# THE VISUAL MULTIPLE SYSTEM CONTAINING BETA LYRAE\*

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#### ABSTRACT

Beta Lyrae is a member of the visual multiple system ADS 11745 and has five companions which range in magnitude from 7.2 to 15 2. Previous and new astrometric measures of the three brightest companions (B, E, and F) yield motions that are probably consistent with physical association with  $\beta$  Lyrae. Spectroscopic measures of these three companions show that one of them (E) is probably not a member. Another companion (B) is a spectroscopic binary, for which orbital elements are derived. Photoelectric measures show that the two faintest companions (C and D) are not physically associated with  $\beta$  Lyrae From the association of companions B and F with  $\beta$  Lyrae we derive a reddening,  $E_{B-\nu}$ , of 0 065 mag., a distance modulus of 7.1 mag., and a luminosity,  $M_{\nu}$ , for  $\beta$  Lyrae of -3 9. This leads to a secondary mass for  $\beta$  Lyrae that is greater than that of the primary.

#### I. INTRODUCTION

Beta Lyrae is an eclipsing binary which has been extensively observed (e.g., Struve 1958) and has been unusually productive of information on gaseous streams in a close binary system. The system consists of, in addition to the gaseous streams, a B8 star of high luminosity and a spectroscopically invisible secondary, often assumed to be of spectral type F.

Definitive values for the radii and masses have not been determined from the lightand radial-velocity curves because of the contamination of the former by the luminous streams. Kuiper (1941) and others have tabulated values of the radii and masses for various assumed values of the mass ratio, and, on the expectation that the primary is the more massive star, Kuiper suggested a luminosity for the primary of  $M_V = -7.6$ . However, he realized that this luminosity might be too high for three reasons: (1) Struve's estimate of the luminosity of the primary from the appearance of its spectrum gives  $M_V$  considerably fainter than -7.6; (2) if the principal visual companion (ADS 11745 B) to  $\beta$  Lyrae is physically associated with the latter, the luminosity of  $\beta$  Lyrae is  $M_V \simeq -4.5$ ; (3) the systemic radial velocity of the  $\beta$  Lyrae system gives no evidence of galactic rotation, although such evidence would be expected if the binary were as distant as corresponds to a luminosity of  $M_V = -7.6$ . More recently Struve (1958) has reiterated the first objection to the higher luminosity. Boyarchuk (1959) estimated the luminosity of the visual companion from the equivalent widths of the H $\gamma$ -H $\epsilon$  lines and derived  $M_V = -2.7$  for the primary of  $\beta$  Lyrae.

Beta Lyrae is a member of a well-known visual multiple system (ADS  $11745 = \beta$  8868) which has components listed in Table 1. If some or all of the five companions to

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 $\beta$  Lyrae are physically associated with it, the color-magnitude diagram for these companions will yield a good distance modulus. It is the purpose of this paper to present new astrometric, spectroscopic, and photometric measures of the companions to ascertain whether this hope can be fulfilled.

## **II. ASTROMETRIC MEASURES**

Early measures in the system ADS 11745 are collected by Burnham (1906) and Aitken (1932). Recent measures are listed in Table 2. Also listed there are photographic measures on plates taken with the automatic double-star camera on the Lick Observatory 36-inch refractor. The internal mean error of the final results, which depend on numerous settings, are 0".006, 0".008, and 0".017 for plates numbered 2337, 2338, and 2356, respectively. These errors are rather large because of the large inequality in magnitudes.

In Figure 1 all known measures or means are plotted with weights proportional to the number of measures averaged, except that recent photographic measures have been weighted 5. Least-squares lines have been determined. The motions in position angle are due primarily to the effect of precession, which amounts to  $-0^{\circ}6525$  per century (Chauvent 1891). After allowing for this, we find the actual changes in position angle ( $\Delta P.A./\Delta t$ ) and separation ( $\Delta \rho/\Delta t$ ) that are listed in Table 3. The astrometric data on stars C

Name	ADS mag	Position Angle with A	Separa- tion from A
β Lyrae β² Lyrae	3 0 6 7	149°	46 <sup>''</sup> 46
BD+33°3222 BD+33°3225	13 0 14 3 9 2 9 0	68 318 19	64 67 86
	β Lyrae $β^2$ Lyrae $BD+33^{\circ}3222$	Name         mag $\beta$ Lyrae         3 0 $\beta^2$ Lyrae         6 7           13 0         14 3           BD+33°3222         9 2	Name         ADS mag         Angle with A $\beta$ Lyrae         3 0 $\beta^2$ Lyrae         6 7         149°           13 0         247            14 3         68           BD+33°3222         9 2         318

TABLE 1

# Components of ADS 11745

TABLE 2	
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**RECENT ASTROMETRIC MEASURES IN ADS 11745** 

Pair	Date	P.A. (Epoch)	Sep	No. Measures	Observer
	(1928 05	148°8	45″.82	33	Komendantoff (1935)
	1929 59	148 5	45 59	3	Aller (1930)
В	∫1949 71	148 70	45 76		Kranjc (1951)
<b>VD</b>	1955 69	148 51	45 733	2	Bottger (1958)
	1959 852	148 697	45 718	61	Present (No 2337)
	1959 869	148 747	45 700	34	Present (No. 2356)
	(1929 59	318 1	66 97	3	Aller (1930)
E	1959 852	317 473	66 997	54	Present (No. 2338)
	1959 869	317 506	66 965	34	Present (No 2356)
	(1929 58	18 3	84 99	2	Aller (1930)
F	1959 852	18 562	86 035	54	Present (No 2338)
	1959 869	18 646	86 042	34	Present (No. 2356)

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and D are insufficient to determine motions, although those of star C are probably roughly the same as those in Table 3.

The proper motion of  $\beta$  Lyrae is given (Jenkins 1952) as 0".7 per century. In Figure 2 we have plotted to scale the multiple system with an arrow on A ( $\beta$  Lyrae) indicating its proper motion and arrows on the other stars indicating their motions relative to A, after allowing for the precession effect. For star B we see that its well-determined motion relative to A is very small compared with the latter's proper motion, and hence it shares in that proper motion. This comparison for stars E and F is less certain. Their motions in position angle relative to A are also very small, but the motions in separation seem to be

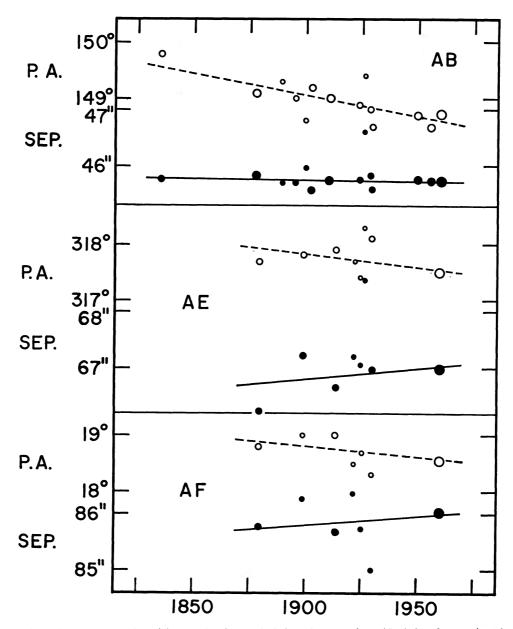


FIG 1—The measures of position angles (*open circles*) and separations (*dots*) for three pairs of components (AB, AE, and AF) in ADS 11745 are plotted with time The weights are indicated approximately by the sizes of the symbols The dotted lines are least-squares relations

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appreciable compared with the proper motion of A. However, Figure 1 shows that the changes in separation are very poorly determined, undoubtedly because of the difficulty in measuring pairs of stars that differ so much (7 mag.) in brightness. The elimination of the 1879 mean separation for AE and the 1929 mean for AF would greatly alter the derived changes in separation of the components.

We can ask whether the motions in Table 3 are reasonable for a multiple system. If we assume (1) the final distance modulus of 7.1 mag., (2) reasonable masses for components A  $(31.2 \text{ } \mathbb{M}_{\odot})$  and B  $(4 \text{ } \mathbb{M}_{\odot})$ , and (3) a circular orbit seen pole-on, then the expected

Star Pair	ΔP A /Δt (Deg/Century)	Δρ/Δt ("/Century)	Star Pair	ΔP A /Δt (Deg/Century)	$\Delta \rho / \Delta t$ ("/Century)
AB AE	$ \begin{array}{r} -0 \ 108 \\ +0 \ 166 \end{array} $	$ \begin{array}{r} -0 & 056 \\ +0 & 383 \end{array} $	AF	+0 285	+0 314

TABLE 3MEAN MOTIONS IN ADS 11745

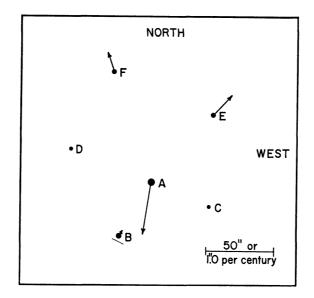


FIG. 2.—The positions and motions of the components of ADS 11745 The scale at the lower right is for both dimensions (in seconds of arc) and motions (in seconds of arc per century). The arrow on component A ( $\beta$  Lyrae) indicates the extrapolated proper motion of that star in a century, while the arrows on components B, E, and F show the motions of those stars relative to A in the same time interval and after removal of the precession effect This shows that if the proper motion of A is well determined, star B, at least, shares it.

motion in position angle for AB is  $0^{\circ}.162$  per century. The observed amount is  $0^{\circ}.108$  per century. On the other hand, if, instead of the third assumption, we assume a circular orbit seen equator-on and with component B at, say,  $45^{\circ}$  to the line of sight, then the expected motion in separation is  $0^{\prime\prime}.077$  per century compared with an observed amount of  $0^{\prime\prime}.056$  per century. Therefore, the motion of B relative to A is reasonable for a double-star system.

The motions of E and F are far more difficult to predict because the assumption of a two-body system is probably not valid for these less massive, more distant stars. However, their general similarity with that of B, after allowance for the poorer determina-

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tions, indicates that their motions may also be attributable to motion in the multiple system.

The conclusions from the astrometric observations are that component B and perhaps C, E, and F may be members of a physical system with A; nothing can be said about component D at this point.

#### III. SPECTROSCOPIC MEASURES

Spectra of stars B, E, and F were obtained with the Mount Wilson 60-inch Cassegrain X-spectrograph and 8-inch camera (dispersion 41 A/mm) for purposes of obtaining spectral classifications and radial velocities. The spectral classifications were estimated as follows. Star B has  $\lambda$  4026 much stronger than the K line and  $\lambda$  4471 somewhat stronger than  $\lambda$  4481. It appears to be a normal B7 V star. The spectrum of star E is mostly like that of an A8 dwarf except that the Sr II lines at  $\lambda$  4077 and  $\lambda$  4215 are stronger than,

Star	Plate No.	Date (UT)	Corrected Velocity (km/sec)	Cycle and Phase
;	$\begin{cases} Xe-4729 \\ 4750 \\ 4775 \\ 4796 \\ 4896 \\ 4919 \\ 4940 \\ 5040 \\ 5067 \\ 5347 \\ 5402 \end{cases}$	1959 Oct. 11 187 12 181 13 149 14 222 1959 Nov. 7 139 8 166 9 154 1959 Dec. 12 097 13 105 1960 Mar. 5 540 7 501	$\begin{array}{c} -38 & 6 \\ -20 & 5 \\ -20 & 7 \\ -32 & 2 \\ -28 & 7 \\ -22 & 3 \\ -24 & 6 \\ -24 & 5 \\ -18 & 9 \\ -26 & 5 \\ -28 & 6 \end{array}$	$\begin{array}{c} 0 \ 155 \\ 0 \ 383 \\ 0 \ 606 \\ 0 \ 853 \\ 6 \ 550 \\ 6 \ 817 \\ 14 \ 393 \\ 14 \ 625 \\ 33 \ 814 \\ 34 \ 265 \end{array}$
C	$\left\{egin{array}{c} 4776 \\ 4897 \end{array} ight.$	1959 Oct 13 192 1959 Nov. 7 177	+ 1 5 + 1 2	· . · .
	4751	1959 Oct. 12.224	-22 7	

TABLE 4

RADIAL VELOCITIES IN ADS 11745

respectively,  $\lambda$  4045 and  $\lambda$  4226. It is classified A8p (Sr). The spectrum of star F is A8–9 V. The radial velocities were calibrated with measures of Procyon and Vega, which yielded a mean spectrograph correction of -6.8 km/sec. The corrected velocities for stars in ADS 11745 are listed in Table 4.

We can probably expect that any component of the multiple system whose measured radial velocity is within about 10 km/sec of that of  $\beta$  Lyrae may be a physical member. Struve (1958) indicates that the systemic velocity of  $\lambda$  Lyrae may be slightly variable between -15 and -20 km/sec. The single radial velocity (-22.7 km/sec) of star F agrees sufficiently well with this, although the two velocities (mean of +1.3 km/sec) of star E do not agree. Unless star E is a velocity variable, these observations cast doubt on its membership in the physical system.

Star B (HD 174664 = Boss  $2777 = BD+33^{\circ}3224$ ) is a spectroscopic binary of small amplitude which was discovered to be variable by H. H. Plaskett (Plaskett, Harper, Young, and Plaskett 1921). The spectrum of the secondary is not seen. The velocities of B in Table 4 were analyzed by the method of Lehmann-Filhes and several least-squares solutions. The final orbital elements are listed in Table 5, and the velocity-curve is shown in Figure 3. The curve is poorly determined because the observations are few in number.

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The mean scatter (p.e.) of 1.4 km/sec in the observations is less than the expected internal error of 3.2 km/sec. No attempt was made to incorporate the four Victoria velocities (Plaskett *et al.* 1921) of 1920 because the period determined is not sufficiently accurate to bridge the interval of time. One of the Victoria velocities (+11.1 km/sec) is outside the velocity range observed here and will not fit these orbital elements, but, since it came from a spectrum of only fair quality, we may be justified in neglecting it.

The systemic velocity (-29.4 km/sec) of this binary is probably sufficiently close to that of  $\beta$  Lyrae for agreement in membership. After substituting the expected primary mass in the mass function, we learn only that for  $i \leq 90^{\circ}$ , the secondary mass is  $\geq 0.22$   $\mathfrak{M}_{\odot}$ . The principal spectroscopic results are collected in Table 6.

#### TABLE 5

# ORBITAL ELEMENTS OF ADS 11745 B P = 4.348 days e = 0.25 $T_0 = JD2436852$ 014 $\omega = 158^{\circ}.6$ $\gamma = -29$ 4 km/sec $a_1 \sin i = 0$ 693 × 10<sup>6</sup> km K = 12 0 km/sec $f(\mathfrak{M}) = 0$ 000702 $\mathfrak{M}_{\odot}$

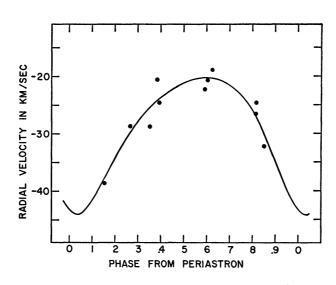


FIG. 3.—The radial-velocity curve of ADS 11745 B

TABLE	6
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## Spectroscopic Results in ADS 11745

Star	Spectrum	Intrinsic Color $(B-V)_0$	Mean Radial Velocity (km/sec)	Membership
B E F	B7 V A8p (Sr) A8-9 V	$ \begin{array}{r} -0 \ 13 \\ + \ 23 \\ +0 \ 24 \\ \end{array} $	$ \begin{array}{r} -29 \ 4 \\ + 1 \ 3 \\ -22 \ 7 \end{array} $	Probable member Possible non-member Probable member

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### IV. PHOTOMETRIC MEASURES AND CONCLUSIONS

Photometric measures on the U, B, V system of the components of ADS 11745 were attempted with the Palomar 20-inch reflector. However, it was found that, with its scale, accurate photometry on the fainter components was inhibited by scattered light from  $\beta$  Lyrae. The fainter components could be measured only with a telescope with a much larger scale during times of moderately good seeing and by interpolating between measures of the background intensity at numerous positions around each star. Even so, the ultraviolet measures of the fainter components could sometimes not be obtained. The photometric results are listed in Table 7.

,								
Date (U T ) Instrument	1959 Oct 2 100″	1959 May 30 20"	1960 June 1 20"	1960 June 16 200"	1960 Sept 28 200"	1960 Sept. 29 200"	Means	
				17				
Star	V $B-V$ $U-B$							$V_0$ $(B-V)_0$ $(U-B)_0$
В	$\begin{cases} 7 & 25 \\ - & 0 & 08 \\ - & 0 & 47 \end{cases}$	$\begin{array}{r} 7 & 22 \\ -0 & 08 \\ -0 & 54 \end{array}$	$\begin{array}{r} 7 & 20 \\ -0 & 08 \\ -0 & 46 \end{array}$				$\begin{array}{r} 7 & 22 \\ - & 0 & 08 \\ - & 0 & 49 \end{array}$	$\begin{array}{r} 7 & 03 \\ - & 0 & 14 \\ - & 0 & 53 \end{array}$
С				$\begin{cases} 13 \ 43 \\ + \ 1 \ 22 \\ + \ 1 \ 05 \end{cases}$	13 44 + 1 23		$13 \ 43 \\ + \ 1 \ 22 \\ + \ 1 \ 05$	$\begin{array}{r} 13 \ 24 \\ + \ 1 \ 16 \\ + \ 1 \ 01 \end{array}$
D			•	$\begin{cases} 15 \ 13 \\ + \ 0 \ 74 \\ + \ 0 \ 18 \end{cases}$	$     \begin{array}{r}       15 & 18 \\       + & 0 & 85     \end{array} $	÷	15 15 + 0 79 + 0 18	$\begin{array}{r} 14.96 \\ + & 0 & 73 \\ + & 0 & 14 \end{array}$
Е	$\begin{cases} 9 & 82 \\ + & 0 & 32 \\ + & 0 & 20 \end{cases}$			9 81 + 0 33 + 0 21	9 81 + 0 36 + 0 21	$\begin{array}{r} 9 & 81 \\ + & 0 & 33 \\ + & 0 & 20 \end{array}$	$\begin{array}{r} 9 & 81 \\ + & 0 & 33 \\ + & 0 & 20 \end{array}$	$\begin{array}{r} 9 & 62 \\ + & 0 & 27 \\ + & 0 & 16 \end{array}$
F	$\begin{cases} 10 \ 17 \\ + \ 0 \ 32 \\ + \ 0 \ 08 \end{cases}$	·		$10 \ 16 \\ + \ 0 \ 34 \\ + \ 0 \ 06$	$10 \ 15 \\ + \ 0 \ 33 \\ + \ 0 \ 07$	$10 \ 16 \\ + \ 0 \ 31 \\ + \ 0 \ 07$	$10 \ 16 \\ + \ 0 \ 32 \\ + \ 0 \ 07$	$\begin{array}{r} 9 & 97 \\ + & 0 & 26 \\ + & 0 & 03 \end{array}$

TABLE 7	
PHOTOMETRIC MEASURES IN ADS 1	1745

The space reddening of the group can best be obtained by a comparison of observed colors and spectral types. The intrinsic colors corresponding (Johnson and Morgan 1953) to the spectral types in Table 6 are listed in the same table. A comparison of these with the observed colors in Table 7 indicates reddenings,  $E_{B-V}$ , of 0.05, 0.10, and 0.08 mag. for stars B, E, and F, respectively. In view of the doubt expressed in the last section on the membership of star E in the physical system, the reddening of the system will be obtained from stars B and F only, namely,  $E_{B-V} = 0.065$  mag. Using the usual relations,  $A_V = 3 E_{B-V}$  and  $E_{U-B} = 0.7 E_{B-V}$ , we obtain the intrinsic magnitudes and colors in the last column of Table 7.

The color-color diagram is shown in Figure 4, together with the curve for mainsequence stars with B - V > 0.2 from Sandage and Eggen (1959) and for the bluer ones from Johnson and Morgan (1953). We see that the  $(U - B)_0$  color of star B places it

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directly on the main-sequence line. This gives slight support for the assumption to be made below that the magnitudes and colors of this spectroscopic binary are not affected by the secondary star. Star D does not fit the main-sequence line, and slightly different values of the interstellar reddening will not change this. The conclusions about the other three stars are uncertain.

The color-magnitude diagram for all stars in ADS 11745 is shown in Figure 5, using magnitudes and colors corrected for interstellar reddening. With a true distance modulus of 7.1 mag., stars B and F fit on the zero-age main-sequence line (Sandage 1957). Since stars only as much as 3 mag. (Sandage 1957) below the upper termination of the main sequence are detectably evolved off the zero-age line and since  $\beta$  Lyrae must be close to this termination, it is reasonable to assume that star B would not have evolved detectably from the zero-age main sequence. Star E may still be a member of the physical system but be above the main sequence because it is an Ap star; Eggen (1957) has shown

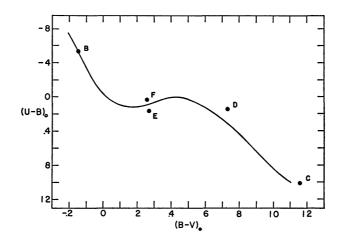


FIG. 4.—The color-color measures (*dots*) for stars in ADS 11745 for an assumed interstellar reddening of  $E_{B-Y} = 0.065$  mag. The curve is for main-sequence stars.

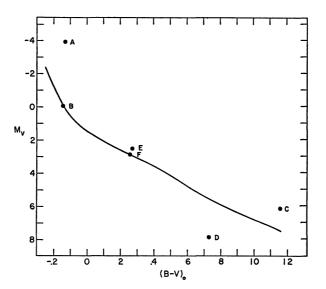


FIG. 5.—The color-magnitude diagram for stars in ADS 11745, assuming an interstellar reddening,  $E_{B-v}$ , of 0.065 mag. and a distance modulus of 7.1 mag.

that such stars are generally above the main sequence. Star D undoubtedly is a background star. Star C may be a star still contracting to the main sequence. However, this system is very similar in age, as judged by the upper termination point, to the Pleiades, and the corresponding position to that of star C in the Pleiades is well above that of the possible contracting stars (Johnson and Mitchell 1958). We conclude that star C is not a member. The conclusions on membership in the physical system are summarized in Table 8, where parentheses indicate some uncertainty.

The observed magnitudes and colors of  $\beta$  Lyrae at maximum light are (Wood and Walker 1960) approximately V = 3.35, B - V = -0.07, and U - B = -0.58. The intrinsic magnitude and colors are then  $V_0 = 3.16$ ,  $(B - V)_0 = -0.13$ , and  $(U - B)_0 =$ -0.62. With a true distance modulus of 7.1 mag., the distance is 260 parsecs, and the luminosity of  $\beta$  Lyrae is  $M_V = -3.9$ . The probable error in the luminosity due to the uncertainty in reddening and in the color-magnitude diagram fitting is about  $\pm 0.2$  mag.

Most investigators (see Struve 1958) consider the primary eclipse of  $\beta$  Lyrae to be annular, since the spectrum of the primary star is seen throughout the eclipse. This leads to a low luminosity for the secondary star relative to the primary; Kopal (1941) obtains

Star		0			
STAR	Astrometry	Radial Velocities	Spectra	Photometry	Conclusions
B C . D	Member (Member)	Member	Member	Member Non-member Non-member	Member Non-member Non-member
E F .	(Member) (Member)	(Non-member) Member	Member Member	(Member) Member	(Non-member) Member

#### TABLE 8

## Conclusions on Membership in $\beta$ Lyrae Physical System

a ratio of 0.136. The luminosity of the primary star alone would then be  $M_V = -3.8$ . This is in agreement with Struve's (1958) estimate based on the appearance of its spectrum. Assuming for the primary (1) a bolometric correction of 1.0 mag. (Arp 1958), (2) a brightening of 1.5 mag. since evolution from the main sequence, (3) application of the mass-luminosity relation (Schwarzschild 1958) for main-sequence stars, and (4) no appreciable loss in mass since evolution from the main sequence, we drive a primary mass of 11  $\mathfrak{M}_{\odot}$ . The observed mass function of 8.5  $\mathfrak{M}_{\odot}$  (Struve 1958) then leads to a secondary mass of 20.2  $\mathfrak{M}_{\odot}$ Thus the invisible secondary is more massive than the primary star.

We are indebted to Dr. D. H. Schulte for performing the least-squares solutions on the Kitt Peak National Observatory Royal-McBee LGP-30 computer.

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