LETTERS TO THE EDITOR

AN INTERPRETATION OF DOUBLE PERIODS IN BETA CANIS MAJORIS STARS*

The beat phenomenon observed in several β Canis Majoris stars, indicating two oscillations with nearly equal frequencies (Ledoux and Walraven 1958), is not satisfactorily understood. Ledoux (1951) has suggested that the two frequencies may correspond to the oscillations of two different non-radial deformations, both having the form of secondorder spherical harmonics. Since the frequencies of such oscillations would be equal in the absence of rotation, Ledoux supposed that the star had a small angular velocity, which was determined by the observed beat frequency. We wish to suggest the alternative hypothesis that *one* of the frequencies corresponds to a second-order harmonic deformation and the other to a radial expansion and contraction.

A convenient formula for the square of the frequency of a radial pulsation is

$$\sigma_R^2 = (3\gamma - 4) \frac{|\mathfrak{M}|}{I}, \tag{1}$$

where \mathfrak{W} is the gravitational energy and I is the moment of inertia about the center, both evaluated for the spherical equilibrium configuration; γ is the ratio of specific heats (the pulsation is supposed to take place adiabatically) and the subscript R identifies the pulsation as radial. Equation (1) can be derived by means of the virial theorem (Ledoux 1945). It is known to be a good approximation for a wide range of central condensations (Ledoux and Pekeris 1941). It has recently been shown (Chandrasekhar and Lebovitz 1962) that a generalization of Ledoux's procedure enables one to find not only formula (1) for radial pulsations but also a formula,

$$\sigma_{NR}^2 = \frac{4}{5} \frac{|\mathfrak{W}|}{I},\tag{2}$$

for non-radial oscillations of surface deformations represented by second-order spherical harmonics. (Unlike the purely radial mode, the non-radial modes belonging to the frequency σ_{NR} are fivefold degenerate.)

If $\gamma = 1.6$, $\sigma_R^2 = \sigma_{NR}^2$. One might therefore expect to observe beats if (1) both kinds of modes are excited and (2) γ is near 1.6. It is noteworthy that the beat phenomenon does *not* appear in the broadening of the spectral lines. That the suggested interpretation is in accord with this observational datum appears as follows: On the basis of the approximation used in deriving equations (1) and (2), it is found (Chandrasekhar and Lebovitz 1962) that the corresponding pressure variations are

$$(\delta p)_R = \text{Constant } \cos(\sigma_R t + \delta)$$
 (3)

and

$$(\delta p)_{NR} = 0 . (4)$$

Consequently, if the line broadening is associated with the pressure variations, only one frequency—that of the radial pulsation—should be observed. (If the star should be

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