PHOTOELECTRIC SPECTROPHOTOMETRY OF A-TYPE STARS

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ABSTRACT

The physical characteristics of a variety of A-type stars have been determined by comparing observations of the energy distributions in the continuous spectra of these stars with the predictions of model atmospheres. The observations were obtained with a photoelectric spectrum scanner at the Newtonian focus of the Curtis Schmidt of the University of Michigan and consisted of approximately ten scans over the wave-length region λλ 3400–6200 A of each of the eighteen program stars. The resolution was 19 A for the brighter stars and 47 A for the fainter stars. The observed energy distributions were corrected for the effects of differential atmospheric extinction and calibrated on the absolute system of Whitford and Code by means of supplementary observations of two standard stars.

The observations were compared with the predicted fluxes of model atmospheres computed by Hunger and by Osawa. From this comparison the effective temperature and surface gravity of each of the program stars were derived. With these data, the radii and masses of the stars were then determined. The values so obtained indicate that reliable masses of single, normal A-type stars may be determined in this manner. An effective temperature scale for the A-type stars is given. This scale is about 1000° cooler at A0 than that given by Kuiper but seems to approach it at about F0. Evidence is given which suggests that the continua of some of the peculiar stars studied show the effects of rapid rotation and of a moderately strong magnetic field. The masses of some metallic-line A-type stars are perhaps more appropriate to F stars. Finally, a brief discussion of evolutionary effects is given.

I. INTRODUCTION

In the past, nearly all determinations of the physical characteristics of stellar atmospheres have been made by analyzing the line spectra of stars. With the development of photoelectric spectrum-scanning devices, however, the difficulties inherent in photographic photometry of the continuum are largely eliminated. It is now feasible to attempt an analysis of the atmospheres of stars by comparing the observed continua with the predictions of model atmospheres. In this paper the results of such an attempt for a group of eighteen A-type stars are given.

The stars chosen for this work and the considerations leading to their choice are given in Section II. The next two sections describe, respectively, the equipment and technique by which the observational material tabulated in this section was obtained and the model atmosphere computations with which the observations are compared. This comparison enables the effective temperatures, surface gravities, radii, and masses of the program stars to be determined. These results are given and discussed in detail in Section V. The paper closes with a summary of the conclusions.

II. THE PROGRAM STARS

Several considerations led to the choice of A-type stars for this study. Their spectra show relatively few absorption lines, so that their continua may be reliably located, and they exist in moderately large numbers in nearby open clusters. Membership in such clusters enables the absolute magnitudes of these stars to be fairly accurately determined. Recently, Osawa (1956) and Hunger (1955) have computed a series of model atmospheres of stars from type A0 to A9 which can serve as the theoretical material with which to compare the observations.

All the color-magnitude data for galactic clusters and associations were examined, and eighteen A stars in five open clusters and one association were selected. The charac-

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teristics of the clusters are given in Table 1, and of the individual stars in Table 2. Most of the columns are self-explanatory. The column headed "E(Sp)" in Table 2 gives the color excess determined by comparing the observed $(B - V)$ index with the intrinsic $(B - V)$ colors for each MK spectral type (Johnson and Morgan 1953; Morgan, Harris, and Johnson 1953). The so-called Q-method of determining color excesses (Johnson and Morgan 1953) is not applicable to most A stars, since Q itself is not well defined for stars later than A0. Only the color excesses for the Pleiades and for the I Persei stars are believed to be real for two reasons: (1) consideration of the four other clusters as a whole indicates that they are not appreciably reddened, and (2) the relation between intrinsic color and MK spectral class is not unique for the A stars, and consequently color excesses determined in the above manner for individual stars may not be entirely reliable.

TABLE 1

THE CLUSTER DATA

Cluster	α	δ	$m-M$	References
I Persei	03^h22^m	+49°45′	6.0	Roman and Morgan (1950); Harris (1956)
Pleiades	03 44	+23 58	5.5	Johnson and Morgan (1953); Mendoza (1956)
M39	21 30	+48 13	7.0	Johnson (1953); Weaver (1953)
Coma Berenices	12 22	+26 23	4.5	Weaver (1952); Johnson and Knuckles (1955)
Hyades	04 24	+15 45	3.0	Van Bueren (1952); Johnson and Knuckles (1955)
Praesepe	08 37	+20 10	6.0	Johnson (1952); Bidelman (1956)

TABLE 2

THE STAR DATA

Cluster	HD or BD No.	Star No.	V	$(B-V)$	$(U-B)$	Sp.	E(Sp)	M_v	Remarks
I Persei	21375	....	7^m47	+0.11	+0.03	A1 V	+0.08	+1 2	
	21479	....	7.28	+ .10	+ .01	A2 V	+ .04	+1.2	
Pleiades	23194	43	8.05	+ .197	+ .05	A5 V	+ .05	+2.4	
	23489	341	7.34	+ .097	+ .12	A2 V	+ .04	+1.7	
	23628	513	7.64	+ .206	+ .12	A4 V	+ .09	+1.8	
	23763	742	6.94	+ .125	+ .09	A1 V	+ .09	+1.1	
M39	....	1	7.35	− .02	+ .01	A0 V	− .02	+0.4	Marked rot. vel.
	....	5	7.85	+ .04	− .01	A0 V	+ .04	+0.8	Spectrum resembles 21 Comae
Coma Berenices	+28°2084	10	6.04	+ .114	+ .106	A4 V	− .01	+1.5	Var. rad. vel.
	+29 2280	68	6.67	+ .177	+ .094	A4	+ .06	+2.2	Met. lines; rot.
	+26 2352	139	6.76	+ .163	+ .096	A4	+ .04	+2.3	Met. lines
	+25 2517	160	5.46	+ .049	+ .104	A3 p	− .04	+1.0	21 Comae; Sr II star; spect. var.
Hyades	27819	47	4.80	+ .158	+ .121	A5 V	+ .01	+1.8	
	27962	56	4.30	+ .049	+ .055	A2 V	− .01	+1.3	Mag. field
	28319	72	3.41	+ .179	+ .132	F0 V	− .12	+0.4	θ^2 Tau
	29388	104	4.27	+ .123	+ .113	A3 V	+ .03	+1.3	c Tau; var. rad. vel.
Praesepe	73711	276	7.54	+ .160	+ .131	A3	+ .07	+1.5	Met. lines
	73819	348	6.78	+0.168	+0.140	A6	+0.00	+0.8	Lines diffuse

The absolute magnitudes of the program stars were determined from the distance moduli given by Johnson (1957) and from the V magnitudes and color excesses (of only the I Persei and Pleiades stars) of Table 2. The ratio of total to selective absorption was taken equal to 3.0 (Hiltner and Johnson 1956). It should be noted that the program stars include many peculiar objects: metallic-line stars (BD+29°2280, +26°2352, HD 73711), Sr II stars (BD+25°2517; No. 5, M39), and a magnetic star (HD 27962).

III. INSTRUMENTS AND OBSERVATIONS

The observations on which this work is based were obtained with a photoelectric spectrum scanner at the Newtonian focus of the 24-inch Curtis Schmidt of the University of Michigan. The spectrum scanner was designed and constructed by Liller (1957), who has described it in detail. Accordingly, only a brief description of it will be given here. The scanner, when used with the Curtis Schmidt, is capable of detecting radiation in the range λλ 3400–6200 of stars of 8.0 mag. and brighter. The dispersing element is a 2 × 2-inch Bausch and Lomb replica grating having 600 lines per millimeter. The dispersion is 32 A/mm in the second order. The wave length region λλ 3400–5600 is observed in the second order, while the region λλ 5800–6100 is observed in the first order. Overlapping orders are eliminated by means of a yellow filter. The grating rotates at such a rate as to give a scanning speed of 270 A/minute in the second order. The Hyades stars and the standard stars (α Lyrae, α Andromedae, η Ursae Majoris) are sufficiently bright to allow the use of a 19-A exit slot. However, it was necessary to use a 47-A exit slot in order to obtain adequate deflections from the fainter stars.

The radiation detector is a twenty-stage Lallemand photomultiplier refrigerated with dry ice. After the signal from the photomultiplier is amplified by a d.c. amplifier of conventional design, it is recorded by a Brown recording potentiometer. The observing schedule on a given night consisted of obtaining spectral scans over the wave-length region λλ 3400–6100 for the program stars and for the extinction star, i.e., the star used to evaluate the effects of differential atmospheric absorption. Generally, three scans were obtained of a program star on a given night, while ten to twenty traces of the extinction star were made through air masses of 1.0 to about 2.1–2.5. During the spring, 1957, series of observations of stars in the Coma cluster, a program star itself (No. 160) served as the extinction star. In the fall, 1957, series, however, α Andromedae (B9p) was used as the extinction star. Since this star is a spectrum variable, it was perhaps an unfortunate choice. However, no light-variations greater than 0.01 mag. have been reported for it. In the spring series, scans were also made of η Ursae Majoris and, in the fall series, of α Lyrae. These auxiliary observations enabled the program stars to be calibrated on the Whitford-Code system. These workers, using equipment and techniques similar to those described above, have determined "monochromatic" magnitudes of forty-two stars of a wide range of spectral and luminosity types. They calibrated all their stars relative to α Lyrae, using their own absolute measures as well as those of Williams (1938), Kienle, Wempe, and Beileke (1940), and Chalonge and Divan (1952) to calibrate Vega. Thus, by comparing a program star with any Whitford-Code star, one may eliminate the effects of the photocathode spectral sensitivity, the transmission of the optics, etc., and calibrate the star on an "absolute" basis.

In the wave-length region considered in this work, Whitford and Code give magnitudes at ten wave lengths, viz., λλ 3400, 3650, 3860, 4040, 4190, 4590, 5060, 5560, 5810, and 6050. These were so chosen that, with the 15-A slot width used by Whitford and Code, no absorption lines with equivalent widths greater than 0.1 A are included. With the exception of the measurement at λ 3860, this is still the case for A stars when the slot width is 47 A, as in most of our measurements. The deflections at λ 3400 are very small and consequently inaccurate, largely because the Schmidt correcting plate transmits only 40 per cent of the light at this wave length. Furthermore, they seem to be systematically too large, compared with the model-atmosphere predictions, suggesting that first-order red radiation may have contaminated the small λ 3400 deflections. Because of this possibility, no use is made of these measures in the analysis. The deflections at λ 6050 are also small because the A stars do not radiate strongly in the red and because the photomultiplier sensitivity is falling off rapidly here. At λ 5560, the observed continuum may include a contribution from the strong airglow and auroral line at λ 5577 [O I]. The measures at the other standard wave lengths are believed to be free from any serious complicating effects.

The positions of the standard wave lengths were established accurately by reference to the hydrogen lines. The magnitude measures at these wave lengths were normalized to $m(\lambda\,4190) = 0.00$ mag. This procedure reduces possible difficulties caused by changes in the amplifier sensitivity or sky transparency as long as these changes are small during a given trace. The deflection at λ 4190, usually the largest, was chosen as the reference deflection for convenience. For each night a least-squares solution was made for the extinction coefficients, and with these coefficients the magnitudes of the program stars at the various standard wave lengths were reduced to outside the atmosphere in the usual manner. The deflections for a given star at each wave length were then averaged and reduced to the Whitford-Code system by means of the corrections derived from observation of the standard stars. Finally, the results for each night were averaged. The energy distributions for the program stars are given in Table 3, along with the mean errors for the magnitudes at the standard wave lengths. In addition, the energy distribution of α Lyrae adopted by Whitford and Code is given. The errors are completely accounted for by the uncertainties in the extinction coefficients, which in turn are probably largely produced by small variations in the sky transparency. The column headed "n" gives the number of nights on which a star was observed.

The observations of the two stars in Perseus and the four stars in the Pleiades must be corrected for the effect of interstellar reddening. The wave-length dependence of the reddening between λ 3500 and λ 10000 is $\lambda^{-0.6}$, according to Greenstein (1951). Whitford's (1958) recent revision of the reddening law does not alter our results obtained with his older measurements. A table of the ratio of observed (reddened) intensities to true (unreddened) intensities at the standard wave lengths was computed for a range of densities in the reddening particles. A guess was then made as to the amount of reddening affecting a given energy distribution, and this distribution was "unreddened" by the appropriate amount, using the table just mentioned. The $(B - V)$ color of the resulting energy distribution was then computed and compared with what was believed to be the star's intrinsic color. This procedure was repeated until a match was obtained, the energy distribution then being adjusted by the indicated amount. The corrections for reddening to be added to the observed energy-curves are given in Table 4.

The relation between $(B - V)$ color and spectral class for A-type stars is not unique; there is a range of several hundredths of a magnitude in the intrinsic color index for a given spectral type. Within this range, that true color was taken which gave the most consistent results in the derived temperature. Thus not all the color excesses given in Table 4 agree with the excesses corrected for in Table 2. Finally, smooth curves were drawn which best fit all the observed points. These smoothed energy distributions were used in the subsequent analysis.

An important observational parameter is the Balmer discontinuity, which cannot be determined directly from a spectral scan. The λ 3650 measurement gives us at once the intensity of the continuum to the short-wave-length side of the Balmer limit (λ 3650^-), but, because of the overlapping of the hydrogen lines, the continuum at λ 3650^+ is not seen. One could simply extrapolate the observed continuum redward of the Balmer limit to λ 3650^+, but this involves an undesirable element of subjectivity. Instead, the value of the deflection at λ 3650^+ has been computed by assuming that the true energy distribution from this wave length to λ 4190 and λ 4590 may be represented by one Planckian curve. The Balmer discontinuities determined in this manner are given

TABLE 3

Observed Monochromatic Magnitudes

Cluster	Star No.	λ 3400	λ 3650	λ 3860	λ 4040	λ 4590	λ 5060	λ 5560	λ 5810	λ 6050	*n*
I Persei....	21375	$+0^{m}36\pm0.04$	$+0^{m}95\pm0.06$	$+0^{m}14\pm0.03$	$+0^{m}08\pm0.03$	$+0^{m}21\pm0.01$	$+0^{m}46\pm0.04$	$+0^{m}57\pm0.02$	$+0^{m}95\pm0.01$	$+0^{m}99\pm0.03$	1
	21479	+0.51 .15	+0.81 .03	+ .03 .04	+ .08 .01	.19 .02	+ .45 .05	+ .57 .05	+0.80 .03	+0.89 .01	3
Pleiades....	43	+0.43 .08	+0.93 .05	+ .23 .07	+ .02 .02	.19 .02	+ .34 .05	+ .50 .03	+0.64 .03	+0.74 .05	3
	341	+0.51 .04	+0.93 .02	+ .16 .03	+ .08 .02	.22 .02	+ .48 .01	+ .61 .03	+0.82 .04	+0.90 .04	3
	513	+0.57 .08	+0.84 .02	+ .19 .02	+ .01 .01	.18 .02	+ .35 .02	+ .46 .01	+0.62 .06	+0.67 .04	3
	742	+0.53 .03	+0.93 .05	+ .20 .03	+ .06 .01	.21 .02	+ .43 .04	+ .59 .04	+0.77 .02	+0.82 .07	4
M39.......	1	+0.64 .04	+0.85 .03	+ .01 .02	+ .10 .02	.26 .01	+ .52 .03	+ .72 .04	+0.91 .03	+1.00 .02	4
	5	+0.62 .06	+0.81 .03	+ .01 .03	+ .12 .02	.24 .01	+ .50 .03	+ .64 .03	+0.80 .03	+0.88 .05	3
Coma Berenices	10	+0.66 .06	+1.02 .08	+ .30 .06	+ .06 .01	.15 .03	+ .39 .06	+ .60 .01	+0.74 .03	+0.90 .01	2
	68	+0.63 .15	+0.98 .05	+ .36 .05	+ .03 .01	.12 .01	+ .41 .01	+ .56 .09	+0.72 .01	+0.79 .03	2
	139	+0.54 .02	+0.98 .08	+ .40 .07	+ .03 .01	.11 .01	+ .33 .01	+ .48 .08	+0.66 .02	+0.78 .06	2
	160	+0.57	+1.04	+ .29	+ .07	.16	+ .43	+ .67	+0.81	+ .92	2, See text
Hyades....	47	+0.64 .04	+0.88 .02	+ .06 .02	+ .05 .01	.16 .01	+ .38 .01	+ .54 .01	+0.70 .04	+0.81 .02	3
	56	+0.66 .07	+0.89 .04	− .01 .01	+ .07 .01	.23 .02	+ .50 .04	+ .73 .04	+0.88 .05	+1.01 .07	3
	72	+0.57 .01	+0.90 .02	+ .04 .03	+ .04 .02	.14 .02	+ .36 .01	+ .53 .02	+0.68 .02	+0.79 .01	3
	104	+0.65 .08	+0.93 .02	+ .04 .01	+ .06 .01	.17 .03	+ .42 .02	+ .60 .03	+0.76 .04	+0.84 .04	4
Praesepe...	276	+1.04 .10	+0.93 .05	+ .20 .07	+ .04 .04	.15 .02	+ .37 .02	+ .54 .03	+0.70 .04	+0.67 .05	1
	348	+0.72 .06	+0.96 .04	+ .19 .03	+ .04 .03	.13 .02	+ .34 .05	+ .49 .05	+0.56 .02	+0.69 .06	1
	α And	+0.09 0.03	+0.24 0.02	− .20 0.01	+ .10 0.01	.26 0.02	+ .58 0.02	+ .82 0.01	+1.03 0.04	+1.16 0.06	4
	α Lyr	+0.88	+0.94	−0.05	−0.09	+0.25	+0.55	+0.81	+0.97	+1.12	See text

in Table 5. Determinations of the discontinuity for six of these stars have been made by Westerlund (1956) and are given in the table. The agreement between the two sets is satisfactory.

IV. THE MODEL ATMOSPHERES

With the assumptions customarily made, a model stellar atmosphere is completely specified by three parameters: the effective temperature, the surface gravity, and the chemical composition. The temperatures encountered in A-star atmospheres are too

TABLE 4

CORRECTIONS FOR INTERSTELLAR REDDENING TO BE ADDED TO OBSERVED ENERGY-CURVES

λ	I PERSEI		PLEIADES			
	HD 21375 $m(E=0.10)$	HD 21479 $m(E=0.04)$	No. 43 $m(E=0.08)$	No. 341 $m(E=0.04)$	No. 513 $m(E=0.08)$	No. 742 $m(E=0.06)$
3400	−0.10	−0.04	−0.10	−0.04	−0.08	−0.07
3650	− .06	− .03	− .06	− .03	− .05	− .05
3860	− .04	− .02	− .04	− .02	− .03	− .03
4040	− .02	− .01	− .02	− .01	− .01	− .01
4190	.00	.00	.00	.00	.00	.00
4590	+ .04	+ .02	+ .04	+ .02	+ .04	+ .03
5060	+ .08	+ .03	+ .08	+ .03	+ .07	+ .06
5560	+ .12	+ .05	+ .12	+ .05	+ .10	+ .09
5810	+ .14	+ .06	+ .14	+ .06	+ .12	+ .10
6050	+0.15	+0.06	+0.15	+0.06	+0 13	+0 11

TABLE 5

THE BALMER DISCONTINUITIES

Cluster	Star No.	Mag.	Mag. (Westerlund)
I Persei	21375	1.24	....
	21479	1.06	....
Pleiades	43	1.18	....
	341	1.23	....
	513	1.07	....
	742	1.23	....
M39	1	1.22	1.18
	5	1.15	1.17
Coma Berenices	10	1.19	1.19
	68	1.12	1.12
	139	1.10	1.10
	160	1.30	1.30
Hyades	47	1.07	....
	56	1.21	....
	72	1.09	....
	104	1.14	....
Praesepe	276	1.11	....
	348	1.12	....

low for helium to contribute to the opacity but are sufficiently high that the free electrons come largely from hydrogen. Therefore, large variations in the abundances of the elements will have little effect on the continuous spectrum, and an A-star model atmosphere is essentially specified by only two parameters—the effective temperature and the surface gravity.

Osawa (1956) computed a set of six model atmospheres appropriate to the A stars. His models are characterized by log $g = 3.5$, 4.0, and 4.5, each with effective temperatures of 7560° and 8900°. He took as the ratio by number of the hydrogen abundance to the metal abundance, A, the value log $A = 3.8$. These models contain no helium. The contributions to the opacity of H, H^-, and H_2^+ were included. Flux constancy of ± 1 per cent was attained.

Hunger (1955) computed four model atmospheres in the A-star region. Since he was attempting to represent the continuum of α Lyrae, he computed four models with what he believed to be the surface gravity of Vega, log $g = 4.3$, and for effective temperatures 8160°, 8660°, 9000°, and 9500°. Hunger's models differ from Osawa's in one important

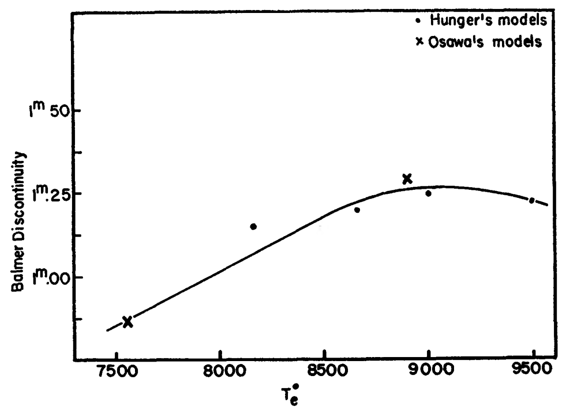

FIG. 1.—The Balmer discontinuities predicted by the Hunger and Osawa models for log $g = 4.3$

respect. In his first approximation, Hunger used the Rosseland mean absorption coefficient computed by Vitense (1951). Vitense assumed the composition of the stellar matter to be 85 per cent hydrogen and 15 per cent helium, and the only source of opacity important for A stars she considered to be due to H and H^-. She did not take into account the effect of the H_2^+ molecule on the absorption coefficients, nor did Hunger in his second approximation. Hunger also attained a flux constancy of ± 1 per cent in his models.

Figure 1 gives the Balmer discontinuities as a function of effective temperature for Hunger's models and for Osawa's models interpolated to log $g = 4.3$. The mean curve given fits all the points quite well with the exception of the point at $T_e = 8100°$. This point—the Balmer discontinuity for Hunger's lowest temperature model—lies about 0.05 mag. above the mean curve. The justification for not fitting the curve to this point is based on the effect of the H_2^+ molecule on the absorption coefficient. Osawa computed a model with $T_e = 7560°$ and log $g = 4.0$, in which he neglected the absorption by H_2^+. This model has a Balmer discontinuity 0.05 mag. larger than that in the corresponding model, which includes the absorption by H_2^+. Osawa states that in this temperature range the absorption by this molecule amounts to about 10 per cent of that due to H and H^-. According to Bates (1952), the continuous absorption coefficient of H_2^+ at the Balmer limit decreases by a factor of about 3 as the temperature is raised from 7000° to 10000°. Furthermore, the increasing dissociation of H_2^+ as the temperature increases causes the temperature dependence of the absorption to be even stronger, so that the Balmer discontinuities of the higher-temperature models should be comparable. Qualitatively, then, one can account for what seems to be a discrepancy in the size of the Balmer discontinuity of Hunger's low-temperature model.

Ideally, the comparison between the computed fluxes and the observations would be carried out by means of sets of magnitude differences at two wave lengths in the continua of the model atmospheres, which are primarily sensitive either to the effective temperature or to the surface gravity. This is possible in the former case, but criteria sensitive only to the surface gravity do not appear to exist. Since the energy distribution in the continuum of an A star to the red of the Balmer limit is essentially Planckian, temperature-sensitive criteria can be found easily. Three sets of magnitude differences were obtained from Osawa's models: $m(\lambda\ 4600) - m(\lambda\ 4190)$; $m(\lambda\ 5600) - m(\lambda\ 4400)$; and $m(\lambda\ 6000) - m(\lambda\ 5000)$. These were chosen so as to utilize as much as possible

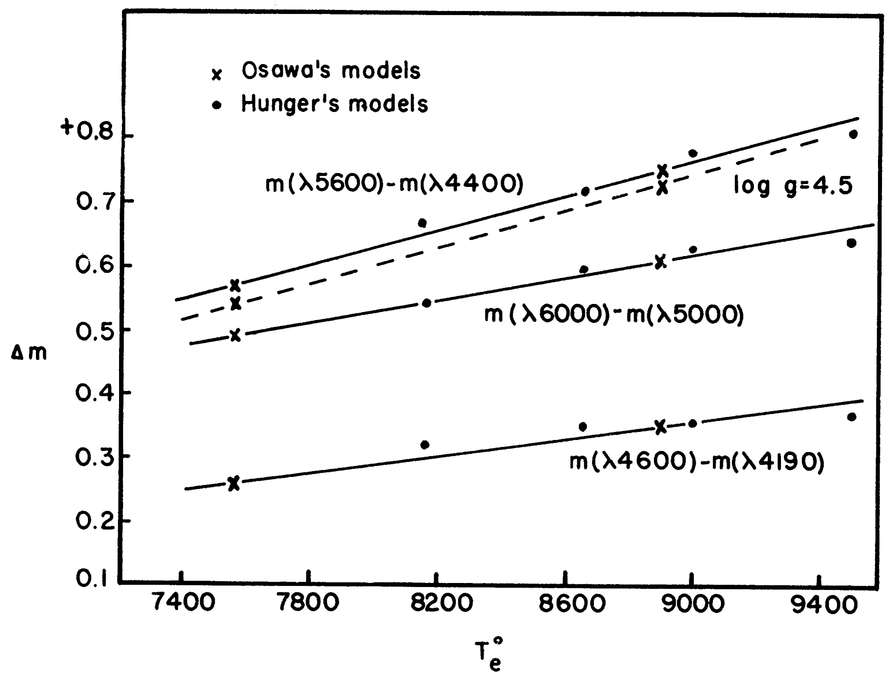

FIG. 2.—Temperature criteria derived from Osawa's log $g = 4.0$ models and from Hunger's log $g = 4.3$ models.

of the visual spectrum to the red of the Balmer discontinuity. Figure 2 shows graphs of these magnitude differences as a function of the effective temperature. In all cases the values of these criteria were found from Osawa's models for log $g = 4.0$. The maximum effect of the surface gravity on these magnitude differences occurs for the $m(\lambda\ 5600) - m(\lambda\ 4400)$ criterion, which is illustrated by the dashed line in the figure. The effect of surface gravity is seen to be small. Also given in this diagram are the corresponding values of these magnitude differences derived from Hunger's models.

Temperature-independent criteria of surface gravity are not so readily found. The Balmer discontinuity itself is quite sensitive to the surface gravity, but it is also sensitive to the temperature, as is shown in Figure 3. Once the temperature is known, however, the discontinuity can be used as a surface-gravity criterion. The interpolation between 7560° and 8900° and the extrapolation to 9500° have been made on the assumption that the relation between the Balmer discontinuity and T_e for log $g = 4.3$, given in Figure 1, is valid for log $g = 3.5$–4.5. Figure 3 shows another surface-gravity criterion, Bal.

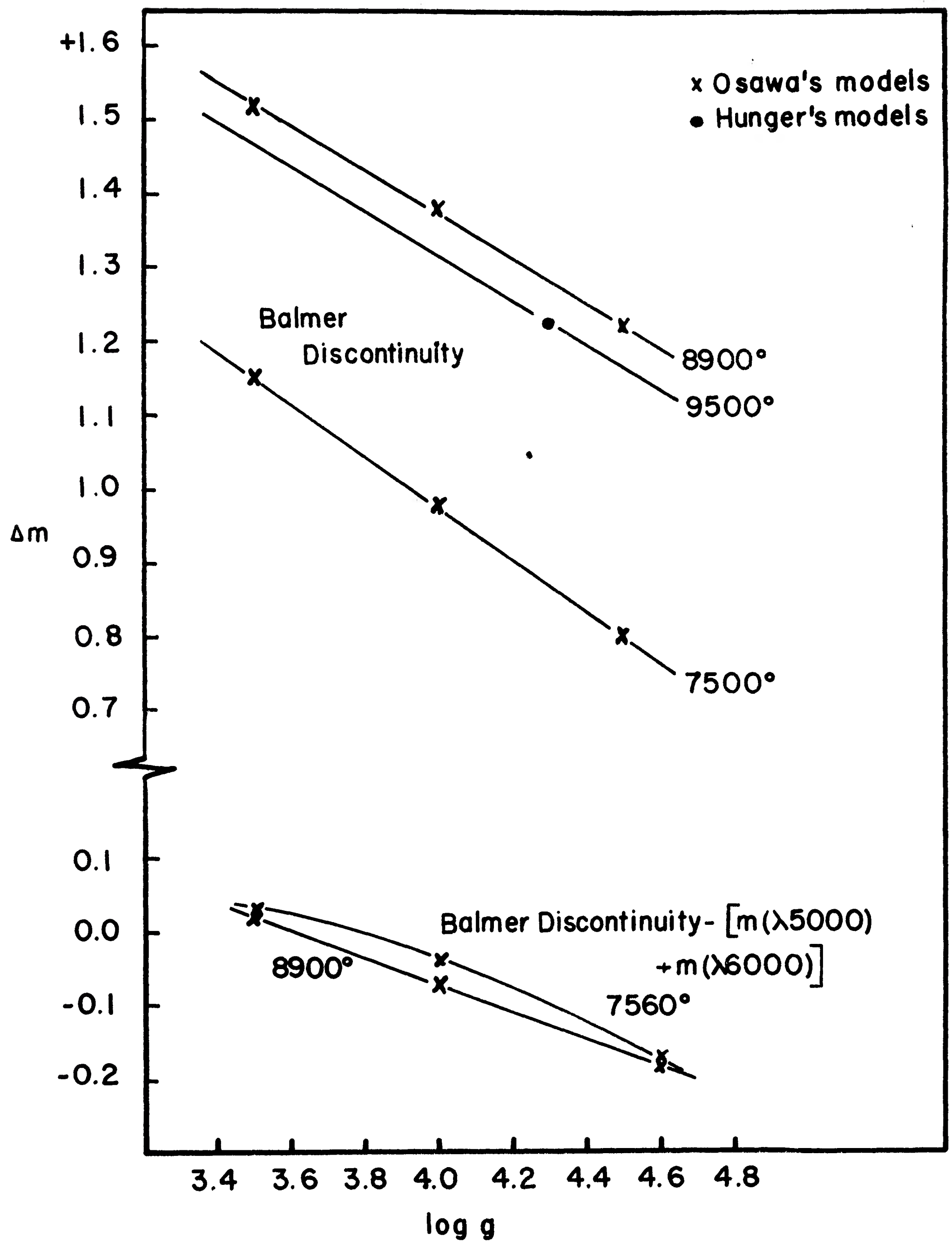

FIG. 3.—Surface-gravity criteria from Osawa's and Hunger's models

Disc. $-[m(\lambda\ 5000) + m(\lambda\ 6000)]$. This quantity is rather insensitive to the temperature, but neither is it so sensitive to the surface gravity as the Balmer discontinuity. However, it will be used as an auxiliary criterion within the limits defined by Osawa's models.

V. THE RESULTS

a) The Temperatures, Surface Gravities, Radii, and Masses

Table 6 gives the values of T_e and of log g derived as described above for all the program stars and for α Lyrae; T_e is the average of the temperature given by the three criteria. In deriving the value of log g, twice the weight was given to the value derived from the Balmer discontinuity alone than that found from the second criterion. No systematic effects depending on the particular criterion used were found for either the temperatures or the surface gravities.

TABLE 6

DERIVED TEMPERATURES, SURFACE GRAVITIES, RADII, AND MASSES OF PROGRAM STARS

Cluster	Star No.	Sp.	$\langle T^{\circ}\rangle$	$R_*/R_\odot$	$\langle \log g\rangle$	$(\log g)_{\mathfrak{M}-L}$	$\mathfrak{M}_*/\mathfrak{M}_\odot$
I Persei	21375	A1 V	9500	2.2	4.25	4.15	3.1
	21479	A2 V	8700	2.5	4.8	4.00	....
Pleiades	43	A5 V	8450	1.6	4.25	4.30	1.7
	341	A2 V	8600	2.0	4.25	4.15	2.7
	513	A4 V	8000	2.0	4.15	4.10	2.2
	742	A1 V	8650	2 5	4 20	4.00	3.6
M39	1	A0 V	8900	3 5	4.50	3.80	....
	5	A0 V	8500	1.3	4.40	3.90	....
Coma Berenices	10	A4 V	8200	2.4	3.85	4.05	1.4
	68	A4	8000	1.8	4.00	4.20	1.1
	139	A4	7850	1.7	3.85	4.20	0.8
	160	A3 p	8550	2 8	3.90	3.95	2 2
Hyades	47	A5 V	7900	2 2	4.05	4.05	1.9
	56	A2 V	8900	2.3	4.50	4.10	....
	72	F0 V	7700	4.4	3 70	3.60	3.3
	104	A3 V	8150	2.6	4 00	3.95	2.5
Praesepe	276	A3	7900	2.5	3.90	4.00	1.8
	348	A6	7300	3.9	3.40	3.65	1.4
	α Lyr	A0 V	9400	3.2	4.00	3.90	3.6

The mass-luminosity relation, though of questionable validity for an individual star, provides a convenient standard against which to compare the derived surface gravities. We have adopted the relation $M_{\rm bol} = 4.73 - 9.5 \log \mathfrak{M}$ obtained by using for the absolute visual magnitude of the sun the value $M_v = 4.84$, given by Stebbins and Kron (1957), and the bolometric correction of -0.11 mag. given by Kuiper (1938). The coefficient of log $\mathfrak{M}$ is the average value obtained from the determinations by Russell and Moore (1940), van de Kamp (1954), Eggen (1956), and Strand (1957).

Now $g \sim \mathfrak{M}/R^2$ and $L \sim R^2 T_e^4$. Thus, for two stars, $\Delta \log g = 0.3\Delta M_{\rm bol} + 4.\Delta \log T_e$. Our standard star will be the sun, for which Goldberg (1953) gives log $g = 4.44$ and log $T_e = 3.76$. The above equation then becomes

$$\log g\,(\text{star}) = 4.44 - 0.30\,[4.73 - M_{\rm bol}\,(\text{star})\,] - 4\,[3.76 - \log T_e\,(\text{star})\,]\,.$$

Absolute magnitudes were obtained from the data in Tables 1 and 2. The bolometric corrections used were the mean values defined by Kuiper's (1938), Osawa's (1956), and Hunger's (1955) values. These corrections are given by Popper (1959) and so will not be listed here. Since the zero point of the scale of bolometric corrections is set at F0, the corrections for the A stars are relatively small. Furthermore, an error in the correction of 0.1 mag. leads to an error of only 0.03 in log g.

The surface gravities calculated with the above information are given in Table 6, in the column headed "log $g_{\mathfrak{M}-L}$." A comparison of the surface gravities derived from the continuum observations and from the mass-luminosity relation is most easily made by means of Figure 4. The four stars showing the largest discrepancies between the surface gravities computed by the two methods display various peculiarities. These stars will be discussed in a later section.

The masses of the program stars are given in Table 6. These values were derived from the relation $M/M_\odot = gLT_\odot^4/g_\odot L_\odot T^4$, using the relevant values given above.

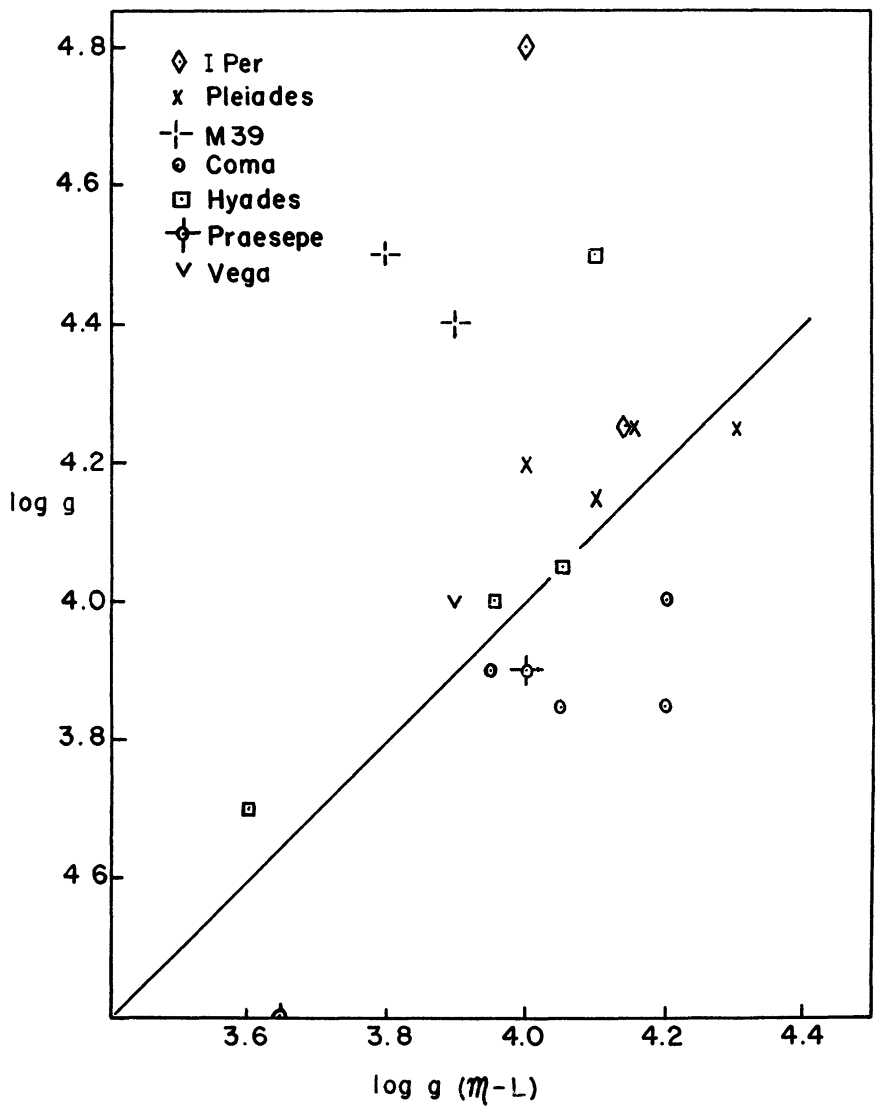

FIG. 4.—Derived log g versus log g ($\mathfrak{M} - L$)

No masses are given for the four stars mentioned in the preceding paragraph. Evidence will be given later which indicates that their Balmer discontinuities may be modified by factors neglected in the model-atmosphere calculations, and therefore surface gravities so derived are not physically meaningful. From the equation for the luminosity of a star, $L = 4\pi R^2 \sigma T_e^4$, the stellar radius may easily be found in terms of that of the sun: $\log R/R_\odot = 8.47 - 0.2M_{\rm bol} - 2 \log T_e$. The radii of the program stars found in this manner are also given in Table 6.

It is difficult to estimate the uncertainty of the derived effective temperatures, surface gravities, masses, and radii. However, errors of ± 0.20 in $\log g$ and of $\pm 400°$ in temperature for the normal stars (i.e., stars which do not violate the restrictions under which model atmospheres are computed) seem likely. The uncertainty in the mass is probably about ± 0.2 in the logarithm, since the temperature enters only logarithmically and the bolometric magnitudes are multiplied by 0.3 in the surface-gravity determinations and by 0.4 in the mass determinations. The errors in the radii are estimated to be ± 0.1 in the logarithm. Errors in the distance moduli, including the effect of cluster depth, will not be very important in our results, with the possible exception of stars in the Coma Berenices cluster. Weaver (1952) estimates that the uncertainty in the absolute magnitude of these stars due to cluster depth alone may be as large as 0.3.

In addition to the random errors of observation mentioned above, systematic errors may be introduced because of inadequacies of the model atmospheres and because of second-order observational effects. In the first category, line absorption, particularly absorption by the Lyman lines, probably affects the temperature distribution and hence the emergent flux in the atmosphere of an A star. At present, however, this problem is only recognized, not solved. This effect would modify the observed energy distribution so that a spuriously high effective temperature would be inferred. In this same category may be included the effects of convection. A detailed discussion by Spiegel (1958) indicates that convection may have an important effect on the continuum of A stars. Spiegel investigated Osawa's 8900° models and found that the atmosphere is unstable against convection from about $\tau = 0.1$ to about $\tau = 12$. Furthermore, layers just below $\tau = 12$ do not rapidly become subadiabatic. One would therefore expect a convection zone of large vertical extent, both geometrically and optically. Spiegel surmises that convective processes at $\tau = 2$, say, would produce observable effects in the continuum of an A star, unless its rotation is sufficient to inhibit these processes. Hunger, however, takes the position that, since an estimate indicates that little energy is transported by convection at $\tau = 1$, convection will produce no observable effects. On the assumption either that the atmosphere is entirely in radiative equilibrium or that in the convection zone complete convective equilibrium maintains, Hack (1956) has computed the maximum effect of convection on the continuous spectrum. Her approximate computations indicate that comparing an observed energy distribution from an atmosphere in which the energy transfer is by convection, with a predicted energy distribution from a model in which the energy transfer is by radiation, will cause the derived temperature to be underestimated by about 400° for an early A-type dwarf. The error decreases for A stars of later type. Since this is the maximum effect expected, it seems unlikely that convection introduces a large uncertainty in the temperatures.

The atmospheric structure of A stars may also be appreciably modified if the star has a magnetic field or if it is in rapid rotation. At present, it is difficult to predict the modifications caused by these complications, since detailed models specifically including these factors are not available. However, a feeling for the orders of magnitude involved may be readily obtained. The magnetic pressure exerted by a field H is given by $P_{\rm mag} = \mu H^2/8\pi$. With a field of 500 gauss and $\mu = 1$, $P_{\rm mag} = 10^4$ dynes/cm², which is comparable with the gas pressure in an A-star atmosphere. Even though we do not know just how the atmosphere would be modified by such a field, it seems evident that the effect is not negligible. Similarly, an A star rotating with an equatorial velocity of 250 km/sec would have a centrifugal acceleration of about 3.6×10^3 cm/sec^2, a value comparable with the gravitational acceleration. The net flux at any point in the star is proportional to the effective gravity (see, e.g., Milne 1930). Therefore, the observed energy distribution would be the integral over the surface of the emergent flux, weighted according to its surface brightness and orientation with the line of sight.

Finally, if a shell surrounds the star, it may emit a sufficient amount of energy in the Balmer continuum to decrease the observed Balmer jump and thereby produce the appearance of a large surface gravity. P Cygni is an extreme example of such a case. Evidence suggesting that these effects are indeed operative in some of the stars in this study will be given later.

Perhaps the most important error in the second category mentioned above is caused by the finite resolving power of the scanning spectrograph. Although no line with equivalent width greater than 0.1 A is included in the band defined by the slit width centered at the standard wave lengths (with the exception of λ 3860), there are, of course, weak lines in the band. These produce a depression of the continuum, which is probably more pronounced toward the blue end of the spectrum. The observed continuum would then appear to correspond to one with a spuriously low effective temperature. However, because of the small blanketing in A stars to the red of the Balmer discontinuity, this effect is probably negligible for our measures.

b) The $(B - V)$ *Effective Temperature Scale*

In Figure 5 we have plotted the effective temperatures of the program stars as a function of their $(B - V)$ color index. The dashed curve gives the adopted values of this temperature calibration, disregarding the four low points, which represent the two stars in M39, No. 348 in Praesepe, and 21 Comae. These stars will be considered in a later section. The temperature scale given by Keenan and Morgan (1951), which is essentially that of Kuiper (1938), is also plotted in Figure 5. The two scales differ by about 1000° at A0 but by only about 300° at A7. Popper (1959) has recently discussed the uncertainties in the classical temperature scale, so that such a discussion need not be repeated here. It is worth noting that the revision suggested by these data is also suggested by the temperature derived from the measurement of the angular diameter of Sirius by Brown and Twiss (1956). Their measurements imply a temperature of 9350° (Popper 1959), while the present scale suggests a temperature of about 9400° for a star of $(B - V) =$ 0.01. This scale thus agrees with that given by Popper at $(B - V) = 0$ but is lower than Popper's by a few hundred degrees for redder $(B - V)$ colors.

A revised temperature scale provides better agreement between the observed Balmer discontinuities and those predicted by Osawa's models than does the classical scale. Osawa found the spectral type of his models from the $(B - V)$ color corrected for the blanketing effect. Thus his 8900° model corresponds to type A3, for which Chalonge and Divan (1952) give as the average Balmer discontinuity 1.22 mag. Assuming that an A-type dwarf has a surface gravity of $\log g = 4.0$, Osawa's model predicts a Balmer discontinuity of 1.37 mag., about 0.15 mag. larger than the average observed value. If, however, the spectral type is derived from our temperature scale, one finds that 8900° corresponds to type A2, for which the observed mean discontinuity is 1.30 mag., in better agreement with the model. The classical scale, however, gives for this temperature a spectral class of A4. The mean discontinuity for this type is 1.19 mag., in poor agreement with Osawa's model. Similarly, the 7560° model predicts a Balmer jump of 0.98 mag., compared with the observed value of 0.93 mag. for an A8 star. Our temperature scale gives a type A7 and therefore a discontinuity of 1.02 mag., so that the discrepancy is as large but in the opposite sense. The classical scale, however, predicts a spectral type of F0 for this temperature, for which the Balmer jump is 0.82 mag. Again, the classical scale gives poorer agreement with the models than does the scale suggested by the present work, which assumes the validity of the model stellar atmospheres.

c) Comparison of (B − V) and "Monochromatic" Photometry

The magnitude difference $m(\lambda\ 5560) - m(\lambda\ 4190)$ corresponds rather closely to the $(B - V)$ color index. It is of interest to compare the two indices for our program stars and for four stars from the Whitford-Code list (1955): β Orionis, α Cygni, δ Cygni, and α Lyrae. The data are plotted in Figure 6 and display the typical non-linear relation found between narrow-band and wide-band colors of the same effective wave length. It is seen that all the points fall about a mean curve with little scatter, except for the four program stars, which fall below the mean effective temperature-curve (Nos. 1 and 5 in M39, No. 348 in Praesepe, and 21 Comae), and α Cygni, β Orionis, and δ Cygni. Now α Cygni and β Orionis are supergiants, and δ Cygni is a giant. Evidently, the departure of a star below the mean curve is associated with high luminosity or, in other terms, with low effective surface gravity. The deviation from the mean curve

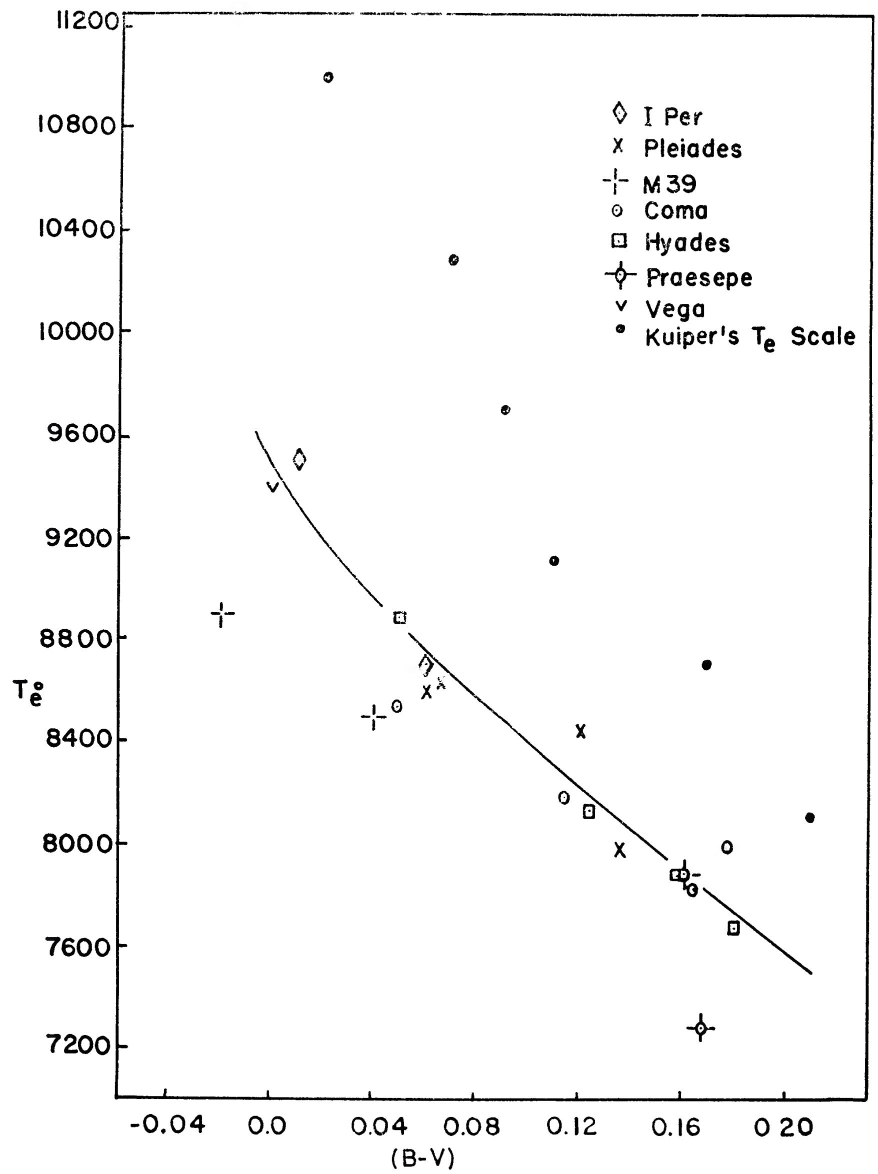

FIG. 5.—The color-temperature relation

probably arises from the effect of gravity on the equivalent widths of the hydrogen lines. In a star with low surface gravity, these lines are relatively narrow and have small equivalent widths. As a consequence, the B magnitude is brighter (less blanketing) in a star of low surface gravity than in a star of large gravity, and the $(B - V)$ index is brighter. (The V magnitude has only the slightest response at Hβ and is consequently unaffected by gravity changes.) Since our measurements are essentially monochromatic compared with the U, B, V filter measures, our results do not show this effect. The argument may account for the seven stars which fall well below the mean curve of Figure 6. As mentioned previously, α Cygni, δ Cygni, and β Orionis have smaller surface gravities than do the main-sequence stars which define the mean curve. In the next section we shall give evidence which indicates that Nos. 1 and 5 of M39, No. 348 of Praesepe, and 21 Comae also have low surface gravities. It should be noted that if the four program stars which lie below the mean curve in Figure 6 had the $(B - V)$ colors predicted by the curve, they would fall on the mean $(B - V)$-temperature curve of Figure 5.

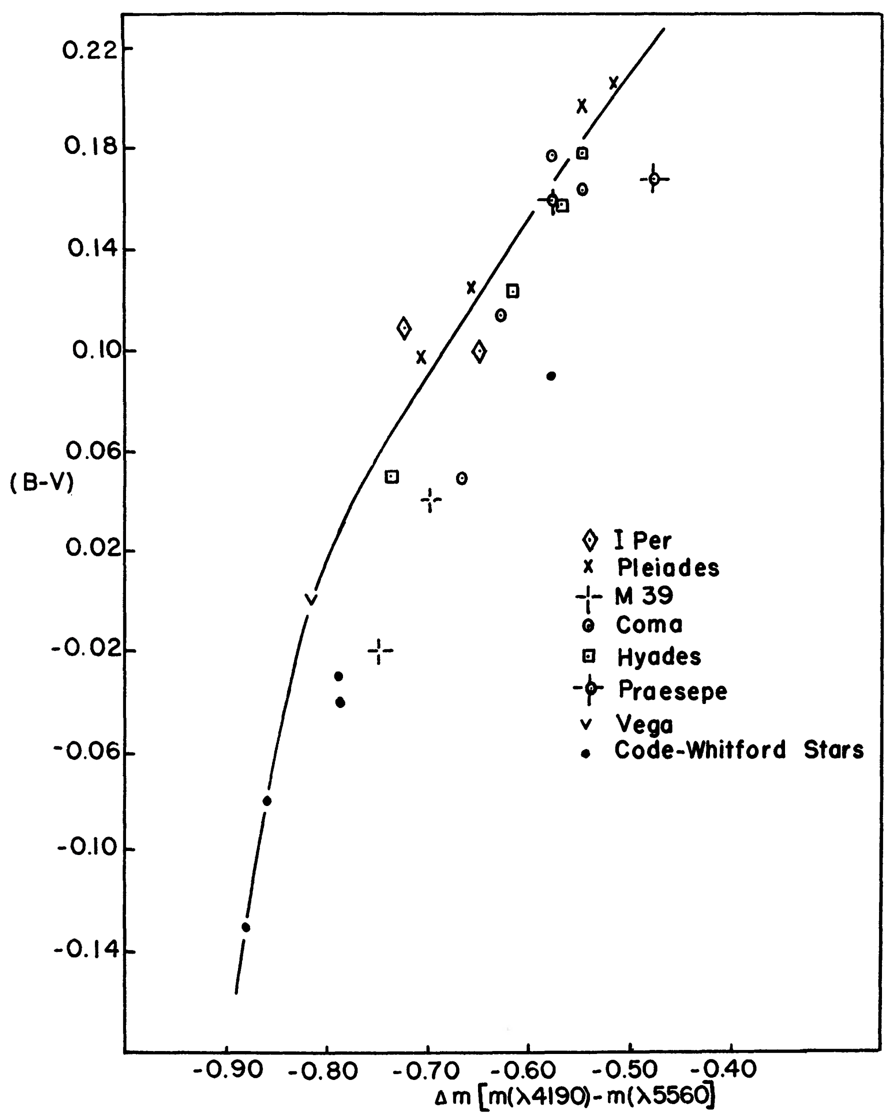

FIG. 6.—The $(B - V) - \Delta m$ relation

d) Discussion of Individual Stars

(i) *HD 21479* (*I Persei*).—This star shows the most marked difference of any of the program stars between the surface gravity derived from the observations and that from the mass-luminosity relation. With the color excess of 0.04 mag. adopted for this star—giving a true $(B - V)$ index of 0.06 mag.—the effective temperature derived fits the adopted curve satisfactorily. However, the observed Balmer discontinuity of 1.06 mag. is about 0.24 mag. smaller than the mean value given by Chalonge and Divan (1952) for an A2 star. The surface gravity derived from this value of the discontinuity would agree with that given by the mass-luminosity law. It is highly unlikely, however, that our observations of the Balmer discontinuity could be in error by even a quarter of this amount.

Since the I Persei association is only about 10^7 years old, HD 21479 may still be surrounded by remnants of the gas cloud from which it originally condensed, and in this cloud the recominbation of hydrogen to the $n = 2$ level may be contributing radiation to that of the star in the Balmer continuum. This would decrease the magnitude of the Balmer discontinuity and thereby produce the appearance of high surface gravity.

(ii) *No. 56*, *Hyades*.—The temperature of this star falls on the adopted color-temperature curve of Figure 5. Assuming the correctness of this scale, the discrepancy in log g of 0.4 (see Fig. 4) must be ascribed again to an abnormally small Balmer discontinuity. In order for the observed surface gravity to agree with that predicted by the mass-luminosity relation, the Balmer discontinuity of this star would have to be 0.15 mag. larger than is observed. This is well outside the possibility of error for such a bright and well-observed star. This object is the bluest star in the list of Hyades stars given by Johnson and Knuckles (1955) and appears to be subluminous by about 0.5 mag. relative to the cluster main sequence.

In his catalogue of magnetic stars, Babcock (1958*a*) lists this star as having a field of 400 gauss. This field exerts a magnetic pressure, $\log P = 3.45$ dynes/cm^2, which is of the same order of magnitude as the gas pressure in an A-star atmosphere. Perhaps this field modifies the atmosphere of this star in such a way as to produce a small Balmer discontinuity and thus the appearance of large surface gravity.

If the above suggestion is correct, then there is a possibility that HD 21479 (I Persei) may also have a magnetic field. If the apparent surface gravity as given by the Balmer discontinuity varies as the magnetic-field strength, the field would have to be stronger here than in the Hyades star.

Conversely, the possibility of Balmer emission in a shell or extended chromosphere of the Hyades star cannot be ruled out. High-dispersion spectrograms of these stars should enable one to detect the effect of large surface gravity on the hydrogen-line profile or the alternate possibility of emission in the hydrogen lines (and therefore in the Balmer continuum).

(iii) *No. 1, M39*.—As was mentioned earlier, this star departs from the mean curve in the $(B - V) - \Delta m$ diagram (Fig. 6) in a manner which implies a low surface gravity relative to that of main-sequence stars. This effect also manifests itself as an abnormally low temperature in the color-temperature diagram (Fig. 5). Interstellar reddening would influence the apparent temperature of the star in this manner, but for several reasons it seems unlikely that the star is reddened. First, its $(B - V)$ color of -0.02 mag. is already slightly blue for an average A0 star; second, there is no color excess, as estimated by the Q-method of Johnson and Morgan (1953); and, third, a color excess of about 0.10 mag. would be required in order for the derived temperature of the star to be 9900° (the value indicated by the mean temperature-curve). The star would then have an extremely blue $(B - V)$ color for its spectral type. Furthermore, reddening of this amount seems unlikely in M39, where Johnson estimates that the average excess is only about 0.02 mag. A cloud causing an excess of 0.10 mag. should be visible on a direct photograph, whereas no spottiness in the distribution of the absorbing material is seen. Errors in the observations of the continuum could hardly account for the low temperature, since the whole curve to the red of the Balmer discontinuity would have to be shifted to fainter magnitudes by 0.05 mag. at λ 4600 and by 0.10 mag. at λ 6000. Such a shift would increase the Balmer discontinuity by nearly 0.1 mag., in clear disagreement with Westerlund's value (Table 5). In addition, this star was observed on four nights, and the values derived on each of the nights agree well with each other. Thus the discrepancy must be real.

Weaver (1953) remarks that the line spectrum of this star shows a marked rotational-velocity effect, indicating that much of the radiation we observe arises from the star's equatorial regions. It is here that the maximum gravity-darkening effect would occur. Perhaps, then, the rotation of the star appreciably modifies the structure of the star so that it appears to have giant-like characteristics—in particular, an abnormally low effective temperature. If this is the case, our original observational result for the surface gravity, $\log g = 4.5$ (see Fig. 4), is meaningless, since the model atmospheres used in this work are not applicable to rapidly rotating objects; furthermore, such an object might develop a shell or extended chromosphere which could decrease the magnitude of the Balmer jump, thus simulating the effect of a large surface gravity.

The measured $(B - V)$ color of this star is -0.02 mag., while the color derived from our measurements of the continuum corresponds to a $(B - V)$ of about $+0.05$ mag., as the low effective temperature would imply. This difference is probably caused by the abnormally low surface gravity which brightens the $(B - V)$ magnitude of the star but leaves our measurements unaffected, as described earlier. The line spectrum should show the effect of the low effective temperature also. The spectral type given by Johnson (1953) is A0, about normal for a $(B - V)$ of -0.02 mag.; Weaver, however, gives the spectral type to be A1, implying a $(B - V)$ of about 0.04 mag., in agreement with our result.

(iv) *No. 348, Praesepe.*—The same rotational effect may be present in this star, which is also below the mean curve in Figure 6, since Bidelman (1956) remarks that the spectral lines are "notably diffuse." Here, however, the possibility of observational error cannot be discounted, since this star was observed on only one night. Regarding the difference between our color and the $(B - V)$ color, the same argument can be made here as was given for No. 1, M39.

(v) *No. 5, M39, and 21 Comae.*—Both these stars fall below the mean curve in the $(B - V) - \Delta m$ graph of Figure 6 and, as a consequence, have temperatures too low for their $(B - V)$ colors. Thus these two stars display giant-like characteristics.

The color of the M39 star is slightly red for its spectral type; furthermore, the Q-method indicates a color excess of 0.06 mag. for it. If this reddening is believed to be real (the Q-method may give incorrect results for the A0 stars), the temperature of the star would be increased to 8900°, which fits the mean curve well (Fig. 5). With this increased temperature, however, the surface gravity derived for the M39 star would deviate even more from that given by the mass-luminosity relation (the discrepancy is already 0.5 in $\log g$; see Fig. 4). The derived surface gravity of 21 Comae, however, agrees well with the value derived from the mass-luminosity relation. According to Weaver (1953), the line spectrum of the M39 star resembles that of 21 Comae, a star with abnormally strong and variable lines of Sr II. If we assume that the M39 star is unreddened, its $(B - V)$ color also would be identical with the color of 21 Comae. We shall therefore assume that this is the case, that the star is not appreciably reddened. With this assumption, the derived temperatures of the two stars agree, as, of course, they should. The stars differ markedly in their Balmer discontinuities, however; the M39 star has a discontinuity 0.15 mag. smaller than does 21 Comae, and this difference accounts for the differences in the surface gravity. The M39 star has a $(U - B)$ color 0 11 mag. brighter than that of 21 Comae, consistent with the difference in the Balmer discontinuities. The Balmer jumps of these two stars have been measured by Westerlund, whose results agree with ours (Table 5).

According to Deutsch (1956), the period of the spectrum variation of 21 Comae is 1.03 days. For spectrum variables as a class, he has found what he believes to be an inverse relation between line width and period, which implies that this period is the rotation period of the star. If this is the case, then the equatorial velocity of 21 Comae is about 100 km/sec. This rotation, while not producing as marked effects as are perhaps seen in No. 1, M39, might account for the giant-like characteristics displayed in 21 Comae. Since No. 5, M39, also exhibits such characteristics and since its spectrum resembles that of 21 Comae, perhaps it, too, is rotating fairly rapidly. The determination of the period of the spectrum variation would be of great interest in this connection. In addition to rotation, we might also expect a shell to be present in the star, filling in the Balmer jump to its abnormally low value.

(vi) *α Lyrae.*—Since an independent determination of the effective temperature and surface gravity of α Lyrae was made by Hunger, it is appropriate to compare his results with ours. From a comparison of his model atmospheres with the observations, Hunger derived, for the temperature of Vega, $T_e = 9500° \pm 300°$, which agrees well with our value of 9400°. However, Hunger's value of the surface gravity, $\log g = 4.4 \pm 0.2$, differs greatly from our determination, $\log g = 4.0$. Hunger derived his value of the surface gravity by two methods: from the mass-luminosity relation and from the profiles of Hγ and Hδ. In the first case, he found $\log g = 4.3$, while we derived $\log g = 3.9$ by the same method. Hunger used the Russell-Moore (1940) mass-luminosity relation in which the sun is overluminous for its mass. In this relation the effective temperature of a normal star of one solar mass is 5000°. We used the more recent relations (mentioned earlier), according to which the sun is a normal star, so that the effective temperature of a star of solar mass is 5700°. This difference contributes 0.24 mag. in $\log g$ to the discrepancy between his gravity determinations and ours. For the absolute magnitude of Vega, Hunger used the value $M_v = 0.6$ given by Schlesinger and Jenkins (1940), whereas we used the value $M_v = 0.44$ given by Johnson and Morgan (1953). Also, Hunger's bolometric correction was 0.05 mag. fainter than ours. Furthermore, the bolometric magnitude of the sun which he quoted is 0.13 mag. brighter than the value used in this work. All these magnitude differences affect the final result in the same sense and contribute about 0.1 in $\log g$ to the discrepancy. This accounts for 0.34 of the 0.4 difference in $\log g$; the remainder is accounted for by Hunger's rounding off the values used. With $\log g = 4.3$, he then computed his models. The model with $T_e = 9500°$ fits the observed continuum of Vega to the red of the Balmer limit (it will be recalled that the surface gravity has little effect on this portion of the spectrum), but it predicts a Balmer discontinuity of only 1.23 mag., whereas the measured value is 1.29 mag. This discrepancy indicates that the surface gravity of the model is too large.

Using the Holtsmark theory (1919) for the line-absorption coefficients, Hunger calculated the line profiles of Hγ and Hδ. A comparison of these profiles with the observed profiles showed that the fit would be improved if the surface gravity were increased to $\log g = 4.4$. It seems well established, however (see, e.g., Girem, Kolb, and Shen 1959), that abnormally large surface gravities are required to fit the hydrogen-line profiles in the Holtsmark line-broadening theory. Therefore, Hunger's value of $\log g = 4.4$ seems to be entirely too large. This becomes particularly evident when one considers the mass implied by Hunger's results, namely, 7.2 solar masses. This is an enormous mass for an A star, compared with the observational value of 2.3 solar masses for Sirius (van de Kamp 1954). The mass of Vega calculated from our results is 3.0 solar masses.

e) Masses and Evolutionary Tracks

The position of the program stars in the HR diagram is given in Figure 7. The number in parentheses by a star gives its $\log g$, and the number outside parentheses the mass, both derived from the continuum observations. The symbols used are *ml* for a metallic-line star; *Sr* for a strontium star; *rot.* for a star in which large rotational effects are believed to be operative; *mag* for a star with a magnetic field; and *p* for peculiarities of other sorts. The shaded band represents Kushwaha's (1957) evolutionary track for a star of 2.5 solar masses.

One general feature of the diagram may be noted, namely, the surface gravities of the stars farther off the main sequence are smaller than those of stars on the main sequence. This is to be expected, of course, and indicates that the derived surface gravities are at least sufficiently reliable to show this trend. Also, if one considers stars in the same cluster, thereby eliminating most of the effects of uncertainties in the distance moduli, one finds that the more luminous the star, the greater its mass, which again is to be expected.

The two faintest Coma stars and the faintest Pleiades stars form a short sequence roughly parallel to the evolutionary track. The masses of these stars should then be approximately equal. The two Coma stars, however, are undermassive compared with the Pleiades star. Both the Coma stars are metallic-line stars, the star of lower mass showing the greater degree of metallicism (Weaver 1952). (Greater significance is attached to the difference in masses of these peculiar objects relative to normal stars

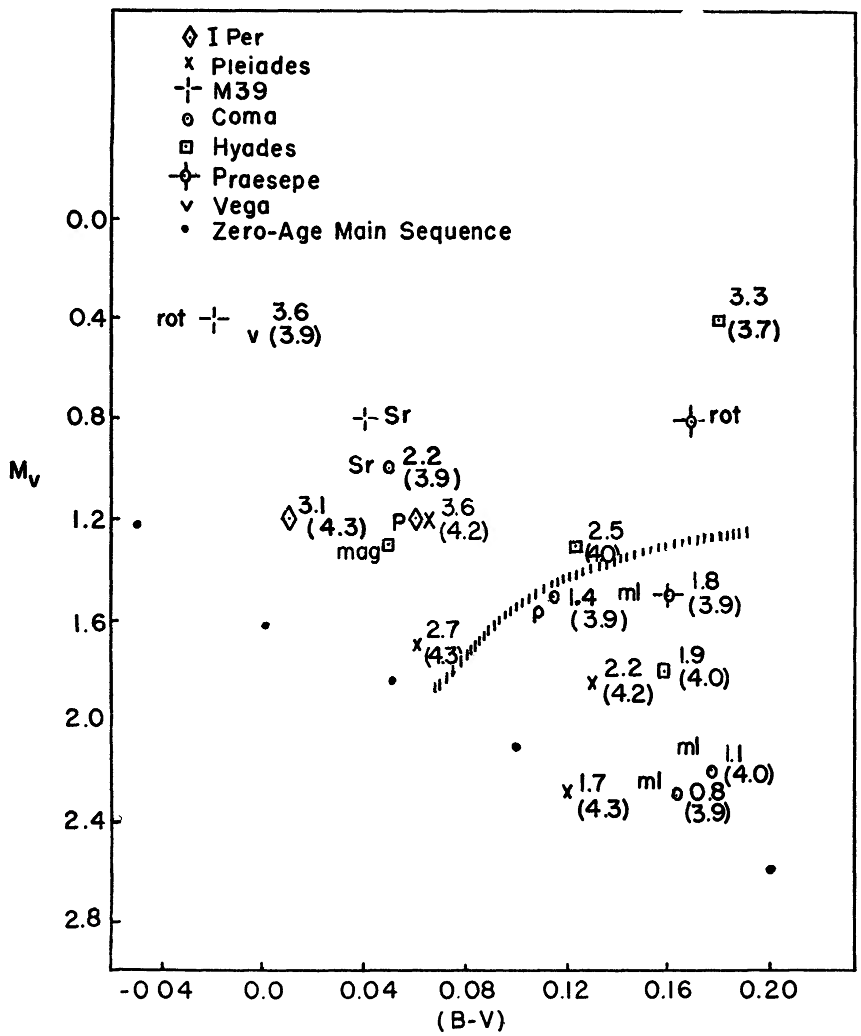

FIG. 7.—Color-magnitude diagram of the program stars

than to the value of the masses themselves.) This suggests that metallic-line stars are systematically less massive than normal stars in the same part of the HR diagram. This notion is further supported by the derived mass of the fainter of the two Praesepe stars, which again is lower than the masses of the normal stars around it. If this suggestion is correct, then it would follow that these objects should be considered F-type stars with abnormally weak calcium lines, rather than A-type stars with abnormally strong metallic lines. This contradicts the appealing suggestion made by Babcock (1958*b*) that the only difference between normal and peculiar A stars is in the orientation of their rotation axes, so that "peculiar" stars are ordinary A stars seen pole-on. In this orientation the peculiarities are not washed out by rotation. However, the large variety of peculiarities observed in the A-type stars also suggests that they are not a single class of objects. Alternatively, the presence of many absorption lines may cause an apparent depression of the continuum, the effect increasing to the blue. If appreciable, this could cause the temperature to be underestimated, causing, in turn, a spuriously low surface gravity to be inferred (see Fig. 3). The continuum depression required is large, however. The Praesepe metallic-line star shows a small $(U - B)$ deficiency, but the Coma stars do not.

Recently Abt (1960) has found that, allowing for orientation effects, all the members of a randomly selected sample of twenty-five metallic-line stars are observed to be spectroscopic binaries. This result implies that all such stars are spectroscopic binaries. If the secondaries of the metallic-line stars included in this study contribute little light to the system and if no mass exchange has taken place, then the suggestions made above would not be invalidated by Abt's discovery. If, however, the secondary contributes appreciably to the radiation of the system and if it is of a later spectral type than the primary, then the observed radiation to the red of the Balmer discontinuity would be redder than that from the primary alone, and a spuriously low effective temperature would be derived. Furthermore, this would lead to an underestimate of the surface gravity and mass, just as in the case of appreciable line blanketing mentioned earlier. On the other hand, radiation from such a secondary would decrease the observed Balmer discontinuity, producing the appearance of a large surface gravity and mass. Clearly, a determination of the relative luminosities of both components of the metallic-line stars studied here is necessary in order to interpret the observations.

The Coma star just to the blue of the Praesepe metallic-line star is peculiar. Its mass of 1.4 solar masses is abnormally low, but it is not listed as a metallic-line star. Weaver (1952), however, does note that the radial velocity of this star is variable.

The three normal stars clustered about Kushwaha's evolutionary track have masses of 2.2 (No. 513, Pleiades), 2.7 (No. 341, Pleiades), and 2.5 (No. 104, Hyades) solar masses and therefore provide a degree of observational confirmation of Kushwaha's evolutionary track for a star of 2.5 solar masses.

VI. SUMMARY OF RESULTS

The results of this work may be summarized as follows:

a) Perhaps the most interesting result of this investigation is that it appears possible to determine the masses of single, normal A stars (and stars of other spectral types as well, probably) by comparing their observed continua with accurate model atmospheres. High-quality observations should enable a precision to be attained of ± 0.10–0.20 in the logarithm of the mass.

b) A scale of effective temperatures for the A stars has been determined. This scale is about 1000° cooler at A0 than that given by Kuiper but seems to reach it at about F0.

c) The possibility has been suggested that the continua of some of the peculiar stars studied show the effects of rapid rotation and of a moderately strong magnetic field.

d) Rather strong evidence indicates that the surface gravity of α Lyrae derived by Hunger is too large by a factor of 2.5.

e) Evidence has been given which suggests that at least some of the metallic-line stars of type A may have masses more appropriate to F stars.

One of the best tests of this method of deriving stellar masses is afforded by Sirius, for which a reliable dynamical mass is available. This star is to be observed as soon as possible. The author also intends to extend this method to late B- and early F-type stars.

It is a pleasure to acknowledge the invaluable aid and encouragement given to me by Dr. William Liller throughout the course of this work. I am also indebted to Drs. A. D. Code, D. E. Osterbrock, and E. A. Spiegel for helpful discussions and suggestions and to the National Science Foundation for financial support.

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