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ABSORPTION SPECTRA OF M-TYPE MIRA VARIABLES

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ABSTRACT

A set of spectrograms of Me variables is reproduced in order to separate the effects produced by several of the physical parameters that characterize their atmospheres.

1. For M giants later than M5, the spectral types will more accurately give a temperature sequence if they are obtained from relative strengths of molecular bands (TiO and VO) rather than from absolute intensities. The reason is that even the vibrational levels of TiO are far enough apart to have appreciably different populations at temperatures of 3000° K and lower.

2 In certain Mira variables later than M5e, consistent differences appear in the ratios of the bands of YO, ScO, AlO, and TiO, which appear to reflect real differences in the relative abundances of these metals. Strengthening of YO and ScO indicates a tendency toward spectral type S; in such stars, AlO is often weak.

3. Mira variables consistently show weaker absorption lines than do other M-type giants of the same types The strong lines of most neutral atoms are the ones most weakened in Miras earlier than M6e at maximum light. In these members of the high-velocity population the line weakening suggests a metal deficiency of order 10^{-2}.

4. In the spectra of the low-velocity population comprising most Mira variables later than M5e at maximum light, the line weakening is generally less conspicuous and may vary greatly from cycle to cycle. While part of this line weakening can be explained by a metal deficiency of the order of 10^{-1}, an important part must also be due to other causes, such as variable opacity due to continuous absorption by TiO and other molecules.

I. INTRODUCTION

Variable stars of the Mira type usually exhibit many strong emission lines in their spectra. The bizarre details of these features have been extensively described; a recent summary and a bibliography are given by Merrill (1960). Somewhat less attention has been paid to the absorption features, although their richness and variety provide significant clues to the problems of the atmospheres of these stars. In particular, the absorption lines put in clear evidence certain systematic abundance anomalies which relate directly to the mode of synthesis of the heavy nuclei and to their fractionation in the galaxy. The spectroscopic clues are to be found in (*a*) systematic *differences* in absorption-line strengths between the Mira variables and the non-Mira giants of similar temperature and luminosity; (*b*) *differences* between individual Mira variables; and (*c*) *changes* in the spectrum of a single variable, whether as a function of phase or from one light-cycle to another.

The complexity of the spectroscopic phenomena and their drastic changes with time reflect the complex and variable physical conditions in these stellar atmospheres. Only crudely quantitative determinations can be attempted for some of the physical parameters of interest. Among the parameters whose effects can be isolated with some success are the following: temperature; luminosity (or pressure or surface gravity); ratio of metals in the Zr abundance peak to those in the Fe abundance peak; ratio of metals

* The manuscript was prepared for publication after Dr Merrill's death by the two of us: P C. K and A. J D

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to H; and at least one additional, unidentified factor which appears to be responsible for a characteristic selective weakening of many lines and bands in certain light-cycles of late-type Miras.

The illustrations have been enlarged from the spectrograms listed in Table 1. The magnitudes and phases of the variables are mostly from the A.A.V.S.O.; values in parentheses are predictions based on the mean light-curves (Campbell 1955). The radial velocities of the Miras are those given by Merrill (1941) for the absorption lines; other radial velocities have been taken from the *Mount Wilson Catalogue* (Wilson 1953). The spectral types of the variables have been estimated directly from the tabulated plates. The periods are those of Kukarkin *et al.* (1958).

In the third column, which shows the plate numbers, the symbol "Pc" refers to plates taken with the 36-inch camera on the coudé spectrograph of the 200-inch telescope (original dispersion, 13 A/mm in the red) and "Pd" to plates taken with the 18-inch camera of the same spectrograph (16 A/mm in the blue and 26 A/mm in the visual region). Spectrograms of the Ce series were taken at the coudé focus of the 100-inch telescope with the 32-inch camera (10 A/mm in the blue and 15 A/mm in the red). The plates of the Ced series were taken with the 16-inch camera, which provides dispersions just half as great. The γ plate in Figure 5 was obtained with the 18-inch camera on the old prism spectrograph at the Cassegrain focus of the 60-inch reflector; the dispersion was 38 A/mm at $H\gamma$. The C-plate was made with a similar instrument on the 100-inch telescope. Figure 6 was enlarged from the little Xf plates (80 A/mm) taken with the new grating spectrograph on the 60-inch reflector.

II. EFFECTS OF TEMPERATURE

Subdivision of all M-type stars into temperature types on the basis of the strengths of the bands of TiO has been generally adopted (cf. Keenan 1961 for general references). At any one luminosity, these types have been found to correlate well with the excitation temperatures indicated by the atomic lines—the stronger the bands, the cooler the star. This classification by absolute band strengths may lead to errors, however, if applied indiscriminately to all bands. When coudé spectrograms of the coolest M-type stars are compared, it becomes clear immediately that even the bands of the one molecule, TiO, do not all vary with temperature in the same way. We illustrate this in Figure 1 for the visual region, where several band systems overlap. The figure shows a set of spectra of giant stars arranged in order of decreasing temperature. In the upper set of spectra the two bands originating by absorption from excited vibrational levels of the α-system at 5759 A (0, 2) and 5810 A (1, 3) become prominent between M3 and M5, but then level off and increase in strength little, if at all, at lower temperatures. In contrast, the 1, 0 bands (note λ 5838) of the γ' system (Duner-Coheur bands), which arise from the lowest vibrational level of the same ground term, continue to increase rapidly through type M8. Note also how the 5736 A band from the ground level of VO gains rapidly on the λ 5759 and λ 5810 bands of TiO and surpasses them in types later than about M6. Although it is dangerous to estimate temperatures by comparison of bands from different compounds, the sensitivity of vanadium oxide to temperature is so great that the green band (λ 5736) and the infrared bands of this molecule are extremely valuable in classification.

The behavior of the yellow α-bands of TiO is due to their origin at levels situated above the lowest vibrational level by 0.25 ev (1988 cm^{-1}) for the λ 5759 (0, 2) band and by 0.37 ev for the λ 5810 (1, 3) band. If the total surface density of TiO molecules remained constant, the Boltzmann formula alone would predict that these bands should go through a maximum near 2500° K. Actually, if we judge from the continued increase in strength of bands from the lowest level, we find that the effective number of TiO molecules (as determined by dissociative equilibrium and opacity together) is evidently

1962ApJ...136...21M

TABLE 1

OBSERVATIONAL DATA FOR SPECTROGRAMS REPRODUCED IN FIGURES 1–12

Star	Figure	Plate	Date, U T 1900+	m_v	Phase (days)	V_r (km/sec)	Spectral Type	Period (days)
72 Leo	1	Ce 12346	29 Dec. 58	4 9		+16	M3 III	
51 Gem .	1	Ce 11002	20 Jan. 57	5 3		− 9	M4 III	
HD 11961	1	Ce 12209	29 Oct. 58	7 2		−45	M5 III	
HD 207076	1	Pc 12208	29 Oct. 58	7 2	.	−37	M7 III	. . .
X Oph	1	Pc 5200	13 July 60	7 2	(−11)	−71	M7e	334
R LMi	1	Pc 5541	27 Dec. 60	(7 2)	(− 3)	+10	M7 5e	372
	4	Ce 12263	30 Nov. 58	(7 2)	(0)	.	M8e	.
R Aur	1, 4	Pc 5441	2 Nov. 60	8 5:	(+20)	+ 8	M8e	459
R Leo	2, *a*, 3, *b*	Pd 4899	16 Dec. 59	(9 7)	−110	+13	M8e	313
	2, *b*	Pd 5504	5 Dec 60	(8 3)	−58		M7e	.
	2, *c*	Ced 14203	31 Jan. 61	5 5	0		M6 5e	.
	5, *e*	Ce 3316	12 Dec. 43	6 6	−18			.
μ Gem . .	3, *a*	Pd 4888	14 Dec. 59	3 2	. . .	+55	M3 III	
	6	Xf 2420	17 Jan. 57	. .	.	.	. .	
W And	4	Ce 13904	29 Sept. 60	6 8	(−9:)	−29:	S6,1e	397
R Hya	4	Ce 12463	25 Sept. 59	5	−10	−10	M7e	386*
	5, *a*	γ 12667	20 May 24	5 9	+63	. .	M7e:	.
	5, *b*	C 7441	20 Jan. 40	5 9	+34	. .	M7e:	. . .
	5, *d*	Ce 3756	27 Apr. 45	6 0	+33	.	M6e:	. .
Mira	5, *a*	Ce 4576	5 Feb. 47	5 5	+58	+64	M7e:	332
	5, *b*	Ce 4915	6 Oct. 47	3 2	−15		M6e	.
	5, *c*, 9, *b*	Ce 5352	14 Oct. 48	3 9	+19		. .	.
	9, *a*, 10, *a*, 11, *b*	Ce 4926	24 Oct. 47	3 1	+ 2	.	M6e	
	10, *b*	Ce 6028	14 Nov. 49	5 8	+81			.
	10, *c*	Ce 5486	12 Dec. 48	6 0	+78	.	.	
χ Cyg.	5, *f*	Ce 12778	24 July 59	7 1	−51	0	Se	407
Y And	6	Xf 2170	14 Nov. 56	9 0	+10:	− 7:	M3e	220
56 Leo	6	Xf 2422	17 Jan. 57	6 0		−13	M5 III	. .
R Tri	6	Xf 2395	15 Jan. 57	7 4	−30:	+67	M5e	206
RT Cnc	6	Xf 2434	23 Jan. 57	8 1		+36	M5	
υ Gem	7, *a*	Pd 4890	14 Dec. 59	4 2	. .	−21	M0 III	. . .
R Ari	7, *b*	Pd 6205	19 Oct. 61	(8 7)	(−15)	+114	M3e	187
W Lyr	7, *c*	Pd 6207	19 Oct. 61	(8 1)	(+10)	−174	M5e	196
R Per	7, *d*	Pd 6206	19 Oct. 61	(9 5)	(+28)	−78	M5e	210
R Lyr	7, *e*, 12, *d*	Pd 4697	14 Aug. 59	4		−28	M4.5 III	
θ Lyr	8, *a*	Pc 3468	20 Sept 57	4 5	. .	−31	KO II	. .
Z Oph	8, *b*	Pc 3238	17 June 57	8 2	(+15)	−81	K4ep:	348
g Her.	11, *a*	Ce 3403	8 Apr. 44	5 0		+ 3	M6 III	
SW Vir .	11, *c*	Ce 11840	5 Apr. 58	7:	.	−15	M7 III	. . .
U Ari	12, *a*	Ced 14035	28 Nov. 60	8:	(− 1)	−37:	M6 5e	371
R Dra	12, *b*	Pd 4465	16 Apr. 59	7 7:	(− 6)	−133	M5e	246
R Aql .	12, *c*	Pd 5152	3 July 60	(6 3)	(+ 4)	+32	M6 5e	300

* R Hya is the best-known case of a Mira variable with continuously decreasing period The period of 386 days as given in the second edition of the *General Catalogue of Variable Stars* presumably refers to the epoch JD2434620 (31 Aug 1953)

increasing down to temperatures of less than 2000° K. Thus the predicted maxima for the excited bands are actually flattened and shifted to lower temperatures.

An even more striking contrast is seen in the lower set of spectrograms in Figure 1, where the singlet bands of the β-system (λλ 5597, 5629, and 5661) can be compared with the bands of the 2, 0 sequence of the γ' system (λλ 5569, 5591, and 5615). The latter arise from the same ground term as the stronger 1, 0 bands discussed above. They evidently have low transition probabilities, for they are scarcely noticeable in stars earlier in type than M5 and are missing from most tables and reproductions of the laboratory spectrum of TiO (note their almost complete absence in Plate 27 of *Molecular Spectra of Diatomic Oxides* [Specola Vaticana, 1957]. They strengthen rapidly with decreasing temperature, becoming about equal to the strongest singlet bands in Figure 1 for stars later than M7.

The behavior of these bands in stars supports the laboratory evidence given by Phillips (1952) that the lowest singlet level lies above the lowest triplet level, which must then be the ground state of the TiO molecule. In the absence of intercombination bands between the singlet and triplet levels, it has been difficult to be certain which are the lowest; but, from spectrograms taken in the electric furnace at a series of temperatures, Phillips estimated that the $v'' = 0$ singlet level was about 580 cm^{-1} above the lowest triplet level. By close comparison of the bands from the several vibrational levels in Figure 1, we can conclude independently that the lowest singlet level is the higher by at least several hundred cm^{-1}, and perhaps by as much as 1000 cm^{-1} (0.12 ev).

The importance of these differential effects for the classification of stellar spectra is clear. In the later subdivisions of type M the *ratios* of TiO bands rather than their absolute intensities should be used whenever possible; in particular, the absolute strengths of the bands from the excited levels should be avoided as likely to give ambiguous types. With dispersions of about 20 A/mm or better, the most sensitive criteria in the regions shown in Figure 1 are 5838/(5810 + 5759), VO 5736/5759, 5664/5661, and (5569 + 5591 + 5615)/5597. Thus 5664 = 5661 at about M6. Even on spectrograms with scales as low as 150–200 A/mm, some of these ratios—such as that of λ 5569 to the blended feature around 5597—can be estimated quite accurately. Similar criteria can be found in other spectral regions.

These criteria can be made objective by using measured absorptions in the bands. Unfortunately, the *total* absorption of a band in the late M stars is almost impossible to measure; for the fluting due to the upper rotational levels is almost always overlapped by the heads of other bands, and the position of even a "local" continuum can be estimated only near the vibrational heads. This is evident in Figure 1, where it can also be seen that, even for such a well-behaved molecule as TiO, some of the heads themselves (e.g., λ 5759 and λ 5810) are not well defined. For comparison of any one band in different stars or for laboratory absorption measurements, the total absorption between the head and the first overlapping feature can sometimes be used. When we wish to measure the ratios of different bands, each having its own structure, however, we believe that the most impersonal measure that can be given is the ratio of the greatest depths $(1 - I_b)$ near the heads, which appears usually as the "break" in the continuum at the heads. The quantity $(1 - I_b)$ corresponds to the central depth of an absorption line; in it, I_b is the residual intensity at the bottom of the band expressed in units of the intensity in the continuum just outside the band head. Even these depths are not always easy to measure on tracings, and they will vary with the linear scales of the spectrograms. The band depths and their ratios given in Tables 2 and 3 were measured on our coudé spectrograms; the depths on small-scale spectrograms would usually be much smaller. Table 2 gives smoothed means of the ratios of depths of the most useful bands shown in Figure 1; the data are tabulated as a function of spectral type for late M giants.

The discussion up to this point applies equally to Mira variables and to most other M-type stars; indeed, we have found it convenient to use typical Me variables to illus-

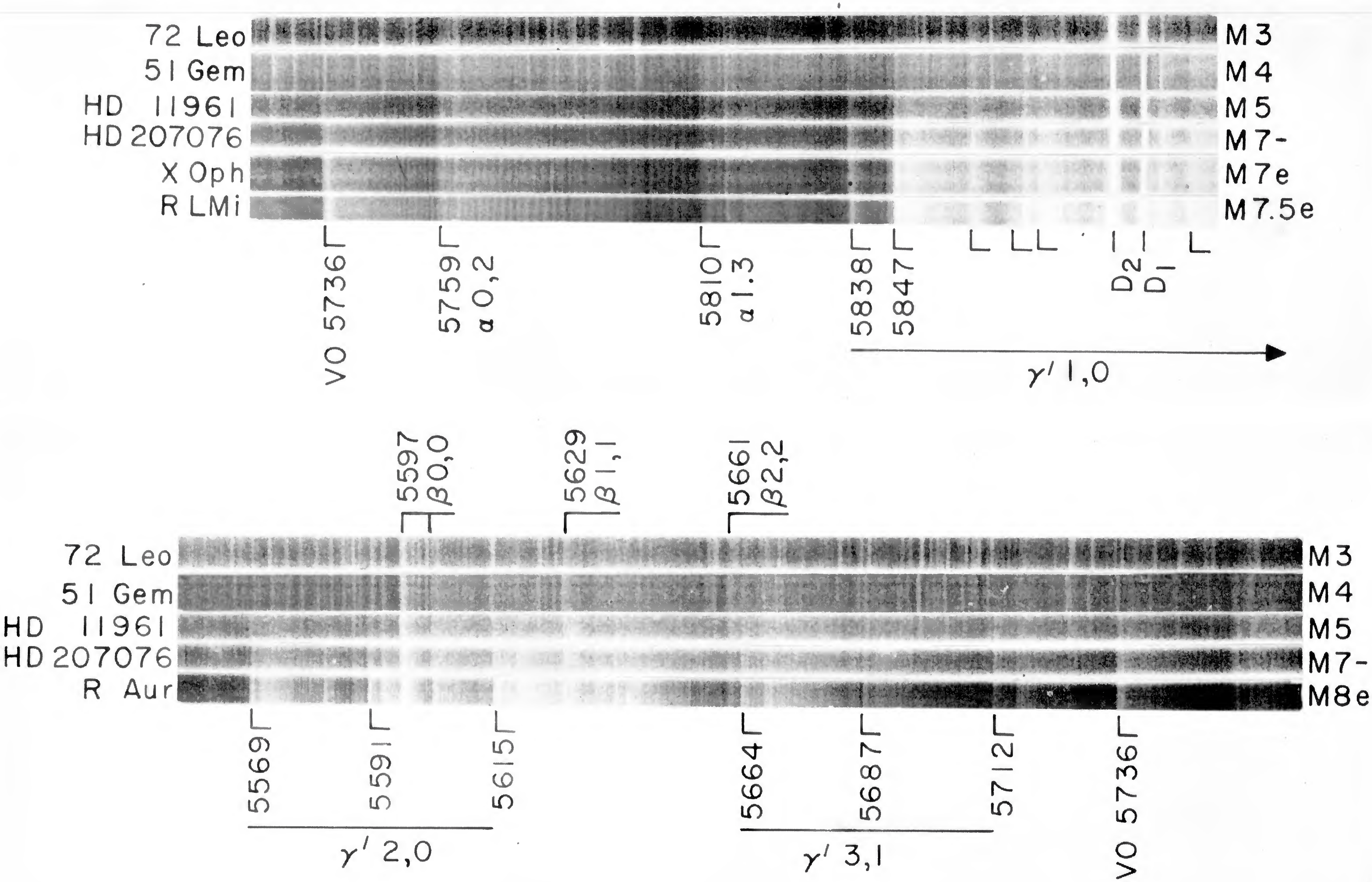

FIG. 1.—Temperature sequence of giants in the visual region, showing the different behavior of TiO bands from the second and third vibrational levels of the α-system, the lowest level of the Duner-Coheur γ'-system, and several vibrational levels of the singlet β-system.

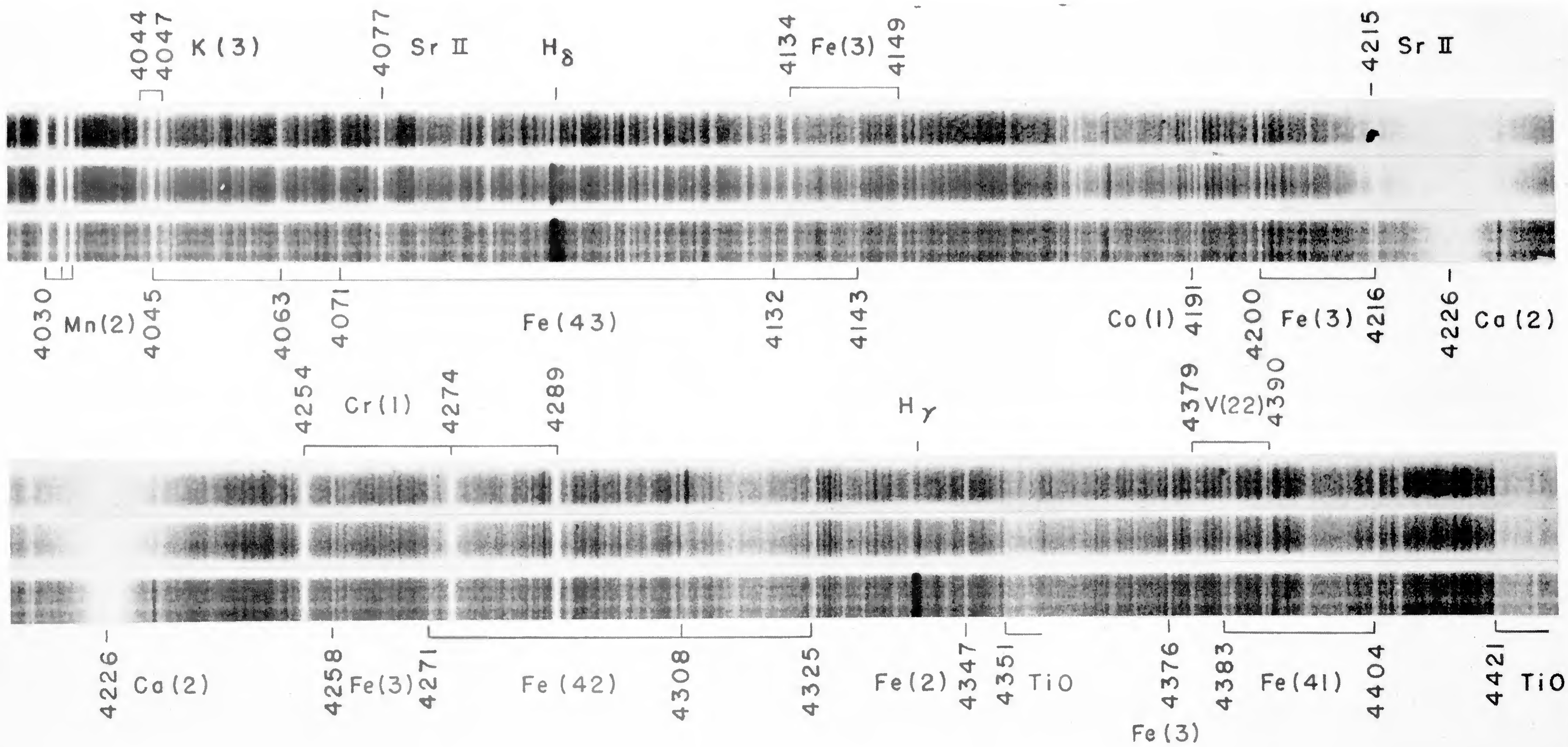

FIG. 2.—Changes in the blue spectrum of R Leo as the star brightens from phase -110^{d} (strips *a*) to maximum light (strips *c*). Note Ca 4226 and K 4044, 47

trate the latest types in Figure 1. The late-type variables, however, may intermittently show differences in their relative band strengths as compared with normal giants. The absorption lines may also be affected; these effects will be detailed in Section V. The most commonly observed difference in the bands is an apparent weakening of the strong bands relative to the weaker ones. In Figure 12 it can be seen that the strong bands in U Ari would give a type of only about M5, but that the weaker bands in the blue, which usually appear only at types M6–M7, are also present. It seems likely that there is some loss of contrast in the weaker bands also, but in any case the band ratios, such as those recommended in the last paragraph, lead to types that seem consistent with the spectral-energy distribution. Thus in Figure 12 the type of U Ari at its maximum should be estimated at about M6.5. An extreme example of this effect in the TiO bands was shown by R Ser at its unusually low maximum in 1960, at the time when AlO was observed in emission (see Fig. 1 in Iwanoska *et al.* 1960).

Since the effect can be described as a general weakening of contrast in the bands, such spectra will have a "washed-out" appearance when observed with low dispersion. Then observers working with slitless spectrograms may assign types that are much too early; but the more extreme cases can be spotted as peculiar even with the lowest dispersions

TABLE 2

BAND RATIOS IN VISUAL REGION FOR CLASSIFICATION OF M-TYPE GIANTS

Type	$\frac{5838}{(5810+5759)}$	$\frac{\text{VO } 5736}{5759}$	$\frac{5664}{5661}$	$\frac{(5569+5591)}{5597}$
M5	1 3	0 2:	0 8:	0 4:
M5.5	0 7	0 5	1 2	0 3
M6	1 2	0 7	1 0	0 5
M7	1 7:	1 2	1 5	0 8
M8	2 2	2 2:	2 0:	1 1:

in use. It is probable that only a few of the published types are systematically in error from this source.

On coudé spectrograms of the blue region, the TiO bands below about 4400 A are relatively weak, and their heads are not sharply defined. For this reason certain atomic resonance lines are the most useful gauges of temperature, even in the cooler phases of the Mira variables. Figure 2 shows that, although the changes in most lines are not great as R Leo brightens by about 3 mag., the pair λ 4044 and λ 4047 from the lowest level of KI weakens steadily. The ratio λ 4044/λ 4045 is the best temperature criterion in this region, probably because of the very low ionization potential (4.3 ev) of K. In the same spectral region, resonance lines of Mn, Ca, and Cr are less sensitive to temperature because of their higher ionization potentials. The changes of subordinate lines due to the Boltzmann factors tend to be distorted by incipient emission or by gravity effects.

At the lowest temperatures observed in M giants, many low-excitation absorption lines of neutral metals become very strong in the ultraviolet. For example, as may be seen in Figure 3, in passing from M3 to M8 one finds a marked strengthening of the lines arising from the ground state of Fe. This must signify that in this spectral region the opacity falls steeply as the temperature decreases, for the concentration of all Fe in the ground state of Fe I is essentially complete at $T = 2800°$ ($\backsim$M3.5 III). At this temperature, only 4 per cent of the Fe atoms lie in excited states, and at the electron pressures to be expected in such stars ($\log P_e \simeq -2.6$), only the fraction 1×10^{-3} of Fe is ionized.

In addition, Figure 3 shows that at very low T the zero-volt lines of Ti weaken relative to those of Fe. This behavior is consistent with equilibrium theory, which predicts that in such cool atmospheres the supply of titanium atoms should be substantially depleted by the formation of TiO (Stanger 1962). Apparently, the recombination of VO proceeds much more slowly, for the lines of V appear to strengthen with decreasing T approximately as the Fe lines do.

III. EFFECTS OF COMPOSITION

It is generally recognized that differences in *chemical composition* distinguish types S and C (=R, N) from type M. Since the similar, but less extreme, spectroscopic differences within type M, particularly among the Me variables, have not been so well defined, we give an illustration in Figure 4. Here the four spectrograms are arranged in order of increasing strength of the λ 6132 band of YO. From the relative strengths of the TiO bands on these spectrograms, we assign types near M7e to R LMi, R Hya, and R Aur on the dates of observation. The top spectrogram shows W And at its 1960 maximum, when it resembled χ Cyg closely in showing ZrO bands in addition to TiO and could be classified as S6, 1e (or approximately M6 Se). Although the YO bands are never strong in stars earlier than M5 and strengthen rapidly with decreasing temperature, the slight differences in temperature between the stars in Figure 4 cannot explain the increase by a factor of 5 in the ratios YO 6132/TiO 6148 from R LMi to W And (see Table 3).

TABLE 3

INTENSITIES OF BANDS IN λλ 6000–6200 REGION

Star	Plate	Type	TiO 6148	YO 6132	YO 6132/ (6036+ 6079) 2	YO 6132/ TiO 6148	(ScO 6036 +6079)/ TiO 6148
W And . . .	Ce 13904	S6, 1e	>0 90	>0 86	. . .	≈1	. . . .
R Aur. .	Pe 5441	M8e	.78	> 66	0 51	>0 85	0 65
R Hya	Ce 12463	M7e	.70	61	48	0 87	68
R LMi. .	Ce 12263	M8e	0 84:	0 18	0 41	0 21	0 49

There is another factor, however, which makes it difficult to estimate how much of the increase of YO 6132/TiO 6148 is actually due to a change in the concentrations of the two molecules. It can be noticed in Figure 4 that the intensity of most lines and bands increases in sequence from bottom to top, being greatest in W And (although the high density of the continuum on Ce 13904 made it impossible to measure the strengths in this region of W And). Thus the ScO bands increase about 20 per cent in depth from R LMi to R Aur. The exception is the λ 6087 band of VO, which, taken in conjunction with the bands in the λλ 5400–5900 region, indicates that W And was observed at a temperature considerably above those of the other three stars. Since the TiO and ScO bands seen in Figure 4 all originate at their lowest energy levels, it must be concluded that W And has lower atmospheric opacity than the other stars. Indeed, it appears that opacity decreases as we go upward through the sequence R LMi → R Hya → R Aur → W And.

In order to estimate the real changes in the relative abundance of YO, we should try to correct for these differences in opacity. If we neglect the effects of the curve of growth and make the thin-atmosphere approximation, the relative concentrations of molecules in two stars are

$$\frac{N_1}{N_2}=\frac{D_1}{D_2}\frac{k_2}{k_1},$$

where D_1 and D_2 are the band depths and k_1 and k_2 are the opacities in the observed spectral region. On the conservative assumption that the apparent changes in ScO are

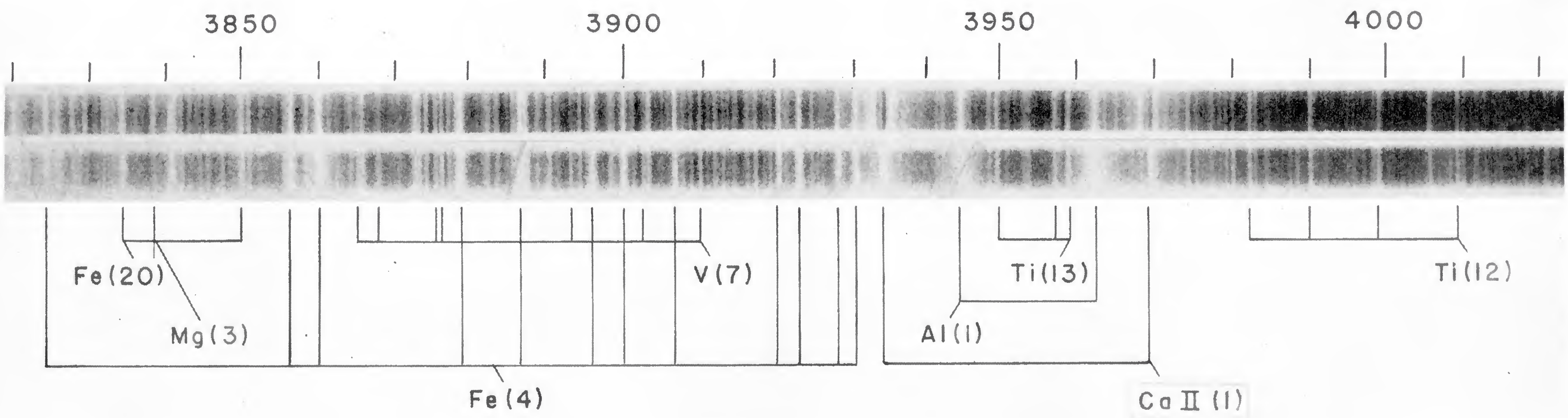

FIG. 3.—The spectra of (*a*) μ Geminorum (M3 III) and (*b*) R Leonis (313^d, M8e). The zero-volt lines of Fe multiplet (4) are stronger in R Leonis, indicating a lower opacity than in μ Geminorum. However, the zero-volt lines of Ti multiplets (12) and (13) are about equally strong in both stars. Probably these lines fail to strengthen with the others from zero volts because Ti is substantially depleted by recombination into TiO.

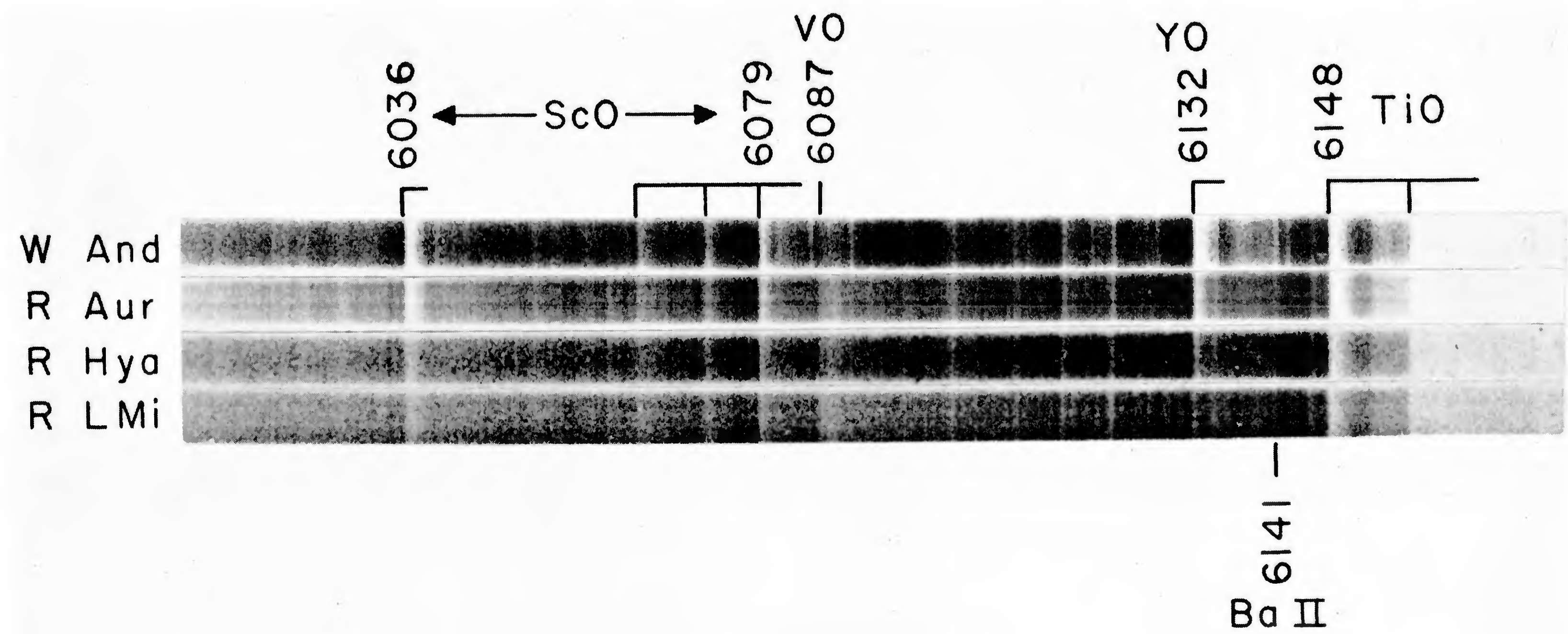

FIG. 4.—The red bands of YO and ScO in stars of comparable temperatures as indicated by their TiO bands. The spectrogram of W And was taken at the 1960 maximum, when W And resembled χ Cyg very closely and was thus representative of stars close to the boundary between types M and S.

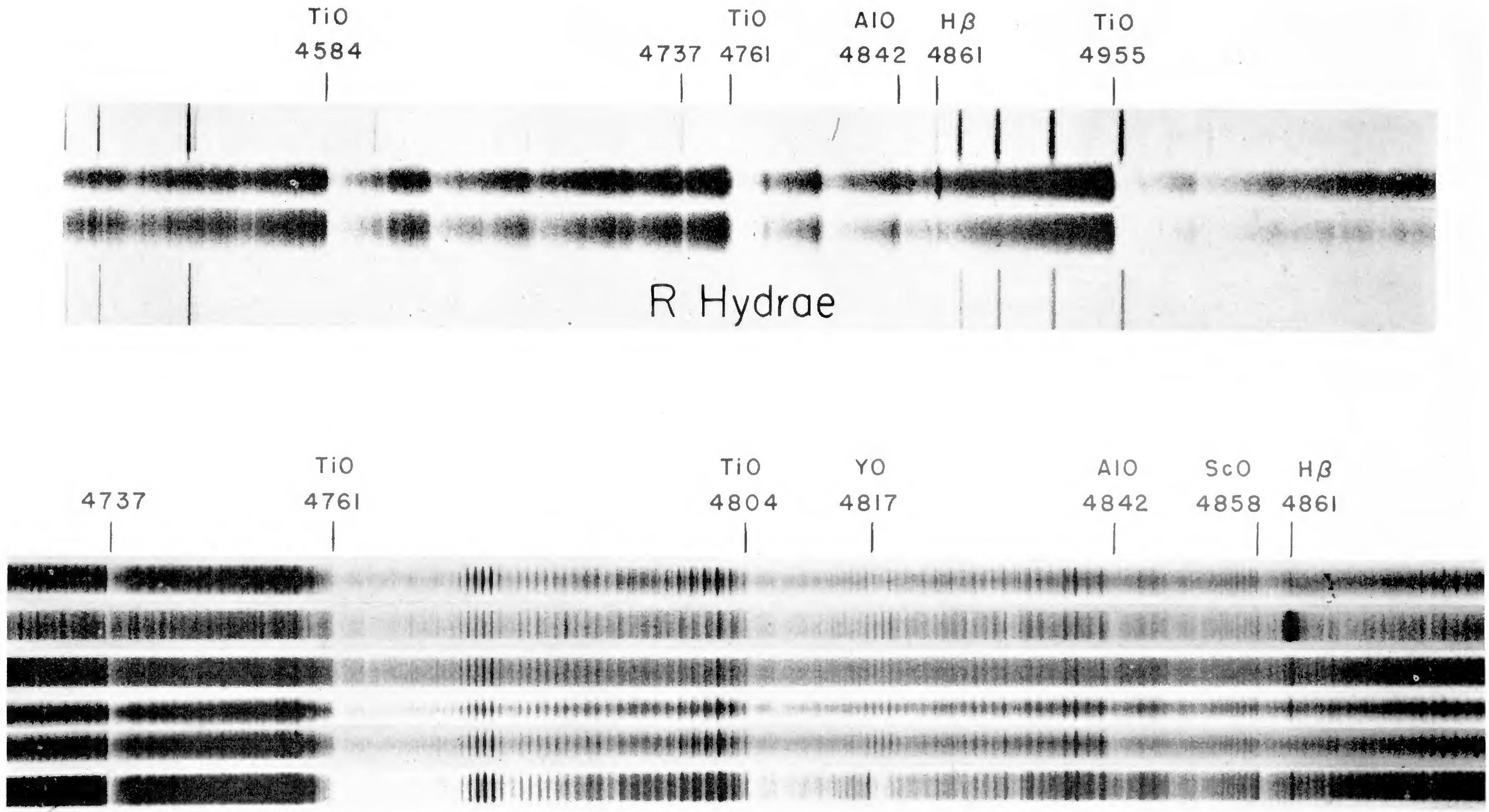

FIG. 5.—Band heads of TiO, YO, AlO, and ScO near Hβ. *Above:* Variation of AlO in R Hya. The λ 4842 band was unusually strong on the lower plate (1940) compared with the upper one (1924). *Below:* (*a*), (*b*), (*c*), *o* Ceti; (*d*), R Hya; (*e*), R Leo; (*f*), χ Cyg.

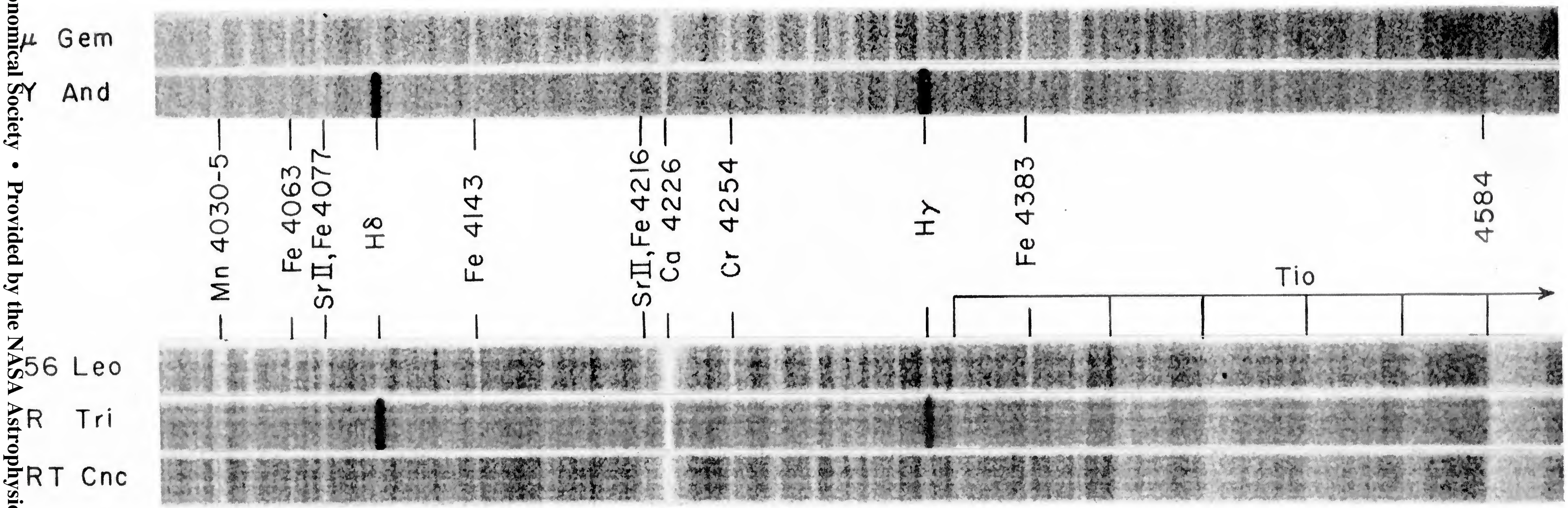

FIG. 6.—Comparison of Mira variables with red giants on spectrograms of low dispersion (80 A/mm). (*a*) Type M3. μ Gem is an M3 III standard. (*b*) Type M5. 56 Leo had a type of M5 III on this spectrogram. The lowest spectrogram shows the SRb variable RT Cnc, which has a mean amplitude of less than 1 mag.

due to opacity, the observed increase of about a factor of 5 in YO should be reduced by something like 20 per cent, suggesting that the relative numbers of molecules $N' = N_{YO}/N_{TiO}$ are

$$\frac{N'(\text{W And})}{N'(\text{R LMi})} \approx 4 .$$

This represents the change in YO in passing from a pure M-type star to a weak S-type star and agrees with the earlier observations that the YO bands appear stronger in S-type spectra (Merrill 1940, p. 36). Within type M, it is particularly interesting to find a threefold increase of YO/TiO between R LMi and R Aur, although no other S characteristics were noticed on our plates of R Aur. The YO bands seem to be the most sensitive indicators of relative metal abundances that are observable in late M stars.

Whether the abundance ratio YO/TiO continues to increase with ZrO/TiO through the stronger S stars or passes through a maximum in intermediate stars like χ Cyg and W And is not yet known. This is partly because most of the coudé spectrograms of pure S stars have been taken near their maxima, when their temperatures were apparently higher than those of the stars in Figure 4. There is also the complication that in at least some parts of the visual spectrum the stronger S stars seem to have very low opacity, if we may judge by the great strength of the sodium D lines at even the brightest (=hottest) maxima. This may be due mainly to the removal of TiO absorption overlying these lines (Bidelman 1954). In the intermediate stars the lowering of opacity throughout the spectrum is striking, and the observer cannot fail to be struck by the extraordinarily high contrast of the band heads on most spectrograms of χ Cyg, even when the dispersion is very low.

The molecules YO and ScO have band systems lying in the blue part of the spectrum also, but, since they fall within the heavy absorption of the +1 sequence of the α-bands of TiO, they are not easily seen or measured in M stars. In Figure 5*f*, the strongest blue band of YO, at 4817 A, is conspicuous only in the spectrogram of χ Cyg. The principal blue band of ScO, at 4858 A, behaves similarly. This band was identified in Mira by D. N. Davis (1940), who noted that it was variable in that star, being stronger on September 29, 1929, at phase $+54^d$ than in any of the spectrograms reproduced in Figure 5, while on October 15, 1926, at -6^d it was not observable. The difference could be due entirely to the higher temperature near maximum. She found the band to be weak in α Her (M5–M6, I*b*–II*a*) and not observable in a number of other M-type stars. In general, we can say only that the evidence from this region is consistent with that from the more favorably placed red bands of YO and ScO.

The strongest observable bands of AlO lie in the region enlarged in Figure 5, where the 0, 0 head at λ 4842 can be seen to vary greatly in Mira and in R Hya, even at corresponding phases in their light-curves. There seems to be no correlation between AlO 4842 and the bands of ScO and YO. The changes in the AlO absorption are probably connected with the tendency of its bands to go into emission on rare occasions (Joy 1926; Iwanowska *et al.* 1960).

One other feature to be noted in Figure 5 is the deep absorption at λ 4737, which has the appearance of a very short band shaded longward. This feature has remained unidentified for more than thirty years (Merrill 1940, p. 37 and Frontispiece). On three spectrograms of Mira, R Leo, and R Hya (of scale 9–10 A/mm), the mean wavelength of the head was measured as 4737.42 ± 0.01 A. This is lower than the value of 4738.297 A in Mira given by Joy (1926) and later revised to 4738.12 (Merrill 1940) because Joy apparently measured the center of the deep absorption. If the feature is interpreted as a band, the shorter wavelength should be used for the head. The feature is consistently strongest in the coolest M-type stars. Thus, if the absorption were atomic, it would need to be a group of resonance lines, which is highly improbable. It is more probably the head of a molecular band, as Joy noted, but its failure to appear in low-temperature sources in the laboratory and the absence of any similar bands nearby remain puzzles. In pure S-type spectra this feature seems to disappear, being replaced by two weaker bands (or lines), of which the stronger was measured in R Cyg at 4737.01 A and the weaker at 4737.81 A. The former is probably the ZrO band observed in the laboratory at 4736.91 A (Afaf 1950). The weaker head coincides with the faint HfO band measured at 4737.8 A (Gatterer *et al.* 1957), but until some of the many stronger heads of HfO are measured in an S-type star the presence of this molecule must remain uncertain.

IV. LINE WEAKENING IN EARLY-TYPE MIRA VARIABLES

Several observers have noticed the weakness of absorption lines in Mira variables, particularly in those with types M5e or earlier at maximum (e.g., Iwanowska 1957; Keenan 1957). Figure 6 illustrates the line weakening at M3e (Y And) and at M5e (R Tri) on spectrograms taken with a scale of 80 A/mm. The SRb variable RT Cnc, M5, which has an amplitude of only about 1 mag. in m_v, is included in the figure to illustrate a tendency noticed on many small-scale spectrograms taken at Mount Wilson or Perkins: a very slight weakening of the stronger absorption lines in small-amplitude variables as compared with non-variable giants of the same spectral type.

The spectrograms of Figure 7, with an original dispersion of 20 A/mm, show the line weakening with more detail in three early-type Mira variables. The following regularities occur in these stars:

1. The line weakening does not vary greatly with wavelength over the observed range, $\lambda\lambda$ 3600–6500.
2. The lines that are most weakened lie on the damping part of the curve of growth. Examples are Ca 4226, Cr 4254, Fe 4045, etc. The weaker lines lie on the flat part of the curve of growth, where they are much less sensitive to changes in the line or continuous absorption coefficient.
3. The strong lines from ions—Ca II H and K, Sr II 4078, Ba II 4554, etc.—show much less weakening than neutral lines of comparable strength.

Good coudé spectrograms are at hand for only a few early-type Mira variables. However, in a survey of small-scale spectrograms of 30 with types M6e or earlier, great line weakening was found in 25 variables and moderate weakening in 5. In none of them were the typical lines of Ca, Fe, and Cr as strong as in M giants of the same types. We can say accordingly that the weakening of lines of neutral atoms is a general characteristic of the group of Mira variables having early Me spectra and correspondingly short period (usually less than 260^d). The differential behavior of different lines suggests that these stars are slightly cooler but more luminous than ordinary giants of the same spectral types. The spectroscopic indications of higher luminosity are borne out by the well-known results obtained from the mean motions of these stars. For the range K5e–M4e in spectrum (mean period = 240^d), Wilson and Merrill (1942) found $\langle Mv \rangle = -1.6$. Since the completion of the McCormick program on proper motions, Osvalds (1961) has obtained a value of $\langle Mv \rangle = -2.1$ for a larger sample of a nearly comparable group having $\bar{P} \simeq 207^d$ and a mean type at maximum of M3.4e. This is to be compared with $\langle Mv \rangle = -0.4$ for ordinary giants (luminosity class III) in the same range of spectral types (Keenan 1960).

The line weakening in early-type Miras is much too large for attribution to errors in classification (see, e.g., Ca 4226 in Fig. 7). Miss Iwanowska (1957)[1] has attempted to explain the line weakening by "the presence of emission which for some lines reaches the level sufficient to change them into emission lines." But emission components are weak or absent in many of the lines which are most weakened (e.g., λ 4226 and λ 4254),

[1] She and others have referred to the line weakening as "veiling" We have avoided the use of this term in the present paper, for it has been introduced sometimes to describe line weakening and sometimes to describe a reduction in the strength of the continuum (as by clouds of dust or smoke) without a marked effect on the lines.

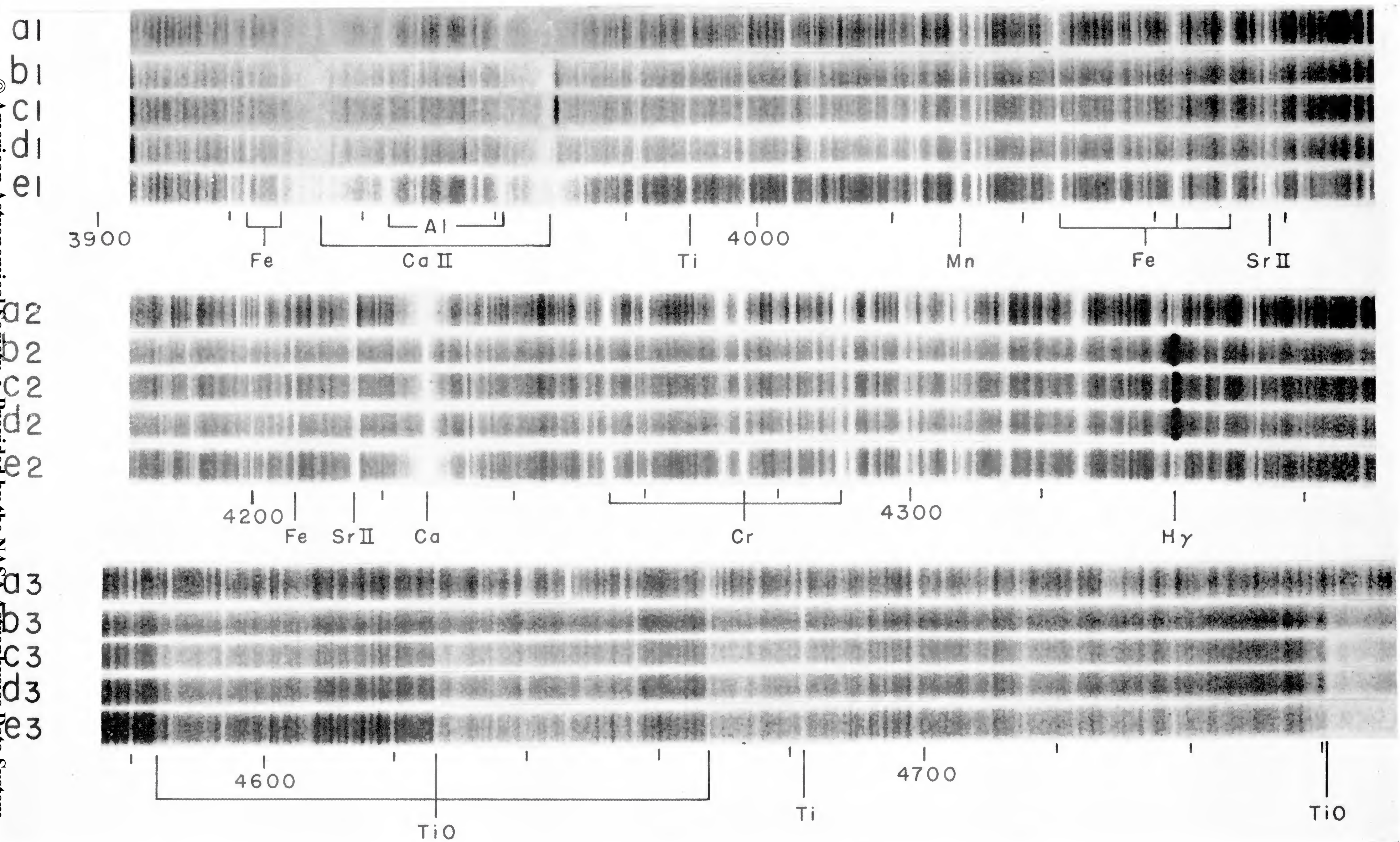

FIG. 7.—Line weakening in three high-velocity Mira variables at maximum light. The variables are *b*, R Ari, M3e; *c*, W Lyr, M5e; and *d*, R Per, M5e. The other stars are *a*, υ Gem, M0 III; and *e*, R Lyr, M4.5 III.

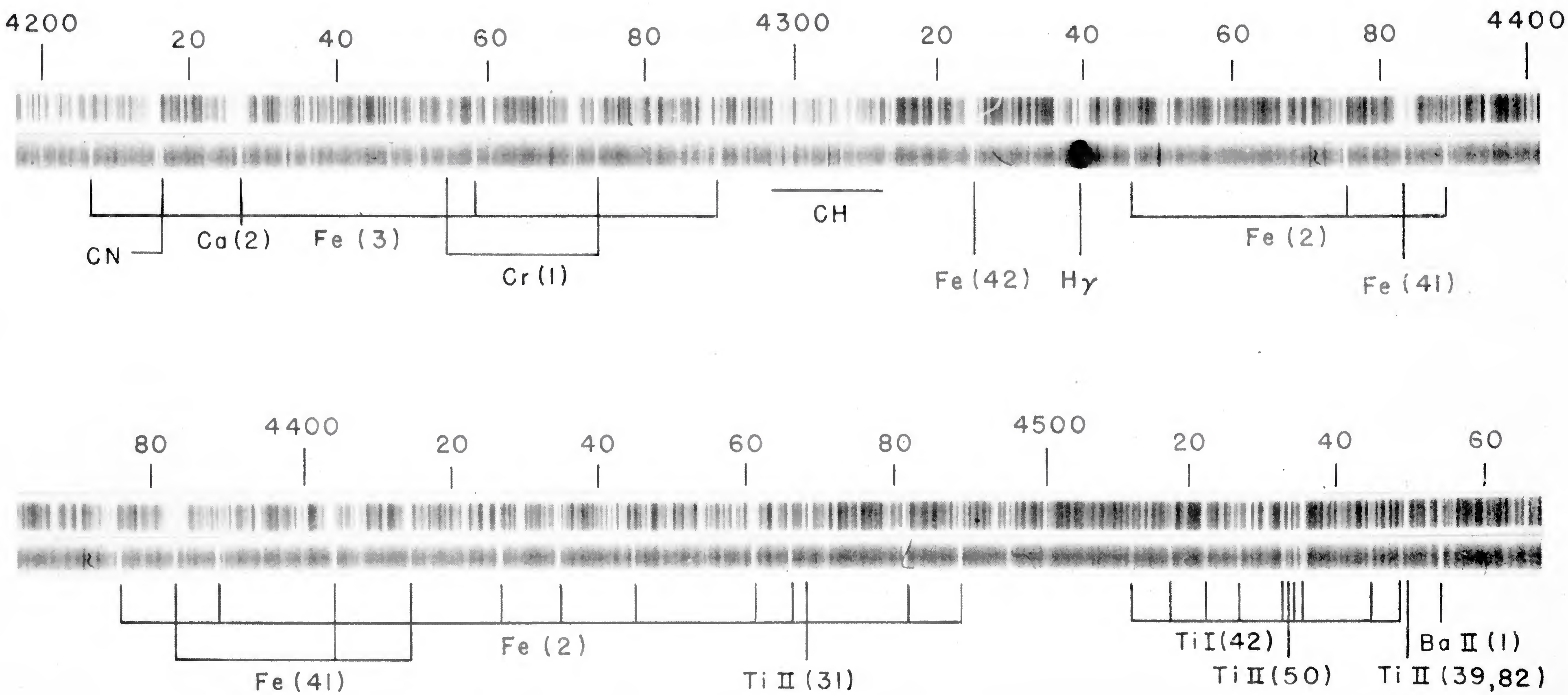

FIG. 8.—Comparison of the K4ep Mira variable Z Oph (1957 maximum) with θ Lyr (K0 II, strips *a*). Note line weakening and absence of TiO bands. Both spectrograms were taken by J. L. Greenstein.

and the emission is strong in many lines which are little weakened (e.g., Fe 4376). Moreover, where emission lines do occur, they are characteristically sharp and are displaced shortward with respect to the absorption lines. But with the relatively high dispersion now available, it appears that severely weakened lines, like λ 4226, retain profiles which are symmetrical and otherwise normal for their equivalent widths. Moreover, the emission lines in Mira variables have intensities which are strong functions of phase, while the line weakening is relatively insensitive to phase. It is evident that emission cannot explain the major part of the observed weakening of lines.

Neither can the phenomenon be ascribed chiefly to band absorption by titanium oxide, in spite of the known suppression of lines near strong bands (cf. Sec. V). The decisive evidence on this point is furnished by the variables in which the TiO bands are so weak near maximum light that they must be assigned a type of M2e or earlier. These include RW Peg, RY Cep, Z Oph, and RT Cyg at some maxima. In all these variables most of the prominent absorption lines are weaker than in even a K-type giant. In Figure 8, Z Oph (K4ep) is shown along with an early K bright giant, in order to emphasize the fact that the line intensities cannot be approximated by merely assuming an earlier temperature type for the variable. The spectrum of Z Oph may be described as presenting an extreme case of many peculiarities shown by the other early Me variables. More than any other variable observed by us, it shows an enhancement of the zero-volt lines of Ti I and Fe I, and also of certain subordinate lines of Ti I (e.g., RMT multiplet 42, E.P. = 0.8 ev). The absence of TiO at several maxima of Z Oph has been confirmed by spectrograms of the visual region. We must reject TiO as the dominant agent in the line weakening that is typical of the early M-type Mira variables.

On the other hand, if the line weakening is due to a real deficiency of the metals in these stars, one might anticipate the occurrence of large temperature errors in their spectral classification; for the spectral types have been assigned principally on the basis of the absolute strengths of the TiO bands in the blue and green, and one might expect these features to be weakened by metal deficiency approximately to the same degree as the strong metal lines are weakened. Band weakening would then lead to the assignment of spectral types that are too early and correspond to temperatures that are too high.

In fact, it appears that the TiO spectral types are in surprisingly good accord with those based on temperature-sensitive line ratios. The TiO bands evidently are much less weakened than the lines in Mira variables when these stars are compared with non-Mira early M giants of the same temperature. It is possible to give a qualitative explanation of this effect. Since the TiO bands overlap, this molecule contributes to the total absorption coefficient on both sides of a band head. If TiO were the only source of opacity, the strength of a band would be nearly independent of the abundance of TiO, for a decrease in this abundance would diminish the total absorption coefficient by the same factor on both sides of the band head. In an intermediate situation, where other particles besides TiO molecules contribute appreciably to the opacity, it is clear that an intermediate degree of band weakening will occur.

We shall now estimate the metal deficiency in the early-type Miras. Let us suppose that shortward from λ 4400 the opacity in their atmospheres is due principally to H^-. Since we often observe Ca II K to be much stronger than Ca I 4226 in these stars, we shall assume that most metal atoms are singly ionized. Let W be the equivalent width of the strong line of a neutral metal, a line which lies on the damping part of the curve of growth. If g is the acceleration of gravity, and A the number of hydrogen atoms per metal atom, we then find, following Strömgren (1940),[2] that in stars of the same temperatures $W \propto (g/A)^{1/4}$. We estimate that, in the early-type Miras, $\Delta \log W$ lies in the range

[2] Similar relations have been given by Schwarzschild *et al* (1951) and by Kinman (1959)

-0.4 to -0.5 for Ca 4226. Since large changes in g are unlikely, it follows that $\Delta \log A$ is between $+1.6$ and $+2.0$.[3]

As may be seen in Figure 7, Ca II H and K are often weakened in the Miras by a much smaller factor than are Ca I 4226 and other strong lines of neutral metals. But this is readily explained. In the simple theory of Strömgren, the strong line of an ion has an equivalent width that is independent of A.

For A equal to its normal value, 1×10^4, Aller (1960) gives $\log P_e \simeq -2.6$ at $T = 2800°$ (M3.5 III). It follows that Ca II/Ca I = 2, and this is consistent with the equivalent widths of K and λ 4226 in the spectrum of a normal M3 giant. In these conditions, most metal atoms are neutral. But $P_e \propto (g/A)^{1/2}$, and if $\Delta \log A \simeq +2$, then $\Delta \log P_e \simeq -1$. Hence $\log P_e \simeq -3.6$, and Ca II/Ca I $\simeq 20$ at the same T and g. At higher luminosity, the K line should gain still more on λ 4226.

These conclusions are clearly in rough quantitative accord with the observations. The results also show that in these stars one must not use the usual ratios of ionized lines to neutral lines as simple luminosity indicators, for a decrease in A affects these ratios in the same way as does a decrease in g. The absolute strengths of the lines must be taken into consideration, as well as their ratios.

As may be seen in Table 1, the variable stars of Figure 7 are all high-velocity objects. In this respect, they are representative of the early-type Miras, for these stars are known to be a high-velocity group. At the Vatican Conference of 1957, for example, they were assigned to "intermediate Population II" (O'Connell 1958). But large and systematic metal deficiencies have recently been established for other high-velocity objects which belong to this population or to the extreme population II or galactic halo. Metal deficiencies within the range 10^{-1}–10^{-3} have been found in the stars of globular clusters (Helfer, Wallerstein, and Greenstein 1959; Kinman 1959), and RR Lyrae variables (Preston 1959, 1961), and the subdwarfs (Aller and Greenstein 1960); and lesser deficiencies probably occur in some of the so-called "weak-line" stars of the solar neighborhood (Schwarzschild *et al.* 1957).

If the weak-line characteristic that signals gross metal deficiency was not recognized earlier in the spectra of the early-type Miras, it is probably because spectroscopic differences were sought between the high-velocity and low-velocity members of this group. But while there are some low-velocity stars among the early-type Miras—as there must be in any high-velocity group—it may be that there are no representatives of the disk population among them. If no early-type Miras exist among the metal-rich stars, the line weakening reveals itself only when the spectra of these variables are compared with the spectra of non-Mira giants of the same spectral types.

Among the other high-velocity objects that have been studied for metal deficiency, a correlation occurs between metal deficiency and kinematic properties. This has the sense that the greatest deficiencies occur in populations that have the largest velocities and the most nearly isotropic velocity distribution relative to an origin at the center of the galaxy. We may expect that further study of the Mira variables will confirm that they, too, show this correlation. However, it will be necessary to proceed cautiously in this endeavor, for the spectra are complex and variable and, as we shall now see, they are sometimes subject to unidentified processes which can simulate some of the effects of metal deficiency.

V. LINE WEAKENING IN LATE-TYPE MIRAS

Most Mira variables per unit volume of space near the sun have spectral types at maximum later than M5e. These stars have a mean speed perpendicular to the galactic plane $\langle |Z| \rangle = 24$ km/sec (Oort 1958). In contrast, the early-type Miras have $\langle |Z| \rangle = 36$

[3] These estimates may be too high because of neglect of (1) Rayleigh scattering by the hydrogen atoms (as was kindly pointed out by R. P. Kraft) and (2) radiative damping In a subsequent paper by one of us (A. J. D.) these difficulties will be detailed (*added in proof*).

km/sec, and the non-Mira M giants $\langle|Z|\rangle = 16$ km/sec. The late-type Miras therefore belong to a population with intermediate kinematic characteristics.

These stars also have an intermediate degree of line weakening, which possibly reflects a metal deficiency of order 10^{-1}. However, we shall now see that in the late-type Miras the line strengths are intermittently subject to large disturbances by additional factors which are not understood. For this reason, it is very difficult to isolate the effects of metal deficiency and to obtain a measure of this important datum.

Mira itself affords the best examples of these effects, for it has been extensively observed at coudé dispersion for many years. In his 1926 discussion of the spectrum of Mira during eleven successive cycles, Joy found that the spectral type correlates strongly with magnitude. The mean relation gave a type of M5.2 at magnitude 3.1, and a type of M9.0 at magnitude 8.9. However, Joy added this remark: "While the general characteristics of the spectra certainly depend mostly on magnitude, there is no escape from the conclusion that many peculiarities of individual lines or regions vary in different cycles."

TABLE 4

MAXIMA OF MIRA

JD of Max. 2400000+	m_v at Max.	Line Strength	JD of Max. 2400000+	m_v at Max.	Line Strength
28830	4 7	1	33153.	2 8	5
29159	4 0	4	33487	3 8	2
29481	2 9	......	33806	3 5	. .
29820	3 7		34151	3 0	
30164	3 6	.. .	34491	3 6	
30498	3 6	.	34821	3 2	
30820	3 1	. .	35140	3 9	.
31152	4 0	.... .	35482	3 7	.
31484	2 7	3	35805	4 2	......
31823	4 2	2	36141	4 1	3
32164	4 3	2	36473	3 2	4
32480	3 1	5	36798	3 4	3
32819	3 7	1	37131	(3 2)	3

Figure 9 illustrates some of the differences that do, indeed, occur between spectrograms made at about the same phase and magnitude, but in different cycles. The principal effect is a notable weakening of many absorption lines, particularly near λ 4350. The line weakening diminishes toward shorter wavelengths and virtually disappears at $\lambda <$ 3900 A.

The degree of line weakness is characteristic of a whole cycle; in particular, it is not a function of phase. Table 4 lists the maxima of Mira since 1936, together with estimates of line strength from the coudé plates that were obtained near these maxima. The line strengths increase with the arbitrary numerical parameter listed in the table. The dates of maxima and the magnitudes are from the A.A.V.S.O. For recent cycles, provisional values were kindly provided by Mrs. Mayall. Where no estimate of line strength is given, no suitable coudé plates are available. The table shows that strong-line spectra tend to occur in cycles with bright maxima and weak-line spectra in those with faint maxima. But there is considerable dispersion about the mean relation. For example, the spectrum showed only a minimum line weakening near the 1958 maximum at $m_V = 4.1$, but the weakening was extreme near the 1948 maximum at 3.7. The TiO bands were comparably strong near both these maxima.

In any wavelength range, the line weakening in the lower spectra of Figure 9 is evidently less for resonance lines (*short markers*) than for subordinate lines (*long markers*). At the low temperature of this reversing layer, the Boltzmann factor will impart a very high temperature sensitivity to subordinate lines. For some of the observed effects we may, therefore, give a qualitative explanation by supposing that the line weakening is associated with a reduction in the mean temperature of the reversing layer. The concentration of TiO then increases, and the atmosphere becomes less transparent throughout the region of the TiO bands. On a Schuster-Schwarzschild model of the atmosphere, this raises the level of the photosphere and leaves fewer atoms per square centimeter above the photosphere. On a Milne-Eddington model, it increases the opacity and therefore diminishes the line strengths, η_0.

It has long been known that such effects occur in the Mira variables and most conspicuously at the D lines (Merrill 1940, p. 59). These lie not far from the well-marked head of the strong (1, 0) band of the Duner-Coheur γ' system, evidently in a region of heavy TiO absorption. In Figure 1 the D lines remain almost constant, actually passing through a slight minimum between M3 and M7.5, over a drop of probably 1000° in temperature. Shortward from λ 4400, however, the band heads are always relatively weak, and one is therefore at first surprised to find evidence for occasional strong TiO absorption even at these wavelengths. These bands, including λ 4395 (8, 3) and λ 4352 (7, 2), arise from intrinsically weak transitions in the α-system, and they become important only when the surface density of TiO grows very large, as in the coolest M stars. In these objects we have seen that many of the stronger bands at $\lambda > 4500$ can no longer serve as sensitive indicators of temperature. Because of the proliferation of band absorption, the reduction of temperature may bring an actual diminution in contrast of the band structure that can be seen at many wavelengths, including both the heads and the rotational structure away from the heads. Figure 5 illustrates the change in contrast of the TiO bands between strong-line cycles (strips *a* and *b*) and weak-line cycles (strip *c*) in Mira. The difference is no mere photographic effect; the bands are actually much softer in the weak-line cycle. This is the effect discussed in Section II.

Although part of this intermittent line weakening must be due to an opacity effect associated with the TiO band strengths, it is probable that other processes play a role. For example, at a weak-line maximum both Ca I 4226 and Ca II H and K have smaller equivalent widths, although they lie outside the region of heavy TiO absorption. In addition, as may be seen in Figure 9, the zero-volt lines of Fe near λ 4350 are subject to much less weakening than the zero-volt lines of V—this despite the fact that some of the Fe lines belong to a multiplet (RMT No. 2) in which many of the lines show emission components near and after maximum light, while no comparable emission occurs in the V multiplet.

The evidence is conflicting with respect to the correlation between absorption-line weakening and emission-line strength. Joy (1954) showed clearly that in Mira the "bright lines are stronger and more numerous in cycles of high maximum brightness." But we find that absorption-line weakening occurs preferentially at faint maxima. Hence it would appear that the absorption lines tend to be strongest when the emission lines are also the strongest. On the other hand, the low-dispersion spectrograms in the Mount Wilson and Perkins collections show that in a number of stars the maxima with very weak absorption lines have very strong emission lines of hydrogen. R Ser at the weak maximum of 1960 is an example (Iwanowska *te al.* 1960). In the same sense, the SRa variables have weaker absorption lines but stronger Balmer emission than SRb or I variables of smaller amplitude.

Strips *a* and *b* of Figure 10 illustrate the changes that occur in the spectrum of Mira following a strong-line maximum. The absorption lines and bands clearly indicate a decrease in temperature. This strengthens the TiO bands, and it weakens most subordinate lines. In addition, many of the lines (especially subordinate ones) appear to become shallower and wider, giving the spectrum a "blurred" appearance (Merrill 1940, p. 51). At the same time, reduced ionization causes strengthening of the zero-volt lines due to Al, Ca I, K, and Cr. But this is accompanied by a slight weakening of the zero-volt

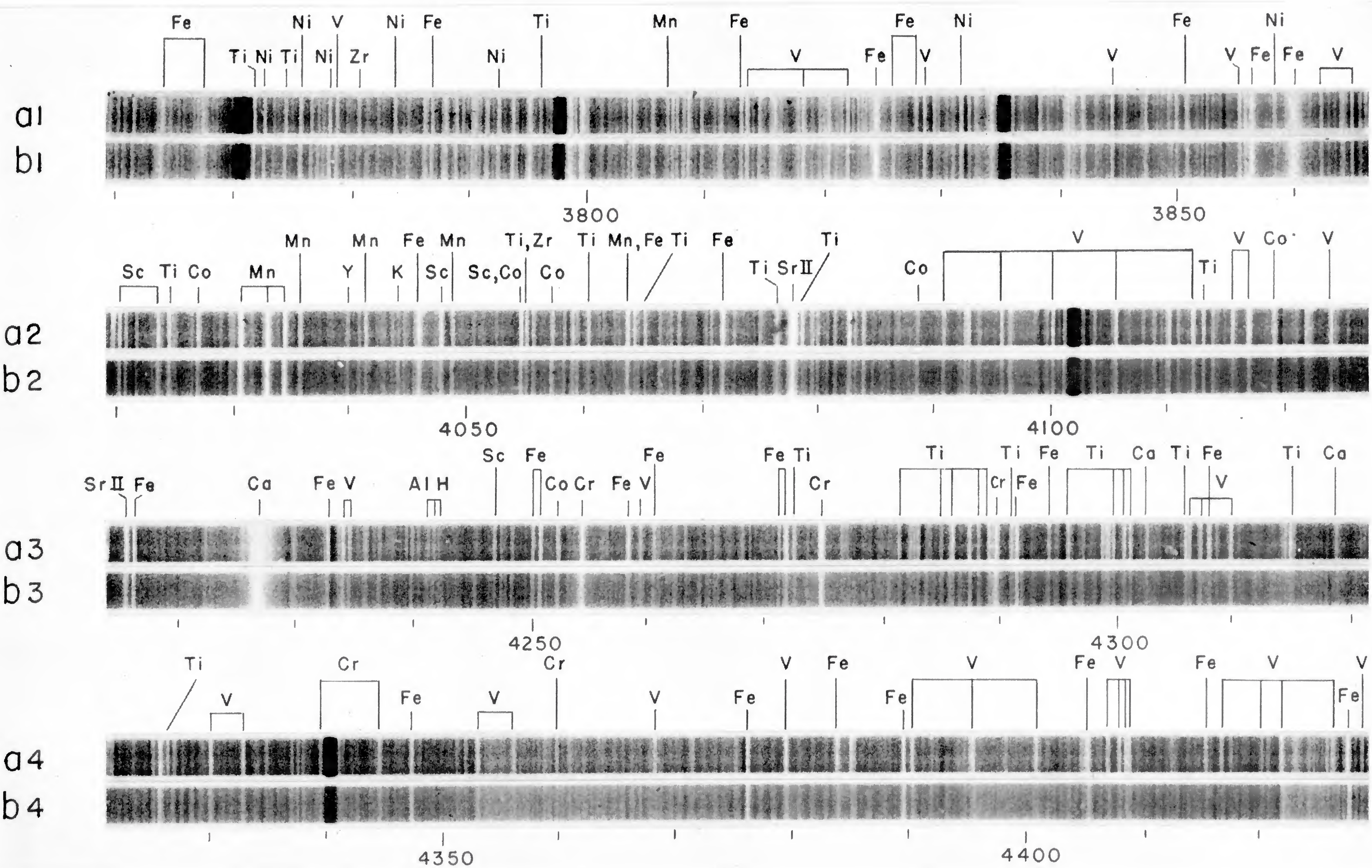

FIG. 9.—The spectrum of Mira (332^d, M6e) at a strong-line maximum in 1947 (*a*) and at a weak-line maximum in 1948 (*b*). Subordinate lines are designated with longer markers than zero-volt lines.

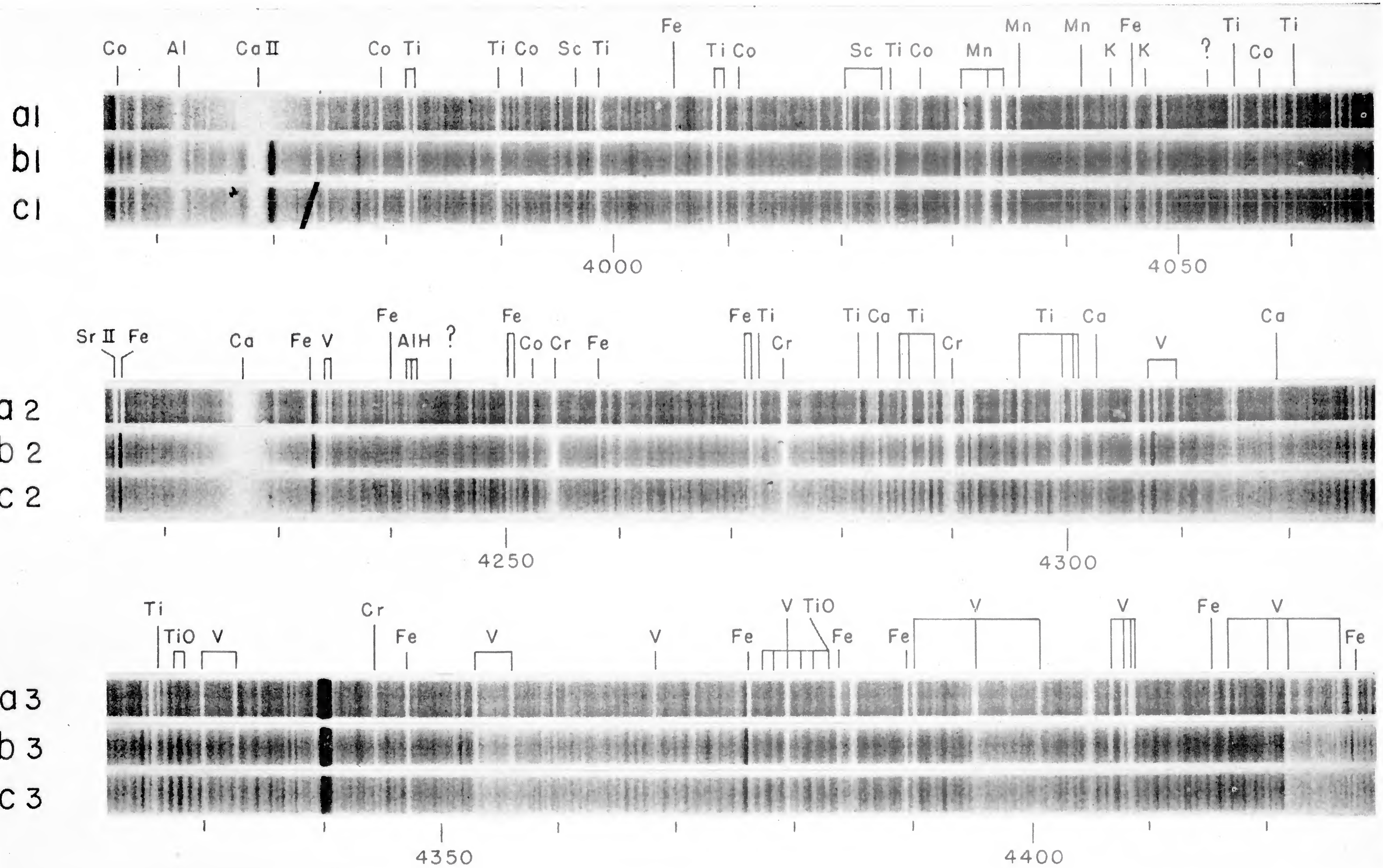

FIG. 10.—Changes in the spectrum of Mira with phase and with light-cycle. Strips (*a*) and (*b*) were taken 2 days and 81 days, respectively, after a strong-line maximum. Strip (*c*) was taken 78 days after a weak-line maximum. Subordinate lines are designated with the longer markers.

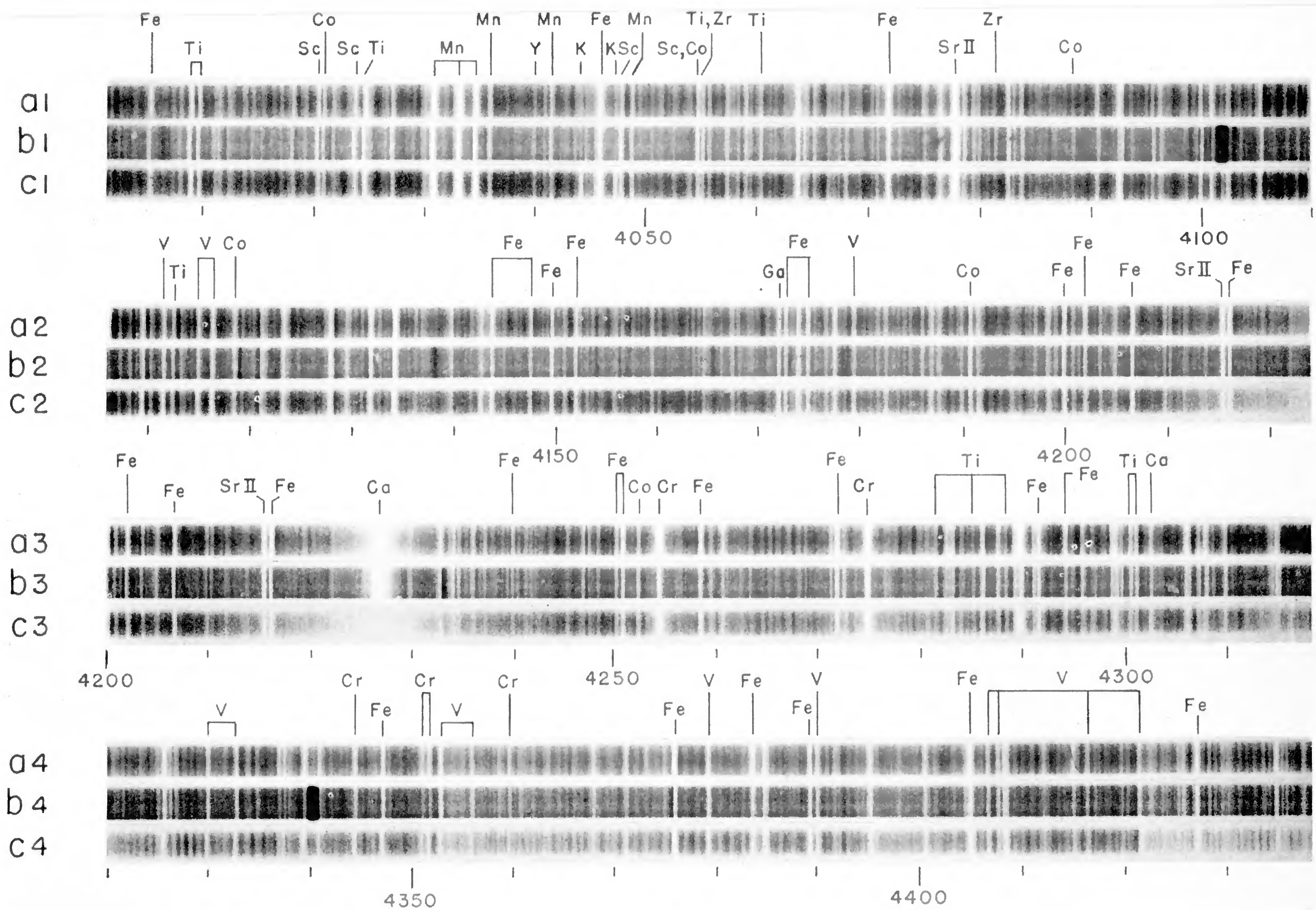

FIG. 11.—Mira at a strong-line maximum (1947) (*b*), compared with the small-amplitude variables 30 g Her (M6 III) (*a*) and SW Vir (M7 III:) (*c*). Subordinate lines are designated with the longer markers.

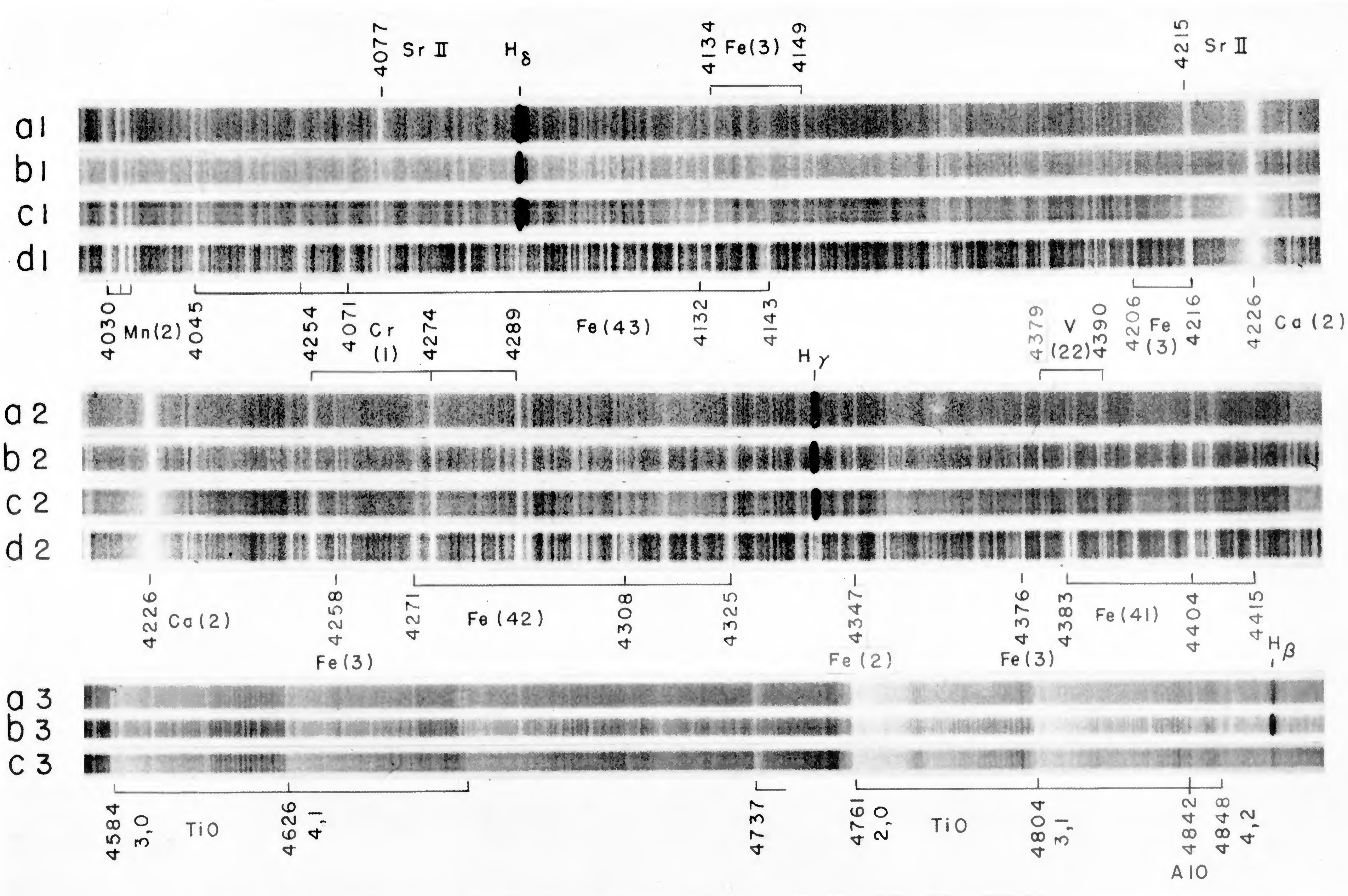

FIG. 12.—Line weakening in three Mira variables: (*a*) U Ari at a weak-line maximum in 1960; (*b*) R Dra at a bright maximum in 1959; (*c*) R Aql at a slightly weak-line maximum in 1959; (*d*) small-amplitude variable R Lyr (M4.5 III). Subordinate lines are designated with the longer markers.

lines due to Sc, Ti, V, Y, and Zr—possibly the result of increased formation of the oxides (cf. Sec. III). The zero-volt lines of Mg, Mn, and Fe represent a special case; they diminish as the temperature drops because emission components are superposed upon them.

Strips *c* of Figure 10 show the postmaximum spectrum in a weak-line cycle. As compared with the spectrum at the weak-line maximum, *all* low-level lines are strengthened, except for those of Mg, Mn, and Fe that have obvious emission components. The blurring of lines does not show much change with phase, nor does the excitation temperature. The differences between the two postmaximum spectra, strips *b* and *c*, are still in the same sense as the differences between the maximum spectra of Figure 9, i.e., the absorption lines are stronger after the strong-line maximum, especially for $\lambda > 4300$ A. However, the differences are smaller than near maximum light.

Buscombe and Merrill (1952) have given the equivalent widths of many absorption lines in Mira at six different phases. From these data they have estimated the electron pressure, the excitation temperature, and the relative opacity. It happens, however, that two of their spectrograms were made in a weak-line cycle and the other four in a strong-line cycle—a circumstance which obscures the interpretation of their results.

The absorption spectrum of Mira most closely resembles that of a non-Mira M giant near the maximum phase in strong-line cycles. For example, Figure 11 shows Mira at such a maximum, placed between two giants of different temperatures. All the usual criteria indicate that at this epoch the temperature of Mira lay between that of 30 g Her (M6 III, strips *a*) and SW Vir (M7 III, strips *c*), but nearer that of g Her. Nevertheless, the general level of line strength is evidently less in Mira than in the other two stars, especially so for the strong lines of neutral metals. Since the line strengths are never appreciably greater in Mira than on this spectrogram, and they are often much smaller, we estimate that the metals are probably deficient by a factor of order 10^{-1}. However, this conclusion is uncertain, for it is possible that H^- absorption no longer dominates the opacity in the atmospheres of the relatively metal-rich Me variables of types later than M5e.

Because of the large number of spectrograms available, we have been able to give a rather detailed description of the absorption spectrum of Mira and its complex changes from cycle to cycle. But there is every reason to believe that similar effects occur in other late-type Mira variables, which have not been so well observed. Representative examples are illustrated in Figure 12; there three Miras at maximum light are compared with the M4.5 giant R Lyr (strips *d*), which is only slightly variable. Strips *a* show U Ari in a weak-line cycle (cf. Fig. 9 of Mira). Both the atomic lines and the band heads that are usually strongest appear more "washed out" than usual, although the type estimated from the *relative* band strengths in the yellow is M6.5e, about as late as at normal maxima (cf. Sec. II). For the reason given in Section IV, we may not assume that the strength of Sr II 4077 is necessarily an indicator of high luminosity. The spectrum of R Aql, in strips *c*, shows many of the same characteristics, but the selective weak-line effects are less pronounced.

Strips *b* of Figure 12 show the spectrum of R Dra. This high-velocity star has an abnormally late spectral type for its period of 246 days. Usually the type at maximum is M6e, but in this cycle it reached M5e. As in other high-velocity Miras (Fig. 7), the strong zero-volt lines of the neutral metals are greatly weakened in R Dra and by considerably more than in the long-period Miras U Ari and R Aql. Also very weak in R Dra are Sr II 4077 and 4216, and this may be an indication that the star has lower luminosity than the other high-velocity variables with weaker bands. However, many other lines are weakened much less in R Dra, particularly longward from λ 4250. This manifests the difference between the selective, wavelength-dependent line weakening that sometimes occurs in the late-type, long-period, disk-population Miras like U Ari and R Aql; and the simpler metal-deficiency line weakening that is characteristic of the early-type, short-period, halo-population Miras.

The authors wish to acknowledge the important contribution to this research by the night assistants at the 100-inch and 200-inch telescopes. Without their co-operation in the sustained and careful guiding of many long exposures, the spectrograms illustrated here would not have been obtained.

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