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BINARY STARS AMONG CATACLYSMIC VARIABLES
I. U GEMINORUM STARS (DWARF NOVAE)

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ABSTRACT

A spectroscopic test for binary motion among several U Gem variables is presented. Five stars are shown to be binaries with $P < 9$ hours (SS Cyg, U Gem, RX And, RU Peg, SS Aur), one spectrum is composite (EY Cyg), and one other (Z Cam) shows evidence of binary motion with an as yet undetermined period. There is no evidence contradicting the hypothesis that all members of this group are spectroscopic binaries of short period.

The peculiar radial velocities are small, and the corresponding statistical parallax leads to $\langle M_V \rangle = +9.5 \pm 1$; the blue stars in these systems are probably white dwarfs. The masses of the red components and their spectra (when photographed) seem consistent with a star of mass $\sim 1\, \mathfrak{M}_{\odot}$. Thus the red components of U Gem variables are seriously underluminous for their masses. Evidence is presented which indicates that the red stars overflow their lobes of the inner Lagrangian surface; the ejected material forms, in part, a ring, or disk, surrounding the blue star. Several lines of argument suggest that the U Gem variables are descendants of the W UMa stars.

I. INTRODUCTION

The cataclysmic variables of type U Geminorum (dwarf novae) are characterized by a small range (2–6 mag.) and by the existence of an "induction time" between outbursts. The latter has a time scale of the order of days or weeks, which can be described as "periodic" when averaged for any one variable over a long enough time interval. Of the roughly 100 known stars of this type, two subgroups are recognized: SS Cygni and Z Camelopardalis stars. The former have a more or less well-defined minimum brightness; the latter do not always descend to the minimum after each outburst but rather maintain some magnitude intermediate between minimum and maximum. The properties of U Gem itself are similar to those of SS Cyg, and this has led to the designation of the whole group, by some authors, as SS Cyg stars; this will not be done here. A few of these variables have been shown to vary irregularly in light at minimum (Walker 1957); this "flickering" has a time scale of the order of 1–10 minutes and a range of a few hundredths to a tenth of a magnitude or more. The variations are similar to those found in various old novae and nova-like objects (Walker 1957).

In 1956 Joy announced that SS Cyg was a spectroscopic binary with $P = 6^{h}38^{m}$ and with components of spectral types dG5 and sdBe. This result, coupled with Walker's (1954, 1956) discovery that Nova (DQ) Her is an eclipsing binary of period $4^{h}39^{m}$, led to the speculation that all cataclysmic variables might be binary systems. The present paper describes a spectroscopic test for binary motion in several U Gem variables; remaining papers of the series will deal with other stars of the group and with certain of the old novae. The results for U Gem stars can be summarized as follows: five are definitely binaries with $P < 9$ hours; one is composite, and one other shows evidence of velocity variation but with an as yet undetermined period. There is at present no evidence in direct contradiction to the hypothesis that all stars of this group are, indeed, spectroscopic binaries of short period.

It should be stated at the outset that U Gem stars are very faint at minimum light; thus observational material is gathered rather slowly even with the 200-inch telescope. The data reported here were obtained over a 30-month interval. Certain fairly definite conclusions can be reached from this limited material, and it was thought best to make a somewhat preliminary report at this time. The possibility remains, of course, that new observations may radically alter some of the conclusions drawn.

II. A SURVEY OF SPECTRA AT MINIMUM LIGHT

A survey of spectra of U Gem variables and related objects was carried out by Elvey and Babcock (1943) at the McDonald Observatory. Since the brightest of these stars is twelfth magnitude at minimum, low dispersion was employed (~340 A/mm) except during the moments of outburst. In most cases the minimal spectra contained rather broad, bright Balmer lines, feebler emission of the triplet He I lines, and bright lines of Ca II (H and K) superimposed on an apparently continuous background having an intensity distribution similar to spectral type G or K. At, or near, maximum light the spectra were found to have a continuous distribution corresponding to type B or A, occasionally crossed by broad, very shallow, absorption lines of H; these sometimes contained faint, narrow emission features. At this dispersion, however, a satisfactory exposure required 2–3 hours (Babcock, private communication)—rather too long to provide sufficient time resolution, as we shall see.

Somewhat earlier, Joy (1940) had indicated that the spectrum of RU Peg at minimum had the absorption lines of a star of type dG3 as well as the usual emission features. Later

TABLE 1

SUMMARY OF OBSERVATIONAL DATA FOR FIGURE 1

Star	Plate No *	Date (U.T.) (Mid-exp.)	Length of Exposure (min.)	Approx. Dispersion (A/mm)	Remarks
U Gem	N1324a	1961 Feb. 10.162	31	180	
EX Hya	B1607a	1960 Feb 23.443	101	180	Sharp emission lines arise from Hg vapor in Los Angeles city lights
Z Cam	β 1758b	1960 Dec. 19 444	122	180	
T Leo	B 925a	1956 Mar. 5 371	150	180	
RX And	N1156b	1960 Aug. 28.320	16	180	
SS Aur	N1164	1960 Aug 30.494	29	180	
RU Peg	N1149a	1960 Aug. 26 329	30	90	
	b	Aug 26 352	30	90	
	c	Aug. 26 375	31	90	
EY Cyg	N1162a	1960 Aug. 30.312	40	180	

* N = 200-inch prime-focus spectrograph; B = Mount Wilson Newtonian focus spectrograph operating at the 100-inch; β = same, operating at the 60-inch

Joy (1956) showed that the spectrum of SS Cyg at minimum was composite and that the star was a binary of short period, as already mentioned. Since the present survey has as its purpose the detection of binary motion, it differs from that of Elvey and Babcock in three important respects: (1) almost the entire effort was put on obtaining spectrograms at minimum light; (2) no dispersion less than 180 A/mm was employed; (3) the program was, to a considerable extent, planned to permit sufficient time resolution.

Several spectrograms have been obtained with the grating nebular spectrograph at Mount Wilson, using both the 60-inch and the 100-inch telescopes; the dispersion was 180 A/mm. Because of the faintness of the stars in question, these spectrograms do not, in general, satisfy the last criterion mentioned above; they must be regarded as exploratory in character. The larger bulk of spectrograms was obtained with the nebular spectrograph at Palomar; these have dispersion of 180 A/mm, except for the plates of RU Peg, which were obtained at a dispersion of 90 A/mm.

Some representative spectra of U Gem and suspected U Gem variables at minimum light are shown in Figure 1. Not all stars observed are included. The spectra are arranged roughly in order of decreasing emission-line width; observational details are summarized in Table 1. All spectrograms were obtained with the 200-inch, except for EX Hya and T Leo (100-inch) and Z Cam (60-inch). It is not certain, therefore, to what extent the wide lines of these three stars may result from integration over significant velocity changes.

The spectra reproduced in Figure 1 confirm the results obtained by Elvey and Babcock and also give some new information. He II (λ 4686) is present in a few spectrograms of U Gem (see Fig. 6); there may also be lines of Fe II (see appendix). A faint trace of the Balmer jump in emission is present in SS Aur and T Leo; however, in all the stars, the Balmer lines converge long before the series limit is reached, and thus they effectively merge with the Balmer continuum. The emission lines of U Gem, EX Hya, and possibly Z Cam are doubled—in the case of U Gem, very conspicuously so. We are not yet certain whether the emitting region is optically thick or thin, so it is not clear whether self-absorption plays a role. The absorption lines of a late-type dwarf star are present in the spectra of RU Peg and EY Cyg; some indication of these lines is found in the spectrum of SS Aur, but completely satisfactory agreement with the features of a G- or K-type dwarf star has not been found.

TABLE 2

ORBITAL ELEMENTS FOR U GEMINORUM VARIABLES

	RX And	SS Aur	U Gem	EY Cyg†	SS Cyg	RU Peg
P (hr., min.)	5 05	3 30(?)	4 10 5		6 38	8 54
P (days)	0 21173	0 15(?)	0 1739825		0 276244	0 3708
K_1* (km/sec)	77 5	~85	265	(?)	122	137
K_2 (km/sec)					115	112
γ (km/sec)	−18	~+45	+42	~−10	−9	−7
Sp. (1)	sdBe	sdBe	sdBe	sdBe	sdBe	sdBe
Sp. (2)	...	...	...	K0 V	dG5	G8 IVn‡
$a_1 \sin i \times 10^{-10}$ (cm)	2 09	~0 88	6 31		4 63	7 02
$a_2 \sin i \times 10^{-10}$ (cm)	...	...	...		4 37	5 73
e	0.40	(?)	0 05		~0	~0
ω	220°	(?)	160°	...	...	...
$\mathfrak{M}_1 \sin^3 i$ (⊙)					0 18	0 27
$\mathfrak{M}_2 \sin^3 i$ (⊙)					0 20	0 32
$\mathfrak{M}_1/\mathfrak{M}_2$					0 90	0 85

* Subscript 1 refers to the blue star, regardless of whether or not it is the more massive.
† Probably viewed nearly pole-on
‡ The luminosity class varies during the cycle, but averages MK class ~ IV

In spite of the considerable variation in width from star to star, the emission-line spectrum of these variables is very characteristic. We have not been able so far to confirm the finding by Elvey and Babcock that AY Lyr has no emission lines at minimum; the star is very faint and is in a crowded field—misidentification is a possibility. However, even if this result should be confirmed, it is reasonable to state that a sufficient condition for class membership is the presence of an emission spectrum similar to those of Figure 1. On this basis we include EX Hya and T Leo in the group (cf. Brun and Petit 1952, 1959; Kukarkin, Parenago, Efremov, and Kholopov 1958; Petit 1959).

III. RADIAL VELOCITIES AND ORBITAL ELEMENTS

The journal of observations and velocity-curves for RX And, U Gem, and RU Peg are given in the appendix. In Table 2 we summarize the orbital elements for these stars, including also the results from Joy's (1956) study of SS Cyg. In addition, preliminary re-

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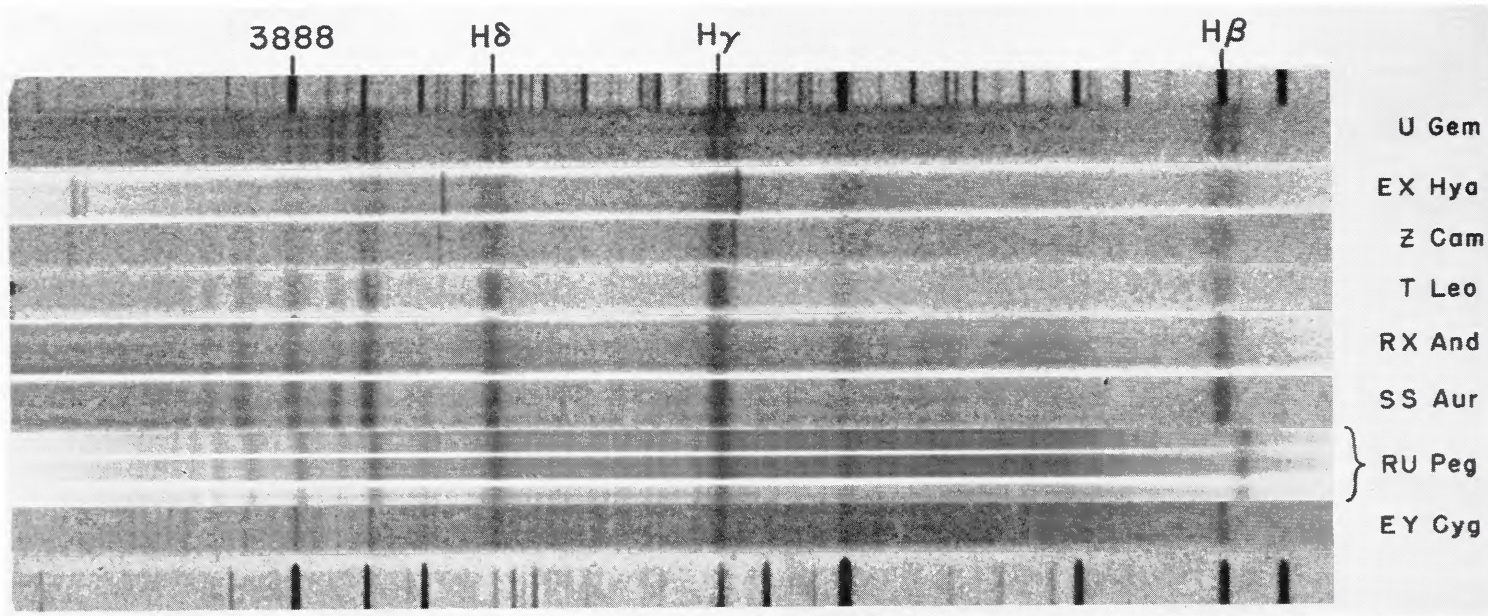

FIG. 1.—Representative sample of spectra of U Gem (and suspected U Gem) variables at minimum light. Observational details are summarized in Table 1. Sharp emission lines of mercury vapor are present in the spectra of EX Hya, Z Cam, and T Leo.

sults for SS Aur and EY Cyg are given; we expect to obtain more spectrograms of the former. EY Cyg may have a small velocity variation, but it is close to the limit of detection; the star may be a system viewed nearly "pole-on." Our assurance of binary characteristics is based on the fact that the spectrum is composite. The spectrum of Z Cam shows evidence of velocity variation, but the period cannot be determined from the available data. A short description of some of the spectral details of these stars is also given in the appendix.

The main conclusions summarizing Table 2 are the following:

1. No U Gem variable so far studied in detail has failed to show evidence of binary characteristics either from velocity variations or from the presence of a composite spectrum.
2. The mean period of the four best-studied stars is $\langle P \rangle = 0.25$ day.
3. In those cases in which a late-type spectrum is detected, the spectral type is dG or dK; the mass ratio is near unity, with the red star slightly more massive than the blue.
4. From the material at hand, it appears that the late-type absorption spectrum is most easily detected in stars having the narrowest emission lines.
5. It is possibly, though not necessarily, significant that the only system of high eccentricity is also the only star belonging to the Z Cam subgroup.

In the cases of SS Cyg and RU Peg, a comparison of the probable radius of the late-type star and the size of the corresponding lobe of the inner Lagrangian surface (given as soon as $\mathfrak{M}_1/\mathfrak{M}_2$ and $a_2 \sin i$ are known) indicates that the star, even if a dwarf, may fill that surface.[1] Following Kuiper and Johnson (1956), if $\mu = \mathfrak{M}_2/(\mathfrak{M}_1 + \mathfrak{M}_2)$, the effective radius r_2 of the larger inner lobe is given by

$$\log \frac{r_2}{a} = -0.335 \log \mu - 0.511 , \tag{1}$$

where a is the separation of the centers of mass. Thus we have $\log r_2/a = -0.396$ and -0.400 for SS Cyg and RU Peg, respectively. The minimum values of a for each star are 9.00×10^{10} cm and 12.75×10^{10} cm; hence the minimum values of r_2 for SS Cyg and RU Peg are 3.6×10^{10} cm and 5.1×10^{10} cm, respectively. But for stars of types dG5 and dG8, we would have $R_{SS} = 6.2 \times 10^{10}$ cm and $R_{RU} = 5.9 \times 10^{10}$ cm. If we set $r_2 = R$, we can estimate the inclination of the orbit. Thus $i = 35°$ and $59°$, respectively, for SS Cyg and RU Peg. Considering the crudity of the argument, these inclinations can scarcely be regarded as more than rough estimates. However, Grant (1955) found that SS Cyg does not eclipse. Since the blue star is quite small (the results of Sec. V show that it is a white dwarf) in terms of the dimensions of the system, this would mean that $i < 76°$; the limiting value would be the same if RU Peg, as well, did not eclipse; this is not known, however. The values of i found above certainly satisfy this limit.

However, there is more compelling evidence that, at least in the case of RU Peg, the late-type component acts as if it filled one lobe of the inner Lagrangian surface. Classification on the MK system is possible, since the dispersion is high enough to resolve the ratio λ 4254/λ 4260. Less satisfactory as a basis for classification is the absolute intensity of λ 4226; it is filled in to some extent by the blue continuum of the hot component. The hydrogen lines obviously cannot be used. On this basis, spectral types are listed in the appendix. The lines are always somewhat diffuse (broadened by rotation?), and the spectral type is roughly constant at G8. However, the luminosity class runs from V to III, apparently depending on aspect, as judged from the strengths of λ 4077 and λ 4215 of Sr II. The highest luminosity corresponds to phases when the stars are in conjunction, red star behind. A star of dwarf dimensions which, at the same time, fills a lobe of the inner Lagrangian surface might be expected to imitate a star of higher luminosity,

[1] Throughout this paper, the subscipt 1 refers to the blue star, whether or not it is the more massive component.

especially at superior conjunction, because of the reduction in effective surface gravity. (The effect would be more pronounced, of course, if i were nearer to 90° than is probably the case in RU Peg.) If the star were, in fact, a normal star of type G8 IV, it would have a radius of 1.5×10^{11} cm, the orbital inclination would therefore be about 18°, and the mass of the red star would be 12⊙ ! This is completely unreasonable, since, as we shall see in Section V, the luminosity is probably near $M_V = +9.5$. Thus it seems likely that the spuriously high spectroscopic luminosity results from the reduction in surface gravity caused by the centrifugal acceleration.

IV. MASSES FOR THE COMPONENTS

The preceding discussion suggests that the model advanced for AE Aqr (Crawford and Kraft 1956), T CrB (Kraft 1958), and DQ Her (Kraft 1959) is probably also applicable to the U Gem stars. We suppose that the late-type component fills its lobe of the inner Lagrangian surface. Material spills out from the inner Lagrangian point and forms a ring or disk around the blue companion. If the emitting region is truly flattened and not spherical, the shape of the emission line will depend on the orbital inclination. Regardless of its optical thickness, the disk will produce a doubled emission if seen in the plane of the orbit, and a single emission if i deviates from 90° to any appreciable extent. We may tentatively suppose, therefore, that U Gem and possibly EX Hya and Z Cam are viewed near the plane of the orbit—they may even be eclipsing binaries. Support for this proposal is found in the presence of conspicuously doubled emission lines arising near the hot component of DQ Her (Greenstein and Kraft 1959), a well-known eclipsing binary.

If the preceding model is correct, we can make a rough estimate of the masses to be expected for U Gem variables. The argument of Section III, coupled with the "single" character of the emission lines of SS Cyg and RU Peg, suggests that the minimum masses listed in Table 2 are rather smaller than the actual masses. Though i is probably near 90° for U Gem, unfortunately the spectrum of the red star has not yet been photographed. Consider, then, the following indirect argument for the determination of masses.

Kuiper (1941) has shown, on the basis of Jacobi's integral, that a particle, in falling from the inner Lagrangian point to the ring around the blue star, approximately conserves its angular momentum with respect to the blue star. In that case, we can derive a relation between the width of the emission lines, the period in the orbit, and the masses of components (Kraft 1959); viz.,

$$\frac{G\mathfrak{M}_1 \sin^3 i}{v \sin i} = \lambda (a_1 \sin i)^2 \left(1.39 \frac{\mathfrak{M}_1}{\mathfrak{M}_2} - 0.39\right)^2 \frac{2\pi}{P}, \tag{2}$$

where $v \sin i$ is the projected half-half-width of the emission line, $\mathfrak{M}_1$ and $\mathfrak{M}_2$ are the masses of the blue and red stars, respectively, and λ is a factor of proportionality describing the fractional change of angular momentum. For both SS Cyg and RU Peg, $v \sin i = 500$ km/sec, as derived from microphotometer tracings. For SS Cyg, the left- and right-hand sides of equation (2) are 5.3×10^{17} and $4.2 \times 10^{17}\,\lambda$, respectively; thus $\lambda = 1.26$. Corresponding values for RU Peg are 8.0×10^{17}, $5.9 \times 10^{17}\,\lambda$, and $\lambda = 1.36$. Thus the values of λ are very similar, and $\langle\lambda\rangle = 1.31$.

If the same value of λ applies to U Gem, we have

$$\frac{G\mathfrak{M}_1}{v} = 1.31\, a_1^2 \left(1.39 \frac{\mathfrak{M}_1}{\mathfrak{M}_2} - 0.39\right)^2 \frac{2\pi}{P}, \tag{3}$$

where $a_2 = 6.31 \times 10^{11}$ cm and $P = 1.50 \times 10^4$ sec. The value of v, however, depends on whether the emitting region is optically thick or thin—the latter leads to $v \sin i = v = 670$ km/sec. Equation (3) can then be balanced reasonably well for $\mathfrak{M}_1/\mathfrak{M}_2$ lying between 0.8 and 1.9, implying 1.9 $\mathfrak{M}_\odot < \mathfrak{M}_1 < 5.5\ \mathfrak{M}_\odot$. Since, as already mentioned, $M_V \sim +9.5$ for these stars, a mass greater than 1.2 $\mathfrak{M}_\odot$ is unlikely because the blue star must be essentially a white dwarf (Schwarzschild 1958). On the other hand, equation (3) cannot be balanced within the error of measuring the width of the emission line if $\mathfrak{M}_1 < 1.0$. Moreover, if the emitting region is optically thick, $v < 670$ km/sec, and the value of $\mathfrak{M}_1$ required to balance the equation is driven up still further. It seems most likely that $\mathfrak{M}_1$ lies in the 0.9–1.2○ range and that the emitting region is optically thin.

If the masses of the red components of SS Cyg and RU Peg are $\sim$1 $\mathfrak{M}_\odot$ as well, the orbital inclinations would be in quite reasonable agreement with those derived by the argument of Section III.

V. STATISTICAL PARALLAX

The systemic velocities and corresponding peculiar radial velocities (i.e., radial velocities corrected for solar motion) are listed in Table 3, along with the proper motions. A rough statistical parallax can be obtained, following Smart (1938), from the radial velocities and proper motions of four stars: U Gem, SS Cyg, RU Peg, and EY Cyg. If

TABLE 3

MAGNITUDES, PROPER MOTIONS, PECULIAR RADIAL VELOCITIES, ETC., FOR U GEMINORUM VARIABLES

Star	V* (km/sec)	μ† (sec of arc/yr)	m_V (min)	Kind	Outburst Period (days)
RX And	−12		13 6	Z	14
SS Aur	+35	. .	14 8	SS	54
U Gem	+37	0 078	14 0	SS	103
EY Cyg	$\sim$+13	062	15 0:	SS	$\sim$1000
SS Cyg	+16	116	12 1	SS	52
RU Peg	+11	0 070	13 1	SS	70

* Radial velocity with solar motion removed
† Mean of values given by Mannino and Rosino (1950) and by Miczaika and Becker (1948)

r is the distance of any star, then, under the assumption that $\langle \log r \rangle \simeq \log \langle r \rangle$, we find $\langle M_V \rangle = +9.5 \pm 0.6$ (m.e.) at minimum light. The quoted mean error is internal; a value of ± 1 mag. would be more realistic.

There are two direct checks on this value of $\langle M_V \rangle$. Strand's (1948) well-determined parallax for SS Cyg gives $M_V = +9.5$ at minimum light. From a study of the color of UZ Ser, Herbig (1944) concluded that the star is in front of an absorbing cloud not farther away than 200 pc. From this he found $M_V \sim +10.1$ at minimum. Both these values agree well with our statistical parallax. It must be admitted, however, that U Gem stars need not all have the same absolute magnitude. We have no way, at present, of determining the dispersion about the mean, if any.

A further indication that $\langle M_V \rangle = +9.5$ is approximately correct is found from a consideration of the colors at maximum light. At a mean distance of only 66 pc, little or no reddening would be expected. At maximum, the spectra of U Gem and SS Cyg are very nearly continuous (Elvey and Babcock 1943). (Occasionally, feeble emission or absorption lines of H and He II are seen [cf. also Adams and Joy 1922].) Thus we make the assumption that SS Cyg and U Gem radiate as black bodies at maximum light. The $U - B$, $B - V$ colors of these stars (Grant and Abt 1959; Wallerstein 1959) may be compared with black-body radiators (Arp 1961). This was done earlier by Wallerstein (1959), but a slightly incorrect black-body trajectory in the color-color plot was used. With Arp's new curve, we have the comparison shown in Figure 2. We conclude that, if

SS Cyg and U Gem radiate as black bodies at maximum light with temperatures of 12000° and 15000° K, respectively, there is no interstellar reddening; this is consistent with $\langle M_V \rangle_{\min} = +9.5$.

This result is not, however, compatible with a spectral type of G5–K0 and $\mathfrak{M} \sim 1.0 \odot$ for the red components of these systems. It would appear that these stars are significantly underluminous for their masses. In this respect and a number of others, the U Gem variables are similar to the stars of type W UMa.

VI. EVOLUTIONARY CONSIDERATIONS

The small peculiar velocities of U Gem stars indicate that they belong to a moderately flattened stellar population—the galactic disk. They are not members of the galactic "halo"; neither can they be described as belonging to Baade's classical population II.

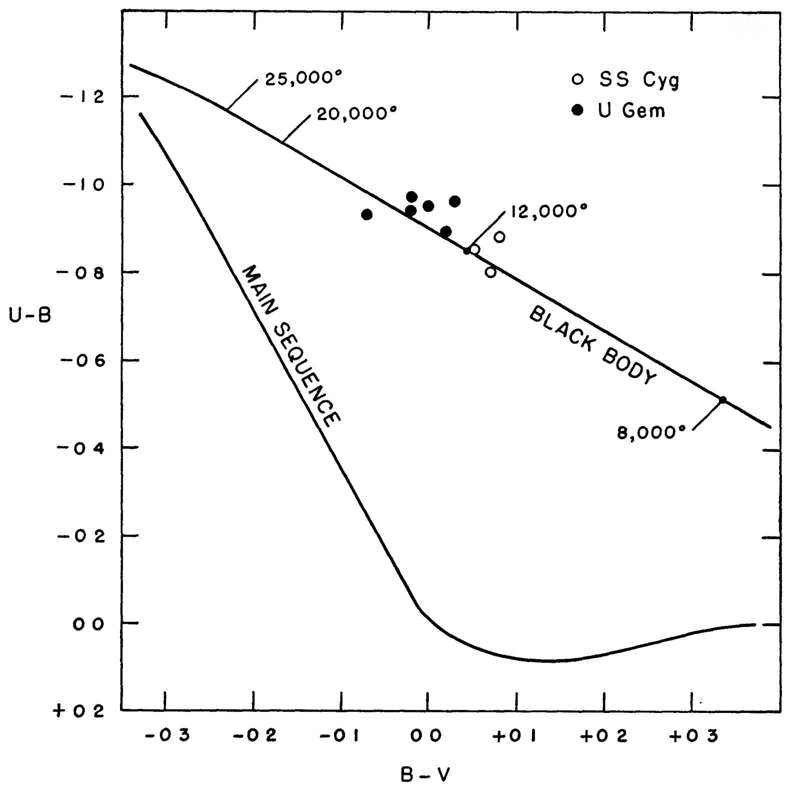

FIG 2.—The $U - B$, $B - V$ colors of U Gem and SS Cyg at maximum light. The black-body locus computed by Arp (1961) is shown.

The only other binaries of comparably short period which also belong to the galactic disk are the W UMa stars (cf. Struve 1950; Kitamura 1959). The remainder of the paper is concerned with evidence supporting the proposition that a *U Gem variable represents a later stage in the evolution of a W UMa star*. This view has been advanced by Sahade (1959) and also independently by the writer for some years.

The physical properties of the U Gem and W UMa systems are summarized in Table 4; data for the latter are taken from the compilation by Kitamura (1959).

In Figure 3 we plot the $z = r \sin b$ distribution for each group. The U Gem stars have been taken to the limit m_{pg} (min.) $= +17.5$; the corresponding limit for the W UMa stars must therefore be $m_{pg} = +12.5$. For each group, 82 and 123 stars, respectively, were found in the *Variable Star Catalogue* (Kukarkin *et al.* 1958). Twenty-five of the former did not have known magnitudes at minimum; the average range of 4 mag. was therefore applied to the magnitudes at maximum. Considering the various uncertainties and particularly the rather different basis for discovery of stars in the two groups, one finds that the z-distributions are extraordinarily similar. This result, coupled with our finding that the total masses and mean peculiar radial velocities are closely alike, strongly suggests a generic relation between the two kinds of stars.

There is, however, also a physical point of similarity not covered in the table. Kitamura (1959) has shown that in the W UMa stars is it likely that the primary overflows its lobe of the inner Lagrangian surface. The star therefore loses mass, which may be collected by the secondary or lost to the system. The main point, however, is that the primary is seriously underluminous for its mass (~3 or 4 mag.). The same is true for the red components of U Gem systems, for, if $M_V \sim +10$ but $\mathfrak{M} \sim 1\odot$, these stars are underluminous by perhaps 5 mag. But if our model is correct, these also lose mass through the inner Lagrangian surface.

Having established the similarity in kinematics and space distribution for the two kinds of variables, we now turn to the question: Are the periods, mass ratios, and total

TABLE 4

PHYSICAL PROPERTIES OF U GEM AND W UMA STARS COMPARED

Property	U Gem	W UMa
$\langle P \rangle$	$0^{d}25$	$0^{d}37$
$\mathfrak{M}_1/\langle \mathfrak{M}_1+\mathfrak{M}_2 \rangle$	~1/2	~1/3
$\langle \mathfrak{M}_1+\mathfrak{M}_2 \rangle$	~1 5–2 0⊙	~1 2–2 5⊙
$\langle M_V \rangle$	+9 5	+4 5
$\langle z \rangle$* = $\langle r \sin b \rangle$	37 pc	40 pc
$\langle V \rangle$	+16 km/sec	−8 km/sec
σ_V	18 km/sec	26 km/sec
No stars†	6	10

* For U Gem and W UMa stars to apparent magnitude limit, $m_{pg}=17$ 5 and $m_{pg}=12$ 5, respectively.

† Used in determination of $\langle V \rangle$ and σ_V only

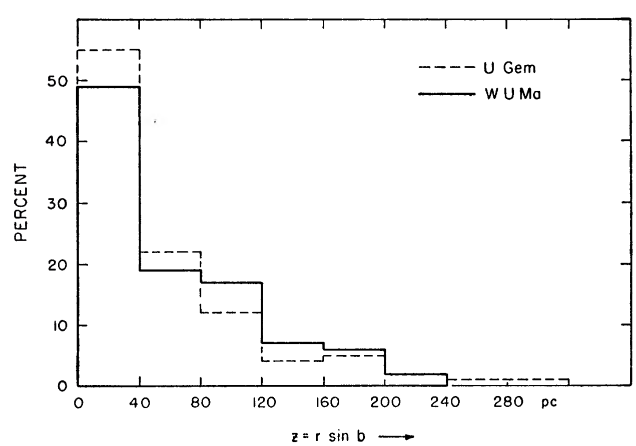

FIG. 3.—The z-distributions of U Gem and W UMa stars compared. The limits of the survey are m_{pg} (min.) = +17.5 for U Gem stars and m_{pg} = +12.5 for W UMa systems.

masses compatible with the proposed evolutionary pattern? If so, we must simultaneously satisfy the following conditions: (1) the total mass remains nearly constant or perhaps slightly decreases; (2) the mass ratio decreases from about 2 to 1; (3) the period decreases from an average value of 0.37 day to 0.25 day. The most important clue suggesting that these conditions are compatible, qualitatively at least, comes from the work of Huang (1956), who showed that if a component of a binary star loses mass to its companion, the period will decrease, but if it loses mass into space, the period will increase.

In the present picture, we imagine that the W UMa primary fills its lobe of the inner Lagrangian surface and loses mass both to the secondary and to outer space. Its evolution is speeded up to the point that it rapidly becomes a white dwarf. Later, the secondary begins to overflow its lobe of the Lagrangian surface, giving the conditions now observed for U Gem variables. If the ejection velocity is small compared with the relative velocity of the components in the orbit and if $e \sim 0$, we can write (cf. Huang 1956)

$$\frac{\delta a}{a}=\frac{1-\alpha}{\alpha}\,\frac{\delta \mathfrak{M}_2}{\mathfrak{M}_1+\mathfrak{M}_2}-2\,\frac{\delta \mathfrak{M}_2}{\mathfrak{M}_2},$$
$$\frac{\delta P}{P}=\frac{2(1-\alpha)}{\alpha}\,\frac{\delta \mathfrak{M}_2}{\mathfrak{M}_1+\mathfrak{M}_2}-3\,\frac{\delta \mathfrak{M}_2}{\mathfrak{M}_2}, \tag{4}$$

where α is the fraction of mass lost by the primary that is collected by the secondary. Integration leads to

$$\frac{a}{a_0}=\left[\frac{(\mathfrak{M}_1+\mathfrak{M}_2)_0}{(\mathfrak{M}_1+\mathfrak{M}_2)}\right]\left[\frac{(\mathfrak{M}_2)_0}{\mathfrak{M}_2}\right]^2,$$
$$\frac{P}{P_0}=\left[\frac{(\mathfrak{M}_1+\mathfrak{M}_2)_0}{(\mathfrak{M}_1+\mathfrak{M}_2)}\right]^2\left[\frac{(\mathfrak{M}_2)_0}{\mathfrak{M}_2}\right]^3, \tag{5}$$

where the zero subscripts indicate initial values. The first terms in equation (4) correspond to loss of mass from the system, and the second to the transfer of mass from primary to secondary.

Let us begin with a W UMa star having $(\mathfrak{M}_1)_0 = 1.43\odot$, $(\mathfrak{M}_2)_0 = 0.76\odot$, $P_0 = 0.37$ day, and $a_0 = 1.98 \times 10^{11}$ cm. Then, if we lose $\mathfrak{M} = 0.41\odot$ from the system and transfer $\mathfrak{M} = 0.20\odot$ to the secondary, we wind up with $\mathfrak{M}_1 = 0.82\odot$, $\mathfrak{M}_2 = 0.96\odot$, $P = 0.28$ day, and $a = 1.93 \times 10^{11}$ cm. These values are quite reasonable for SS Cyg. On the other hand, if no mass were lost from the system and we transferred $\mathfrak{M} = 0.3\odot$ from primary to secondary, the final values would be $\mathfrak{M}_1 = 1.13\odot$, $\mathfrak{M}_2 = 1.06\odot$, $P = 0.14$ day, and $a = 1.03 \times 10^{11}$ cm. These values give a reasonable fit to U Gem. The numbers all become invalid, however, if the velocity of ejection from the primary becomes large; observational evidence on this point is not readily obtainable, however.

Though the preceding discussion seems plausible enough, it leaves unanswered a number of perhaps more serious questions, among them, the following:

1. How can a star of mass $\mathfrak{M} \sim 1.4\odot$ lose one-fourth to one-third of its own mass and become a white dwarf in the process?

2. Why is the U Gem phenomenon observed only when the mass ratio achieves a value near unity? In other words, why do we not observe intermediate cases between W UMa stars and U Gem variables?

3. What is the relation between the binary characteristic and the presence of outbursts in U Gem variables?

4. How can a star losing mass have a spectrum corresponding to a mass of the order of $1\odot$, yet have a luminosity 5 mag. fainter than expected for this mass?

With regard to the last, we might suppose that the outflow of energy is taken up mostly by increasing the potential energy of the ejected particles, leaving little left over for the luminous flux. For a W UMa star, we would then have

$$\frac{1}{2}\left(\frac{dm}{dt}\right) v^2 = L \cong 10^{35} \text{ erg/sec} . \tag{6}$$

With an ejection velocity of (say) 10 km/sec, the equation can be balanced if $dm/dt = 2 \times 10^{23}$ gm/sec $= 6 \times 10^{30}$ gm/yr. This would mean that W UMa stars last for only 500 years—a result unacceptable not only because of the excessively large number of W UMa stars it would imply but also because the change in period per unit time would greatly exceed that observed unless the amounts lost to the system and collected by the secondary were very closely related.

Evidence of the time scale that might be expected comes from the work of Morton (1960), who showed that the mass loss must proceed on the Kelvin time scale. In this case, $t_K = 1.5 \times 10^6$ years. Thus, even if the ejection velocity were 100 km/sec, we could not satisfy the Kelvin time scale and at the same time maintain the validity of equation (6).

However, the Kelvin time is quite short compared with the age of Praesepe, viz., 1–5 $\times 10^8$ years; there is strong evidence that Praesepe contains the W UMa star TX Cnc (Haffner 1937; Eggen 1961). This presumably means that the components of TX Cnc have only recently begun the activity presently identified with the W UMa characteristic. The mass of the primary is about 1.6⊙ (Kitamura 1959). However, the masses of the stars now breaking off the main sequence and evolving to the right in the HR diagram are about 2.0⊙. If the lifetime of TX Cnc is, indeed, significantly less than the age of the cluster, one could imagine that the original system consisted of an A or F primary of mass ~2⊙, together with a much fainter and less massive secondary. At first, neither star filled its lobe of the inner Lagrangian surface. As evolution proceeded, the primary expanded to fill its lobe in a time of the order of 10^8 years; loss of mass has brought it rapidly to its present position in the cluster HR diagram. If this scheme is to be maintained, it will be necessary to explain how the primary can evolve along a line nearly parallel to the main sequence itself.

APPENDIX

Journal of Observations, Velocity-Curves, and Discussion of Radial Velocities for Certain U Geminorum Variables

I. *RX And.*—The emission spectrum of hydrogen is strong and shows the Balmer jump in emission. He I (λ 4771, λ 4026) and Ca II are present in emission; there is a faint indication of bright He II (λ 4686). No trace of an absorption-line spectrum has been found.

The journal of observations is given in Table 5, and the velocity-curve plotted in Figure 4. All spectrograms have dispersion ~180 A/mm. The star is unusual in having an orbit of high eccentricity. The distortion of a sine-wave velocity-curve by streams of gas cannot, of course, be ruled out.

II. *SS Aur.*—Only nine spectrograms (dispersion ~180 A/mm) are available. The emission lines of hydrogen are strong and have relatively sharp edges; the Balmer jump is in emission. Ca II (K) is prominent in emission, but He I is weak. The underlying continuum is irregular and may correspond to the spectrum of a late-type star, but the match is not completely free from ambiguity.

The journal of observations is found in Table 6. A period of about 3½ hours fits the velocities, but more observations are needed.

III. *U Gem.*—The hydrogen emission lines and Ca II (K) are distinctly double. λ 4471 and λ 4026 of He I are feebly present, and there is a bright trace of λ 4686 of He II. The difference in velocity at opposite elongations is illustrated in Figure 7. As was the case in the spectrum of DQ Her (Greenstein and Kraft 1959), the *V/R* ratio reverses at opposite elongations. When the spectra are lined up, as in Figure 8, some faint emission features at λλ 4172, 4232, 4301, 4512(?), and 4582 emerge which may correspond to the Fe II lines λλ 4173, 4233, 4303, 4508 and 4522 (blended), and 4584.

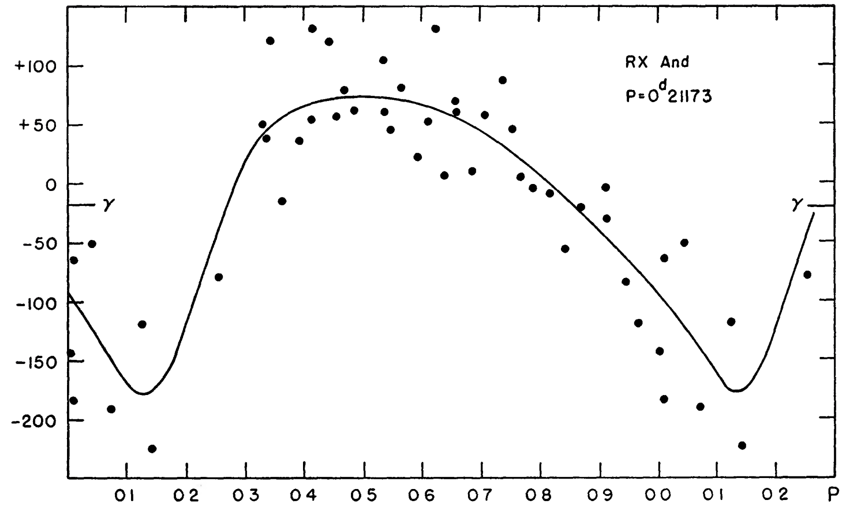

FIG. 4 —The radial-velocity curve (emission lines) of RX And

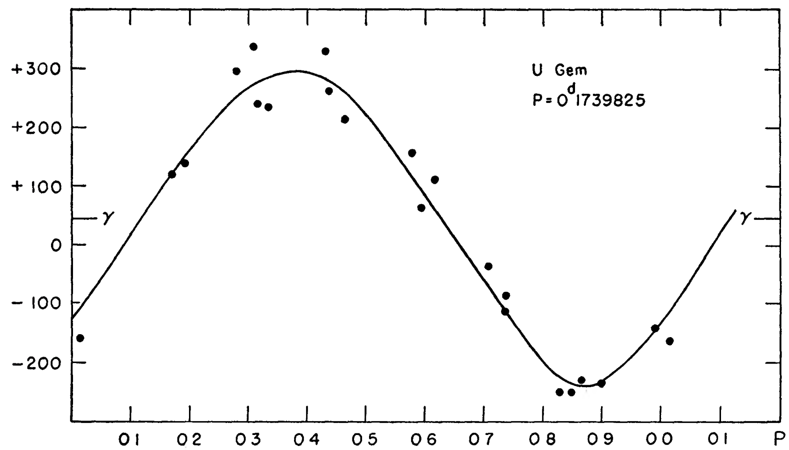

FIG. 5.—The radial-velocity curve (emission lines) of U Gem

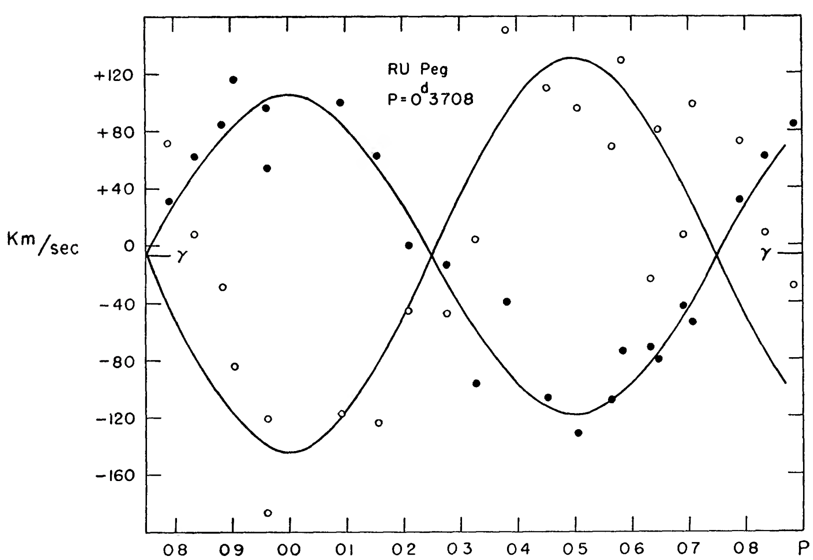

FIG. 6.—The radial velocities of RU Peg Filled and open circles refer to absorption and emission, respectively.

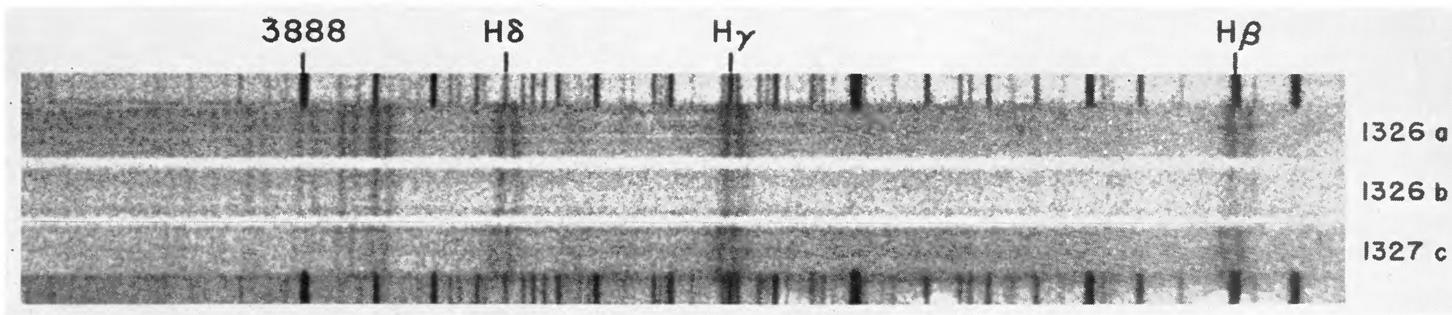

FIG. 7.—The difference in radial velocity of U Gem at opposite elongations

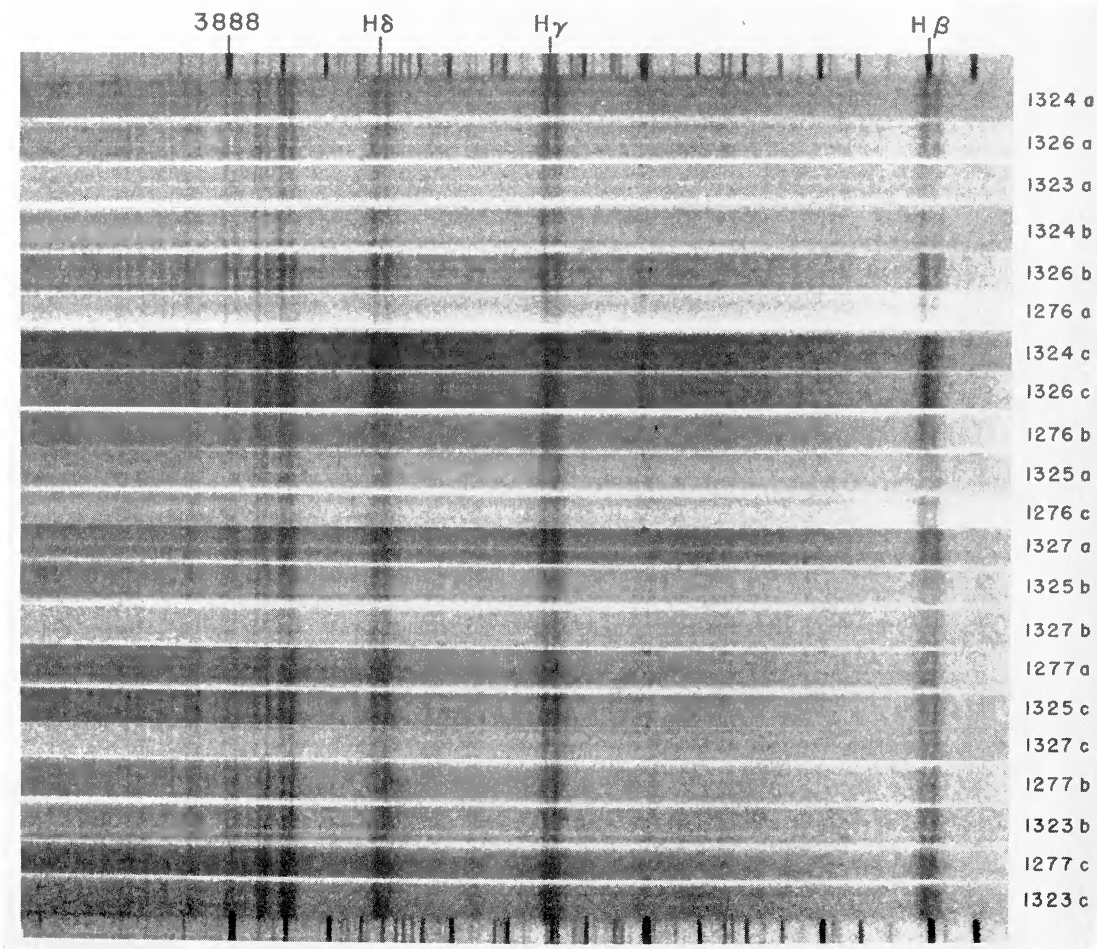

FIG. 8.—Spectra of U Gem around the $4^h10^m.5$ cycle. No attempt has been made to show the radial velocity variation. Notice the change in visibility of the higher members of the Balmer series.

TABLE 5

JOURNAL OF OBSERVATIONS FOR RX AND

Plate No.	J. D. (Helioc.) (mid-exp)	V (km/sec)	Phase* (P)
	2437000+		
N 1151 a	173.8284	− 5	0.912
b	173.8520	− 65	.011
c	173.8735	−139	.126
N 1152 a	173.9235	− 16	.362
b	173.9437	+ 57	.457
c	173.9610	+ 61	.539
N 1153 a	173.9819	+ 6	.638
b	173.9971	+ 57	.709
N 1154 c	174.7479	− 79	.256
N 1155 a	174.7667	+121	.344
b	174.7820	+131	.417
c	174.7931	+ 78	.469
N 1156 a	174.8097	+ 45	.547
b	174.8229	+ 51	.610
c	174.8334	+ 58	.659
N 1157 a	174.8535	+ 45	.754
b	174.8671	− 11	.818
c	174.8778	− 22	.869
N 1158 a	174.8938	− 84	.945
b	174.9063	−144	0.003

Plate No.	J. D. (Helioc.) (mid-exp)	V (km/sec)	Phase* (P)
	2437000+		
N 1205 a	221.6256	+ 68	0.658
b	221.6423	+ 87	.737
c	221.6534	− 6	.790
N 1206 a	221.6909	−120	.967
b	221.6999	−184	.009
N 1207 a	222.6163	+ 38	.338
b	222.6326	+ 54	.415
c	222.6479	+ 62	.487
N 1208 a	222.6645	+ 80	.565
b	222.6770	+130	.624
c	222.6902	+ 8	.687
N 1209 a	222.7076	+ 4	.769
b	222.7236	− 57	.844
c	222.7381	− 31	.913
N 1210 a	222.7562	− 52	.041
b	222.7722	−192	.074
c	222.7867	−225	.142
N 1211 a	222.8270	+ 51	.332
b	222.8402	+ 37	.395
c	222.8506	+120	.444
N 1212 a	222.8701	+104	.536
b	222.8826	+ 22	0.594

*Zero phase taken arbitrarily at J.D.$_{\odot}$ 2437173.0000

TABLE 6

JOURNAL OF OBSERVATIONS FOR SS AUR

Plate No.	Date (U.T.) (Mid-exp)	V (km/sec)
N 1164	1960 Aug 30.494	+ 108
N 1165 a	Aug 31.381	+ 77
b	31.409	+ 99
N 1217 a	Oct 17.389	+ 15
b	17.420	+ 101
c	17.448	+ 130
N 1218 a	17.479	- 21
b	17.502	- 45
c	17.520	- 9

TABLE 7

JOURNAL OF OBSERVATIONS FOR U GEM

Plate No.	J. D. (Helioc.) (mid-exp)	V (km/sec)	Phase* (P)
	2437000+		
N 1276 a	266.9270	+ 235	0.328
b	266.9499	+ 213	.460
c	266.9722	+ 58	.588
N 1277 a	266.9978	− 86	.735
b	267.0201	− 228	.863
c	267.0416	− 143	.987
N 1323 a	339.6428	+ 290	.277
b	340.6206	− 237	.897
c	340.6407	− 160	.013
N 1324 a	340.6675	+ 117	.167
b	340.6914	+ 332	.304
c	340.7129	+ 325	.428
N 1325 a	340.7383	+ 153	.574
b	340.7609	− 39	.704
c	340.7820	− 252	.825
N 1326 a	340.8456	+ 135	.190
b	340.8668	+ 238	.312
c	340.8876	+ 259	.432
N 1327 a	340.9188	+ 108	.611
b	340.9397	− 116	.731
c	340.9595	− 250	0.845

*Zero phase taken arbitrarily at J. D.$_\odot$ = 2437266.0000

TABLE 8

JOURNAL OF OBSERVATIONS FOR RU PEG

Plate No.	J.D. (Helioc.) (mid-exp)	V (km/sec) em.	V (km/sec) abs.	Phase* (P)	Sp. Type
	2437000+				
N 1143 a	171.7137	+ 68	−108	0.566	G8 IV n
b	171.7380	− 24	− 71	.633	G8 IV n
c	171.7613	+ 6	− 43	.691	G8 IV n
N 1144 a	171.8123	+ 7	+ 62	.836	G8 V n
b	171.8360	− 84	+116	.906	G8 V n
c	171.8585	−187	+ 96	.961	K0 V n
N 1145 a	171.9085	−118	+100	.092	G8 IV n
b	171.9311	−124	+ 62	.157	G8 IV n
c	171.9533	− 46	0	.210	G8 III n
N 1146 a	171.9818	− 48	− 14	.277	G8 IV n
b	171.9995	+ 3	− 97	.328	K0 IV n
N 1148 a	172.7596	+150	− 40	.382	G8 V n
b	172.7842	+109	−107	.453	G8 V n
c	172.8061	+ 95	−132	.506	G8 V n
N 1149 a	172.8346	+129	− 74	.584	G8 V n
b	172.8582	+ 80	− 80	.648	G8 IV n
c	172.8807	+ 98	− 54	.708	G8 IV, V n
N 1150 a	172.9099	+ 72	+ 31	.790	G8 IV n
b	172.9419	− 29	+ 84	.885	G8 IV n
c	172.9748	−121	+ 54	0.963	G8 IV n

*Zero phase taken arbitrarily at J.D. 2437171.873, the moment of max. positive absorption-line velocity.

The cyclical behavior of the spectrum is illustrated also in Figure 8, and the velocity-curve is shown in Figure 5. Elongation (1) with the blue star receding corresponds to plates near N 1324 b and c; superior conjunction (blue star behind) to N 1276 c; elongation (2) with blue star approaching to N 1325 c and N 1327 c. The visibility of the higher members of the Balmer series varies in the cycle; they are strongest near elongation (1) and virtually disappear at superior conjunction. There seems to be a tendency for the appearance of duplicity (among the higher members) to be more pronounced, the stronger the emission. The large changes in continuum intensity from plate to plate, though enhanced in the process of reproduction, are at least partially due to changes in stellar brightness.

The journal of observations is given in Table 7. The radial velocities have been corrected (slightly) for the effect of exposure time, which is a small, but significant, fraction of the period (cf. Herbig 1960).

IV. *EY Cyg*.—From five spectrograms, there is no significant velocity variation. For N 1162 a, b, c, the mean velocities of emission ($H\gamma + H\delta$) and absorption (λ 4045 and λ 4226) are +26 and −81 km/sec, respectively. For N 1163 a, b, we have +18 and −8 km/sec, respectively. The mean errors of measurement are large enough that the difference in absorption-line velocity between the two plates is probably not significant.

The emission lines are quite narrow for a U Gem variable. The quoted velocity is a mean between absorption and emission velocities.

V. *RU Peg*.—With the exception of SS Cyg, this is the only other star showing the absorption spectrum of a late-type star in the photographic region (II*a*-O plates). MK spectral types as well as radial velocities are listed in Table 8. The emission lines have amorphous edges at 90 A/mm and are difficult to measure. Ca II (K) sometimes appears doubled; He I is rather weak in emission. The higher Balmer lines converge, and the Balmer continuum is probably in emission. The velocity-curve is illustrated in Figure 6.

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