# STELLAR ROTATION IN GALACTIC CLUSTERS* 

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#### Abstract

Rotational velocities are derived for the brighter stars in the I Lacerta and I Orion associations and the $a$ Persei and Pleiades clusters. These, as well as published data on four other clusters, show that each group has a distinctive dependence of mean rotational velocity on luminosity. For field stars the irregularities in the dependence of mean rotational velocity on luminosity and its dips at B2 and A2 are seen as being due to large admixtures of stars from certain groups which have distinctive rotational velocities, such as the Ursa Major stream and the Cassiopeia-Taurus stars There is evidence in four groups for a strong inverse correlation between binary frequency and mean rotational velocity.


## I. INTRODUCTION

A considerable amount of research has been done (see Treanor 1960 for a bibliography) on the stellar axial rotational velocities of field stars which are on or slightly evolved off the main sequence. As a consequence, we can assume that the general dependence of projected rotational velocities on spectral type is known for a large sample of such stars. The question raised in this paper is whether small samples of stars, such as individual galactic clusters, conform to this mean dependence.

We already have evidence that in some clusters the mean rotational velocities for all the stars, or at least for those of certain spectral types, are higher or lower than for field stars of similar types. Struve (1945) came to the conclusion that the B stars in the Pleiades have unusually large rotational velocities. McNamara (1960) found the B0.5-B3 stars in the I Orion association to have about two-thirds the mean rotational velocities of field stars of the same types. Treanor (1960) found that, whereas the A stars in the Hyades and Praesepe clusters rotate less rapidly than field stars, the F stars in these clusters rotate more rapidly than similar field stars. On the other hand, Deutsch (1955) and Meadows (1961) found the rotational velocities of stars in IC 4665, M39, and the Ursa Major clusters to be normal. Why should clusters differ in this characteristic, and upon what does it depend?

To help answer these questions, additional material was obtained in the form of moderately high-dispersion spectra of the brighter stars in four clusters or associations, namely, the I Lacerta and I Orion associations and the a Persei and Pleiades clusters. A description of the spectrographic material, the method of reduction, and a comparison with measures by others are given in the following section. The results are described and discussed in Section III.

## II. OBSERVATIONS AND REDUCTIONS

Spectrograms $18 \mathrm{~A} / \mathrm{mm}$ in dispersion and 0.5 mm in width were obtained on Eastman Kodak II $a$-O plates with the C camera of the McDonald 82 -inch coudé spectrograph of 29 O9-B3 stars in I Lacerta, 42 O9.5-B5 stars in I Orion, 22 B3-A1 stars in the a Persei cluster, and 23 B6-A1 stars in the Pleiades. The stars were selected in the following way.

[^0]All members of the I Lacerta association according to Blaauw and Morgan (1953) were observed. From the list of I Orion association stars by Sharpless (1952), we selected the first 42 stars of spectral type B5 and earlier, although spectra were not obtained for HD 35899 , 36151, $36429,36629,36842$, 36936, and 36958. Among the a Persei cluster members as listed by Roman and Morgan (1950), all but one (HD 21942) of the stars of spectral type A0 and earlier were measured, and one later star (HD 20391, A1 V) was included. From Mendoza's study (1956) of the Pleiades, we selected all stars classified as A1 or earlier.

Normally, only one good spectrum was obtained of each star. The lines $\lambda \lambda 4026,4471$, and 4481 were traced, whenever they were present, with a Hilger and Watts L470 density-recording microphotometer at a magnification of $\times 101$. Half-widths were measured and plotted against Slettebak (1954) and Slettebak and Howard's (1955) values of $V \sin i$ for the 22 stars that they measured in these groups. The resulting calibration-curves, plotted for each line and for three spectral ranges, were used to obtain the projected rotational velocities for the remaining stars. For the 74 stars for which both He I lines were measured, the resulting rotational velocities from $\lambda 4471$ were $0.8 \pm 17.8$ (p.e. per star) $\mathrm{km} / \mathrm{sec}$ smaller than for $\lambda 4026$.

As a check against accidental errors, effects of occasional soft focus on the tracings, and blends with nearby lines, each spectrum was compared visually on a Hilger and Watts Judd Lewis spectrum comparator with that of one of the stars measured by Slettebak and Howard.

The projected rotational velocities are listed in Tables 1-4. The spectral types for stars in I Lacerta are by Blaauw and Morgan (1953), for I Orion by Sharpless (1952) or Morgan, Code, and Whitford (1955) or Slettebak and Howard (1955), for a Persei by Roman and Morgan (1950), and for the Pleiades by Mendoza (1956).

The following comparisons can be made between various sets of rotational velocities: (1) for 22 stars the difference between the present velocities and those of Slettebak and Howard, against which these have been calibrated, is $+1.8 \pm 7.4$ (p.e. per star) $\mathrm{km} / \mathrm{sec}$; (2) for 21 stars in I Orion (not including HD 35912) in common with McNamara and Larsson (1962), the present velocities are larger by $+2.9 \pm 17.4$ (p.e. per star) $\mathrm{km} / \mathrm{sec}$; (3) for seven Pleiades stars, Smith and Struve's estimated rotational velocities are $+12.9 \pm 28.3$ (p.e. per star) $\mathrm{km} / \mathrm{sec}$ larger than Slettebak's, while van Dien's measures (1948) are $33.3 \pm 28.9$ (p.e. per star) $\mathrm{km} / \mathrm{sec}$ smaller than Slettebak's.

## III. RESULTS AND DISCUSSION

Because of the unknown inclination of any individual star, all work on rotational velocities must be statistical in nature. As such, one wishes to include as large a number of stars as possible in the means to obtain greater accuracy. However, probably the principal result of this paper is that in the past this averaging has been carried too far, that the unique dependence of $V \sin i$ on luminosity or spectral type for each cluster has been missed in averaging over the cluster as a whole. One would wish to use extremely rich clusters, e.g., a cluster with 50 B1-B3 stars, 75 B5-B7 stars, etc. However, instead we must work with the real clusters that happen to be nearby, because only for those can we obtain the moderately high-dispersion spectra required. If a cluster, like the Pleiades, contains only 23 B6-A1 stars, to subdivide this sample several times incurs the danger of obtaining results that are statistically meaningless, but not to subdivide it incurs the danger of missing the result that the B stars have unusually high rotational velocities, while those of the A stars are unusually low. We shall compromise by using samples of at least 8-9 stars, whenever possible, but bear in mind that the means do not have high accuracy.
.The cluster rotational velocities will be compared with those for field stars by, appropriately, Slettebak (1949, 1954, 1955) and Slettebak and Howard (1955), except for the F6-F8 V sample, which was measured by Herbig and Spalding (1955) from higher-
dispersion spectra. The term "field stars" is, in part, a misnomer because many of the stars in these samples are members of clusters or associations; we shall use the term to represent the miscellaneous cross-sectional sample by these authors of stars generally brighter than 5.0 mag. apparent. The field-star mean velocities are plotted in Figure 1, where averages over spectral ranges of three spectral subclasses were taken in nearly every case and where the sizes of the samples (15-65 stars, average of 40 stars) are represented by the sizes of the circles; they are plotted at their weighted mean spectral types. A smooth curve is drawn, even though Boyarchuk and Kopylov (1958) present evidence for the existence of dips at B2 and A2. We shall return to this point below in Subsection $a$.

The normal points for the four clusters or associations are plotted in Figures 2 and 3.
TABLE 1
I Lacerta Association

| HD | Other <br> Designation | Spectral Type | $V \sin i(\mathrm{KM} / \mathrm{SEC})$ |  | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Slettebak and Howard | Present |  |
| 209961 | HR 8427 | B2 V |  | 160 |  |
| 211835 |  | B3 Ve |  | 265 |  |
| 212883 | HR 8549 | B2 V |  | 25 |  |
| 212978 | HR 8553 | B2 V |  | 115 |  |
| 213420 | 6 Lac | B2 IV | 75 | 70 |  |
| 213976 |  | B1 5 V |  | 135 |  |
| 214167 | 8 Lac B | B2 V |  | 30 |  |
| 214168 | 8 Lac A | B1 Vne |  | 365 |  |
| 214240 | HR 8606 | B3 V |  | 60 |  |
| 214263 |  | B2 V |  | 125 |  |
| 214432 |  | B3 V |  | 185 |  |
| 214652 |  | B2 V |  | 115 |  |
| 214680 | 10 Lac | O9 |  | 25 |  |
| 214993 | 12 Lac | B2 III | 80 | 70 |  |
| 215191 | HR 8651 | B1 V |  |  | Probable double-lined spec binary* |
| 216092 |  | B1 V |  |  | Possible double-lined spec. binary $\dagger$ |
| 216200 | 14 Lac | B3 IV |  | 225 |  |
| 216534 |  | B3 V |  | 65 | Double-lined spec binary; velocity refers to stronger component $\ddagger$ |
| 216684 |  | B3 V |  | 125 |  |
| 216851 |  | B3 Vn |  | 310 |  |
| 216916 | 16 Lac | B2 IV |  | 20 |  |
| 217101 | HR 8733 | B2 IV-V |  | 150 |  |
| 217227 |  | B2 IV |  | 30 |  |
| 217543 | HR 8758 | B3e |  | 370 | Shell spectrum |
| 217811 | HR 8768 | B2 V |  | 25 |  |
| 218325 |  | B3 V |  | 220 |  |
| 218344 |  | B2 V |  | 95 |  |
| $\begin{aligned} & 218407 \\ & 218674 \end{aligned}$ | HR 8800 | $\begin{aligned} & \text { B2 V } \\ & \text { B3 IV } \end{aligned}$ |  | 160 |  |
| 218674 |  | B3 IV |  |  | nary§ |

* On November 8 161, 1957 (U T ), the velocities from five line pairs were primary, $+139 \pm 67$ (pe ), secondary, $-1208 \pm$ $91 \mathrm{~km} / \mathrm{sec}$
$\dagger$ Curious structure in $\lambda 4026$ and $\lambda 4471$ on December 12 076, 1957
$\ddagger$ On December 1 136, 1957, the velocities were $-1615 \pm 30$ and $+16 \pm 38 \mathrm{~km} / \mathrm{sec}$, where the stronger component of each of six line pairs was consistently the negative one
§ On November 13 165, 1957, the velocities from three line pairs were primary (sharper lines), $+511 \pm 192$; secondary, $-1247 \pm 106 \mathrm{~km} / \mathrm{sec}$

TABLE 2
I ORion Association

| HD | $\begin{gathered} \text { Other } \\ \text { Designation } \end{gathered}$ | Spectral Type | $V \sin i(\mathrm{Km} / \mathrm{sec})$ |  |  | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Slettebak and Howard | $\left\|\begin{array}{c} \text { McNamara } \\ \text { and } \\ \text { Larsson } \end{array}\right\|$ | Present |  |
| 30836 | $\pi^{4}$ Ori | B2 III | 40 | 40 | 35 |  |
| 31237 | $\pi^{5}$ Ori | B2 III | 90 |  | 90 |  |
| 34511 |  | B5 IV |  |  | 35: |  |
| 34748 | HR 1748 | B1 5 V |  | 280 |  | Double-lined spec binary* |
| 34959 | HR 1761 | B5p |  |  | 445 |  |
| 34989 | HR 1763 | B1 V |  | 50 | 55 |  |
| 35007 | HR 1764 | B3 V |  |  | 40 |  |
| 35039 | 22 Ori | B2 IV | 20 | 0 | 0 |  |
| 35079 |  | B3 V |  |  | 150 |  |
| 35149 | 23 Ori A | B1 V | 280 | 260 | 280 |  |
| 35299 | HR 1781 | B1 V |  | 0 | 30 |  |
| 35407 | HR 1786 | B5 V |  |  | 450 |  |
| 35411 | $\eta$ Ori | B0 5 V |  | 35 |  | Peculiar line profiles |
| 35439 | 25 Ori | B1 Vpe | 295 | 350 | 300 |  |
| 35502 |  | B5 V |  |  | 290 | Shell spectrum |
| 35575 |  | B3 V |  | . | 120 |  |
| 35588. | HR 1803 | B3 V |  |  | 170 |  |
| 35715. . | $\psi$ Ori | B2 IV |  | 140 |  | Double-lined spec. binary on Perkins plates $\dagger$ |
| 35730. |  | B5p |  | 50 | 75: |  |
| 35762 | . | B2 V |  | 170 | 140 |  |
| 35777 |  | B2 V |  | 275 | 335 |  |
| 35792 |  | B3 V |  |  | 65 |  |
| 35912 | HR 1820 | B2 | . | 5 | 85: | Shell characteristics? |
| 30613 |  | B1 5 V |  | 295 | 300 |  |
| 36133 |  | B2 V |  | 255 | 210 |  |
| 36166 | HR 1833 | B1 5 V |  | 185 | 165 |  |
| 36267 | 32 Ori | B5 V | 190 |  | 195 |  |
| 36285 | HR 1840 | B1 5 V |  | 0 | 20 |  |
| 36351 | 33 Ori | B1 5 V |  | 30 | 40 |  |
| 36392 |  | B3 V |  |  | 45 |  |
| 36430 | HR 1848 | B2 V |  | 10 | 40 |  |
| 36486 | $\delta$ Ori | O9 5 V |  |  | $150 \ddagger$ |  |
| 36512 | $v$ Ori | B0 V | 10 | 15 | 20 |  |
| 36591 | HR 1861 | B1 V |  | 20 | 20 |  |
| 36646 | HR 1863 | B3 V |  |  | 215 |  |
| 36695 | VV Ori | B1 V |  | 190 | 170: |  |
| 36741 | HR 1871 | B2 V |  | 200 | 195 |  |
| 36779. | HR 1873 | B3 V |  |  | 200 |  |
| 36824 | . . | B3 V |  |  | 175 |  |
| 36954 |  | B3 V |  |  | 180 |  |
| 36959 | HR 1886 | B1 V |  | 5 | 35 |  |
| 36960 | HR 1887 | B0 Vp |  | 30 | 25 |  |

[^1]Again, the sizes of the spectral ranges are nearly identical with those for the field stars, and the sizes of the circles indicate the numbers of stars (6-23, average of 12) per mean; these means are plotted at their weighted mean spectral types. Thus the data for the field and for the cluster stars were treated in effectively identical ways. Also plotted (with $x$ 's) with the Pleiades measures are Smith and Struve's estimates, decreased by 6.8 per cent in accordance with the above comparison of B stars in that cluster with Slettebak and Howard's measures. The diagrams include Treanor's measures (1960) for the II Persei association and the Hyades and Praesepe clusters. These two clusters are fairly similar and have been averaged, although they may not be identical; Praesepe seems to deviate more from the normal curve than do the Hyades. Meadows' measures

TABLE 3
a Persei Cluster

| HD | Other <br> Designation | Spectral Type | $V \mathrm{SIN} \boldsymbol{i}(\mathrm{KM} / \mathrm{sec})$ |  | Remaris |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Slettebak and Howard | Present |  |
| 20365 | 29 Per | B3 V | 145 | 145 |  |
| 20391 |  | A1 V |  | 290 |  |
| 20418 | 31 Per | B5 V | 320 | 320 |  |
| 20809 | HR 1011 | B5 V | 250 |  | Line structure and possible var. line widths* |
| 20961. |  | A0 V |  | 25 |  |
| 21071 | HR 1029 | B6 V |  | 70 |  |
| 21091 |  | A0 V |  | 340 |  |
| 21181 |  | B9 V |  | 345 |  |
| 21278 | HR 1034 | B3 V | 50 | 75 |  |
| 21362. | HR 1037 | B6 V |  | 385 |  |
| 21398 |  | B9 V |  | 135 |  |
| 21428 | 34 Per | B5 V | 180 | 200 |  |
| 21455 | HR 1047 | B5 V |  | 150 |  |
| 21481 |  | A0 V | . | 290 |  |
| 21551 | HR 1051 | B8 IV | - | 380 | Shell spectrum |
| 21641 |  | B9 V |  | 215 |  |
| 21672 |  | B8 V | . | 225 |  |
| 21699 | HR 1063 | B8 III |  | 50 |  |
| 21931 |  | B9 V |  | 205 |  |
| 22136 |  | B8 V |  | 25 |  |
| 22192 | $\psi$ Per | B5 en | 390 | 385 |  |
| 22401 |  | A0 V |  | 25 |  |

* Campbell and Moore (1928) report possible line doubling on two spectrograms
(1961) for M39 and the Ursa Major clusters indicate, insofar as their few members allow, that these groups are very similar. Straight lines were drawn in Figures 2 and 3 whenever a lack of more points prevented a better determination of the shapes of the curves. Metallic-line stars were excluded in all the data used in Figures 1-3.

Finally, the curves in Figures 1-3 are repeated in Figure 4 for a comparison of cluster and field stars. From this we conclude the following: (1) the dependence of mean projected rotational velocity, $\langle V \sin i\rangle$, on luminosity or spectral type is distinct or nearly distinct for each group; (2) the function $\langle V \sin i\rangle\left(M_{v}\right)$ for each group seems to cross that for field stars, i.e., in each group the stars of some luminosities rotate more slowly on the average than do field stars of similar types, while those of other luminosities rotate more rapidly; (3) the curves $\langle V \sin i\rangle\left(M_{v}\right)$ for the seven groups show no apparent consistant pattern. These results immediately raise several questions which are discussed in the following three subsections.

## a) Rotational Velocities of Field Stars

If the function $\langle V \sin i\rangle\left(M_{v}\right)$ for field stars is a composite for a limited number of nearby groups with distinct dependences of mean $V \sin i$ on $M_{v}$, might not the irregularities in that function be due to the predominance of certain groups? In other words, can the dips found at B2 and A2 in Slettebak and Howard's data or by Boyarchuk and Kopylov be due to the predominance of stars of certain clusters in their averages? After a study of Slettebak and Howard's material we find that the dip at A2 can be attributed entirely to the admixture of the Ursa Major stream stars (Roman 1949), which seem to have nearly the same dependence of mean $V \sin i$ on luminosity as the Ursa Major cluster stars studied by Meadows (1961). Slettebak's sample of 53 B8-A0 and 15 F0-F2 dwarfs contain, respectively, only three and one Ursa Major stream stars and are therefore not significantly affected by them; however, the 12 slowly rotating A1-A3 (<V

TABLE 4
Pleiades Cluster

| HD | $\begin{gathered} \text { Other } \\ \text { Designa- } \\ \text { tion } \end{gathered}$ | Spec- <br> tral <br> Type | $V \sin i(\mathrm{KM} / \mathrm{SEC})$ |  |  |  | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Slettebak | Smith and Struve | Van <br> Dien | Present |  |
| 23288 | 16 Tau | B7 IV | 235 | 250 | 219 | 235 |  |
| 23302 | 17 Tau | B6 III | 245 | 200 | 158 | 230 |  |
| 23324. | 18 Tau | B8 V |  | 150 | 178 | 235 | Possible line structure |
| 23338 | 19 Tau | B6 V | 140 | 150 | 95 | 140 |  |
| 23387 |  | A1 V |  | 0 | 99 | 20 |  |
| 23408 | 20 Tau | B7 III | 30 | 25 | $\leq 30$ | 35 |  |
| 23410 |  | A0 V |  | 100 |  | 185 |  |
| 23432 | 21 Tau | B8 V |  | 150 | 119 | 210 |  |
| 23441 | 22 Tau | B9 V |  | 200 | 256 | 295 |  |
| 23480... | 23 Tau | B6 IV | 315 | 300 | 224 | 320 |  |
| 23512 |  | A0 V |  | 50 |  | 155 |  |
| 23568 |  | B9 5 V |  | 150 | 169 | 280 |  |
| 23629 |  | A0 V |  | 100 |  | 155 |  |
| 23630 | $\eta$ Tau | B7 III | 210 | 250 | 178 | 215 |  |
| 23632 |  | A1 V |  | 150 |  | 260: |  |
| 23642 |  | A0 V |  | 0 |  | 30: | Estimated; refers to pri mary |
| 23753 | HR 1172 | B8 V |  | 300 | 250 | 305 |  |
| 23763 |  | A1 V |  | 50 |  | 100 |  |
| 23850 | 27 Tau | B8 III | 160 | 250 | 198 | 185 |  |
| 23862. | 28 Tau | B8p |  |  |  | 380: |  |
| 23873 |  | B9 5 V |  | 25 | 85 | 85 |  |
| 23923 | HR 1183 | B9 V |  | 250 | 304 | 300: | Estimated |
| 23964 |  | A0 V |  | 0 | 146 | 20 |  |

$\sin i\rangle=62 \mathrm{~km} / \mathrm{sec}$ ) and 5 rapidly rotating A4-A7 $\langle V \sin i\rangle=134 \mathrm{~km} / \mathrm{sec}$ ) Ursa Major stream stars in Slettebak's sample of 49 and 25 stars of these types, respectively, do affect his means significantly. The dots in Figure 1 at A2 and A5.6 show Slettebak's means after removal of the Ursa Major stream stars; the dip at A2 has disappeared. In the same way, the dip at B2 can be attributed as being due partly to the admixture in Slettebak and Howard's sample of 65 B1-B3 stars of 21 members (Blaauw 1956) of the Cassiopeia-Taurus group; the dot near the B2.4 circle in Figure 1 shows what happens when these stars are removed from the mean. The $\mathrm{O}-\mathrm{B} 0$ and $\mathrm{B} 5-\mathrm{B} 7$ means do not include significant numbers of Cass-Tau stars. (The 13 definite and 9 possible members of the I Orion and I Lacerta stars in Slettebak and Howard's data do not contribute much to the $\mathrm{B} 1-\mathrm{B} 3 \mathrm{~V}$ mean but do lower the mean $V \sin i$ for the $\mathrm{B} 1-\mathrm{B} 3$ III + IV stars.) This


Fig. 1 -Slettebak and Howard's and Herbig and Spalding's measures of mean rotational velocities of O-G0 main-sequence stars are plotted against absolute visual magnitude (Arp 1958) or spectral types. The sizes of the circles indicate the number of stars incorporated in the means, namely, 15-65 stars. The dots near the A1-A3 and A4-A7 means represent the means after removal of the Ursa Major stream stars from this sample of field stars, indicating that the low rotational velocities of A0-A3 stars and high rotational velocities for A4-A7 stars in the Ursa Major streams are the cause of the apparent dip at A2. Similarly, the removal of the 21 Cassiopeia-Taurus group stars from the sample of 65 B1-B3 field stars partially removes the apparent dip at B2 (see dot at B2).


Fig. 2.-Mean rotational velocities for main-sequence stars in the I Orion association and the Pleiades cluster The sizes of the circles represent the number of stars (6-16) averaged in the means. The $x$ 's represent estimates for Pleiades stars by Smith and Struve


Fig. 3.-Mean rotational velocities for main-sequence stars in the II Persei and I Lacerta associations and the a Persei, M39 + Ursa Major nucleus, and Hyades + Praesepe clusters. The sizes of the circles represent the numbers of stars (6-23) averaged in the means.


Fig. 4.-The dependence of mean projected rotational velocity on luminosity or spectral type is shown for main-sequence field stars (dashed line) and for main-sequence members of seven clusters or associations (solid lines and the $x$ ).
explanation for the irregularities in a diagram like Figure 1 probably also holds for Boyarchuk and Kopylov's material, although this has not been investigated.

## b) Total Cluster Angular Momenta

A second question raised by Figure 4 is the following: if a cluster like the Pleiades has a different dependence of $\langle V \sin i\rangle$ on luminosity from the field stars, does it have a higher or lower total angular momentum per unit mass than field stars of similar types, i.e., does the excess of angular momentum in the B stars more than or less than compensate for the deficiency among the A's? To answer this, we must compute the total angular momentum for stars of various luminosities and multiply it by the number of stars of


Fig 5 -The lower curve shows the decrease in angular momentum per unit linear rotational velocity with decreasing luminosity The top curve is the initial luminosity function by Salpeter and Sandage. The middle curve shows the decrease in total angular momentum per magnitude interval for a cluster of stars.
each luminosity, which is the luminosity function. We can predict that the larger amounts of angular momentum among early-type stars, due to their larger masses and radii, will be at least partially offset by their lower frequency.

For a rigid rotator in which the density is a function only of radial distance outward, the total angular momentum is

$$
\begin{equation*}
L=\frac{8 \pi V}{3 R} \int_{0}^{R} \rho_{r} r^{4} d r \tag{1}
\end{equation*}
$$

where $V$ is the equatorial linear rotational velocity. Numerical integrations were performed for the models tabulated by Schwarzschild (1958) of 10, 2.5, 1 , and 0.603 solar masses; these give, respectively, values for $L / V$ of $4.98,0.375,0.107$, and $0.0793 \times 10^{44}$ gm cm . A curve of these values is plotted in Figure 5, where Schwarzschild's form (1958) of the mass-luminosity relation was used. Also plotted is Sandage's re-evaluation (1957) of Salpeter's initial luminosity function (1955), $\psi\left(M_{v}\right)$. The initial luminosity function is
appropriate here, rather than the observed general luminosity function, $\phi\left(M_{v}\right)$, because we are concerned with the distribution of angular momentum among all the original stars in clusters, not just among those left at the present evolutionary stages. [The numerical values of $\psi\left(M_{v}\right)$ are, of course, relative to the stellar density within a cluster, but this multiplicative constant drops out in discussions of the relative contributions from various magnitude intervals within a cluster.] Using the mean velocities for field stars as shown in Figure 1 and correcting for a mean projection factor $\langle\sin i\rangle=\pi / 4$ (Chandrasekhar and Münch 1950), we obtain the product $\log L \psi$ as shown in Figure 5. This quantity rises fairly rapidly with increasing luminosity, indicating that the contribution per magnitude interval to the total angular momentum in an initial cluster of stars increases rapidly with luminosity, e.g., all the stars with $M_{v}=-2.0 \pm 0.5 \mathrm{mag}$. have about 16.7 times the angular momentum of all the stars with $M_{v}=+2.0 \pm 0.5$ mag.

With reference to the Pleiades, we can compute $L \psi$ from its mean projected rotational velocities (Fig. 2) for stars with $-1.0 \leq M_{v} \leq+2.5$; this quantity becomes $5.08 \times 10^{48}$ $\mathrm{gm} \mathrm{cm}^{2} / \mathrm{sec} \mathrm{psc}^{3}$, compared with $4.94 \times 10^{48}$ for the same range and number of field stars. Thus, well within our accuracy, we can say that the excess angular momentum among the unusually rapidly rotating remaining $B$ stars in the Pleiades just compensates for the deficiency of angular momentum among the unusually slowly rotating A stars. For the observed stars in the a Persei cluster ( $-2.0 \leq M_{v} \leq+0.5$ ) and the M39 and Ursa Major clusters ( $0.0 \leq M_{v} \leq+2.5$ ), the rotational velocities in Figure 3 produce the same total angular momentum per unit mass within 15 per cent as for comparable samples of field stars. Similar calculations cannot be carried out for the I Orion and I Lacerta associations at present because models for stars with masses greater than 10 solar masses have not been published. Furthermore, similar calculations were not carried out for the Hyades and Praesepe clusters because the rotational velocity distributioncurve is so steep in the region of the F stars that it becomes very sensitive to evolutionary increases in luminosity and to errors in spectral types.

From these calculations for four clusters and from the character of the remaining cluster curves in Figure 4, we are tempted to draw the following conclusion: that, whereas individual groups of stars of common origin seem to exhibit unique or nearly unique dependences of mean angular momenta on luminosity, the total angular momenta per unit mass is not unique; in functional notation, $L\left(M_{v}\right)=f$ (cluster), but in similar spectral ranges

$$
\frac{\sum_{M_{v}} L\left(M_{v}\right)}{\sum_{M_{v}} \mathfrak{M}\left(M_{v}\right)}
$$

However, we doubt that this hypothesis can be true in every case; for example, if we had happened to observe the Pleiades at a later evolutionary stage when all the B stars had evolved into white dwarfs, we would have concluded that the cluster was relatively deficient in angular momentum. In view of the distinctive character of the curves in Figure 4, we cannot include or predict the rotational velocities for those stars which have now evolved into late-type supergiants or white dwarfs, and we do not have data for less luminous main-sequence stars than those plotted. From Figure 5 we see that the less luminous stars will not contribute much to the total cluster angular momentum, particularly after about F5, unless, as happens in the solar system, planetary motion is present and significant. In the initial cluster, the OB stars contribute the bulk of the total cluster angular momentum, so this quantity cannot be determined after these stars have disappeared. Therefore, we are unable to say at present whether these fragmentary indications that the total angular momenta of individual galactic clusters may not be unique have any significance.

## c) Correlation with Binary Frequencies

There is some scattered evidence (Rodgers 1952; Abt 1961) indicating that slowly rotating stars tend to be more frequently members of binary systems than do rapidly rotating ones. If so, we would predict that the frequency of binaries among, for instance, the B stars in the Pleiades should be much less than among the B1-B3 I Orion and I Lacerta stars.

There have been no thorough studies of the binary frequency in galactic clusters, although this quantity, like the rotational velocities, probably varies from cluster to cluster and within a cluster. For example, Smith and Struve (1944) concluded from an average of three spectra for each of 71 Pleiades stars that there was a marked scarcity of large-amplitude binaries in that cluster. Until more detailed data are available for a number of clusters, we must resort to using available radial-velocity measures for those cluster stars which happen to have been measured in more general radial-velocity programs. Aside from spectroscopic binaries with known orbital elements, it is sometimes difficult to decide whether the velocity dispersions reported for individual stars are due to measuring errors in broad-lined spectra or to intrinsic variability. Using a fairly conservative approach, we shall consider only those stars with six or more velocity

TABLE 5
Radial-Velocity Character for Cluster Stars

| Ro- <br> tational <br> Velocity | Group | Spectral Range |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |

measures and decide that those with velocity ranges greater than $10+(V \sin i) / 10$ $\mathrm{km} / \mathrm{sec}$ or with velocity dispersions greater than $3+(V \sin i) / 100 \mathrm{~km} / \mathrm{sec}$ are probably intrinsically variable.

In Table 5 are collected the data on radial-velocity character for the four groups measured. The stars either have orbital elements (orbits), have variable radial velocities or double lines but no orbital elements as yet (variable), or are assumed to have constant velocities (constant). From this table we see that those stars with unusually low rotational velocities in the I Lacerta, I Orion, and a Persei groups have a binary frequency of at least 47 per cent, while, for the Pleiades B stars with unusually high rotational velocities, the equivalent frequency is only 8 per cent. It is extremely unlikely that this can be attributed to a spectral dependence of binary frequency or to difficulties in detecting binaries among broad-lined stars. Furthermore, if the ratio of double- to single-lined binaries is normal, this difference cannot be explained by the difficulty in detecting double-lined binaries among broad-lined stars. This strong correlation between low binary frequency and high rotational velocity is probably due to an original partition of angular momentum between rotational and orbital motion in binary systems, whereas in single stars the available angular momentum must go into rotation only.

There is much that needs to be done along the lines of this investigation. It would be well to have rotational-velocity data for additional clusters, to see whether any pattern emerges from diagrams such as Figure 4. The rotational velocities for longer sections of the main sequence should be obtained in several young clusters, to see whether the total cluster angular momentum per unit mass is indeed normal. Old clusters should be
measured to find out whether the original cluster angular momenta are a function of age. Much more radial-velocity data are needed in clusters to investigate the binary frequency and its correlation with mean rotational velocity. Such data are also of interest in determining the location in color-magnitude diagrams of the main-sequence because its position is effected by duplicity.

It might be pointed out that at the present time the pattern of rotational-velocity dependence on luminosity and perhaps the binary-frequency dependence on luminosity may be the only distinguishing astrophysical characteristics of members of different clusters of similar ages and chemical compositions.

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## REFERENCES

Abt, H. A. 1961, Ap. J. Suppl., 6, 37.
Arp, H. C. 1958, Encyclopedia of Physics, ed. S Flügge (Berlin: Springer-Verlag), 51, 75.
Blaauw, A. 1956, Ap. J., 123, 408.
Blaauw, A., and Morgan, W. W 1953, Ap. J, 117, 256.
Boyarchuk, A. A., and Kopylov, I. M. 1958, Astr. Zhur, 35, 804; trans Soviet Astr, 2, 752, 1959.
Campbell, W. W., and Moore, J. H 1928, Pub. Lick Obs., Vol 16.
Chandrasekhar, S., and Münch, G. 1950, Ap. J., 111, 145
Deutsch, A. J. 1955, Principes fondamentaux de classification stellaire ("Colloques internationaux du Centre National de la Recherche Scientifique" [Paris, 1953]), p 25.
Herbig, G. H., and Spalding, J. F., Jr. 1955, Ap. J, 121, 118
Luyten, W. J., Struve, O., and Morgan, W. W 1939, Pub Yerkes Obs , 7, 251.
McNamara, D. H. 1960, A J., 65, 493.
McNamara, D. H., and Larsson, H. J. 1962, Ap. J, 135, 748.
Meadows, A. J. 1961, A p. J., 133, 907.
Mendoza, E. E., V. 1956, Ap. J., 123, 54.
Morgan, W. W., Code, A. D., and Whitford, A. E. 1955, Ap. J. Suppl , 2, 41.
Rodgers, R. F. 1952, J.R.A.S. Canada, 46, 147.
Roman, N. G. 1949, Ap. J., 110, 205.
Roman, N. G., and Morgan, W. W. 1950, Ap J, 111, 426
Salpeter, E. E. 1955, Ap. J., 121, 161.
Sandage, A. 1957, Ap. J., 125, 422.
Schwarzschild, M. 1958, Structure and Evolution of the Stars (Princeton: Princeton University Press)
Sharpless, S. 1952, Ap. J., 116, 251.
Slettebak, A. 1949, Ap. J., 110, 498.
——. 1954, ibid, 119, 146.

- 1955, ibid, 121, 653.

Slettebak, A., and Howard, R. F. 1955, Ap. J, 121, 102.
Smith, B., and Struve, O. 1944, Ap. J., 100, 360
Struve, O. 1945, Pop. Astr., 53, 259.
Treanor, P. J., S.J. 1960, M.N., 121, 503.
van Dien, E. 1948, J.R.A.S. Canada, 42, 249.


[^0]:    *Contributions from the Kitt Peak National Observatory, No. 18, and Contributions from the McDonald Observatory, University of Texas, No. 370.
    $\dagger$ Summer assistant, Kitt Peak National Observatory, 1961; currently at the University of Arizona.
    $\ddagger$ Operated by the Association of the Universities for Research in Astronomy, Inc., under contract with the National Science Foundation.

[^1]:    * On November 7 295, 1957, the velocities from three line pairs were stronger component, -997士02; weaker component, $+920 \pm 106 \mathrm{~km} / \mathrm{sec}$
    $\dagger$ Slettebak and Howard (1955)
    $\ddagger$ Luyten, Struve, and Morgan (1939) report, for this spectroscopic binary, faint secondary components at $\lambda 4471$ on some plates This component was not apparent on the McDonald plate

