A COARSE ANALYSIS OF THREE RR LYRAE STARS

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ABSTRACT

Coarse analyses of spectrograms of three Bailey type-a RR Lyrae stars obtained at phases on the descending branches of the light-curves lead to estimates of metals/hydrogen ratios that range from the solar value for DX Del to values approximately 20 and 500 times smaller for RR Lyr and X Ari, respectively. The uncertainties of the abundance determinations are of the order of a factor of 3 or 4. With one or two possible exceptions, the abundances of the metals relative to iron are similar to those in Procyon. The excitation temperature derived for X Ari ($P = 0\overset{d}{.}65$) is lower than that of DX Del ($P = 0\overset{d}{.}47$) by approximately 500°. Photoelectric observations on the U, B, V system are presented for DX Del and X Ari. A difference of the order of 500° is also inferred from the color indices when the effects of interstellar reddening and differential line blanketing are taken into account Since this temperature difference is very nearly equal to the value predicted by the color-index versus period relation for the RR Lyrae stars in M3, which are presumably chemically homogeneous, it is possible that all RR Lyrae stars obey a single relation between period and effective temperature, regardless of chemical composition.

I. INTRODUCTION

Subgroups of field RR Lyrae stars, defined by a metallic-line strength criterion, have been found to possess different galactic rotations, concentrations toward the galactic plane, and mean periods of light-variation (Preston 1959). These correlations are characteristic of, and may be taken to define, the transition from the "halo" to the "disk" of the Galaxy. In view of the results of abundance studies of other classes of old stars, it is plausible to attribute the range in line strengths to a range in metals/hydrogen ratios. This interpretation is supported by the fact that at a given period the $B - V$ and $U - B$ color indices of field RR Lyrae stars increase with line strength in accord with the expectations of a simple blanketing theory (Preston and Spinrad 1959). However, because of a lack of information on the degree of similarity of the atmospheric parameters of the strong-lined and weak-lined stars, no quantitative estimate of the range in abundances has been made. The present study is a preliminary reconnaissance of these problems.

DX Delphini, RR Lyrae, and X Arietis, Bailey type-a variables with strong, intermediate, and weak metallic absorption lines, have been intercompared spectroscopically and photometrically. For each star a curve-of-growth analysis has been used to estimate $\theta_{\rm exc}$ and, under the assumptions made in Section III, $\theta_{\rm ion}$, P_e, and the abundances of some elements relative to the sun. Rough determinations of color excesses have been used to estimate intrinsic color indices. Line-blanketing measurements have been used to obtain "line-free" color indices and also to estimate the color indices that these stars would have, were their metals/hydrogen ratios equal to the solar value.

II. OBSERVATIONS AND REDUCTIONS

a) The Observational Material

All spectrograms were obtained with the coudé spectrograph of the 100-inch reflector of the Mount Wilson Observatory. With the exception of single spectrograms of RR Lyrae and X Arietis obtained with the 73-inch camera (4.5 A/mm) and the 32-inch camera (10 A/mm), respectively, all observations were obtained with the 16-inch

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camera (20 A/mm). Baked Kodak IIa-O emulsion was used throughout. Calibration spectrograms, obtained with a wedge-slit spectrograph in the 100-inch dome, were developed simultaneously with the stellar spectrograms. Exposure times of the order of 3 hours near mean light and in good seeing were required for spectrograms widened 0.4 mm for DX Del and X Ari. The time resolution available was insufficient to observe these stars effectively near maximum light. Therefore, attention was necessarily restricted to the properties of the stars near and after mean light on the descending branches of the light-curves, where the variations in atmospheric parameters with phase are less rapid. A journal of the spectroscopic observations is given in Table 1. Phases for the observations of DX Del and X Ari were determined from photoelectric measures obtained with the 60-inch reflector on Mount Wilson in the same observing season. Phases for RR Lyrae were computed by using a time of light-maximum observed by Kron (1958). The photoelectric data are presented in Section IV. Reproductions of portions of the spectra of the three variable stars and an F-type standard star are shown in Figure 1.

TABLE 1

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Star	No	Ce	A/mm	$JD_\odot$ 2436000+	$\langle\phi\rangle$	$\Delta\phi$
DX Del	1	12722	20	737 82	0 26	0 15
	2	12723	20	737 91	45	20
	3	12727	20	738 81	.36	11
	4	12728	20	738 90	54	23
	5	12897	20	846 68	61	.28
RR Lyr	6	12698	20	722 99	44	.02
	7	12816	4 5	809 71	43	.30
X Ari	8	12815	20	808 96	42	16
	9	12821	20	810 95	47	19
	10	13096	10	898 86	0 47	0 29

b) The Equivalent Widths

Lines free of serious blending in the sun and in typical F-type stars were chosen from tabulations by Swensson (1946) and Greenstein (1948). Lines that were seriously blended on our spectrograms were discarded from this list. Equivalent widths were determined by planimetry of lines measured as triangles on direct-intensity tracings made with a microphotometer at the California Institute of Technology. The continuum drawn on each tracing is the smooth curve through those two angstrom intervals of the observed continuum where no line with intensity greater than zero exists in Swensson's list of lines for Procyon. The procedure is very nearly equivalent to the use of the "line-free windows" of Canavaggia and Chalonge (1946) in the solar spectrum. Since it would be difficult to prove that weak, unresolved lines are absent from any region of the spectrum, the position of the continuum must be regarded as uncertain by an amount that depends on the optimism of the investigator. For DX Del, the most difficult case, we note that if the adopted continuum were raised by 5 per cent, it would lie above the observed continuum everywhere on the interval λλ 4000–4800, while if it were lowered by 5 per cent it would pass through absorption features that can be recognized on tracings of several spectrograms. Therefore, we estimate that the position of the continuum is uncertain by no more than 5 per cent, although, as noted in Section III, the uncertainty varies with wave length. The corresponding errors in equivalent width

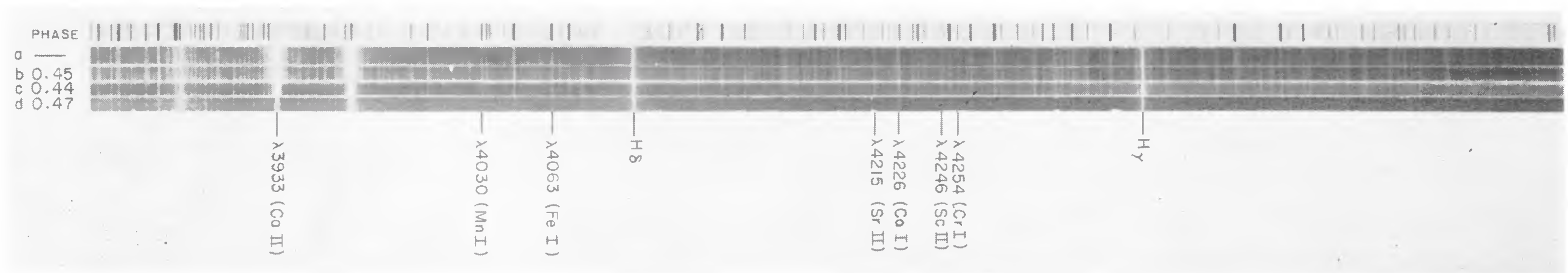

FIG. 1.—Reproduction of portions of the spectra of (*a*) 40 Leonis (F6 IV), (*b*) DX Delphini, (*c*) RR Lyrae, and (*d*) X Arietis. Line strengths in 40 Leo and DX Del are comparable. The relative enhancement of lines of ionized metals in the spectra of the RR Lyrae stars illustrated by line ratios such as λ 4215 (Sr II)/λ 4226 (Ca I), λ 4233 (Fe II)/λ 4235 (Fe I), and λ 4246 (Sc II)/λ 4254 (Cr I) indicates that the level of ionization is higher in these stars than in 40 Leo. A detailed comparison with standard stars is difficult in the case of RR Lyr and virtually impossible in the case of X Ari because of the weakness of the absorption lines. The abnormal values of the ratio λ 4030 (Mn I)/λ 4071 (Fe I) in RR Lyr and X Ari indicate the possibility of the deficiency of manganese relative to iron discussed in the text. Note the great range of the ratio λ 3933 (Ca II)/Hδ and the approximate equality of the hydrogen lines in the three RR Lyrae stars.

could be as large as 10 per cent for the strongest lines and considerably more for the weakest lines that we have measured. Differences of this order exist between the pair of plates Ce 12722 and Ce 12727 and the pair Ce 12728 and Ce 12897 for DX Del. These differences may be in part photometric in origin. The equivalent widths for RR Lyr derived from the 20 and 4.5-A/mm plates also differ systematically by about 5 per cent, with a scatter that increases steadily with decreasing equivalent width. In all these comparisons the systematic and random errors are comparable to the measured quantities for $W_\lambda \approx 0.05$ A. Therefore, the curves of growth are poorly defined and subject to distortion below $\log W/\lambda \approx -4.8$. Measures of individual plates have been combined wherever phases permit to reduce random errors. The line measurements are presented in Table 2.

III. THE ANALYSIS OF THE SPECTRA

a) Curves of Growth

For the abscissae of the curves of growth we have used the quantity $\log \eta$, defined (Aller 1953) by

$$\eta = \frac{N\alpha_0}{\kappa}, \tag{1}$$

where N is the number of atoms in the level from which a given transition arises, κ is the mass continuous absorption coefficient, and α_0, the "fictitious line absorption coefficient," is a quantity proportional to the transition probability and inversely proportional to the most probable speed of the atoms. In the present analysis we are content to estimate abundances relative to the sun. Therefore, we represent the stellar $\log \eta$ value for each line by the sum of (1) the solar $\log \eta$ value, (2) a Boltzmann correction to $\log \eta_\odot$ due to the difference between the excitation temperatures in the sun and the star, and (3) a constant for lines of a given ion which involves the abundances, opacities, and Doppler parameters of the sun and the star,

$$\log \eta_* = \log \eta_\odot + \chi(\theta^\odot_{\text{exc}} - \theta^*_{\text{exc}}) + \log \frac{(N/\kappa V)_*}{(N/\kappa V)_\odot}. \tag{2}$$

The quantity χ is the excitation potential of the lower level for each transition.

Values of $\log \eta_\odot$ were determined by entering the solar curve of growth of Goldberg and Pierce (Aller 1953) with the equivalent widths contained in the *Preliminary Photometric Catalogue of Fraunhofer Lines* (Utrecht 1960) and reading out the corresponding $\log \eta_\odot$ values.

Curves of growth were constructed for several trial values of θ_{exc} and the adopted values of θ^*_{exc} were chosen, by visual inspection of the curves, as those that minimize systematic differences in $\log \eta$ between the high and low excitation lines. The value $\theta^\odot_{\text{exc}} = 1.04$ was assumed. For DX Del it was found that the lines shortward of λ 4200 lay slightly above the curve of growth for lines longward of this wave length, presumably because the continuum was drawn too high at shorter wave lengths. Preliminary curves of growth were constructed for the lines in the two wave-length regions. Small systematic corrections were then applied to the equivalent widths of lines shortward of λ 4200 to bring the two curves into coincidence. The excitation temperature derived from the two regions combined did not differ from the preliminary value, but the procedure permits the use of lines shortward of λ 4200 in the abundance study. A problem of a different nature arises in the case of X Ari. There are too few measurable lines on $\lambda\lambda$ 4000–48000 to construct a determinate curve of growth. Therefore, we have used the 0- and 1-volt Fe I lines of multiplets 4, 20, 22, and 45 of the *Revised Multiplet Table* (Moore 1945) in the near ultraviolet, excluding all lines within 10 A of the cores of hydrogen lines. As a check on this procedure, the same lines in the spectrum of RR Lyrae

TABLE 2—EQUIVALENT WIDTHS

λ	Element	log $\eta_\odot$	χ	−log W/λ, Plate No., DX Del: 1, 3	DX Del: 2, 4, 5	RR Lyr: 6	RR Lyr: 7	X Ari: 8, 9, 10
3815 84	Fe I (45)	4 56	1 48	...	...	4 25	...	4 44
20 43	Fe I (20)	4 84	0 86		...	4 20		4 29
24 44	Fe I (4)	4.30	0 00	...	...	4 11	...	4 40
49 97	Fe I (20)	4 17	1 01	...	...	4 36		4 56
50 82	Fe I (22)	3 50	0 99			4 44	...	...
58 30	Ni I (32)	3 47	0 42	...	...	4 35	...	4 69
59 91	Fe I (4)	4 68	0 00		...	4 22		4 32
62 59	Si II (1)	1 13	6 83	...		4 67		...
65 53	Fe I (20)	3 99	1 01		...	4 29		4 52
72 50	Fe I (20)	4 17	0 99	...		4 26	...	4.56
78 02	Fe I (20)	4 25	0 95	...		4 28	...	4 46
99 71	Fe I (4)	3 95	0 09	...	...	...	...	4 46
3900 55	Ti II (34)	2 97	1 13		...	...		4 38
05 53	Si I (3)	4 29	1 90	...	...	4 27	...	4 46
13 46	Ti II (34)	2 94	1 11	...		4 35		4 46
17.18	Fe I (20)	3 00	0 99	...	...	4 63		...
20 26	Fe I (4)	3 77	0 12			4 35	...	4 56
22 91	Fe I (4)	3 91	0 05		...	4 33	...	4 43
27 92	Fe I (4)	4 01	0 11			4 30	4 23	4 45
30 30	Fe I (4)	4 17	0 09			...	...	4 43
40 88	Fe I (20)	2 90	0 95			4 78	4 76	...
42 44	Fe I (364)	2 27	2 83	...		4 82		
43 34	Fe I (72)	1 81	2 19			5 26	...	...
44 01	Al I (1)	3 99	0 00	...	...	4 34	4 39	4 60
48 10	Fe I (562)	2 37	3 23	...		4 68		
49 95	Fe I (72)	2 59	2 17			4 77		
50 35	Y II (6)	1 34	0 10			4 69	...	
51 16	Fe I (661)	2 37	3 26			4 79	4 96	
51 97	V II (10)	1 44	1 47			4 65	4 82	...
61 52	Al I (1)	4 29	0 01			4 39	4 40	4 59
82 59	Y II (6)	1 37	0 13			4 75	...	
83 96	Fe I (277)	2 06	2 72			4 73	4 63	
86 75	Mg I (17)	3 53	4 33	...		4.70	4 96	...
87 63	Ti II (11)	1.13	0 60	...	...	4 82	...	...
96 00	Fe I (279)	1 61	2 72	4 53	4 51	...		...
98 05	Fe I (276)	2 32	2 68	...	...	4 73		...
98 64	Ti I (12)	2 37	0 05	4 39	4 40	4 89		...
4001 67	Fe I (72)	1 81	2 17	4 51	4 56	...		
02 07	Fe II (29)	0 59	2 77	4 58	4 88			
02 94	V II (9)	1 17	1 42	4 66	4 75	...	...	...
05 25	Fe I (43)	3 84	1 55		...	4 31		4 52
05 71	V II (32)	1 74	1 81	...	...	4.76		...
08 93	Ti I (12)	1 31	0 02	4 67	4 92	...		
21 87	Fe I (278)	2 32	2 75			4 74	...	...
28 33	Ti II (87)	1 68	1 88	...	...	4.58	4 51	5 08
33 07	Mn I (2)	3 30	0 00	4 19	4 17	4 57	4 39	5 01
34 49	Mn I (2)	3 11	0 00	4 24	4 32	4 69	4 72	5 10
36 78	V II (9)	0 60	1 47	4 79	4 80	...		
40 65	Fe I (655)	1 50	3 29	4 49	4 77	5 15		
41 36	Mn I (5)	2 15	2 11	4 32	4 44	...	...	
44 61	Fe I (359)	2 15	2 82	4 43	4 56	5 02	5 25	...
45 82	Fe I (43)	4 53	1 48	3 80	3 90	4 17	4 10	4 34
63 60	Fe I (43)	4 29	1 55	4 04	4 00	4 26	4 12	4 50
67 98	Fe I (559)	2 55	3 20	4 47	4 43	4 80	4 80	
70 77	Fe I (558)	1 76	3 23	4 67	4 57	...	...	...
71 74	Fe I (43)	4 21	1 60	4 12	4 11	4 37	4 41	4 46
72 52	Fe I (698)	1 31	3 42	4 61	4 75	...	...	...
77 71	Sr II (1)	3 77	0 00	4 00	3 97	4 22	4 17	4 45
82 94	Mn I (5)	1 74	2 17	4 65	4 63	...		
4128 74	Fe II (27)	0 94	2 57	4 44	4 57	5 06	...	...
32 06	Fe I (43)	3.81	1.60	4.15	4 17	4.35	4 38	4 64

TABLE 2—*Continued*

λ	Element	log $\eta_\odot$	χ	−log W/λ: Plate No.: DX Del: 1, 3	DX Del: 2, 4, 5	RR Lyr: 6	RR Lyr: 7	X Ari: 8, 9, 10
4132 90	Fe I (357)	2 27	2 83	4 39	4 37	4.75	4 98	...
33 87	Fe I (698)	1 80	3 35	4 55	4 62	...	...	...
37 00	Fe I (726)	1 90	3 40	...	...	4 86		...
39 93	Fe I (18)	1 57	0 99	4 90	4 97	...		...
43 42	Fe I (523)	2 80	3 03			4 54		...
43 87	Fe I (43)	3 89	1 55	...	...	4 44	...	4 49
47 67	Fe I (42)	2 01	1 48	4 40	4 48	4 71	4 87	...
57 79	Fe I (695)	2 34	3 40	4 46	4 58	4 95	4 72	...
67 27	Mg I (15)	3 18	4 33	4 41	4 38	4 61		5 20
74 92	Fe I (19)	1 83	0 91	4 49	4 57	4 99	...	...
75 64	Fe I (354)	2 01	2 83	4 47	4 47	4 90	4 77	...
78 86	Fe II (28)	1 43	2 57	4 17	4 26	4 51	4 61	5 09
84 90	Fe I (355)	1 77	2 82	...	...	4 88	4 95	...
87 04	Fe I (152)	3 19	2 44	4 31	4 33	4 56	4 52	5 03
99 10	Fe I (522)	3 05	3 03	4 32	4 33	4 55	4 47	5 03
4213 65	Fe I (355)	1 61	2 83	4 45	4 61	...	...	...
15 52	Sr II (1)	3 30	0 00	4 06	4 00	...		4 54
22 22	Fe I (152)	3 03	2 44	4 46	4 37			...
26 73	Ca I (2)	4.58	0 00	3 99	3 96	...		4 34
35 94	Fe I (152)	3 71	2 41	4 18	4 23	...	...	...
38 03	Fe I (689)	2 01	3 40	4 48	4 54	...		...
45 26	Fe I (352)	2 23	2 85	4 41	4 44	...		
46 09	Fe I (906)	1 70	3 63	4 65	4 65	...	...	...
46 83	Sc II (7)	2 97	0 31	4 19	4 22		...	4 52
47 43	Fe I (693)	2 86	3 35	4 29	4 33		...	
50 12	Fe I (152)	3 61	2 46	4 22	4 31	...		
50 79	Fe I (42)	3 71	1 55	4 19	4 23	...		...
52 62	Cr II (31)	0 56	3 84	4.50	4 74	...	...	...
54 35	Cr I (1)	3 72	0 00	4 23	4 25	...	...	4 77
60 48	Fe I (152)	3 99	2 39	...	...	4 31	...	4 71
61 92	Cr II (31)	1 12	3 85	4 35	4 43	4 99	4 79	...
64 21	Fe I (692)	1 70	3 35	4 69	4 90			...
65 26	Fe I (993)	1 35	3 91	4 87	4 87		...	...
67 83	Fe I (482)	2 06	3 10	4 61	4 50	...	...	...
71 16	Fe I (152)	3 26	2.44	4 15	4 21	4 55	...	4 79
71 76	Fe I (42)	4 16	1 48	4 10	4 12	4 35	4 31	4 53
74 80	Cr I (1)	3 12	0 00	4 16	4 19	4 41	4 46	...
76 68	Fe I (976)	1 02	3 86	4 92	4 91	...	...	...
82 41	Fe I (71)	2 63	2 17	4 28	4 33	4 51	4 53	4 93
83 01	Ca I (5)	2 36	1 88	4 38	4 40	4 67	4 82	...
94 77	Sc II (15)	1 12	0 60	...	...	4 99	...	...
96 57	Fe II (28)	1 84	2 69	4 25	4 26	4 65	4 67	5 11
4300 05	Ti II (41)	2 89	1 18	4 00	4 06	4 23	4 28	4 53
01 93	Ti II (41)	2 68	1 16	4 13	4 08	4 23	...	...
02 53	Ca I (5)	2 87	1 89	...	...	4 45	...	...
03 17	Fe II (27)	1 80	2 69	4 32	4 17	4 52	...	...
12 86	Ti II (41)	2 75	1 18	4 18	4 26	4 39	4 43	4 87
16 81	Ti II (94)	0 81	2 04	...	...	5 04	4 86	...
18 65	Ca I (5)	2 06	1 89	4 39	4 47	4 68	4 74	...
25 01	Sc II (15)	2 28	0 59			4 46	4 53	5 06
52 74	Fe I (71)	2 54	2 21	...	...	4 79	4 77	...
71 28	Cr I (22)	1 90	1 00	4 45	4 58	...		...
74 46	Sc II (14)	1 90	0 62	...	...	4 55	...	...
75 93	Fe I (2)	2 68	0 00	4 40	4 45	4 66	4 62	...
79 24	V I (22)	1 97	0 30	4 69	4 78	...	...	...
83 55	Fe I (41)	4 33	1 48	4 06	4 07	4 26	4 32	4 35
85 38	Fe II (27)	1 40	2 77	...	...	4 56	4 54	...
87 90	Fe I (476)	1 26	3 06	4 56	4 58	...	...	...
88 41	Fe I (830)	1 77	3 59	4 56	4 63	5 28	...	...
89 97	V I (22)	1 44	0 27	4 73	4 70	...	...	...
94 06	Ti II (51)	1 38	1 22	4 35	4 41	4 60	4 65	...
95 03	Ti II (19)	2 37	1 08	4.12	4.12	4 24	4 30	4 58

TABLE 2—*Continued*

λ	Element	log $\eta_\odot$	χ	−log W/λ, Plate No., DX Del: 1, 3	DX Del: 2, 4, 5	RR Lyr: 6	RR Lyr: 7	X Ari: 8, 9, 10
4395 85	Ti II (61)	1 17	1 24	...	...	4.76	4 81	...
99 77	Ti II (51)	2 02	1 23	4 31	4 31	4.53	4 43	4 94
4400 36	Sc II (14)	1 46	0 60	4 28	4 35	4 60	4 56	5 16
04 75	Fe I (41)	4 17	1 50	4 12	4 11	4 31	4 29	4 48
15 12	Fe I (41)	3 72	1 60			4 47	...	4 62
15 56	Sc II (14)	1 63	0 59	...	...	4 62	...	5 00
16 82	Fe II (27)	1 31	2 77	4 34	4 34	4.66	4 66	...
17 72	Ti II (40)	1 70	1 16	4 29	4 24	4 47		4 77
25 44	Ca I (4)	2 45	1 87	4 59	4 49	4 82	...	
27 31	Fe I (2)	2 67	0 05	4 33	4 39	4 59	4 59	...
30 62	Fe I (68)	1 98	2 21	...	...	4 88	...	...
35 69	Ca I (4)	2 23	1 88	4 39	4 56	4 77	...	...
42 34	Fe I (68)	2 88	2 19	...	...	4 72	...	...
43 80	Ti II (19)	2 36	1 08	4 20	4 18	4 36	...	4.68
47 72.	Fe I (68)	2 86	2 21	4 49	4 46	4 82	4 72	...
55 89	Ca I (4)	...	1 89	4 56	4 49	4 72	...	...
66 55	Fe I (350)	2 15	2 82	4 28	4 34	4 84	...	...
68 49.	Ti II (31)	2 26	1 13	4 20	4 21	4 36	4 26	4.57
81 2.	Mg II (4)	1 83	8 83	4 06	4 10	4 39	...	
84 23	Fe I (828)	1 43	3 59	4 49	4 65	5 13		...
85 68	Fe I (830)	1 18	3 67	4 72	4 84	...		...
89 18	Fe II (37)	1 05	2 82	...	...	4 75	...	...
91 40	Fe II (37)	1 20	2 84	4 39	4 44	4 74	4.74	
94 57	Fe I (68)	2 83	2 19	4 35	4 45	4 66	...	...
4501 27	Ti II 31)	2 23	1 11	4 11	4 25	4 32	4 35	4 57
02 22	Mn I (22)	0 92	2 91	5 07	5 10	...	...	...
08 28	Fe II (38)	1 31	2 84	4 34	4 39	4 54	4 54	
15 34	Fe II (37)	1 35	2 83	4 34	4 38	4 61	4 65	
20 22	Fe II (37)	1 24	2 79	4 25	4 36	4 58	...	
22 63	Fe II (38)	1 40	2 83	4 17	4 20	4 39	4 40	
28 62	Fe I (68)	3 03	2 17	4 20	4 17	4 48	4 46	
41 52	Fe II (38)	1 01	2 84	4 42	4 54	4 75		...
45 14	Ti II (30)	0 84	1 13	4 58	4 64	...		...
45 96	Cr I (40)	1 31	0 94	4 73	4 87	...	...	...
54 03	Ba II (1)	2 80	0 00	4 17	4 27	4 33	4 38	
58 66	Cr II (44)	1 18	4 06	4 32	4 39	4 60	4 64	...
63 76	Ti II (50)	2 15	1 22	4 24	4 29	4 34		4 72
64 59	V II (56)	0 08	2 26	4 72	4 83	...		...
71 97	Ti II (82)	2 27	1 56	4 19	4 26	4 34		4 55
76 33	Fe II (38)	0 95	2 83	4 33	4 49			
78 56	Ca I (23)	1 20	2 51	4 54	4 98	...		...
82 84	Fe II (37)	0 84	2 83	4 44	4 48	...	...	...
83 83	Fe II (38)	1 80	2 79	4 15	4 19	4 40	4 38	...
85 87.	Ca I (23)	1 77	2 51	4 59	4 76	5 12	...	...
88 22	Cr II (44)	1 12	4 05	4 36	4 55	4 65	4 71	...
91 39	Cr I (21)	1 01	0 96	4 66	4 85	...	...	...
92 09	Cr II (44)	0 76	4 06	4 29	4 55	5 02		...
4602 00	Fe I (39)	1 01	1 60	4 68	4 83	...	...	...
02 94	Fe I (39)	1 70	1 48	4 48	4 44	4 77		...
11 28	Fe I (826)	1 74	3 64	4 60	4 67	5 09		
16 14	Cr I (21)	1 28	0 98	4 74	4 64			
16 64	Cr II (44)	0 63	4 05	4 63	4 72		...	
20 51	Fe II (38)	0 82	2 82	4 62	4 67	...	...	...
25 05	Fe I (554)	1 31	3 23	4 68	4 80	...	...	...
34 11	Cr II (44)	0 88	4.05	...	...	4 75	4 87	...
4702 99	Mg I (11)	3 55	4 33	...	...	4 52		...
31 44	Fe II (43)	1 26	2 88	...	...	4 88	...	...
36 78	Fe I (554)	2 37	3 20	...	...	4.86	...	...
4805.10	Ti II (92)	1.55	2 05	...	...	4.57	...	...

were measured on the tracing of Ce 12816. The fit with the curve of growth for lines longward of λ 4000 is satisfactory. Examples of the resulting curves of growth are shown in Figure 2. The solid curves are theoretical curves of growth by Wrubel (1949) for $B_0/B_1 = \frac{4}{3}$. The curve parameters are summarized in Table 3. The uncertainties in θ^*_{exc} are between ± 0.05 and ± 0.10 in all cases; they are larger than the changes in effective temperature inferred from the color variation over the phase intervals that correspond to the exposure times. Our estimate of T_{exc} for X Ari is lower than that for DX Del by

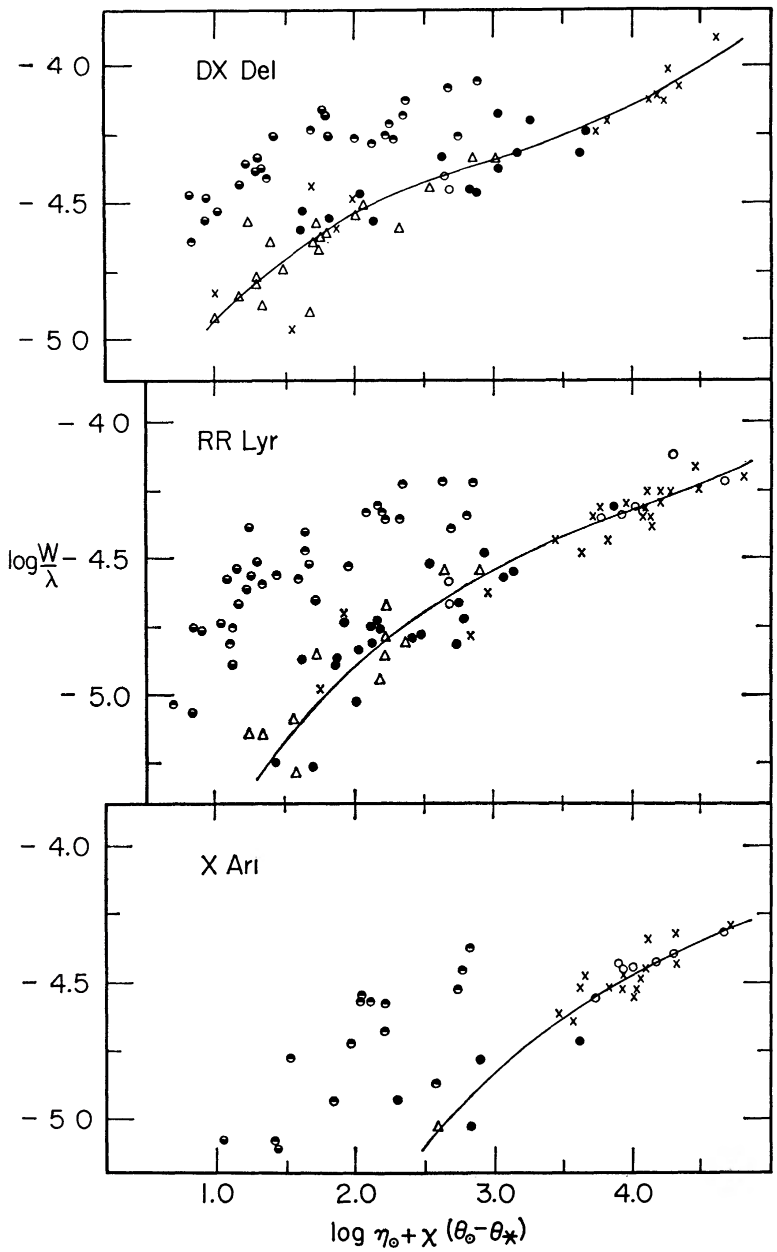

FIG. 2.—Curves of growth for DX Del, RR Lyr, and X Ari. The solid curves are theoretical curves of growth. The coding for Fe I is as follows: *open circles*, 0.0–0.2 ev: *crosses*, 0 8–1.6 ev: *filled circles*, 2 0–3.0 ev: *triangles*, 3.0–4.0 ev. Upper half-filled circles and lower half-filled circles represent Ti II and Fe II lines, respectively.

600°. This difference is approximately equal to the sum of the uncertainties of the two determinations. In our opinion it is probably real. The intermediate value of T_{exc} for RR Lyrae does not differ significantly from either extreme. The Doppler parameters for all three stars lie between 3 and 4 km/sec.

b) *Estimates of Opacities and Abundances*

We use the notation of Helfer, Wallerstein, and Greenstein (1959),

$$[X] \equiv \log \frac{X_*}{X_\odot}. \tag{3}$$

From the vertical and horizontal displacements of the solar and stellar curves of growth we estimate $[V]$ and $[N_{\text{Fe I}}/\kappa V]$. Since

$$[N_{\text{Fe I}}] = \left[\frac{N_{\text{Fe I}}}{\kappa V}\right] + [V] + [\kappa], \tag{4}$$

TABLE 3

CURVE OF GROWTH AND ATMOSPHERIC PARAMETERS

	PLATE NO				
	DX Del		RR Lyr		X Ari
	1, 3	2, 4, 5	6	7	8, 9, 10
$\langle\phi\rangle$ . .	+0 30	+0 53	+0 44	+0 43	+0 46
log a	−2 8	−2 6	−2 4	−2 4	−2 2
V(km/sec)	+3 5	+3 6	+3 6	+3 3	+3 5
θ_{exc}	+1 04	+1.04	+1 09	+1 09	+1 19
θ_{ion} . .	+0 89	+0 89	+0 94	+0 94	+1 04
log P_e .	−0 35	−0 30	−0 65	−0.75	−1 45
log κ_{4300}.	−1 90	−1 85	−2 05	−2 10	−2 45
W_λ (K Ca II) .	+8 7	+8 0	+4 2	+5 7	+2 0

the relative abundance of Fe I may be estimated if we can evaluate $[\kappa]$. Then, by means of the displacement between the stellar curves of growth for Fe I and any ion j, we estimate $[N_j/N_{\text{Fe I}}]$ and hence $[N_j]$. The opacity, which is due primarily to the negative hydrogen ion in middle F-type stars, is a known function of θ_{ion} and P_e. Greenstein (1948) has pointed out that if the abundances of spectroscopically prominent elements, such as iron and titanium, bear the same ratio to hydrogen in the sun and in an F-type star, then the stellar opacities can be obtained directly from $[\kappa] \approx -[N_{ion}/\kappa]$. Opacities determined in this manner impose the second condition on θ_{ion} and P_e required to solve for both quantities. While this procedure cannot be employed in an abundance analysis, it provides valuable information on the relative values of θ_{exc} and θ_{ion} for F-type stars. For ten F- and G-type stars analyzed in this manner (Greenstein 1948; Kraft *et al.* 1959; Abt 1960), the difference $\theta_{exc} - \theta_{ion}$ lies between 0.1 and 0.2. On the basis of these data, we have assumed the relation $\theta_{ion} = \theta_{exc} - 0.15$ for the RR Lyrae stars.

These θ_{ion} and the $[N_{\text{Fe II}}/N_{\text{Fe I}}]$ obtained from the curves of growth have been used in Saha's equation to obtain estimates of P_e and hence $[\kappa]$. The values $\log P_e^\odot = 1.30$ and $\theta_{ion}^\odot = 0.89$ for a representative optical depth $\tau = 0.35$ were taken from the model solar atmosphere of Aller and Pierce (1952). The opacity at λ 4300, calculated as the sum of contributions due to the negative hydrogen ion and to the bound-free absorption and

Rayleigh scattering by neutral hydrogen, was taken as representative of its values over the wave-length interval employed in the analysis.

The logarithmic abundances of several elements relative to the sun and to Procyon (Greenstein 1948), estimated by the procedure described above, are given in Table 4. Ionization correction factors relative to the sun are negligible for ions and lie between 10 and 100 for neutrals. Final values for each element are means of the values derived from neutral and singly ionized elements. The numbers of lines used and a quality estimate for each determination are also given in Table 4. Abundances based on only two or three lines must be regarded as very uncertain. The relative logarithmic metals/hydrogen ratios at the bottom of Table 4 are averages for the two best abundance determinations (Fe and Ti).

By calculating abundances for different assumed values of θ_{exc} (and hence different values of θ_{ion}) for DX Del, we find that a change of 0.05 in θ_{exc} results in a change in the

TABLE 4

LOGARITHMIC RELATIVE ABUNDANCES*

ELEMENT	LOG $N/N_{\odot}$			
	DX Del	RR Lyr	X Ari	α CMi†
Al.	. .	−1 5 (2, 0, *b*)	−3 0 (2, 0, *b*)	−0 3
Ca	0 0 (6, 0, *b*)	−1 0 (5, 0, *b*)	−2 4 (1, 0, *c*)	+ 2
Sc	− 5 (0, 2, *c*)	−1 7 (0, 5, *b*)	−3 1 (0, 4, *b*)	− 2
Ti	− 1 (2, 13, *a*)	−1 3 (2, 18, *a*)	−2 6 (0, 13, *a*)	0
V	− 7 (2, 2, *c*)	−1 8 (0, 2, *c*)	. . . .	− 2
Cr	0 (6, 5, *b*)	−1 1 (1, 5, *b*)	−3 1 (1, 0, *c*)	+ 1
Mn	+ 2 (5, 0, *b*)	−1 5 (2, 0, *c*)	<−3 2 (2, 0, *c*)	+ 2
Fe	0 (51, 15, *a*)	−1 2 (62, 15, *a*)	−2 8 (28, 2, *a*)	+ 3
Sr	− 6 (0, 2, *c*)	−2 0 (0, 1, *c*)	−3 5 (0, 2, *c*)	− 1
Y	. .	−1 2 (0, 2, *c*)		0
Ba	−0 8 (0, 1, *c*)	−1 8 (0, 1, *c*)	. .	−0 6
[*m/H*]	−0 05	−1 25	−2 70	+0 2

* The quantities in parentheses are, in order, (1) no of neutral lines measured; (2) no of ion lines measured; (3) a quality estimated: *a* = good, *b* = fair, *c* = poor

† The abundances for α CMi are taken from Greenstein (1948)

logarithmic abundance of 0.3. Uncertainties of this order may be expected in the determination of θ_{exc} and in the constant which relates θ_{exc} and θ_{ion}. These, combined with possible systematic errors in the equivalent widths, result in an unavoidable uncertainty of a factor of 3 or 4 in the derived abundances. Additional systematic errors may be present if the atmospheric structures of the RR Lyrae stars differ from those of normal F-type stars in such a way that $[N_{Fe\,II}/N_{Fe\,I}]$ cannot be inferred from the displacements between the Fe I and Fe II curves of growth and/or the assumption about θ_{ion} is incorrect.

With this commentery on errors in mind, we summarize the results on abundances: (1) the metals/hydrogen ratio in DX Del does not appear to differ from that in the sun, while in RR Lyr and X Ari it is lower by factors of the order of 20 and 500, respectively; (2) with the exception of manganese and strontium, which may have greater than average abundance deficiencies in RR Lyr and X Ari, the abundances relative to iron of the few elements that we can study are surprisingly similar to those derived for Procyon by Greenstein (1948).

The results obtained for DX Del confirm the impression gained from visual inspection of spectrograms of that star (see Fig. 1). The extremely low estimates of metal abundances in X Ari are due in part to the extreme weakness of the metal lines and in part to the relatively low excitation temperature and hence low opacity that we have obtained. It should be noted that, for the $\theta_{\rm ion}$ and P_e we have adopted for X Ari, Rayleigh scattering by neutral hydrogen is a significant contributor to the opacity. Had it been ignored, our abundance estimates for this star would be lower by a factor of 2.

The abundance estimate for manganese in X Ari is actually an upper limit based on lines that are marginally visible on our spectrograms. A manganese deficiency has been found by Wallerstein and Helfer (1959) for the high-velocity dwarf 85 Peg and is interpreted by them as due to the formation of *e*-process elements at a temperature different from that which results in solar abundances. With the exception of inconclusive determinations for vanadium, we are unable to determine abundances of other elements that could be used to test this hypothesis.

IV. LINE BLANKETING, INTERSTELLAR REDDENING, AND INTRINSIC COLORS

a) Metallic Line Blanketing

Metallic line-blanketing fractions have been measured in order to estimate the effect of variations in line strengths on the intrinsic colors of RR Lyrae stars.

The ratio of the observed relative intensity to the adopted stellar continuum has been averaged over 50-A intervals from λ 3650 to λ 4800 on single spectrograms of each of the program stars. The absorption due to hydrogen lines was not included in the measurements. The resulting empirical functions were weighted by the energy distribu-

TABLE 5

BLANKETING CORRECTIONS

Star	ΔU_M	ΔB_M	ΔV_M^*	$\Delta(U-B)_M$	$\Delta(B-V)_M$	$\Delta(U-B)$	$\Delta(B-V)$
DX Del	−0 40	−0 18	(−0 06)	−0 11	−0 12	−0 22	−0 10
RR Lyr	− 12	− 06	(− 02)	− 06	− .04	− 06	− 04
X Ari	−0 05	−0 02	(−0 01)	−0.03	−0 01	−0.03	−0 01

* Estimated by the procedure described in Sec IV

tion of π^3 Ori (Code 1959) and the cell-filter response-curves published by Johnson and Morgan (1951). The integrals of these products, converted to magnitudes, are the corrections, ΔU_M and ΔB_M, to the *U* and *B* magnitudes. The program stars are too faint to be observed effectively with the 100-inch telescope in the green-yellow region of the spectrum. Therefore, the ΔV_M corrections could not be measured. We have assumed that the ratio of the blanketing fractions in the *B* and *V* regions of the spectrum are are the same for ξ Peg (F7 V) (Burbidge, Burbidge, Sandage, and Wildey 1960) and the RR Lyrae stars. The results are presented in Table 5. The quantities $\Delta(U-B)_M$ and $\Delta(B-V)_M$ are the changes in color indices due to the corrections to *U*, *B*, and *V* described above. Decreases in color temperature that are expected to occur as a consequence of the removal of the line absorption lead to small increases in the color indices. These corrections, taken from Table 12 of Melbourne's (1960) paper, were applied to $\Delta(U-B)_M$ and $\Delta(B-V)_M$ to obtain the final corrections $\Delta(U-B)$ and $\Delta(B-V)$. Our values of ΔU_M are uncertain for two reasons: (1) the integrations could be carried out over less than half the response-curve of the *U* filter, and (2) in the case of DX Del the continuum could not be located with any confidence shortward of λ 4000.

b) Interstellar Reddening

DX Del, RR Lyr, and X Ari are located at galactic latitudes −20°, +11°, and −39°, respectively, and at distances ranging approximately from 300 to 1000 parsecs, precise values depending on assumptions about their luminosities and interstellar absorption. From the distribution of interstellar matter given by Binenndijk (1952) it appears that any or all of the three program stars may be reddened by amounts that are comparable with the blanketing corrections. To obtain rough estimates of the interstellar reddening, single, low-dispersion slit spectrograms (130 A/mm at Hγ) and U, B, V color indices of a number of stars in the regions of DX Del and X Ari were obtained with the 36-inch refractor and the Crossley reflector telescopes, respectively, of the Lick Observatory. The spectrograms were classified by comparison with MK standards, and color excesses were formed as the differences between the observed color indices and the intrinsic $B - V$ color indices appropriate for the spectral types. The observations, including data for two metallic-line stars discovered in the course of the observing, are presented in Table 6. We have adopted as color excesses for DX Del and X Ari the average color excesses of the field stars in the two regions, excluding the three stars (Nos. 3, 6, and 9) nearer than

TABLE 6

COLOR INDICES, SPECTRAL TYPES, AND COLOR EXCESSES OF STARS IN FIELDS OF DX DEL AND X ARI

Region	No	BD	Sp.	V	$B-V$	E_{B-V}	Ang Dist from Var. (deg)
DX Del ..	1	+12°4410	F0 II*	9 63	+0 37	+0 07	2 2
	2	+13°4506	B9 V	7 52	+ 05	+ 10	1 3
	3	+12°4451	F2 V	9 32	+ 34	− 03	0 8
	4	+12°4457	A2 III	7 96	+ 20	+ 13	0 3
	5	+11°4380	A2 V	10 22	+ 12	+ 05	0 3
	6	+12°4490	F3 V	8 78	+ 34	− 06	1 6
X Ari	7	+ 9°381	F5 II	8 94	+ 45	+ 07	2 8
	8	+10°415	F5 III	10 50	+ .57	+ 13	0 3
	9	+ 9°397	A5 V	7 94	+ 30	+0 15	0 6
	10	+11°4352	Am	8 95	+ 32	.	
	11	+10°425	Am	7 98	+0 33		

* The K line may be slightly weak for the spectral type

200 parsecs. The resulting values of E_{B-V} are +0.08 and +0.10 for DX Del and X Ari, respectively. The procedure is admittedly inaccurate, but it is necessary, since it appears that neither field is free of interstellar reddening.

According to Oke and Bonsack (1960), Strömgren has found RR Lyrae to be little reddened. Eggen and Sandage (1959), by comparison with the M3 variables, estimate that it is reddened by 0.05 mag. TU Ursae Majoris, a Bailey type-a variable with a period and spectrum that are identical with those of RR Lyrae is bluer than RR Lyrae by 0.03 mag. and is located at galactic latitude +73° (Preston, Spinrad, and Varsavsky 1961). If we assume that the intrinsic color indices of RR Lyrae and TU Ursae Majoris are the same and note that reddening of objects in the north galactic polar cap is not unknown, then the estimate of Eggen and Sandage is reasonable. We assume that $E_{B-V} = 0.03$ for RR Lyrae.

Evidence for the existence of gaseous interstellar matter along the lines of sight to RR Lyrae and X Ari is provided by interstellar lines. Sharp components of H and K of Ca II are visible on a 4.5-A/mm spectrogram of RR Lyrae obtained during the quarter-cycle immediately following maximum light, when the stellar lines are weak and violet-shifted. A stationary component of K is also visible on several 20-A/mm spectrograms of X Ari obtained in this phase interval. The equivalent widths in both cases are roughly 0.15 A.

c) Photoelectric Observations of DX Del and X Ari

Photoelectric observations of DX Del and X Ari were obtained on five nights in 1959 with a set of U, B, V filters, a refrigerated 1P21 photocell, and the photometer of the 60-inch reflector on Mount Wilson. Differential magnitudes and color indices of the variable stars with respect to nearby field comparison stars were determined from alternating series of sets of deflections for each variable and its comparison star. The magnitude and color indices of the comparison star were determined by means of transformations from the instrumental system to the U, B, V system derived from observations of U, B, V standard stars made each night. Extinction coefficients were determined from the series of observations of the comparison stars. The results are presented in Table 7 and Figures 3 and 4. Phases were computed by means of the periods published in the second edition of the *General Catalogue of Variable Stars* (Kukarkin *et al.* 1958). Zero points were arbitrarily placed at the phases of the V light-maxima. Similar data for RR Lyrae are already available (Hardie 1955).

The light- and color-index-curves of X Ari and DX Del are typical of RR Lyrae stars. In both cases there is a short-lived decrease in $U - B$ during rising light, followed closely by a change in slope of the light-curves. The photometric behavior of DX Del

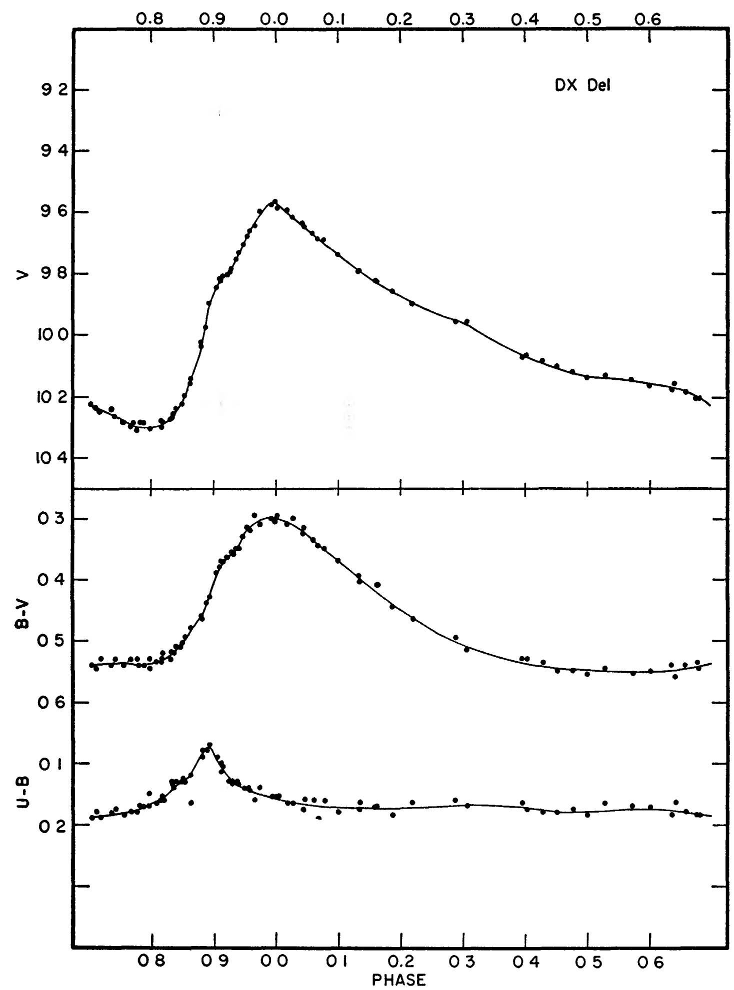

FIG 3.—The variation of V, $B - V$, and $U - B$ with phase for DX Delphini

TABLE 7

PHOTOELECTRIC OBSERVATIONS

JD⊙ 2436000+	ϕ	V	$B-V$	$U-B$	JD⊙ 2436000+	ϕ	V	$B-V$	$U-B$
				DX Delphini					
837 718	0 637	10 175	0 540	0 185	839 730	0 894	9 895	0 430	0 070
728	658	10 185	.540	180	738	.911	9 820	380	100
739	681	10 210	.545	165	740	915	9 810	370	105
.753	711	10 235	.545	.180	744	.924	9 805	365	130
765.	736	10 240	540	180	746	928	9 790	360	130
774	756	10 285	540	185	752.	941	9 730	350	135
784.	.777	10 310	530	180	756	949	9 705	330	140
794	798	10 305	545	150	759	956	9 680	315	140
803 .	.817	10 300	.535	160	.768.	975	9 600	.310	140
.810	.832	10 275	530	130	777	.994	9 575	300	.155
818	.849	10 220	.510	130	780	000	9 565	305	155
838 682	677	10 205	.535	.185	782	004	9 585	.295	.155
.695	704	10.225	.540	190	793	028	9 615	.300	165
702.	.719	10 250	.530	.190	.802	047	9.645	315	160
713.	.742	10 260	530	175	.809	.061	9 670	335	160
.725.	768	10 300	530	180	817	078	9 690	350	.160
731	780	10 285	.540	170	789 692	019	9 590	310	165
.739	797	10 280	530	170	703	043	9 635	325	175
.744 .	808	10 280	535	.165	715	068	9 685	345	190
.749	.819	10 280	520	155	.730	100	9 735	365	180
.756	.833	10 260	.520	135	746 .	134	9 790	395	175
759	840	10 240	510	130	.760	163	9 825	.410	170
766	854	10 195	495	130	819	288	9 955	.495	160
770 .	863	10 160	480	120	.870	396	10 070	.530	165
.778..	.880	10 035	460	.080	.885	428	10 080	.535	180
.782..	.888	9 970	440	.080	896	451	10 110	550	180
.790.	905	9 845	.390	090	908	476	10 120	550	175
.793.	.912	9 825	370	115	919.	.500	10 135	555	185
.801.	.929	9 780	.355	.130	790 692	135	9 790	.405	.165
805	937	9 755	350	130	704 .	161	9 820	410	170
815	.958	9 660	320	145	.717	188	9 855	445	185
819 .	967	9 640	295	160	732.	.220	9 895	.465	165
839 680	788	10 285	540	170	773	307	9 960	515	.170
.694 .	818	10 280	530	160	819	404	10 065	530	.175
702	835	10 260	520	140	878 .	529	10 135	545	165
709	.850	10 220	505	125	899	573	10 150	555	170
716	864	10 145	540	165	912	601	10 165	550	170
724	0.882	10 020	0 465	0 090	931 .	0 641	10 160	0 560	0.165
				X Arietis					
837 850	0 790	9 940	0 560	0 225	837 958	0 956	9 295	0 400	0 150
900	867	9 955	600	120	960	959	9 250	395	160
906	876	9 975	590	120	965	967	9 190	365	175
915	890	9 970	560	080	974	981	9 085	340	195
924	904	9 875	550	085	977	985	9 075	325	200
931	.915	9 740	535	040	985	997	9 040	325	190
934 .	919	9 670	525	015	988	002	9 040	315	.200
.936.	922	9 610	.500	000	998	017	9 040	325	210
.943.	933	9 445	460	040	838 003	025	9 075	325	210
947	939	9 415	445	060	012	039	9 110	315	230
949	0 942	9 390	0 440	0 080	.020 .	0 051	9 155	0 315	0 230

TABLE 7—*Continued*

$JD_{\odot}$ 2436000+	ϕ	V	$B-V$	$U-B$	$JD_{\odot}$ 2436000+	ϕ	V	$B-V$	$U-B$
				X Arietis					
838 930	0 449	9 820	0 580	0 040	839 945 .	0 008	9 030	0.320	0 220
940 .	464	9 845	580	.105	947	011	9 035	.315	.225
951 .	481	9 845	580	.105	949 .	014	9 045	320	220
963	500	9 870	580	.080	951	017	9 055	315	220
974	516	9 880	580	120	959 .	029	9 060	335	215
839 884	914	9 750	530	005	960	031	9 075	330	210
.893	928	9 505	465	005	962 .	033	9 080	330	215
897	934	9 430	475	030	972	049	9 140	330	.220
899	937	9 445	415	050	974	052	9 150	330	225
900 .	939	9 415	.420	065	.975.	054	9 160	.340	220
907.	949	9 345	400	115	985	069	9 205	345	.220
908	951	9 325	410	.120	987	.072	9.210	350	.210
910	954	9 305	380	140	990	077	9 230	345	215
914	960	9 245	380	150	840 001	094	9 255	370	.190
915	962	9 230	370	.150	004	098	9.275	370	195
922	.972	9 110	340	170	014 .	.114	9 300	380	190
925	977	9 100	335	180	017	118	9 315	390	.175
926	978	9.085	335	.185	789 974 .	262	9 565	500	120
933	989	9 040	320	.210	985	.279	9 585	.505	.120
935.	992	9 035	.325	210	790 972 .	795	9 940	590	.090
937	0 995	9 035	0 325	0.215	.982 .	0 810	9 960	0 570	0 120

differs from that of RR Lyrae and X Ari in that (1) there is no depression in the $U - B$ curve at maximum light and (2) there is a shoulder on the rising branch of the $B - V$ curve. Whether or not the shoulders on the rising branches of the light-curves of DX Del come and go on some longer period, as in the case of RR Lyrae, is not known. In any event, the interpretation of the phenomena that occur in RR Lyrae stars during rising light constitutes a problem beyond the scope of this paper. We are concerned here with the values of $B - V$ and $U - B$ that correspond to the phases of our spectroscopic observations. These are plotted as filled circles in Figure 5, *a*, where we indicate the manner in which the relative positions of the three stars in the $U - B$, $B - V$ plane are affected by interstellar reddening. The positions corrected for interstellar reddening are indicated by open circles at the upper termini of reddening lines. The "metal-line-free" positions lie diagonally up and to the left by the amounts $\Delta(U - B)$ and $\Delta(B - V)$ given in Table 5. Inasmuch as these positions cannot readily be compared with those of other high-luminosity stars, we have also made rough estimates of the color indices that the RR Lyrae stars would have, were their metals/hydrogen ratios equal to the solar value. On the basis of the results of Section III no adjustment of the intrinsic colors of DX Del is required. For RR Lyrae and X Ari we increase the color indices by the difference between the blanketing corrections measured for them and the blanketing corrections appropriate for their surface temperatures. We have approximated the latter by

$$\Delta CI_{*} = \Delta CI_{\mathrm{DX\ Del}} + \frac{d\Delta CI}{dT_e}\,\Delta T_e\,. \qquad (5)$$

We have assumed that ΔT_e, the difference between the effective temperatures, is equal to the difference in excitation temperatures and have used the slopes of the blanketing correction versus effective temperature relations near $T_e = 6000°$ for main-sequence

1961ApJ...134..633P

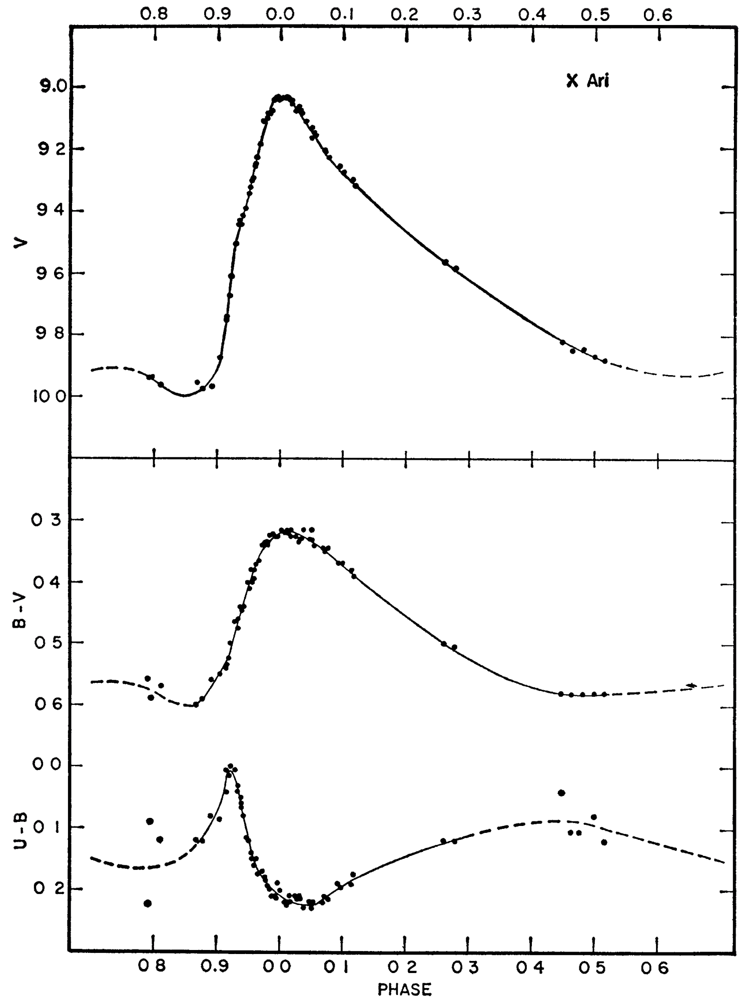

FIG. 4.—The variations of V, $B-V$, and $U-B$ with phase for X Ari

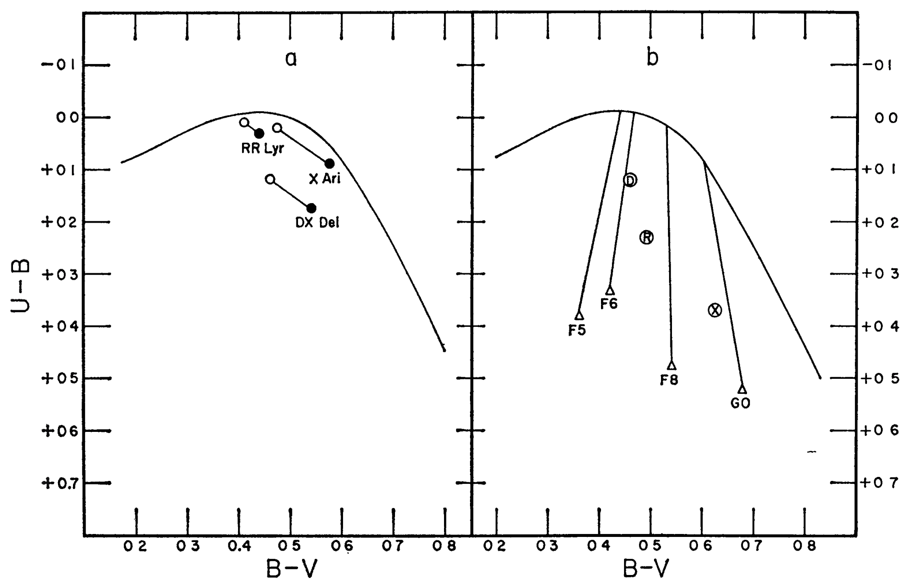

FIG. 5.—The location of the program stars in the $U-B$, $B-V$ plane. The observed colors at the mean phases of the spectroscopic observations are indicated by filled circles. The positions corrected for interstellar reddening are indicated by open circles. The positions that the stars would occupy, were their metals/hydrogen ratios equal to the solar value, are indicated in *b* by open circles, in which the letters *D*, *R*, and *X* denote DX Del, RR Lyr, and X Ari, respectively. The positions of F-type main-sequence and I*b* supergiant stars (*triangles*) are also shown.

stars (Melbourne 1960) to evaluate $d\Delta CI/dT_e$. The resulting positions are indicated in Figure 5, *b*. The atmospheric properties implied by the latter color indices correspond to those of giants in the spectral-type interval F6–G0, as can be seen by comparison with main-sequence and supergiant stars plotted in Figure 5, *b*. The main-sequence color indices are those of Johnson and Morgan (1953). The color indices of the supergiants are means for each spectral type obtained from the six-color measurements by Kron and Svolopoulos (1959), converted to the U, B, V system by means of transformations given by Bahng (1958). Small corrections indicated by recent work of Kraft (1960) were applied. The difference in effective temperatures implied by the color indices of DX Del and X Ari is about 500°. The difference in excitation temperatures is also of this order. The fact that the temperature difference between DX Del ($P = 0^{d}.47$) and X Ari ($P = 0^{d}.65$), inferred from the period versus color-index relation for the M3 variables (which presumably constitute a chemically homogeneous group of stars) is also approximately 500° (Preston 1961), suggests that all RR Lyrae stars may obey a single relation between effective temperature and period regardless of chemical composition. A spectroscopic analysis of pairs of stars with a given period and different chemical compositions would provide a test of this possibility.

d) The Equivalent Width of the K Line as an Abundance Indicator

The results of the present investigation indicate that the observed range in strength of the resonance lines of Ca II in the spectra of RR Lyrae stars is a consequence of ranges in both metal abundances and atmospheric properties. The equivalent widths of the K line given in Table 3 show the decrease with abundance expected from simple curve-of-growth considerations [$W \propto (N/\kappa)^{1/2}$].

An order-of-magnitude estimate of the range in metal abundances in RR Lyrae stars was obtained previously (Preston 1959) on the assumption that the atmospheric parameters of all Bailey type a's were identical. For that case ($W \propto N^{1/2}$) the observed ratio of equivalent widths of the K line in DX Del and X Ari implies a range of only a factor of 15 in abundances. Aller and Greenstein (1960) have found a range in metal abundances one order of magnitude larger than this in high-velocity dwarfs. Since there can be few Bailey type a's brighter than $m_{pg} = 12$ with K lines weaker than that of X Ari (Preston 1959) and since the volume of space in which the spectroscopic survey of RR Lyrae stars has been made is hundreds of times larger than that in which dwarfs have been observed spectroscopically, we are led on general grounds to one of two conclusions: either the ranges in chemical composition of dwarf stars and field RR Lyrae stars differ by an order of magnitude, or the weakest-lined RR Lyrae stars are systematically cooler than their strong-lined counterparts. The latter conclusion is consistent with the results of our spectroscopic and photometric analyses.

Failure to detect this range in temperature from the appearance of the hydrogen lines requires an explanation. On the basis of compilations by Greenstein (1948) and Wright (1951), the equivalent widths of Hγ and Hδ in spectra of normal stars on the interval from F5 to G2 decrease from about 6 to about 3 A and depend little, if at all, on luminosity. The lower limit of the range, based on values for the sun and γ Cyg (F8 I*b*) by Wright (1951), may be underestimated, since Wright comments that the appearance of the wings of his line profiles for these two stars indicates that his continua were drawn too low. Thus the range may be even less than 3 A. We suggest that the increased visibility of the Balmer wings in stars like X Ari that are virtually free of metallic absorption lines may tend to mask this decrease in equivalent width and lead to systematically early hydrogen spectral types. A similar discrepancy has been encountered in the high-velocity dwarfs which have the hydrogen lines of middle F-type stars, according to Roman (1954), but which have the atmospheric parameters of G-type stars, according to Aller and Greenstein (1960). Plausible as such arguments may appear, it should be noted that we may be attempting to explain away a real spectroscopic anomaly that has long been recognized in classical cepheids. It is conceivable that the enhancement of the Balmer lines is characteristic of all cepheid-like stars and that the other peculiarities in the spectra of the RR Lyrae stars have disguised this phenomenon heretofore.

V. SUMMARY

A study of three Bailey type-a variables leads to estimates of metal abundances that range from a solar value for DX Del to a value hundreds of times smaller for X Ari. The atmospheric properties near and after mean light on the descending branches of the light-curves correspond approximately to those of giants or bright giants in the spectral type interval F6–G0. It is possible that these variable stars obey a single period-temperature relation, independent of metal abundances. Since the kinematic properties of the galactic subsystems to which these stars belong indicate only that they must all be very old, our results provide little evidence either for or against the notion that there has been an increase in the heavy-element content of interstellar matter with time.

It should be emphasized that while DX Del is a Bailey type-a variable with a period that occurs in known globular clusters, it is spectroscopically similar to numerous other Bailey type a's with shorter periods that are never found in globular clusters. The most promising clusters, such as NGC 6838 and NGC 6356, with strong metallic lines in their integrated spectra do not contain any RR Lyrae stars at all. Inasmuch as there is no evidence that RR Lyrae stars with high metal content exist in any other kind of cluster, these variables remain homeless at present. They may be representatives of a family of stars formed in a region of the Galaxy and/or in an interval of time in which clusters either were not formed or were so star-poor that they were disrupted long ago. The subject is greatly in need of clarifying ideas.

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