# A STUDY of THE II SCORPII ASSOCIATION 

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#### Abstract

Eighty B stars in the II Scorpii region lying north of declination $-30^{\circ}$ were observed photoelectrically in three colors, and fifty of these were measured in $\mathrm{H} \beta$ photometry as well. Special efforts were made to reduce systematic errors so that other observers may use our bright stars as localstandards with adequate confidence. The majority of the stars were found to be reddened by interstellar absorption by an average of 0.12 mag. in ( $B-V$ ), although several stars near $\rho$ Oph are considerably more reddened. Membership considerations based on radial velocities, proper motions, and any other available data suggest that fifty of the stars studied are probably members of the association. The thirty non-members fall largely into two groups: background stars at a considerably greater distance and stars closer to the galactic plane than to the association. Least-squares solutions were used to calibrate the corrected apparent magnitudes, $V_{0}$ of the stars in terms of $(U-B)_{0}$ and $\beta$. The estimated mean errors for these are 0.32 or 0.50 mag. per star, depending on whether the recognized multiple stars are excluded or included in the solutions. Applying the known distance modulus of the group, we derive a calibration for the absolute visual magnitudes, $M_{V}$, as presented in Tables 7,8 , and 9 . The slight uncertainty in the distance increases the above mean errors to 0.37 and 0.54 mag. per star. An examination of the chief sources of error suggests that a residual of 0.26 mag . is due to cosmic scatter.


## I. INTRODUCTION

Among the nearer stellar associations the Scorpio-Centaurus group is of special interest, inasmuch as its convergent point and its distance have been determined (Bertiau 1958) and it thus provides a basic means of calibration of absolute luminosities. A number of investigators have examined the stars in this group, and an exhaustive study was done by Blaauw (1946). For a bibliography, the reader is referred to the papers by Blaauw (1946) and Bertiau (1958). As yet a thorough photometric investigation in three colors is lacking, since earlier photometry was generally done in one or two colors and in various systems. The present investigators have assembled their $U, B, V$ observations of stars of spectral type earlier than A 0 (HD type) lying in the northernmost portion of the association which is observable at the Dyer and McDonald Observatories. Although this is only a small part of the entire group, it is virtually the whole of the important subsection which Blaauw refers to as the Upper Scorpius division. Particular attention to atmospheric extinction is necessary to make possible an over-all accuracy commensurate with work at high altitudes. It is anticipated that these studies will be extended to the A stars, by means of photometry and objective-prism work, and it is hoped that observers who can more easily reach the southern stars will extend the studies throughout the rest of the association.

In addition to the three-color photometry, the present article includes photoelectric measures of the $\mathrm{H} \beta$ line intensity, which provides an essential parameter related to intrinsic luminosity and age, as demonstrated by Strömgren (1956). The measures are in part made up of those already published by Crawford (1958), and to these have been added additional measures made more recently.

## II. OBSERVATIONS

The $U, B, V$ observations were obtained with the 24 -inch reflector of the Dyer Observatory and with the 36 - and 13-inch reflectors of the McDonald Observatory.

[^0]The $\mathrm{H} \beta$ observations were made either with a conventional sequential photometer or with a simultaneous two-color photometer (Crawford 1958) attached to the 36- and 82inch reflectors of the McDonald Observatory. The work of the two authors was done entirely independently, and only after completion was it combined. In this way, systematic errors due to observing technique or instrumental differences were minimized.

At the latitudes of the two observatories, photometric work on the Scorpius region is particularly difficult, as only a few of the stars ever reach an altitude greater than $30^{\circ}$ above the horizon, so that the air mass through which observations may be made is of the order of 2 or greater. Consequently, a choice of two general alternative observing methods was necessary to minimize the errors due to extinction uncertainties: we had either to make use of only the nights of highest transparency or to determine the extinction coefficients accurately each useful night. Experience has shown us that the second alternative is necessary, particularly in Nashville, as many nights of good quality are available during which the transparency is sufficiently constant, though it varies considerably with the seasons and thus precludes the use of mean extinction coefficients. Details of a method for determining coefficients and zero points accurately from a few observations are described elsewhere (Hardie 1959). Prior to adopting this method, an entire observing season was devoted to the measurement of the Scorpius stars at the Dyer

TABLE 1
Observational Errors per Single Measure


Observatory, but the results were generally so poor that it was soon recognized that mean extinction coefficients were not to be trusted while working at very large air masses. The only exception to this observation were those instances in which unknown stars were referred to standard stars carefully chosen to have the same altitude, a well-recognized procedure which was adopted for calibrating the local standards during that season.

After adopting the better extinction procedure, all work, including the calibration of the local standards, was recommenced. For every night, extinction coefficients were determined to about 3 per cent accuracy. The average air mass for the observations was 2.2., while the extinction coefficients for Nashville ranged through the following values: $k_{v}: 0.2$ to 0.6 mag ., $k_{b v}: 0.1$ to 0.3 mag ., $k_{u b}: 0.2$ to 0.4 mag . In order to visualize the critical nature of the extinction control in this work, it is only necessary to note that, on the average, only 20 per cent of the $U$ radiation succeeded in passing through the atmosphere, while in the extreme cases, the figure was 4 per cent. For the McDonald observations the coefficients found were similar to those determined by Hiltner (1956).

In Table 1 we have summarized the data relating to the estimated accuracy of the three-color work. There are forty-three stars in common in the lists of both observers having two or more observations by each, and these were used to determine the systematic differences between the two sets of data; these are tabulated in the first column. On the basis of these, corrections were applied to any observations made by only one of the observers, so that all observations have been rendered homogeneous and, it is hoped, relatively free of systematic effects. The data on the forty-three stars in common were also used to estimate the external accuracy of our work; the estimated external mean errors, per single observation, after application of the systematic corrections, are tabu-
lated in the second column. In the third and fourth columns, the internal mean errors are given for the work at each observatory, and these are also in terms of a single observation; these are of the same order of magnitude as the external values, except for the $U-B$ color. Such a discrepancy might be expected on account of the critical nature of the shape of the $U$ transmission band, which includes the Balmer discontinuity.

The results of the observations are assembled in Table 2. All the stars were selected from the HD catalogue having types B9 and earlier and lying within a zone defined by these 1900 co-ordinates: $15^{\mathrm{h}} 30^{\mathrm{m}}$ to $16^{\mathrm{h}} 45^{\mathrm{m}},-30^{\circ}$ to $-17^{\circ}$. This zone actually exceeds the eastern boundary of the association and was so chosen for completeness. As will be pointed out later, the few stars east of $\tau$ Sco appear to be non-members. The average number of observations for all stars except the bright local standards was four, while for the standards there were at least twelve observations per star. The columns of Table 2 are arranged in the following order: HD number; star name (if any) or bright star catalogue number (Schlesinger 1940); $V$ magnitude; $B-V$ and $U-B$ colors; the number of three-color observations $n$ (for $\beta$ Sco C the published value of Johnson and Morgan is used); the ultraviolet color corrected for reddening, $(U-B)_{0}$; the visual magnitude corrected for absorption, $V_{0}$; the MK spectral type; the $\mathrm{H} \beta$ parameter $\beta$; the weight $W t$., of this parameter; and, finally, the absolute visual magnitude, $M_{v}$, for those stars whose membership is probable according to a discussion in a later section. An asterisk beside an HD number refers to the remarks following the table.

The correction for reddening to $U-B$ was applied as described by Morgan and Harris (1956), suitably modified to take into account the curvature in the reddening line. The procedure used was as described by Crawford (1958) on pages 192 and 193 of his article. Justification for the use of $(U-B)_{0}$ instead of the more customary ( $B-$ $V)_{0}$ stems from its wider range of variation between B 0 and A0 (over 1 mag. as compared with 0.3 mag.), so that $(U-B)_{0}$ is a more sensitive indicator of temperature. In correcting for reddening in either the $B-V$ or the $U-B$ color, a star with emission lines or one of high luminosity will generally be overcorrected (Harris 1955; Morgan and Harris 1956); more will be said of this in a later section. It is sufficient to note here that the majority of the stars included are of luminosity class V and only two have been noted to have emission.

The visual absorption was computed from the relation $A_{v}=3 E_{(B-V)}$. The MK spectral types used in this table were obtained from Bertiau (1958) or in some cases from Morgan (1958) where these differ from Bertiau's. The parameter $\beta$ is defined by Crawford (1958 and 1960) and is a measure of the strength of the $\mathrm{H} \beta$ line. The weight, $W t$., depends on the number of observations made (which was in all cases two or more) and on the particular system of filters used. For the absolute visual magnitude $M_{v}$, we have applied to $V_{0}$ a distance modulus correction of 6.18 mag. according to Bertiau's mean parallax of 0 " 0058 for the stars in the Upper Scorpius division. The corresponding distance is 172 parsecs.

## III. MEMBERSHIP

In order to derive a useful calibration, it is important to select carefully the probable members of the association. For this purpose, all the observations of Table 2 have been used to plot a color-magnitude diagram (Fig. 1) and a $\beta$ versus $(U-B)_{0}$ diagram (Fig. 2). Particular care must be exercised to avoid rejecting stars which deviate slightly from a narrow main-sequence in the color-magnitude diagram on this basis alone. For certain stars of questionable membership, it will be desirable to supplement the photometric data with other relevant information before rejecting a star. Such additional information may include any of the following: spectral type or luminosity class, parallax, proper motion, radial velocity, known or suspected multiplicity, position in the sky, and interstellar reddening.

Before considering the membership of individual stars, let us recall the usefulness of the

TABLE 2
PHOTOMETRIC DATA FOR B STARS IN THE UPPER SCORPIUS REGION

|  | HD | Star | V | B-V | U-B | n | $(\mathrm{U}-\mathrm{B})_{0}$ | $\mathrm{V}_{0}$ | MK | $\beta$ | Wt | $\mathrm{M}_{\mathrm{v}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 139 | 094 |  | 7.37 | +0.08 | -0.28 | 5 | -0.42 | 6.79 | B8IV | - |  |  |
|  | 160 | 5801 | 6.18 | +0.01 | -0.42 | 6 | -0.53 | 5.72 | B8V | 2.747 | 2 | -0.46 |
|  | 365* | $\tau$ Lib | 3.67 | -0.17 | -0.66: | 4 | -0.67: | 3.64 | B3V | 2.691 | 5 | -2.54 |
|  | 486 |  | 7.65 | +0.02 | -0.09 | 4 | -0.13 | 7.49 | B9.5v | $2.866^{\prime \prime}$ | 2 | +1.31 |
| 140 | 543 |  | 8.92 | +0.01 | -0.89 | 2 | -1.11 | 7.99 | B0.51II |  | - | - |
| 141 | 180 |  | 8.28 | -0.04 | -0.26 | 2 | -0.29 | 8.17 |  | - | - | - |
|  | 404 |  | 7.70 | +0.13 | +0.02 | 4 | -0.09 | 7.24 | B9V | 2.816 | 4 |  |
|  | 637 | 1 Sco | 4.61 | -0.03 | -0.72 | 3 | -0.92 | 3.96 | B2V | 2.640 | 7 | -2.22 |
|  | 774 |  | 7.70 | +0.09 | -0.12 | 4 | -0.23 | 7.25 | B9V | 2.825 | 2 | +1.07 |
| 142 | 096* | $\lambda \mathrm{Lib}$ | 5.02 | -0.02 | -0.58 | 5 | -0.70 | 4.51 | B3V | 2.701 | 5 | $-1.67$ |
|  | 114* | 2 Sco | 4.60 | -0.07 | -0.64 | 4 | -0.73 | 4.22 | B3V | 2.684 | 7 | -1.96 |
|  | 165 | 5906 | 5.37 | -0.02 | -0.40 | 5 | -0.48 | 5.04 | B6V | 2.741 | 5 |  |
|  | 184 | 5907 | 5.40 | -0.04 | -0.62 | 5 | -0.74 | 4.93 | B2V nn | 2.668 | 3 | $-1.25$ |
|  | 250 | 5910 | 6.13 | -0.07 | -0.44 | 5 | -0.49 | 5.95 | B7V | 2.742 | 4 | -0.23 |
|  | 301 | 3 Sco | 5.86 | -0.06 | -0.58 | 6 | -0.67 | 5.50 | B7IV: | 2.691 | 11 |  |
|  | 315 |  | 6.86 | +0.04 | -0.20 | 5 | -0.21 | 6.57 | B9V | 2.822 | 8 | +0.39 |
|  | 378* | 47 Lib | 5.96 | 0.00 | -0.53 | 5 | -0.66 | 5.43 | B5V: | - | - | -0.75 |
|  | 669 | P Sco A | 3.88 | -0.22 | -0.82 | STD | -0.82 | 3.88 | B2V | 2.648 | 9 | -2.30 |
|  | 805 |  | 7.14 | +0.16 | +0.03 | 5 | -0.10 | 6.58 | B9V | 2.818 | 2 |  |
|  | 883 | 5934 | 5.84 | +0.02 | -3.48 | 5 | -0.61 | 5.28 | B3:V | 2.720 | 4 | -0.90 |
|  | 884 |  | 6.80 | +0.01 | -0.45 | 2 | -0.57 | 6.31 | -- | 2.737 | 2 | +0.13 |
|  | 990 | 5942 | 5.42 | -0.09 | -0.65 | 5 | -0.73 | 5.10 | B3:V | 2.682 | 9 | -1.08 |
| 143 | 018* | $\pi \mathrm{Sco}$ | 2.89 | -0.18 | -0.91 | STD | -0.97 | 2.64 | B1V | 2.612 | STD | $-3.54$ |
|  | 275* | S Sco | 2.32 | -0.11 | -0.91 | STD | -1.03 | 1.81 |  | 2.595 | STD | $-4.37$ |
|  | 567 |  | 7.19 | +0.08 | -0.10 | 5 | -0.19 | 6.79 | B9V | 2.845 | 5 | +0.66 |
|  | 600 |  | 7.33 | +0.10 | -3.06 | 5 | -0.16 | 6.90 | B9V | 2.857 | 3 | $+0.72$ |
| 144 | 217* | $\beta$ Sco AB | 2.61 | -0.05 | -0.87 | STD | -1.04 | 1.92 | B0.5V | 2.624 | 4 | $-4.26$ |
|  | 218 | $B \mathrm{Sco} \mathrm{C}$ | 4.92 | -0.02 | -0.70 | $\square$ | -0.84 | 4.29 | B2V | 2.678 | 6 | -1.89 |
|  | 334 | 5988 | 5.92 | -0.08 | -0.56 | 5 | -0.62 | 5.66 | B9:III | 2.722 | 9 |  |
|  | 470 | $\omega^{\prime}$ Sco | 3.97 | -0.04 | -0.83 | 9 | -0.99 | 3.29 | BlV | 2.623 | 11 | -2.89 |
|  | 661 | 5998 | 6.32 | -0.06 | -0.52 | 5 | -0.59 | 6.02 | B7IV: | 2.710 | 6 | -0.16 |
|  | 844* | 6003 | 5.88 | +0.02 | -0.32 | 5 | -0.38 | 5.51 | B9V | 2.793 | 8 | -0.67 |
|  | 947 |  | 10.11 | +0.05 | -0.71 | 2 | -0.93 | 9.22 | - | - 70 | - | - |
| 145 | 102 |  | 6.58 | +0.07 | -0.17 | 5 | -0.27 | 6.15 | 39 Vp | 2.796 | 2 | -0.03 |
|  | 353 |  | 6.89: | +0.10 | -0.09 | 2 | -0.20 | 6.43: | B9V | 2.816 | 2 |  |
|  | 482 | 13 Sco | 4.57 | -0.17 | -0.76 | STD | -0.79 | 4.44 | B2V | 2.660 | 7 | -1.74 |
|  | 483* | 12 Sco | 5.66 | 0.00 | -0.20 | 5 | -0.25 | 5.46 | B9V | 2.839 | 4 |  |
|  | 501* | $\nu$ Sco CD | 6.23 | +0.12 | -0.39 | 1 | -0.59 | 5.40 |  | 2.735 | 5 | -0.78 |
|  | 502* | , Sco $A B$ | 4.31 | $+0.05$ | -0.65 | 6 | - -0.85 | 3.18 | B2IV-V | 2.677 | 7 | -3.00 |
|  | 519 |  | 7.98 | +0.25 | +0.01 | 3 | -0.23 | 7.04 | - | 2.815 | 2 | +0.86 |
|  | 554 |  | 7.64 | $+0.14$ | -0.09 | 5 | -0.23 | 7.03 | B9V | 2.836 | 4 | +0.85 |
|  | 556 |  | 8.90 | +0.07 | -0.39 | 2 | -0.54 | 8.25 | - | - | - | - |
|  | 631 |  | 7.58 | +0.16 | -0.04 | 5 | -0.19 | 6.95 | B9.5V | 2.859 | 2 | +0.77 |
|  | 792* | 6042 | 6.41 | $+0.04$ | -0.46 | 5 | -0.60 | 5.80 | B7IV | - | - | -0.38 |
| 146 | 001 | 6054 | 6.04 | +0.04 | -0.37 | 4 | -0.49 | 5.52 | B8IV | 2.753 | 8 | -0.66 |
|  | 029 |  | 7.38 | +0.07 | -0.08 | 6 | -0.16 | 7.04 | B9V | 2.860 | 4 | +0.86 |
|  | 284 |  | 6.70 | +0.16 | -0.16 | 5 | -0.34 | 5.95 | B8V | 2.768 | 8 | - |
|  | 285 |  | 7.94 | +0.23 | $-0.10$ | 5 | -0.32 | 6.99 | B8 | - 6 | - | +0.81 |
|  | 332* |  | 7.59 | +0.20 | -0.36 | 4 | -0.62 | 6.49 | B5II: | 2.663 | 4 |  |
|  | 416 | 6066 | 6.60 | +0.02 | -0.16 | 5 | -0.21 | 6.37 | B9.5:V | 2.832 | 7 | +0.19 |

TABLE 2- CONTINUED


REMARKS

| 139365 | ( $\tau \mathrm{Lib}$ ) |
| :---: | :---: |
| 142096 | ( $\lambda$ Lib) |
| 142114 | ( 2 Sco ) |
| 142378 | (47 Lib) |
| 143018 | ( $\pi \mathrm{Sco}$ ) |
| 143275 | ( $\delta \mathrm{Sco}$ ) |
| 144217 | ( $\beta$ ScoAB) |
| 145483 | (12 Sco) |
| 145501 | ( * Sco CD) |
| 145502 | $(\sim$ Sco AB) |
| 145792 | (BS 6042) |
| 146332 |  |
| 147165 | ( $\sigma$ Sco A) |
| 147701 |  |
| 147888 | ( $P$ Oph D) |
| 147890 |  |
| 147933/4 | ( P Oph AB) |
| 148184 | ( $x$ Oph) |
| 149367 |  |
| 144844 | (BS 6003) |

144844 (BS 6003)

Variable radial velocity
Variable radial velocity
$3^{\prime \prime}$ Visual binary, $\Delta \mathrm{m}=2.5 \mathrm{mag}$.
$1 / 2^{\prime \prime}$ Visual binary, $\Delta m=2$ mag.
Spectroscopic binary
Spectroscopic binary
Spectroscopic binary and $1^{\prime \prime}$ c.p.m. companion
$4^{\prime \prime}$ Visual binary, $\Delta m=3 \mathrm{mag}$.
$2^{\prime \prime}$ Visual binary, $\Delta \mathrm{m}=1 \mathrm{mag}$.
" Visual binary, $\Delta m=2$ mag.
A has variable velocity; $\pi^{\prime}=0.020 \pm 8$
$11 / 2^{\prime \prime}$ Visual binary, $\Delta m=4 \mathrm{mag}$.
$9^{\prime \prime}$ Visual binary, $\Delta \mathrm{m}=7 \mathrm{mag}$.
Variable of $\beta$ CMa type
$3^{\prime \prime}$ Visual binary, $\Delta m=5$ mag.
"Visual binary, $\Delta \mathrm{m}=1 \mathrm{mag}$.
$4^{\prime \prime}$ Visual binary, $\Delta m=6 \mathrm{mag}$.
$3^{\text {th }}$ Visual binary, $\Delta m=.7 \mathrm{mag}$.
Variable radial velocity and light.
$1 / 2^{\prime \prime}$ Visual binary, $\Delta \mathrm{m}=1 \mathrm{mag}$.
Variable radial velocity


Fig. 1.-The color-magnitude diagram for all stars in Table 2. Solid dots represent stars having MK types and are mostly of class V except as noted. Open circles represent stars not having known MK types. Bars denote stars of known or suspected multiplicity.


Fig. 2.-The $\left[\beta,(U-B)_{0}\right]$-diagram for all stars for which $\beta$ was measured. The symbols are as in Fig. 1. One star has an anomalous $(U-B)_{0}$ and probably should be shifted to the position marked by a cross.
$\beta$ versus $(U-B)_{0}$ diagram in this problem. In this diagram, both parameters are intrinsic properties of the stars, and therefore the positions of the points are independent of distance, insofar as the reddening corrections to $(U-B)$ have been correctly applied. The diagram is capable of distinguishing stars of various luminosities and evolutionary ages, as Crawford (1958) has demonstrated. In general, stars lying above the mean relation defined by most of the stars are brighter intrinsically and have evolved further. They are therefore likely to be field stars. Those lying below the mean relation are fainter intrinsically and are probably younger. The position of the mean relation defined in the diagram is dependent on age; Crawford (1958) has shown that the $\zeta$ Persei association members define another mean relation lying somewhat lower than that of the Scorpius stars, while field stars, which are presumably older, generally define a higher mean relation.

Crawford (1958) has shown that, in general, double and multiple stars do not contribute significantly to the scatter in such a diagram, since the shifts due to duplicity are rightward and downward, generally displacing a star along the relation, but not much away from it. This behavior is seen to hold for those stars in this association which are known or suspected doubles, as few of the barred points deviate substantially from the mean relation.

TABLE 3
Stars Recognized as Non-Members

|  |  |  |
| :--- | :--- | :--- |
| 140543 | 149168 | 150035 |
| 14180 | 149367 | 150347 |
| 14334 | 149387 | 150514 |
| 144941 | 149464 | 151310 |
| 145556 | 149827 | 151831 |
| 146332 | 149883 | 151865 |
| 148499 |  |  |

Possible sources of scatter in such a diagram, apart from intrinsic brightness and accidental errors, are emission and ultraviolet excess. The first will tend to weaken the $\mathrm{H} \beta$ absorption measure and will therefore raise the star vertically in the diagram, while ultraviolet excess will shift the star leftward.

Certain obvious non-members which can be eliminated from further consideration are listed in Table 3. Of the stars in this table, the six stars up to and including HD 146332 are excluded as obvious background objects solely on the basis of their colors, magnitudes, and in some cases the MK classifications. The thirteen stars following HD 146332 lie closer to the galactic plane, near the east or southeast boundary of the observed area, and many of these are undoubtedly background stars.

Stars whose position in Figures 1 and 2 suggest questionable membership are listed below along with other relevant data, and they are accordingly classed as members or non-members:
$\lambda L i b$ : Variable radial velocity suggests multiplicity; all other data consistent with membership; consider as member.
$\tau L i b$ : Variable radial velocity suggests multiplicity; proper motion and mean radial velocity consistent with membership; consider as member.
$\chi$ Oph: Emission lines; no $\beta$ measures; possible variable (Ashbrook 1949); variable radial velocity suggests multiplicity; proper motion and mean velocity consistent with membership; intrinsic color probably should be -0.98 mag., as measures influenced by emission in spectrum; consider as member.
$\beta S c o A B$ and $\beta S c o C$ : Both stars appear to be subluminous or to have ultraviolet
excess in Figure 2; AB includes a spectroscopic binary; all other information suggests membership; consider as members.
$\delta S c o$ : Spectroscopic binary; position in Figure 2 and all other information suggest membership; consider as member.
$\nu S c o A B, C D$ : Position in Figure 2 suggests subluminous or ultraviolet excess; radialvelocity and proper motion consistent with membership; interstellar reddening suggests membership; photometric data on companion, $\nu$ Sco CD, suggest membership; trigonometric parallax $0.020 \pm 8$ (Jenkins 1952); consider as member.
$\pi S c o$ : Spectroscopic binary; proper motion and mean radial velocity consistent with membership; consider as member.
$\sigma \operatorname{Sco} A$ : Luminosity class III; $\beta$ CMa variable; position in Figure 2 and all other information consistent with membership; consider as member.
$\omega^{1}$ Sco and 1 Sco: Position in Figure 2 suggests possibly subluminous or ultraviolet excess; all other data consistent with membership; consider as members.

2 Sco: Binary; possible additional bright companions; proper motion consistent with membership; radial velocity possibly not consistent; consider as member.

3 Sco: Luminosity class IV; proper motion possibly not consistent with membership; consider as non-member.

12 Sco: Proper motion probably inconsistent with membership; consider as nonmember.

HD 139094: Luminosity class IV; proper motion probably inconsistent with membership; no radial velocity or $\beta$ available; consider as non-member.

HD 141404: No radial velocity or proper motion available; position in Figure 2 strongly suggests field star; position in sky not close to other members; consider as non-member.

HD 142165: Radial velocity possibly inconsistent with membership; position in Figure 2 suggests field star; consider as non-member.

HD 142315: Position in Figure 2 suggests possible field star; no radial velocity available; proper motion consistent with membership; consider as member.
$H D$ 142805: Positions in Figures 1 and 2 suggest field star; proper motion possibly inconsistent with membership; consider as non-member.

HD 142884: No radial velocity available; proper motion may be slightly inconsistent; position in Figure 2 consistent; consider as member.

HD 144844: Position in Figure 2 normal; proper motion consistent with membership; Buscombe (1960) reports variable radial velocity; consider as member.

HD 145102: Position in Figure 2 suggests possible field star; proper motion possibly consistent with membership; consider as member.
$H D$ 145353: Position in Figure 2 suggests field star; proper motion inconsistent with membership; consider as non-member.

HD 145519: Position in Figure 2 suggests possible field star; no proper motion or radial velocity available; consider as member.

HD 146284: Position in Figure 2 suggests field star; proper motion possibly inconsistent with membership; consider as non-member.

HD 146285: No $\beta$, proper motion, or radial velocity available; consider as member.
HD 146416: Position in Figure 2 consistent with membership; proper motion consistent; no radial velocity available; consider as member. (Possible spectroscopic binary?)

HD 147196: Position in Figure 2 strongly suggests field star; proper motion probably inconsistent with membership; spectral class does not agree with $(U-B)_{0}$; consider as non-member.
$H D$ 147701: No $\beta$, radial velocity, proper motion, or spectral type available; heavily reddened; near $\rho$ Oph and HD 147889 and HD 148579, which are considered members; consider as member.

HD 147889: Position in Figure 2 suggests subluminous or excessive ultraviolet; no
proper motion or radial velocity available; heavily reddened. This star is intimately associated with a dense knot of nebulosity in a dark lane. Photographs of this area suggest that this obscuration is either at the distance of the association or closer, but undoubtedly not further away. Thus the star would not be expected to be below the main sequence in Figure 1 due to being a background object. Since it does lie below, however, we conclude either that it is intrinsically fainter or that the ratio of non-selective to selective absorption is higher than 3. Blaauw; in a comment on page 206 of a paper by Whitford (1958), leans to the former possibility in a similar problem. Furthermore, there may be an anomaly in the color for this star, since its spectral type is estimated as B2 V and B1.5 V, according to Hiltner (1956) and Bertiau (1958), respectively. On this basis, the expected $(U-B)_{0}$ would be about -0.93 mag., a value which improves its anomalous position somewhat in the color-magnitude diagram. The measured


Fig. 3.-The color-magnitude diagram for stars considered as probable members. The symbols are as in Fig. 1. Two stars having anomalous colors should be shifted to the positions marked by crosses. The solid curve represents a fourth-degree least-squares solution for $V_{0}$ as a function of $\left.(U-B)\right)_{0}$ for all stars; the dashed curve represents a similar solution for only those stars thought to be single.
colors seem reliable, however, and they agree well with those of Hiltner (1956). We shall consider this star as a member.

HD 148579: No $\beta$, radial velocity, or proper motion available; close to 22 Sco, but more heavily reddened; consider as member.

HD 148860 and HD 151346: Located in eastern part of region, probably outside association; no $\beta$, radial velocity, proper motion, or spectral type available; consider as non-members.

The stars which we have retained as probable or possible members have been used to form the color-magnitude diagram in Figure 3, the HR diagram in Figure 4, and the $V_{0}$ versus $\beta$ diagram in Figure 5. Since $(U-B)_{0}$ and $\beta$ are so highly correlated, and by a virtually linear relation for stars of a given age, Figure 5 appears quite similar to the color-magnitude diagram of Figure 3. In Figure 5 we find the multiple stars deviating upward, as expected. Since the stars which are represented in this diagram are predominantly of luminosity class V , the diagram provides a suitable means of calibrating the parameter $\beta$ in terms of absolute magnitude for main-sequence stars having an age of the Upper Scorpius stars. Further studies on other important stellar groups will be
required to check the dependence of $\beta$ on both luminosity and $(U-B)_{0}$ before we feel confident that $\beta$ and $(U-B)_{0}$ measures alone will allow determination of $M_{v}$.

## IV. DISCUSSION OF ERRORS

It is of interest to consider all the sources of error or scatter and their relative influence in the calibration of luminosities, and for this purpose we shall consider the color-magnitude diagram in Figure 3. The principal sources of scatter in this diagram are (a) the inclusion of field stars whose non-membership is not obvious and the exclusion of member stars; (b) the inclusion of unresolved double or multiple stars; (c) the departure of the individual stars from a mean distance; (d) observational errors in $V, B-V, U-B$;


Fig. 4.-The HR diagram for members having MK types. The latter are those of Bertiau. The star at A0 is a peculiar A star. The symbols are as in Fig. 1.


Fig. 5.-The $V_{0}, \beta$ diagram for members. The symbols and curves are as in Fig. 3
(e) errors introduced in the correction for reddening and for visual absorption; and $(f)$ dispersion in age and chemical composition and hence in luminosity for stars of a given mass. Some amplifying remarks on these and some estimates of the order of magnitude of the scatter introduced by them follow in the next few paragraphs.

## a) Field Stars

For the non-member stars which may remain after the elimination of the most obvious cases, little can be added to the discussion of the previous section, other than to note that, in spite of efforts to be objective, there probably are a few cases of non-members which have been retained and members which have been rejected. There are always borderline cases where agreement cannot be reached, but if these are relatively few in number, the over-all aspect will not be substantially influenced.

## b) Multiple Stars

Of the fifty stars considered as possible members which are plottted in Figure 3, sixteen have known or suspected multiplicity and are accordingly marked by bars in the diagram. Of these, all but six lie significantly higher than neighboring points; the six which do not share this behavior are probably simple doubles without additional bright companions. It is of interest to note that all but two of the known multiple stars are earlier than $(U-B)_{0}=-0.6$. Recent considerations of a statistical nature by Blaauw (1960) suggest that about one-third of all the stars considered here would be multiple. Accordingly, one would not expect many of the remaining thirty-four stars to be multiple.

Multiplicity can cause discrepancies in both vertical and horizontal directions in the color-magnitude diagram, and these may range from insignificance for cases of a bright early-type star whose companion is faint and late to very substantial significance for cases of bright stars in triple or quadruple systems. Therefore, it is not practical in general to apply corrections in cases of unknown complexity. One effect of the retention of such stars in the calibration of the absolute magnitudes is that, in spite of a systematic error, the use of such magnitudes in subsequent work on other associations and clusters containing unrecognized multiple stars may lead to more reliable distances; this is simply an instance of compensating errors. We shall summarize our final results with and without the multiple stars so that either may be used according to the needs of any situation.

## c) Distance Spread

For the group under consideration, it is possible to estimate the physical extent. The Upper Scorpius stars occupy a strip about $13^{\circ}$ wide, corresponding to a width of about 40 parsecs, and if we assume a depth of the same extent, then the distances of individual stars may range from about 150 to 190 parsecs. The corresponding distance moduli range from 5.9 to 6.4 mag., leading to a range in the vertical scatter of 0.5 mag. This effect can be regarded as being about equivalent to a mean error of $\pm 0.15 \mathrm{mag}$.

## d) Observational Errors

Since the average number of observations for stars retained as members was 5.6 per star, the expected mean errors in our tabulated values of $V, B-V$, and $U-B$ are of the order of $0.010,0.007$, and 0.010 mag ., respectively, based on the external mean errors from Table 1.

The observational errors may derive from accidental causes; systematic errors due to cell, filters, and other instrumental causes; extinction variation; and unrecognized stellar variability. The preceding mean errors would presumably cover all these sources of error, since they are based on external errors derived from two independent programs, carried
out by two observers using different methods and equipment. Among the stars in Table 2 are two known variables: $\sigma$ Sco, whose amplitude is sufficiently small not to affect substantially its usefulness here, and $\chi$ Oph, whose variability is suspected but not certain. The mean values of $V$ are given to the nearest hundredth for both as for other stars, though these should not be taken as seriously.

## e) Reddening Correction Uncertainties

It is unfortunately all too common for observers to assume that the same high accuracy of photoelectrically determined magnitudes and colors also holds for the corresponding quantities corrected for reddening and especially for absorption. In the first place, the correction for reddening involves assuming certain intrinsic colors for a given type, or, what is equivalent, that all stars can be extrapolated back along a reddening path to that straight line in the $(U-B),(B-V)$ plane which defines the mean relation for main-sequence stars, given by $(B-V)_{0}=0.27(U-B)_{0}$. Examination of the original data from which this relation derives (Morgan, Harris, and Johnson 1953; Morgan and Harris 1956) reveals a scatter which can be characterized by a mean error of about 0.014 in both $(B-V)_{0}$ and $(U-B)_{0}$. Thus the values derived for $(B-V)_{0}$ and $(U-B)_{0}$ for a reddened star have two sources of uncertainty: the observational errors and the correction errors. In the data for the stars of Table 2, therefore, the mean errors of $(B-V)_{0}$ and $(U-B)_{0}$ will, in general, be about 0.016 and 0.018 mag., respectively. Accordingly, the horizontal scatter for main-sequence stars in the color-magnitude diagram, in which the abscissa is $(U-B)_{0}$, is characterized by a mean error of close to 0.02 mag .

For emission stars or luminous stars brighter than MK class III, the reddening correction derived as above may be too much, since, in the first instance, the color measures are contaminated with emission effects and, in the second instance, the intrinsic color $(U-B)_{0}$ is richer in ultraviolet. For the emission stars, a preferable value of ( $U-$ $B)_{0}$ may be obtained by using the mean value for stars of the same spectral type. None of the stars under consideration is known to be brighter than MK class III, although one at least ( $\chi \mathrm{Oph}$ ) is known to have emission and possibly also the star HD 142184, according to Bertiau (1958). For $\chi$ Oph, a more reliable value of its intrinsic color $(U-B)_{0}$ is given by its spectral type, B1, namely, -0.98 mag. For HD 142184, the $(U-B)_{0}$ values derived both ways are in sufficient agreement.

In additon to introducing some uncertainty in the horizontal sense, the reddening corrections affect the vertical uncertainties even more strongly because of the absorption correction, $A_{v}$, which is applied to $V$. In determining the absorption correction, we generally use 3 as the ratio of non-selective to selective absorption, in accordance with Hiltner's and Johnson's results (1956). However, because of the difficulty in determining this ratio by statistical means, there is a rather large uncertainty in its value; the mean error, 0.3 , is about 10 per cent of the value itself.

Moreover, the ratio is thought not to be constant on theoretical grounds (Blanco 1956; Divan 1956; Rozis-Saulgeot 1956; Canavaggia 1959; Wilson 1960) due to filter band width among other things. They show that the ratio increases as absorption increases and that it depends on the stellar temperature. Observationally, Sharpless (1952) demonstrated that, for some of the Orion B stars, the ratio was closer to 6 than to 3 , and, although the evidence for this is not considered strong (see, e.g., Blaauw's comment on p. 206 in a paper by Whitford 1958), evidence for the constancy of the ratio if not strong either. These comments need not imply any variation in the reddening law, the uniformity of which is in general agreement (Stebbins and Whitford 1943; Schalén 1951, 1960; Divan 1954), although even here there may be some local difference in Cygnus, according to Hiltner and Johnson (1956).

We shall therefore treat the factor 3 with due regard to its uncertainty, and, accordingly, the uncertainty in $A_{v}$ will be composed of a 10 per cent mean error in the factor

3 and a 0.017 mag. mean error in general for $E_{(B-V)}$. Table 4 summarizes the errors in $A_{v}$ for various values of $E_{(B-V)}$. For the majority of the stars of Table 2, the $V_{0}$ values have a mean error of about 0.1 mag.; for the stars close to the dense clouds near $\rho$ Oph, the mean errors are apt to be closer to 0.22 mag . If the color excess for the heavily reddened star HD 147889 is 1.13 mag., as computed, the mean error in its value of $V_{0}$ is about 0.35 mag.

Since the colors of $\chi$ Oph and possibly HD 147889 are abnormal, the color excess in each case is subject to further uncertainty, so that the values of $V_{0}$ are even more uncertain than is indicated by the mean errors given above.

## f) Cosmic Dispersion

In Figure 3 are shown two mean relations-the solid curve, pertaining to all stars, including the multiples, and the dashed curve, pertaining to those not known to be multiple. The manner of deriving these relations will be discussed later. In the case

TABLE 4
Estimated Mean Errors for Reddening Correction

| $E_{(B-V)}$ | $A_{v}$ | Mean Error | $E_{(B-V)}$ | $A_{v}$ | Mean Error |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.10 | 0.30 | $\pm 006$ | 060 | 180 | $\pm 019$ |
| 20. | 060 | $\pm .08$ | 070 | 210 | $\pm 022$ |
| 30. | 090 | $\pm 11$ |  |  |  |
| 40. | 120 | $\pm 13$ | 115 | 3.45 | $\pm 035$ |
| 050. | 150 | $\pm 0.16$ |  |  |  |

TABLE 5
Summary of Errors

of the dashed-line relation holding for the single stars, the standard deviation of a point is 0.32 mag. Since we have estimated that the contributions to this made by distance dispersion and by reddening errors are about 0.15 and 0.10 mag., respectively, there is a mean error of 0.26 mag. left to be accounted for by all other sources, including unrecognized duplicity, cosmic scatter, etc.

The standard deviation of a point from the solid curve is 0.50 mag., and we may conclude that the inclusion of multiple stars therefore introduces an additional mean error of 0.39 mag .

When the data discussed here are used for the calibration of luminosities, it is necessary to recognize the inaccuracy in the distance. Bertiau estimates a relative probable error of 6.0 per cent in the mean parallax, which implies that the distance modulus, 6.18 mag., has a mean error of 0.19 mag. Thus the mean error in the value of $M_{v}$ derived from $(U-B)_{0}$ or $\beta$ for any given star is either 0.37 or 0.54 mag., depending on whether the calibration is derived from the relation for single stars alone or for stars including multiples. Table 5 summarizes the estimated errors in absolute magnitude according to the foregoing discussion.

## V. DISCUSSION OF LUMINOSITIES

It will be of interest to compare our results with those of other investigators, and for this purpose Table 6 has been arranged to contain the general calibration of Keenan and Morgan (1951), the Scorpius calibrations of Bertiau (1958) and Petrie (1958), and our present results (HC). Our data are for only those stars having MK classification, viz., the stars which appear in Figure 4. Results based on all these stars are written in roman type under the heading $a$, while the results based on only the single stars are written in italics under the heading $b$. The figures in parentheses denote the numbers of stars used for the corresponding calibration. Of the two sets of luminosities which Bertiau provides, we have selected those which are uncorrected for duplicity because his corrections are very uncertain.

The agreement between Bertiau's values and our $a$ values is good, as is to be expected, since we have used his spectral types. The large systematic difference at B2 is due largely to his inclusions of many stars from the older Centaurus region, while the slight differences at other spectral types is due to our improved reddening corrections which are

TABLE 6
Calibration of Absolute Magnitudes by Spectral Classes

| Spectral Type | MK | Bertiau | Petrie | HC $a$ | $\begin{gathered} \mathrm{HC} \\ b \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| B0 V | $-40$ | -4 0 (2) | -32 | -3 9 (2) | -3 5(1) |
| B1 V | -3 3 | -3 8 (4) | -26 | -3 6 (4) | -29 (1) |
| B2 V | $-27$ | -2 5 (16) | -1 5 | -2.1 (9) | $-18(7)$ |
| B3 V. | $-21$ | -17 (5) | $-10$ | -15 (6) | $-10(2)$ |
| B5 V | -14 | . . . |  | -0 8 (1) |  |
| B7 V | -1 1 | . | . | -0 3 (3) | -02 (2) |
| B8 V | -0 6 | . |  | -01 (3) | $-01(3)$ |
| B9 V | $-01$ |  | . | +04 (10) | +05 9 ) |

photometrically determined. The agreement between our $b$ values and Petrie's suggests that he has used single stars for this purpose. The work of Petrie is described only briefly in the article referred to, and details relating to the stars selected are lacking.

A more significant form of calibration than is obtained by grouping according to spectral type is available, however, viz., the calibration of $M_{v}$ as a function of a color index, such as $(U-B)_{0}$, and of the $\beta$ strength. Among the stars at hand, there is only one star more luminous than classes IV and V, so that it is unfortunately not possible to determine the relation in a form such as $M_{v}=f\left[(U-B)_{0}, \beta\right]$. Therefore, we shall determine $M_{v}$ as a function of $(U-B)_{0}$ and of $\beta$ independently for these stars, which presumably are of common age and composition.

For this purpose we have determined the relationships shown by the curves in Figures 3 and 5 by a least-squares method. Though this may appear unnecessarily elaborate, it is a trivial matter to do so on an IBM 650 computer. Accordingly, a curve-fitting program (IBM No. 6.0.006) was used which fits the best first-, second-, third-, and fourthdegree polynomials to the given data. The data were handled in two ways: one in which all stars (including the class III star) were included and one confined to those stars thought to be single. However, included among the latter were the several double stars whose companions are distinctly faint. In all cases the third- and fourth-degree solutions were barely better than those of second degree, as evidenced by only a slight diminution of the sum of the squares of the residualls. While we hạve no physical basis to expect any
particular mathematical form for the solutions, the curve-fitting process is an accepted form of analysis of empirical data. However, one is not justified in extrapolating the solutions beyond the range of the data used. With these qualifying comments then, we list the solutions as follows:

All stars:
$M_{v}=-154.98+40435 \beta+10723 \beta^{2}+3.7505 \beta^{3}-2.0220 \beta^{4} \pm 050$,
$M_{v}=1.49+6.65(U-B)_{0}+16.73(U-B)_{0}^{2}+25.64(U-B)_{0}^{3}+10.48(U-B)^{4}$

$$
\pm 0.50
$$

Single stars:
$M_{v}=-135.33+33.783 \beta+10.964 \beta^{2}+2.7316 \beta^{3}-1.7042 \beta^{4} \pm 0.32$,
$M_{v}=133+4.73(U-B)_{0}+10.45(U-B)_{0}^{2}+16.89(U-B)_{0}^{3}+6.96(U-B)_{0}^{4}$ $\pm 0.32$.

A more conventional form of expressing the results is to be found in Tables 7 and 8, which will be more useful for practical purposes. In these tables the headings $a$ and $b$ differentiate between the solutions based on all stars and single stars, respectively, as in Table 6. Also tabulated in Table 7 are mean zero-age values taken from Sandage (1957)

TABLE 7
Least-Squares Solution for $M_{v}$ as Function of $(U-B)_{0}$ For Upper Scorpius Stars

| $(U-B)_{0}$ | $M_{v}(a)$ | $M_{v}\left({ }^{(b)}\right.$ | $\begin{gathered} \text { SJ } \\ \text { Zero Age } \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| -100 | -36 | -29 |  |
| -0 90. | -27 | -22 | -21 |
| -0 80 | -20 | $-16$ | -16 |
| -0 70 | -12 | $-10$ | -10 |
| -0 60 | -07 | -0 5 | -0 5 |
| -0 50. | -02 | -01 | 00 |
| -0 40. | +01 | +02 | +03 |
| -0 30 | +04 | +04 | +08 |
| -0 20 | +06 | +06 | +10 |
| -0 10 | +09 | +09 | +13 |

TABLE 8
Least-Squares Solution for $M_{v}$ as Function of $\beta$ for Upper Scorpius Stars

| $\beta$ | $M_{v}(a)$ | $M_{v}(b)$ | $\beta$ | $M_{v}(a)$ | $M_{v}(b)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2600 | -39 | $-32$ | 2750 | $-04$ | -02 |
| 2625 | $-32$ | $-26$ | 2775 | 00 | +02 |
| 2650 | -2 4 | $-20$ | 2800 | +04 | +04 |
| 2675 | -18 | $-15$ | 2825 | +06 | +06 |
| 2700 | -13 | $-10$ | 2850 | +08 | +08 |
| 2725. | $-08$ | $-0.6$ | 2875 | $+09$ | +0.9 |

and Johnson (1957) for comparison purposes. It will be seen that, for the earlier stars, the zero-age calibration is in substantial agreement with our single-star values, but a systematic difference exists among the later B stars, in the sense that our results for the Scorpius stars are brighter than the zero-age values.

The reasons for this systematic difference are not evident at the present time but would appear to justify further work, particularly aimed at including fainter stars. Since our faint stars are not too near the limiting magnitude of the HD catalogue, we have no reason to expect that some selection effect has caused us to include only B8 and B9 stars brighter than typical because of multiplicity, for example. Nor have we at the present time any reason to suspect that the B8 and B9 stars are spatially distributed

TABLE 9
Mean Relation between $M_{v},(U-B)_{0},(B-V)_{0}$, and $\beta$ For Upper Scorpius Stars

| $M_{v}$ | $(U-B)_{0}$ | $(B-V)_{0}$ | $\beta$ |
| :---: | :---: | :---: | :---: |
| -3 5. | $\{-099$ | -0 27 | $\begin{aligned} & 2613 \\ & 2590 \end{aligned}$ |
| $-30$ | $\begin{cases}-0 & 94 \\ -1 & 02\end{cases}$ | -.25 $-\quad 28$ | $\begin{aligned} & 2630 \\ & 2619 \end{aligned}$ |
| -25 | $\begin{cases}-0 & 87 \\ -0 & 94\end{cases}$ | $-\quad 24$ $-\quad 26$ | 2648 2629 |
| $-20$ | $\begin{cases}-0 & 81 \\ -0 & 87\end{cases}$ | $-\quad 22$ $-\quad 24$ | 2668 2651 |
| -15. | $\begin{cases}-0 & 74 \\ -0 & 79\end{cases}$ | - 20 $-\quad 21$ | 2689 2673 |
| -10. | $\left\{\begin{array}{rl}-0 & 66 \\ -0 & 70\end{array}\right.$ | -18 -19 | $\begin{aligned} & 2713 \\ & 2690 \end{aligned}$ |
| -05. | $\begin{cases}-0 & 57 \\ -0 & 60\end{cases}$ | -15 -16 | $\begin{aligned} & 2741 \\ & 2728 \end{aligned}$ |
| 00 | $\begin{cases}-0 & 44 \\ -0 & 47\end{cases}$ | -12 -13 | $\begin{aligned} & 2772 \\ & 2761 \end{aligned}$ |
| +05 | $\left\{\begin{array}{lll}-0 & 27 \\ -0 & 27\end{array}\right.$ | $\begin{aligned} & -\quad 07 \\ & -0 \quad 07 \end{aligned}$ | $\begin{aligned} & 2814 \\ & 2808 \end{aligned}$ |

closer to us than the earlier types. Until further data are on hand, it would be premature to discuss this matter more fully, and we therefore leave the subject with a final compilation of our data in Table 9. Here, a summary of the intrinsic parameters $M_{v},(U-B)_{0}$, ( $B-V)_{0}$, and $\beta$ for the Upper Scorpius B stars are listed, the values in roman type being based on all stars, those in italics on single stars only.

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