.}10 \geq \delta(U - B) \geq 0\overset{\mathrm{m}}{.}06$ for data *B;* $0\overset{\mathrm{m}}{.}15 \geq \delta(U - B) \geq 0\overset{\mathrm{m}}{.}11$ for data *C;* and $0\overset{\mathrm{m}}{.}25 \geq \delta(U - B) \geq 0\overset{\mathrm{m}}{.}16$ for data *D*. It is seen immediately from Figure 1 that the stars in the groups of increasing ultraviolet excess fall progressively fainter in the $(M_V, B - V)$ diagram, which confirms our previous result (Sandage and Eggen 1959) that $\delta(U - B)$ and the magnitude difference from the Hyades main sequence are correlated. The mean magnitude differences, taken in the sense star *minus* Hyades, read at the same $B - V$, are $\langle \Delta M_V \rangle = -0\overset{\mathrm{m}}{.}04 \pm 0\overset{\mathrm{m}}{.}04$ for data *A;* $\langle \Delta M_V \rangle = +0\overset{\mathrm{m}}{.}22 \pm 0\overset{\mathrm{m}}{.}05$ for data *B;* $\langle \Delta M_V \rangle = +0\overset{\mathrm{m}}{.}56 \pm 0\overset{\mathrm{m}}{.}11$ for data *C;* and $\langle \Delta M_V \rangle = +1\overset{\mathrm{m}}{.}10 \pm 0\overset{\mathrm{m}}{.}14$ for data *D*. The quoted errors refer to the final $\langle \Delta M_V \rangle$ and were found from the relation $0.8453\, \Sigma |r_i| / n(n - 1)^{1/2}$, where n is the total number of stars in the sample and r_i is the residual of the magnitude difference of star i from $\langle \Delta M_V \rangle$. Some stars in data samples *A* and *B* are obviously evolved and are probably of luminosity class IV, although they were not so called by the spectroscopic observers. The 16 highly evolved stars, brighter than $M_V = +4\overset{\mathrm{m}}{.}1$, in data *A* were excluded from the analysis of the mean values of $\langle \Delta M_V \rangle$. Eleven stars were excluded for the same reason from data *B*. This exclusion does not completely remove evolutionary effects from the data because some evolution must occur for stars at least as faint as $M_V = +5^m$, but it does eliminate those stars which are highly evolved. If these objects had not been excluded, the range of $\langle \Delta M_V \rangle$ over the various ultraviolet-excess groups would have been even greater than given above.

Blanketing corrections for M_V and for $B - V$ were obtained for each star in our four data groups from the values given by Wildey, Burbidge, Sandage, and Burbidge (1962, Table 4), who used the observed ultraviolet excess as an index of line weakening. The values of $\delta(U - B)$ were determined relative to the Hyades $(U - B, B - V)$ diagram (Sandage and Eggen 1959, Table III). Figure 2 shows the resulting $(M_V, B - V)$ diagram for all our northern stars after blanketing corrections have been applied. It is evi-

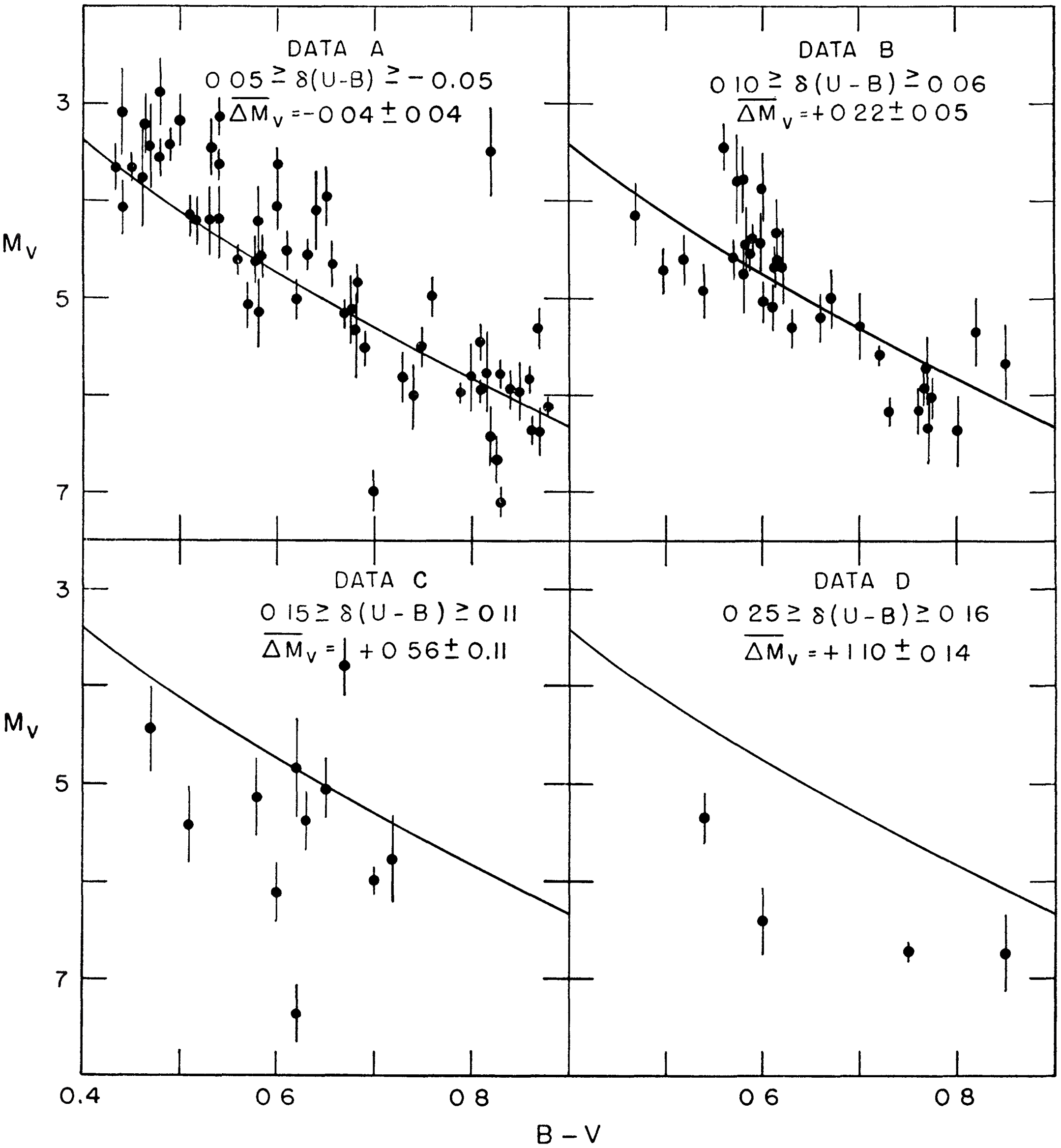

FIG. 1.—The M_V, $B - V$ diagram for stars with Allegheny parallaxes $\geq 0''.035$, separated into four ultraviolet-excess groups. No blanketing corrections have been applied. The mean deviations $\langle \Delta M_V \rangle$ from the Hyades (shown as a solid line) are shown after excluding stars which are obviously highly evolved. The sense of the differences ΔM_V is star *minus* Hyades read at the same $B - V$.

dent that the systematic correlation of $\langle \Delta M_V \rangle$ with $\delta(U-B)$ has disappeared. The extreme subdwarfs of data *D* do not correct exactly to the Hyades main sequence but, in the mean, remain fainter by only $0^m\!.15 \pm 0^m\!.11$, whereas stars in data groups *A*, *B*, and *C* are slightly overcorrected, with $\langle \Delta M_{V(\mathrm{Corr})} \rangle = -0.07 \pm 0.04$ (data *A*); $-0^m\!.20 \pm 0^m\!.05$ (data *B*); and -0.10 ± 0.10 (data *C*). In forming the values of $\langle \Delta M_{V(\mathrm{Corr})} \rangle$, those stars in data groups *A* and *B* whose advanced evolution has already been discussed, were again excluded.

The results for the southern stars are shown in Figures 3 and 4. Three instead of four data groups are used because only one southern star (HD 134439) with available photometry that also meets the parallax requirements shows an ultraviolet excess greater than $+0^m\!.15$; this object is plotted with the data group *C* stars as an open circle in Figures 3 and 4. Five highly evolved stars were discarded in data group *A* before the mean

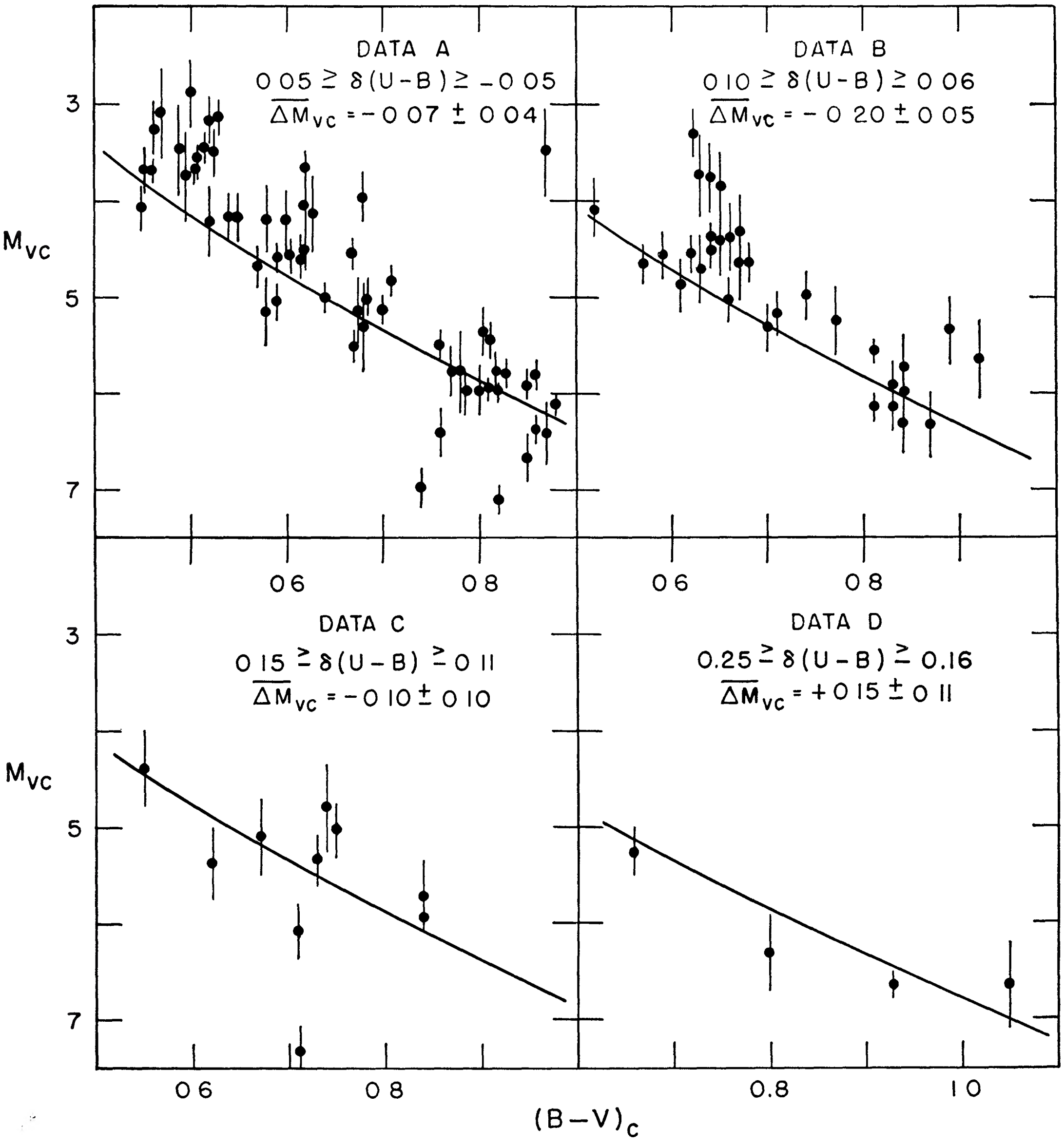

FIG. 2.—Same as Fig. 1 but with blanketing corrections applied to each star

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values of $\langle \Delta M_V \rangle$ and $\langle \Delta M_{V(\mathrm{Corr})} \rangle$ were formed. Figure 3 shows the same trend in the correlation of $\langle \Delta M_V \rangle$ with $\delta(U - B)$ as Figure 1, and again this correlation practically disappears in Figure 4, where the blanketing corrections have been applied.

From data groups *B* and *C* in Figures 2 and 4 it is evident that "subdwarfs" exist in the observed $(M_V, B - V)$ diagram only because of the effects of weak Fraunhofer lines on the observed values of *V* and $B - V$. However, data group *D*, which is of the greatest interest for the globular-cluster problem, contains only five stars in the combined northern and southern data, and the question of whether or not the blanketing corrections completely account for the departure of these stars from the Hyades main sequence is not satisfactorily answered from these data alone. We have therefore added to the stars in data group *D* the members of three high-velocity groups (Eggen 1962)—Groombridge 1830 group, Kapteyn's star group, and the group with (U, V) near $(-150, -300)$. The resulting sample of 16 stars has an average $\delta(U - B)$ of $+0^{\mathrm{m}}.21$, which is near the upper limit

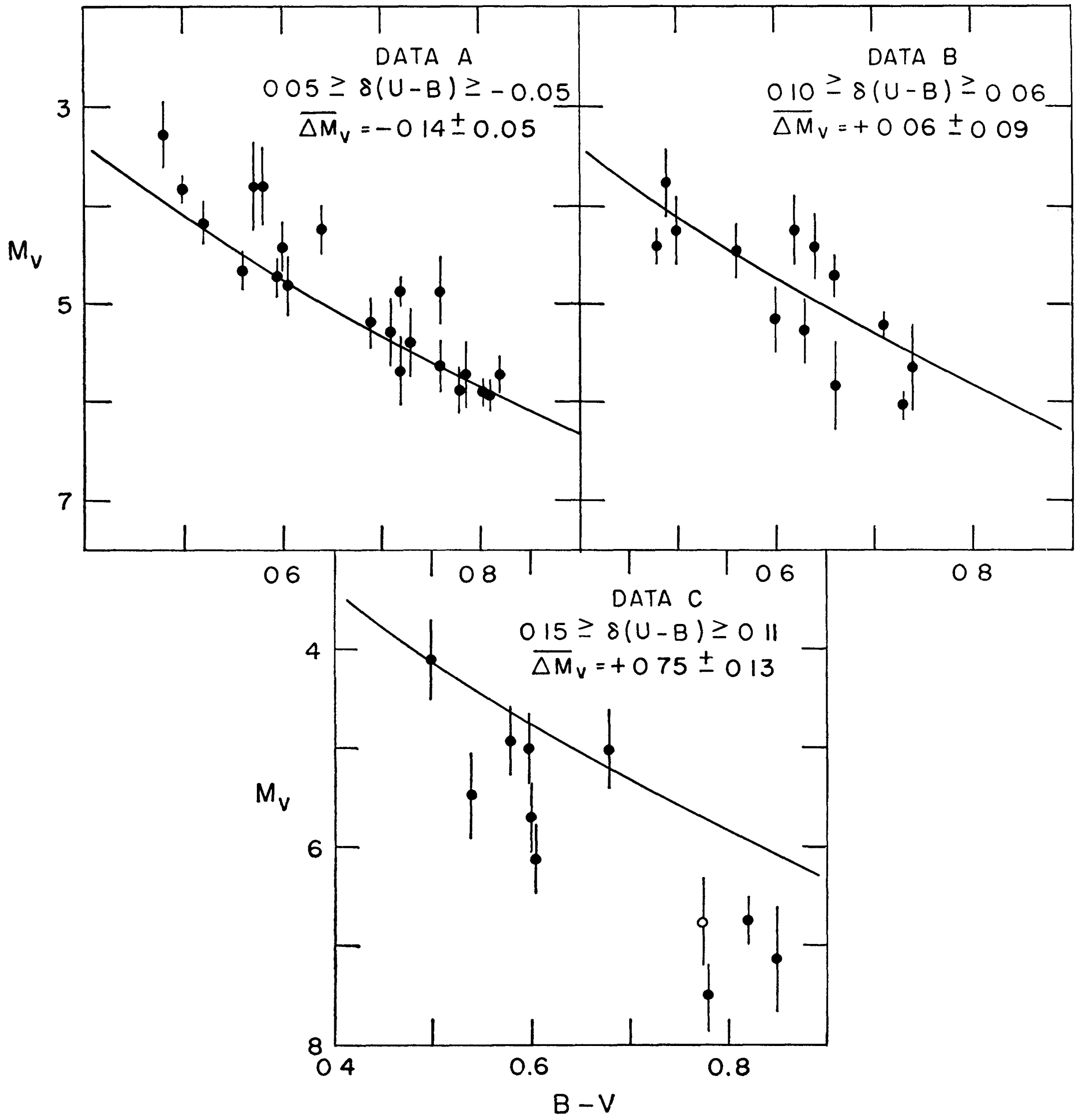

FIG. 3.—Same as Fig. 1 but for the southern stars using Cape and Yale parallax values

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for the extreme subdwarfs. Table 1 lists the relevant data, and we now take these as a sample of stars similar in all known characteristics (photometric and kinematic) to the main-sequence stars in globular clusters. Figure 5 shows the results for these objects before and after correction for blanketing effects. The filled squares in the figure represent the five parallax stars, whereas the crosses, triangles, and circles represent, respectively, members of the Groombridge 1830 group, Kapteyn's star group, and the group with (U, V) near $(-150, -300)$. Before blanketing corrections are applied, the 16 stars lie $1\overset{\mathrm{m}}{.}05 \pm 0\overset{\mathrm{m}}{.}04$ below the Hyades main sequence, whereas after correction they show only a small scatter about that sequence, with an average difference of $+0\overset{\mathrm{m}}{.}03 \pm 0\overset{\mathrm{m}}{.}05$.

The data in Figures 1, 3, and 5 are summarized in Figure 6, where $\langle \Delta M_V \rangle$ is plotted against $\langle \delta(U - B) \rangle$ for the four data groups from the Allegheny parallax material (*circles*), the three data groups from the Cape and Yale parallax material (*squares*), and the data of Figure 5 (*triangle*). Two theoretical relations are drawn, representing the ex-

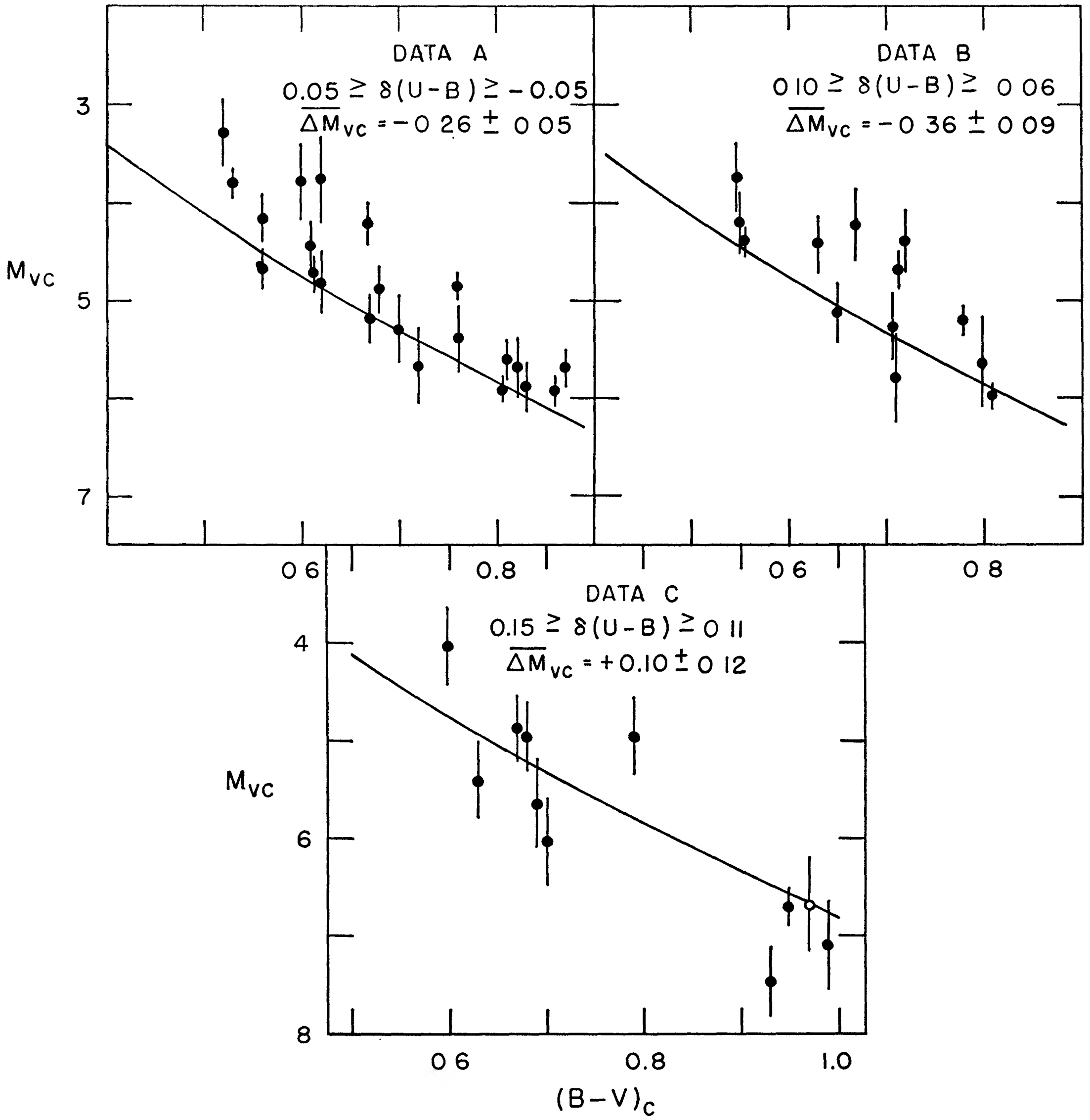

FIG. 4.—Same as Fig. 3 but with blanketing corrections applied

pected correlation between $\langle \Delta M_V \rangle$ and $\langle \delta(U - B) \rangle$ if all stars corrected exactly to the Hyades main sequence. The theoretical relations were obtained from blanketing lines in the $(U - B, B - V)$ diagram at (1) $B - V = +0^{\text{m}}.6$ and (2) $B - V = +0^{\text{m}}.8$ with the appropriate slopes, $\Delta(U - B)/\Delta(B - V)$, given by equation (9) of Wildey *et al.* (1962). These blanketing lines, in conjunction with the Hyades $(U - B, B - V)$ diagram, give a relation between $\delta(U - B)$ and $\Delta(B - V)$ that is found by reading the two quantities at the appropriate places along the blanketing lines. Finally, $\Delta(B - V)$-values, together with ΔV from Table 4 of Wildey *et al.* (1962), define two blanketing lines in the $(M_V, B - V)$ diagram for which ΔM_V-values can be obtained, corresponding to any given $\delta(U - B)$. Figure 6 indicates that the mean points from the data used here deviate slightly from the theoretical relations. The four points representing the data groups with the smallest ultraviolet excess all lie above the theoretical relations, and this is very likely an effect of evolution on the stars with $M_V = +3^{\text{m}}$–$+5^{\text{m}}$; this point is discussed below in more detail.

TABLE 1

THE MAIN SEQUENCE FOR STARS WITH $\delta(U - B) \geq 0\ 16$

NAME*	π(0 001)	M_V	$(B-V)$	$(U-B)$†	$\delta(U-B)$	$M_{V(\text{Corr})}$	$(B-V)_c$
			Trig π Stars				
Yale 763.	54±6	5 34±0 24	0 54	−0 09	0 16	5 28±0 24	0 66
Yale 1857.	41±7	6 40± 37	60	−0 12	25	6 33± 37	0 80
Yale 2745‡	111±6	6 72± 12	75	0 15	19	6 65± 12	(0 93)
Yale 3425	35±8	6 77± 50	78	(1 75)	21	6 70± 50	(0 97)
Yale 4524	38±7	6 74±0 40	0 85	0 34	0 20	6 67±0 40	(1 05)
			Moving Groups Group at (—150, —300)				
HD 74000		4 55	0 41	(1 46)	0 21	4 47	0 54
+29°2091		5 34	49	−0 20	23	5 26	65
W 7296 ..		4 78	43	−0 21	21	4 70	57
HD 108177		4 68	43	−0 25	25	4 59	60
−35°14849		4 57	0 40	(1 45)	0 23	4 49	0 54
			Kapteyn's Star Group				
W 5757	..	5 27	0 58	−0 12	0 23	5 19	0 76
HD 106038		4 78	48	− 18	20	4 70	63
W 8296	.	5 68	0 60	−0 10	0 23	5 61	0 78
			Groombridge 1830 Group				
+72°94		4 74	0 41	−0 21	0 22	4 66	0 55
Gm 1830‡.		6 72	75	0 15	19	6 65	(93)
HD 163810		5 92	0 60	(1 59)	0 20	5 86	0 76

* Yale numbers refer to the Yale *Catalogue of Parallaxes* (1952 ed) W numbers refer to the Wilson *Catalogue of Radial Velocities* ("Carnegie Institution of Washington Publications," No 601)

† Values in parenthesis are on the Cape refractor system.

‡ Groombridge 1830 appears in both the parallax and the monvig-group list but is counted only once in the calculations

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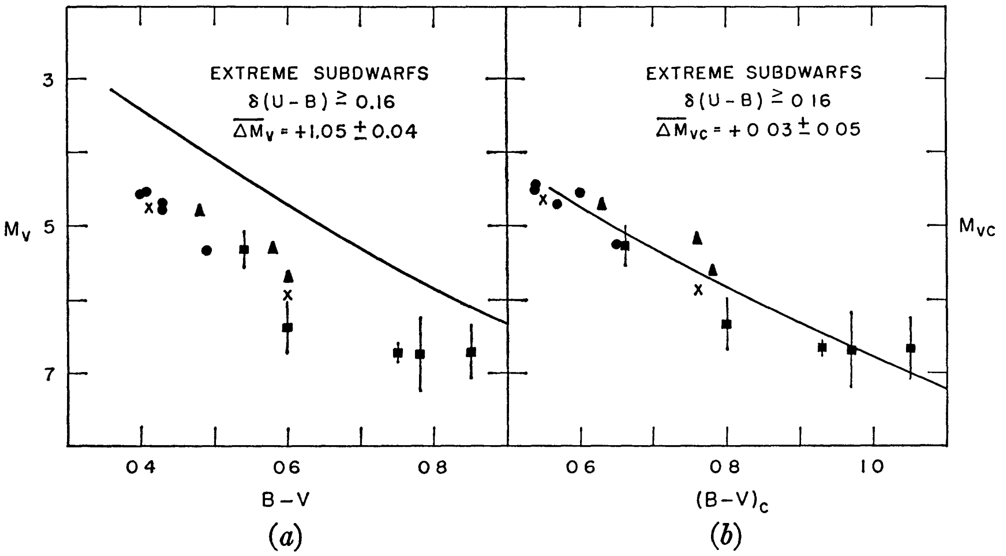
The conclusion from Figures 1–6 is that the position of main sequences in the (M_V, $B - V$) diagram, after corrections for line blanketing, is not greatly affected by the large variations in chemical composition of the stars in our sample and that the faint Hyades main sequence can be used as a standard when the blanketing corrections of Wildey *et al.* (1962, Table 4) are applied. In particular, Figure 5 suggests that photometric moduli of the extreme subdwarfs can be determined with a systematic error of less than $0^{\mathrm{m}}.1$ by using the Hyades main sequence and the corrections for line blanketing.

Note added in proof, September 18, 1962.—In principle, these results permit the deter-



(*a*) (*b*)

FIG. 5.—(*a*) The uncorrected data for the stars in Table 1. Squares represent the five parallax stars, crosses are the Groombridge 1830 group, triangles are the Kapteyn's star group, and circles are for the −150, −300 group. The Hyades line is also shown. (*b*) Same as *a* but with blanketing corrections applied.

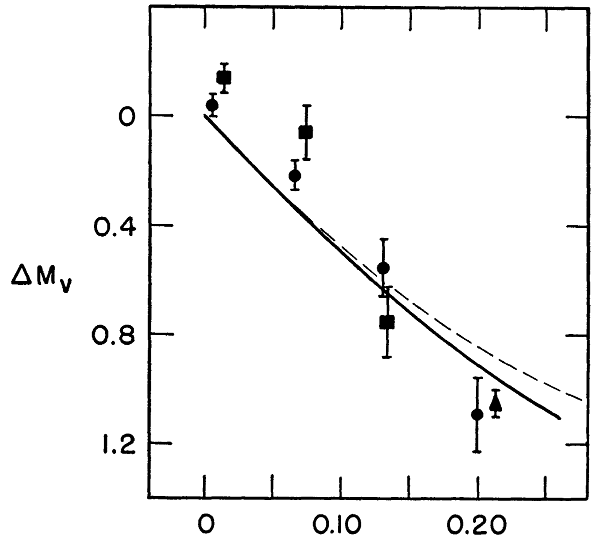


FIG. 6.—The correlation of the mean ΔM_V and the mean $\delta(U - B)$. Circles are the four data groups from the Allegheny data shown in Fig. 2; squares are the three data groups from the Cape plus Yale data; and the triangle is the extreme subdwarf sample of Fig. 5. Two theoretical lines show the expected relation if all stars correct to the Hyades exactly and if no evolutionary effects occur.

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mination of the helium content of the subdwarfs relative to Hyades-like stars. It is well known that if the metal abundance of the stars is decreased, keeping the hydrogen and helium content the same, then a true subdwarf sequence will occur in the M_{bol}, log T_e plane. But if Y is decreased at the same time that Z is decreased, then a compensation will occur in the position of the main sequence in the M_{bol}, log T_e plane. Exact compensation, as our present data require, can only be accomplished if the change in Z is related to the change in Y in a certain unique way (see, e.g., Sandage and Eggen 1959, p. 292, for an illustrative example using homology arguments). Homology arguments can give the direction of the expected changes in the H-R diagram for changes in Z and Y, but the exact values must come from detailed stellar models. The stellar model calculations of Demarque (1960) have been used in a preliminary way together with our present data to estimate what the helium content of the stars in Table 1 must be for exact compensation to occur. The result is that if $X = 0.60$, $Z = 0.04$ for the Hyades stars, then $X \simeq 1.0$, $Y \simeq 0.0$ for the extreme subdwarfs. This suggests that the extreme subdwarfs, which are the oldest stars we know, have a very small helium content and, therefore, that the primeval helium abundance in the Galaxy may have been close to zero. However, more work on this important point is needed before the result is certain.

III. COMPARISON WITH CODE'S RESULTS

The results of the previous section depend critically on the validity of the blanketing corrections. If equation (9) and therefore Table 4, of Wildey *et al.* (1962), are incorrect, perhaps because of systematic errors in the spectrophotometry used in determining the blocking coefficients, $\epsilon(\lambda)$, then our corrected values of $(B - V)$ will be in error and the conclusions therefrom vitiated. Code's (1959) discussion of the $(M_V, G - I)$ diagram, mentioned above, would indeed suggest that our blanketing corrections may be too large. This critical point has been reinvestigated by using unpublished six-color observations of 31 subdwarfs very kindly made available by R. Sears and A. E. Whitford. The majority of these subdwarfs have available (U, B, V) photometry, so that no transformations from the six-color to the (U, B, V) system are involved. The observed values of $\delta(U - B)$ for the Sears-Whitford stars range from $+0^{\mathrm{m}}.02$ to $+0^{\mathrm{m}}.27$, with a mean of $\langle\delta(U - B)\rangle = +0^{\mathrm{m}}.14$.

The test of the blanketing theory consists of the following steps. (1) We obtain a value of M_V for each of the Sears-Whitford stars by a force-fit to the $(M_V, B - V)$ diagram of the Hyades main sequence after the observed values of $B - V$ are corrected by an amount appropriate for the observed value of $\delta(U - B)$ as given by Table 4 of Wildey *et al.* (1962). (2) We then plot the values of M_V, resulting from step 1, against the values of $[G - I]$ observed by Sears and Whitford. If our adopted corrections to $B - V$ are systematically wrong, then the luminosities obtained by the force-fit to the Hyades $(B - V, M_V)$ diagram will also be wrong, and the stars will be systematically displaced from the Hyades $(M_V, G - I)$ diagram. This follows because Sandage and Smith (1963) have shown that the blanketing corrections to $[G - I]$ are vanishingly small.

The results of step 2 above are shown in Figure 7, where the average displacement of the Sears-Whitford stars below the Hyades $(M_V, G - I)$ diagram is only $0^{\mathrm{m}}.04 \pm 0^{\mathrm{m}}.03$. The Hyades main sequence shown in Figure 7 was obtained from extensive six-color observations of Hyades stars, also by Sears and Whitford. We therefore do not confirm Code's result that $\Delta M_V \simeq +1.0 \pm 0.4$ in the $(M_V, G - I)$ plot but rather conclude that our adopted $\Delta(B - V)$-values, based on Table 4 of Wildey *et al.*, are systematically valid and therefore that a difference may exist between Sears and Whitford's six-color data and Code's transformation of his $G - I$ colors to that system.

IV. THE INTRINSIC WIDTH OF THE MAIN SEQUENCE

Most modern studies of the $(M_V, B - V)$ diagram for nearby stars (Eggen 1955; Sandage 1958; Stoy, quoted by Arp 1958) show that the main sequence has a spread of at least 1 mag. at $B - V$ near $+0^{m}.6$, even when stars with parallaxes of the highest weight are used. It seems clear that the largest part of this dispersion is caused by neglect of the blanketing corrections to the observed $B - V$ colors for stars of low metal abundance. This is demonstrated in Figure 8, where all the stars discussed above have been plotted with the observed values of $B - V$ (panel *a*) and after correction for blanketing (panel *b*). As already noticed above in connection with Figures 2, 4, and 6, the stars bluer than $B - V$ near $+0^{m}.7$ fall systematically brighter than the Hyades, as might be expected from a consideration of the initial evolution from the age-zero main sequence. Figure 9 shows the same stars as those contained in Figure 8, *b*, but with the schematic (M_V, $B - V$) diagrams of the old galactic clusters M67 and NGC 188 superposed (Sandage 1962*a*, *b*). Three features of Figure 9 are noteworthy: (1) the scatter of the stars redder than the "turnoff" of the NGC 188 main sequence is very nearly symmetrical about the

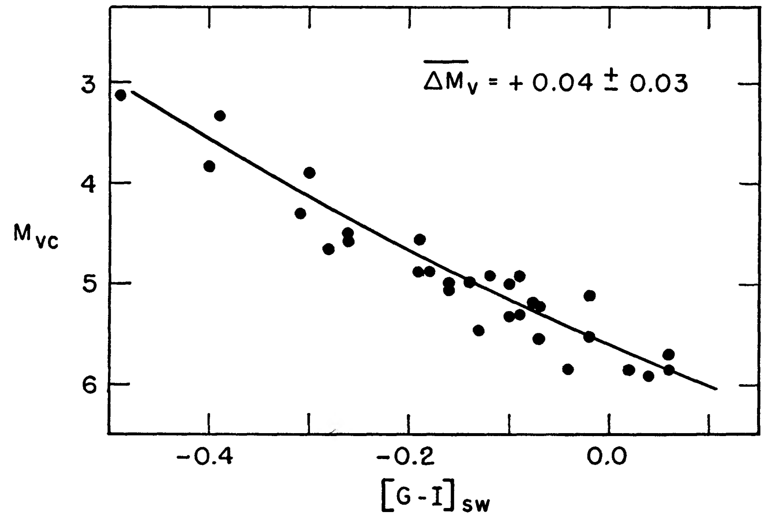

FIG. 7.—M_V(Corr), $[G - I]_{SW}$ diagram for 31 subdwarfs observed by Sears and Whitford. The M_V(Corr) were obtained by force-fitting each star to the M_V, $B - V$ Hyades line after blanketing corrections were applied. If the blanketing values ΔV and $\Delta(B - V)$ were incorrect, the stars should deviate from the Hyades line in this diagram.

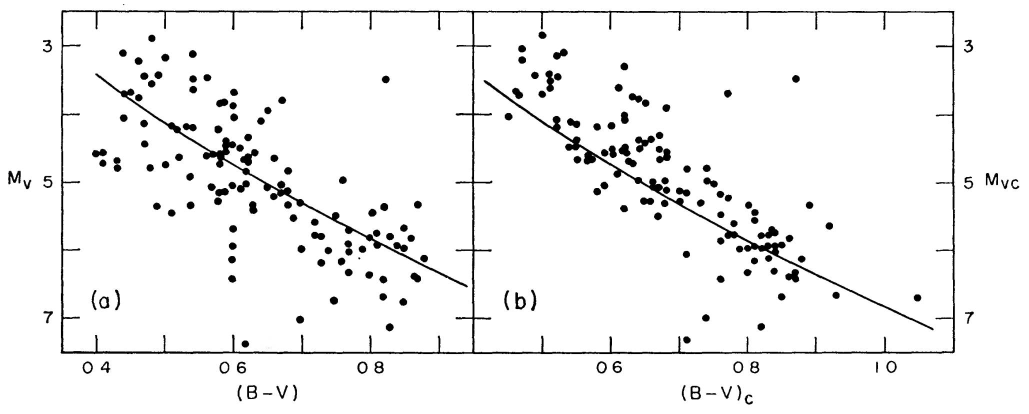

FIG. 8.—(*a*) All the stars shown in Figs. 1–5 plotted together. No blanketing corrections are applied. The Hyades main sequence is shown. (*b*) Same as *a* but with blanketing corrections applied to each star.

Hyades main sequence, with a total range to the scatter of about $0^m.3$, as might be expected from the probable errors of the parallaxes alone; (2) the position of the field stars between the M67 and NGC 188 termination points is consistent with an evolutionary picture in which stars of all ages exist up to, but not exceeding, the age of NGC 188; and (3) the sequence of evolving subgiants in NGC 188 forms the lower boundary for the most obvious subgiants among the parallax stars.

V. APPLICATION TO THE GLOBULAR CLUSTERS

The results of Section II, especially of Table 1 and Figure 5, have a direct application to the problem of the distances of globular clusters. The procedure which has recently been used to obtain the distance of these objects is to determine the blanketing corrections ΔV and $\Delta(B - V)$ for each main-sequence star and then to fit the resulting, corrected $(V, B - V)$ diagram to the $(M_V, B - V)$ diagram of the Hyades main-sequence stars. This procedure now seems to be fully justified by Figure 5. However, it is extremely difficult accurately to measure $(U - B)$ colors for faint globular-cluster stars. The error in the derived distance modulus is about six times the error in $\Delta(B - V)$, corresponding to an error of about five times the error in the observed ultraviolet excess due to faulty photometry. Except in the one or two most favorable cases, errors in the observed values of $U - B$ can easily be as much as $0^m.05$ for a given star fainter than the twentieth magnitude, which leads to errors of at least $0^m.2$ in the final distance modulus due to inaccuracies in the applied blanketing corrections alone.

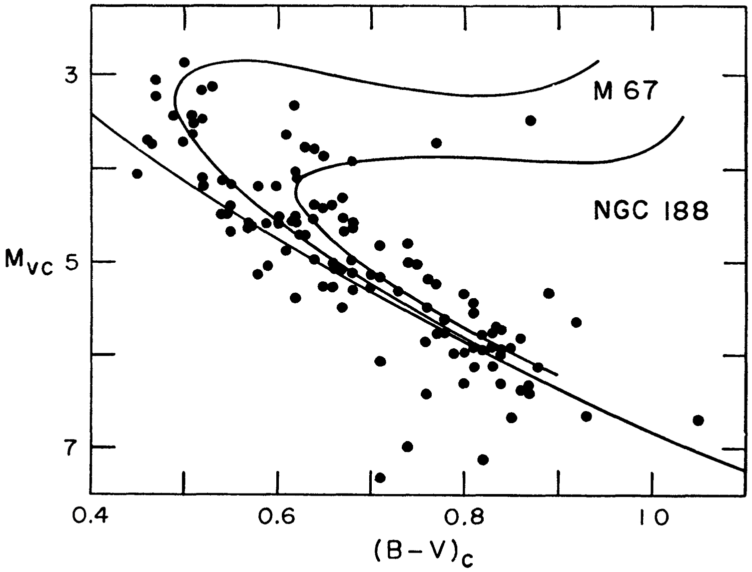

FIG. 9.—Same as Fig. 8, *b*, but with the schematic sequences for M67 and NGC 188 superposed

For the present, it may be better to circumvent the blanketing corrections entirely and to fit the observed color-magnitude arrays to the uncorrected sequence of Figure 5. This procedure, of course, runs the danger of neglecting differences in $\delta(U - B)$ from cluster to cluster because the procedure requires that the mean $\langle\delta(U - B)\rangle$ for the cluster stars be near the value $+0^m.21$, obtained from stars in Table 1. The data now available do not permit a conclusive answer to the question of whether or not the ultraviolet excesses of the main-sequence stars in such clusters as M3, M13, M15, and M92 are the same. Until the problem of accurate $(U - B)$ photometry of many faint cluster stars is solved, moduli found from the "standard" sequence defined by the 16 stars in Table 1 are probably as accurate as the values obtained from the application of the blanketing theory to the individual stars. Figure 10 illustrates the use of this procedure in the case of M13, where no blanketing corrections have been applied. The mean value of the ultraviolet excess for the M13 stars (Baum *et al.* 1959) is $+0^{m}.23$, which is close enough to the value of $+0^{m}.21$ for our "standard" sequence to justify the procedure. Figure 10 shows that the shapes of the main sequences are similar and that a good fit is possible. The resulting modulus is

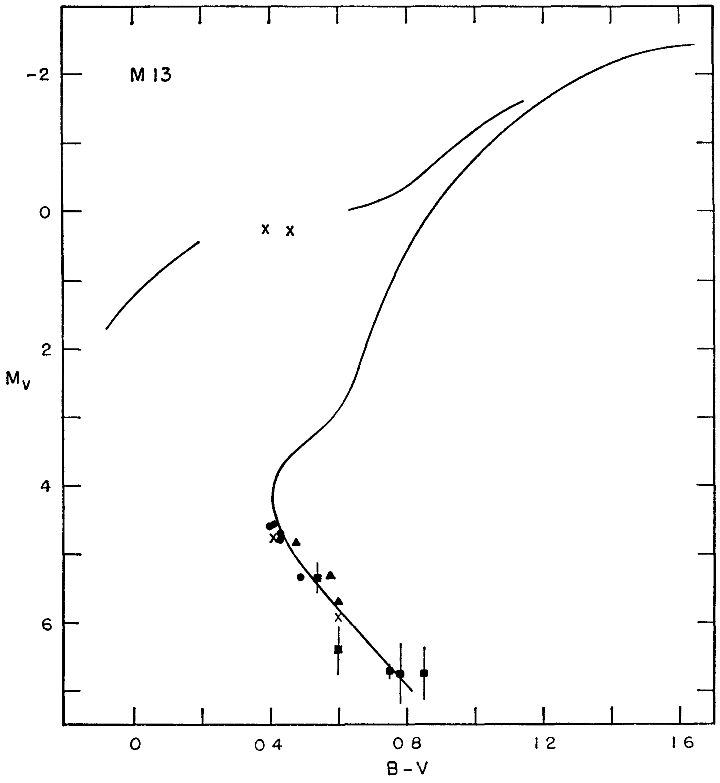

FIG. 10.—The M13 diagram fitted to the "standard sequence" of Table 1. The symbols are the same as in Fig. 5.

$m - M = 14^{m}.30$ (no reddening), which puts two of the RR Lyrae stars at $M_V = +0^{m}.25$ and $+0^{m}.29$ if we adopt, with Arp (1955), visual magnitudes of $14^{m}.55$ and $14^{m}.59$, respectively. The modulus and absolute magnitudes are almost identical with those obtained previously by Eggen and Sandage (1959), using only the Groombridge 1830 group, and by Baum *et al.* (1959), who used the full blanketing theory.

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