THE ABUNDANCE OF LITHIUM AND CONVECTIVE MIXING IN STARS OF TYPE K*

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ABSTRACT

The abundance of lithium relative to vanadium, or its upper limit, has been determined for forty-six normal stars of spectral types from G8 to M0, inclusive. These results are based upon high-dispersion spectrograms obtained with the coudé spectrograph of the 100-inch telescope. The abundance ratio for the sun has also been determined, using an equivalent width for lithium measured by Greenstein and Richardson and measurements of vanadium lines in the Utrecht *Atlas*. In addition, temperatures, electron pressures, and turbulent velocities in the stellar atmospheres have been derived. A range of up to a factor of 100 in the abundance ratio is found among stars of similar surface characteristics. The maximum abundance ratio observed among similar stars declines with surface temperature. It is not likely that a significant part of these variations is due to changes in the vanadium abundance. Greenstein and Richardson have proposed that the litnium in the solar surface has been depleted by convective mixing to hotter regions. The possibility that this hypothesis may explain both the trend and the variations in the cooler stars is suggested and discussed.

I. INTRODUCTION

The light elements deuterium, lithium, beryllium, and boron pose a special problem for any theory of the origin of the elements which proposes that all the elements are built up from hydrogen in the stars. Such a theory has recently been described in detail by Burbidge, Burbidge, Fowler, and Hoyle (1957). The difficulty arises because the lifetimes of these elements against proton capture, at the temperatures and pressures at which most stellar matter exists, are short compared to the stable lifetimes of stars. These elements then cannot be produced in stellar interiors unless they are transported rapidly to the surface, and if they are produced at the surface, non-equilibrium processes must be involved. Further, they can exist in significant quantities at the surface only in the absence of rapid mixing to the interior. Thus an investigation of the abundances of these elements in stellar atmospheres bears both upon questions of nucleogenesis and upon astrophysical problems of mixing and atmospheric disturbances.

Of these four elements, lithium is the most amenable to spectroscopic observation. Boron has no lines in the astronomically accessible regions; the lines of deuterium are masked by the strong lines of hydrogen; and the most favorable lines of beryllium are the resonance doublet of Be II at λ 3131, a region which can be photographed with sufficient dispersion to resolve the crowded lines in only a few very bright, hot stars. On the other hand, the resonance doublet of Li I at λ 6708 may be expected to be present in stars of type K and cooler and is near the maxima of the energy distributions of these stars in a part of the spectrum little affected by crowding of lines or by atmospheric absorption.

The only star for which a quantitative result on the abundance of lithium has been obtained is the sun. Here the doublet is very weak; the total equivalent width on the solar disk was measured by Greenstein and Richardson (1951) as only 3.7 mA. Comparison of this value with the equivalent width of the Ca I line at λ 6573 permitted determi-

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nation of the abundance of lithium relative to calcium. Greenstein and Richardson concluded that the abundance of lithium on the sun is approximately one hundred times less than on the earth. Unsöld (1955) criticized this analysis and determined the abundance relative to potassium by using the same equivalent width for Li I. He also concluded that the solar abundance is less than the terrestrial by a factor of 100.

The Li I doublet has been observed in many other stars, but no quantitative analyses have even made. McKellar and Stilwell (1944) and Sanford (1950) have found features with equivalent widths as large as 10 A in cool carbon stars. In discussing the spectra of thirty such stars, McKellar and Stilwell found the strength of the Li I feature highly variable and not correlated with the strength of the analogous Na I transition, the D lines. Relatively weak Li I features have been seen in S stars (Keenan and Teske 1956; Merrill and Greenstein 1956; Teske 1956), and Hunger (Herbig 1958) has identified strong Li I features in the spectra of T Tauri and RY Tauri.

The purpose of the present study is to derive results on the abundance of lithium in the atmospheres of a significant sample of normal stars of types near K. The restriction to normal stars (excluding carbon stars, S stars, Ba II stars, and the like) minimizes the number of variables on which any variation in the abundance might depend. In hotter stars increased ionization and increased continuous opacity render the doublet too weak to be observed at stellar dispersions, and in cooler stars increased blending due to molecular features makes quantitative results quite difficult to obtain.

II. OBSERVATIONS AND MEASUREMENTS

a) The Observations

A list of stars for observation was selected according to the above criteria from the tables of stars classified on the MK system presented by Johnson and Morgan (1953), Keenan and Morgan (1951), and Roman (1952). The stars chosen were brighter than the sixth apparent visual magnitude and were of spectral types G8–M0, inclusive. All luminosity classes were included. As many of these stars were observed as time and the weather permitted, but usually no attempt was made to secure more than one good spectrogram of each star.

The spectrograms, which covered the wave-length range λλ 5300–6800 with a dispersion of 6.8 A/mm, were obtained with the coudé spectrograph of the 100-inch Hooker telescope. Kodak II*a*-F or 103*a*-F(3) plates were used for the region longward of λ 6000, and II*a*-D or 103*a*-D plates were exposed to the shorter wave lengths. It was found that spectrograms made on the II*a* emulsion and widened to 0.5 mm required approximately the same exposure time and yielded about the same noise level in the subsequent microphotometer tracings as spectrograms made on 103*a* emulsion and widened to 1.5 mm. The brighter stars on the program were observed with at least this degree of widening; the fainter stars were recorded on 103*a* plates with widths ranging down to 0.5 mm. The width of the slit image at the plate was 19 μ (0.13 A) in all cases.

Table 1 lists the forty-six stars for which spectrograms of good photometric quality were obtained, giving the number in the *Henry Draper Catalogue*, the name, the spectral class, the apparent visual magnitude, the co-ordinates for the epoch 1950, the emulsion type, and the spectrum width in millimeters.

Figure 1 presents sections of the spectra of representative stars in Table 1. The illustration covers the wave-length region λλ 6600–6720. The Li I doublet and a selection of other lines arising from neutral atoms are identified. All spectra, with the exception of the lower one, are of giant stars, arranged in order of decreasing surface temperature from the upper to the lower spectrum. The lines show the regular increase in strength and the changes of relative strength due to decreasing excitation from the earlier to the later classes which would be expected. The lithium doublet, however, shows a most erratic behavior not duplicated by any other line shown or by any line in the photo-

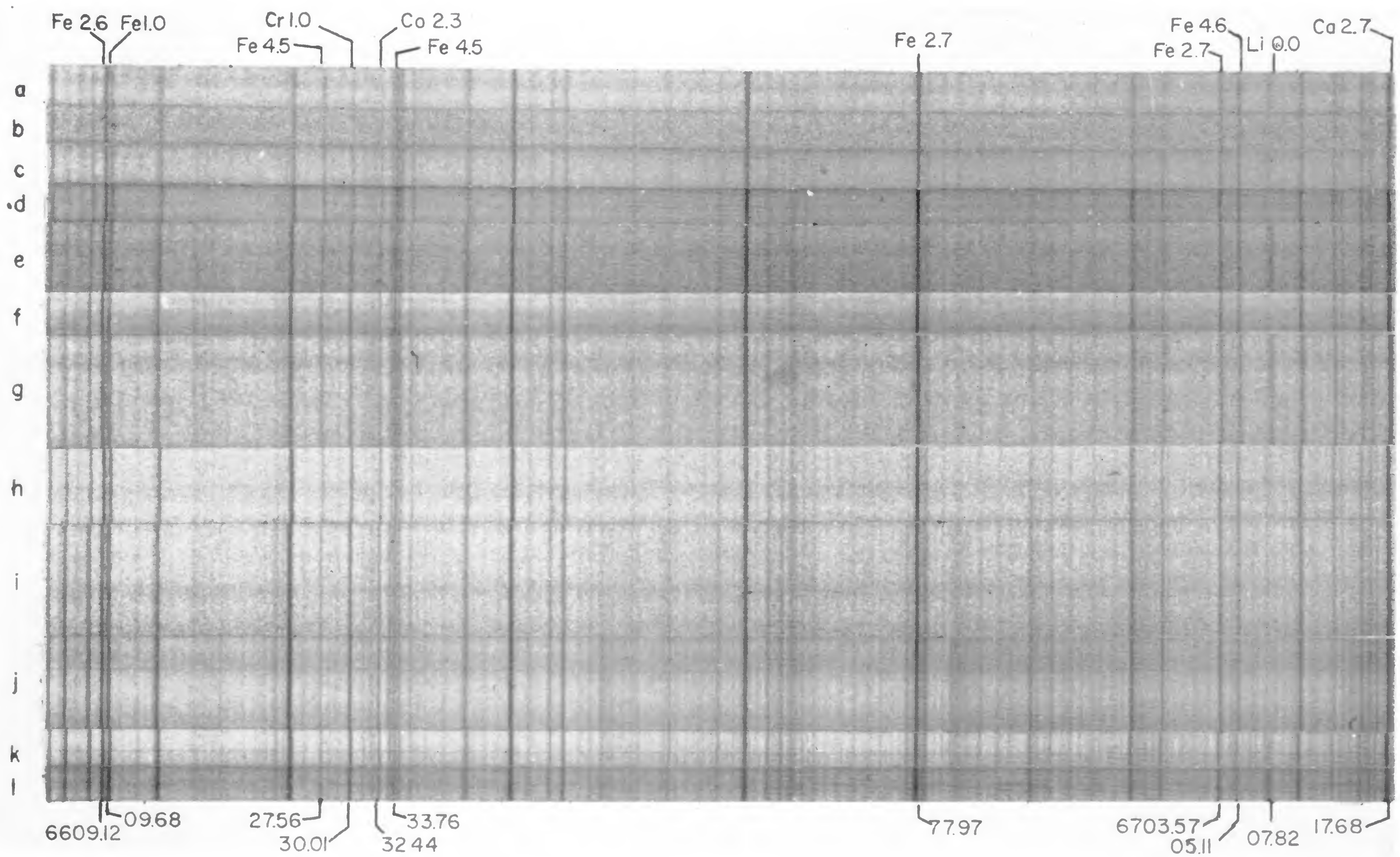

FIG. 1.—Sections of twelve sample spectra. Lines of neutral atoms are identified by element and excitation potential above and wave length below. The stars are (*a*) HD 9270, G8 III; (*b*) HD 104979, G8 III; (*c*) HD 27697, K0 III; (*d*) HD 197989, K0 III; (*e*) HD 96833, K1 III; (*f*) HD 12929, K2 III; (*g*) HD 6805, K2 III; (*h*) HD 98262, K3 III; (*i*) HD 90432, K4 III; (*j*) HD 29139, K5 III; (*k*) HD 189319, K5 III; and (*l*) HD 200905, K5 Ib.

TABLE 1

STARS OBSERVED

HD	NAME	TYPE	m_v	(1950) α	(1950) δ	EMULSION	WIDTH (mm)
3627	δ And	K3 III	+3.5	0h36m6	+30°35′	IIa	1.0
4128	β Cet	K0 III	+2.2	0 41.1	−18 16	103a	1.5
6805	η Cet	K2 III	+3.6	1 06.1	−19 27	103a	1.5
6860	β And	M0 III	+2.4	1 06.9	+35 21	IIa	0.5
8512	θ Cet	K0 III	+3.8	1 21.5	−08 26	103a	1.1
9270	η Psc	G8 III	+3.7	1 28.8	+15 05	IIa	0.5
						103a*	0.5
10476	107 Psc	K1 V	+5.2	1 39.8	+20 02	103a	0.5
11909	ι Ari	K1p	+5.2	1 54.6	+17 34	103a	0.5
12533	γ And A	K2 III	+2.3	2 00.8	+42 05	IIa	0.5
12929	α Ari	K2 III	+2.2	2 04.3	+23 14	IIa	0.5
20468	HR 991	K2 II	+4.9	3 15.6	+34 02	103a	0.5
20644	HR 999	K3 II–III	+4.7	3 17.3	+28 52	103a	0.8
22049	ε Eri	K2 V	+3.8	3 30.6	−09 38	103a	1.5
27371	γ Tau	K0 III	+3.7	4 16.9	+15 31	103a	0.8
27697	δ Tau	K0 III	+3.9	4 22.6	+17 49	IIa	0.5
28305	ε Tau	K0 III	+3.6	4 25.7	+19 04	103a	1.5
29139	α Tau	K5 III	+2.2	4 33.0	+16 25	IIa	1.2
30504	HR 1533	K4 II	+5.1	4 46.5	+37 24	103a	0.6
35620	φ Aur	K3p	+5.3	5 24.3	+34 26	103a	0.5
50778	θ CMa	K4 III	+4.2	6 51.9	−11 58	103a	1.5
62345	κ Gem	G8 III	+3.7	7 41.4	+24 31	IIa	0.5
69267	β Cnc	K4 III	+3.8	8 13.8	+09 20	103a†	1.2
76294	ζ Hya	K0 III	+3.3	8 52.5	+06 08	103a	1.5
81797	α Hya	K3 III	+2.2	9 25.1	−08 26	103a	1.5
90432	μ Hya	K4 III	+4.1	10 23.7	−16 35	103a	1.5
90537	β LMi	G8 III–IV	+4.5	10 25.0	+36 58	103a	0.6
96833	ψ UMa	K1 III	+3.2	11 06.9	+44 46	103a	1.5
98262	ν UMa	K3 III	+3.7	11 15.8	+33 22	103a	1.0
98430	δ Crt	G8 III–IV	+3.8	11 16.8	−14 30	103a	1.5
104979	o Vir	G8 III	+4.2	12 02.7	+09 01	103a	0.5
108381	γ Com	K1 III–IV	+4.6	12 24.4	+28 33	103a	0.8
124897	α Boo	K2 IIIp	−0.1	14 13.4	+19 27	IIa	1.5
165341	70 Oph A	K0 V	+4.3	18 02.9	+02 31	IIa	0.5
168723	η Ser	G8 IV	+3.3	18 18.7	−02 55	IIa	0.5
176670	λ Lyr	K3 II	+5.1	18 58.1	+32 04	103a	0.5
186791	γ Aql	K3 II	+2.8	19 43.9	+10 29	IIa	0.5
189319	γ Sge	K5 III	+3.7	19 56.5	+19 21	IIa	0.5
196321	70 Aql	K5 II	+5.2	20 34.1	−02 43	103a	0.5
197989	ε Cyg	K0 III	+2.4	20 44.2	+33 47	IIa	0.5
200905	ξ Cyg	K5 Ib	+4.1	21 03.1	+43 44	IIa	0.5
201091	61 Cyg A	K5 V	+5.2	21 04.7	+38 30	103a	0.6
201092	61 Cyg B	K7 V	+6.0	21 04.7	+38 30	103a	0.5
201251	63 Cyg	K4 II	+4.9	21 04.9	+47 27	103a	0.6
206778	ε Peg	K2 Ib	+2.4	21 41.7	+09 39	IIa	0.5
219615	γ Psc	G8 III	+3.8	23 14.6	+03 01	IIa	0.5
225212	3 Cet	K3 Ib	+5.2	0 01.9	−10 47	103a	1.2

* Obtained by W. G. Melbourne.

† Two spectrograms.

graphed, but not illustrated, spectra. On the basis of this figure alone, one would expect variations in the abundance of lithium among these stars.

b) Photometry

Each spectrogram was calibrated for photometry by means of twelve continuous spectra of known relative intensity impressed on the plate through the step-slit arrangement which is part of the coudé spectrograph. In addition, for many spectrograms separate wedge-slit calibration spectrograms were made. Comparison of the characteristic curves derived from these two systems for several plates indicated no systematic differences.

In order to provide a reference for the lithium abundance and to indicate physical conditions in the stellar atmospheres, equivalent widths of 35 lines of V I, 4 lines of Sc I, and 3 lines of Sc II were measured in each star. The reasons for these choices will be given in the next section. The equivalent width of each line was obtained by assuming the profile to be Gaussian, measuring the central depth and total width at half-intensity (half-width), and computing the equivalent width, W, from

$$W = \Delta\lambda \left(\frac{F_0 - F_c}{F_0}\right) \frac{\sqrt{(\pi \ln 2)}}{2} = 1.064 \Delta\lambda \left(\frac{F_0 - F_c}{F_0}\right). \tag{1}$$

Here F_0 is the flux in the continuum, F_c is that at the line center, and $\Delta\lambda$ is the half-width. The validity of this approximation was tested by comparing equivalent widths calculated from equation (1) with values obtained by numerical integration of the apparent profiles. This test was carried out for all the lines measured in HD 28305 (K0 III) and HD 206778 (K2 I*b*) and for strong, unblended lines in HD 81797 (K3 III). Analysis of the half-widths suggested the presence of damping wings in the latter star. The results from the two methods were found to be insignificantly different in all cases. The root-mean-square difference between equivalent widths was 13 mA for all lines compared, and there was no correlation between the difference and line strength. Apparently, the expected damping wings are concealed by turbulence in the stellar atmospheres and the width of the instrumental profile.

Some of the lines used in the analysis were partially blended in the stars so that the half-width could not be measured. To supply these quantities, a mean relation between the half-width and the central depth was established from the other lines measured for each star. In the hotter stars these quantities were uncorrelated, and a single mean half-width was used, but in the cooler stars a strong correlation, apparently due to unobserved damping wings, was found.

The location of the continuum was, with few exceptions, not a problem in these measurements. Generally a region of undisturbed continuum could be found within a few angstroms of any line of interest, but for a few lines in the cool stars it was necessary to look as far as 25 A. In this study, errors attributable to uncertainties in the continuum are probably small.

Few comparisons of equivalent widths with those of other investigators are possible; the most significant of these is one with measurements made in the spectrum of HD 124897 (α Boo) by Wright (1951*b*). Using a dispersion similar to that used in this study, Wright obtained equivalent widths by fitting empirically determined standard profiles. Wright's values for the twenty-nine lines common to the two studies are approximately 10 per cent greater than those derived here; this difference is probably due to the large wings in his empirical profiles which have no counterparts in the tracings used in this study.

For two of the stars studied, two spectrograms were available, making tests for systematic errors possible. The equivalent widths in HD 9270 (the second spectrogram of which was kindly loaned the writer by W. G. Melbourne) did not differ significantly in the mean between the two measurements, but those in HD 69267 were slightly larger on one plate than on the other. In the latter case the spectrogram with the weaker lines was in somewhat poorer focus; possibly the Gaussian approximation is not appropriate in this case. In addition, there may have been real changes in the atmosphere of this K4 III star in the eight months between the epochs of the two plates. The differences were, however, not significant with respect to the final results of this study.

c) The Li I Doublet

The identification of lithium in the spectrum of a star studied here depended only upon the unresolved Li I doublet at λ 6707.74 and λ 6707.89. No other lines of Li I are strong enough to be observed, and Li II has no observable lines. The identification depended, therefore, on wave-length coincidence and the apparent profile of the doublet, which is broader than that of a single stellar line of similar depth. The wave lengths of the Li I lines were located to within a few hundredths of an angstrom on each tracing by referring to five strong lines in the vicinity. In most of the stars, the doublet is blended with a weak unidentified feature at λ 6707.52. This was noticed and allowed for by Greenstein and Richardson (1951) in the sun. It was always possible to free the Li I profile from the effects of the blend by simple extrapolation, since the blending feature was weak and did not contribute significantly over most of the doublet.

Greenstein and Richardson took note of the considerable isotope shift in the lithium spectrum. The strong line of the less abundant isotope, Li^6, is approximately coincident with the weak line of Li^7. The wave lengths given in the present study refer to Li^7. These authors found no evidence in the sun for an isotope ratio, Li^6/Li^7, in excess of the terrestrial value, 0.08. Here no particular allowance is made for Li^6, but if it contributes strongly to the observed profile, its contribution is included in the measurement and the derived abundance. In the strongest profiles observed, no significant contribution by Li^6 could be seen.

The total equivalent width of the blended Li I doublet was measured by direct application of equation (1) whenever possible. Since this formula actually is simply that for the area of a triangle with a numerical correction for rounded corners, it is not inappropriate for the basically asymmetrical profile of the Li I feature. In those stars in which the feature was so weak that only the central depth or its upper limit could be measured, the corresponding equivalent width was computed on the assumption that the Gaussian absorption profiles of the two lines added. The resultant equivalent widths for the Li I doublet are given in Table 3.

d) The Sun

Since lithium has been observed in the sun, it is of interest to include the sun in the present investigation in order to put its abundance on the same scale as that for the cooler stars. Because of the relatively high temperature and opacity in the solar atmosphere, the vanadium lines used would be very weak at 6.8 A/mm, and the weaker lithium feature would be unobservable. It seemed preferable, therefore, to use the much higher dispersion available in the intensity-scale tracings of the Utrecht *Atlas* (Minnaert, Mulders, and Houtgast 1940) to determine the vanadium and scandium equivalent widths and to adopt the equivalent width of the Li I feature given by Greenstein and Richardson. As will be shown later, no important systematic differences between the sun and the other stars seem to have arisen from the different source of data.

III. ANALYSIS

a) Introduction

The results of this investigation are presented as ratios of the abundance of lithium to that of vanadium in each star considered. This form has a greater physical significance than the absolute abundance of lithium; furthermore, it can be obtained with relatively high accuracy from a minimum amount of labor per star. This latter point is quite significant in attempting a survey.

Vanadium was chosen as a reference element for several reasons. First, it is a member of the "iron group" of elements, which, on the theory presented by Burbidge *et al.* (1957), would be expected to show a minimum variation in abundance among themselves from star to star; second, no fluctuation of the vanadium line strengths relative to other lines of similar spectroscopic properties is noticeable in the spectrograms; and, finally, a number of relatively weak vanadium lines originating in levels of low excitation potential are found in the wave-length region λλ 5600–6300, which may be conveniently photographed with the region of the Li I doublet. Table 2 gives the wave length, multiplet number, excitation potential, and a quantity log η_0' related to the abscissa of the curve of growth for each of the V I lines used and the first three quantities for the Li I doublet and for the Sc I and Sc II lines used to determine the state of ionization.

b) The Relative Abundance of Neutral Atoms

The abundance of Li I relative to V I was determined by using the standard curve-of-growth technique, utilizing the lines of V I listed in Table 2, with the exception of λ 5632, to establish the curve for each star. Relative transition probabilities have been determined in the laboratory for these lines (King 1947). Since the Li I doublet arises from the ground state, a curve of growth for ground-state lines is needed for the analysis. The V I lines fall into two groups, one including lines of multiplets 19 and 20 with mean excitation potential 0.28 ev and the other including lines of multiplets 34, 35, 36, and 37 with mean excitation potential 1.06 ev. From these two groups an excitation temperature was deduced and used to correct the abscissae of the plotted points to the value for the ground state. This small extrapolation probably gave rise to no serious errors because of the rather low excitation of even the higher-level lines and the fact that the derived temperature includes not only true temperature effects but also any influence of physical stratification in the atmosphere.

The abscissa of the preliminary curve of growth used to determine the excitation temperature was the quantity log η_0' given in Table 2. This quantity is defined by

$$\log \eta_0' = \log \frac{g f' \lambda}{\alpha(1.4)} - 25 , \tag{2}$$

in which g is the statistical weight of the lower level, f' is the relative transition probability, λ is the wave length in angstroms, and $\alpha(1.4)$ is the absorption coefficient of the H^- ion per neutral hydrogen atom per unit electron pressure at the wave length of the line and for the temperature $\theta = 1.4$ ($\theta = 5040/T$). The values of α were taken from the tables of Chandrasekhar and Breen (1946). Because, at the temperatures of the surfaces of these stars, stimulated emission is negligible, the wave-length, temperature and pressure variations occur as separate factors in α; thus in comparing two lines in the same star, only the wave-length variation of the opacity has any influence. The other factors cancel, as does the abundance of neutral hydrogen atoms. For this reason it was possible to compute a single value of log η_0' for each line, using an arbitrarily chosen temperature, and apply it in each star studied.

Having determined the excitation temperature parameter θ (exc.), an empirical curve of growth for the ground state was derived by correcting the abscissae to zero excitation. Sample curves for three stars are given in Figure 2 (the new abscissae are labeled log η_0''). Here points from the lines of the two different excitation-potential groups are distinguished by different symbols. It is clear that the two groups of lines were always sufficiently similar in strength that the excitation temperature could be determined without reference to a theoretical curve of growth.

TABLE 2

SPECTRAL LINES USED

λ	Ident.	Mult. (RMT)	E.P. (ev)	log η'
5604.943	V I	37	1.04	5.57
26.014	V I	37	1.04	5.55
27.628	V I	37	1.08	6.52
32.469	V I	1	0.07	3.52
40.971	Sc II	29	1.49	...
46.112	V I	37	1.05	5.64
57.449	V I	37	1.06	5.84
57.870	Sc II	29	1.50	...
68.369	V I	37	1.08	5.73
70.827	V I	36	1.08	6.42
71.805	Sc I	12	1.44	...
5703.562	V I	35	1.05	6.64
06.973	V I	35	1.04	6.82*
17.30	Sc I	12	1.43	...
27.024	V I	35	1.08	6.85
27.662	V I	35	1.05	5.98
37.040	V I	35	1.08	6.10
76.670	V I	36	1.08	5.31
6039.690	V I	34	1.06	5.95
58.113	V I	34	1.04	5.23
81.421	V I	34	1.05	6.00
90.184	V I	34	1.08	6.54
6111.622	V I	34	1.04	5.90
19.505	V I	34	1.06	6.28
35.36	V I	34	1.05	5.85
50.132	V I	20	0.30	5.06
89.350	V I	20	0.27	3.88
99.202	V I	19	0.29	5 16
6210.676	Sc I	2	0.00	...
13.874	V I	20	0.30	4.80
16.368	V I	19	0.27	5.34
24.507	V I	20	0.29	4.84
33.187	V I	20	0.27	4.78
45.214	V I	20	0.26	3.69
45.629	Sc II	28	1.50	...
51.83	V I	19	0.29	5.39
56.906	V I	19	0.27	4.65
66.32	V I	20	0.27	4.56
74.670	V I	19	0.27	5.00
85.185	V I	19	0.27	5.16
92.858	V I	19	0.29	5.18
6305.671	Sc I	2	0.02	...
6707.74 .89	Li I	1	0 00	...

* Derived from stellar W, not laboratory gf'.

The continuous curves plotted in Figure 2 are theoretical curves of growth which have been fitted to the empirical points. The theoretical curves are necessary for extrapolating the results from the relatively strong V I lines to the region of the often vanishingly weak Li I feature. In addition, the difference between the ordinates of the theoretical and empirical curves yields v, the most probable velocity of small-scale motions in the atmosphere; this quantity is needed in the subsequent analysis. The theoretical curves are chosen from those computed by Wrubel (1949) from Chandrasekhar's exact solution of the radiative-transfer equation in the Milne-Eddington approximation. Probably the Schuster-Schwarzschild model would apply more accurately to low-excitation lines of neutral atoms, but, in comparing similar transitions, the difference in the models is probably negligible. Wright (1951*b*) found no significant differences in the

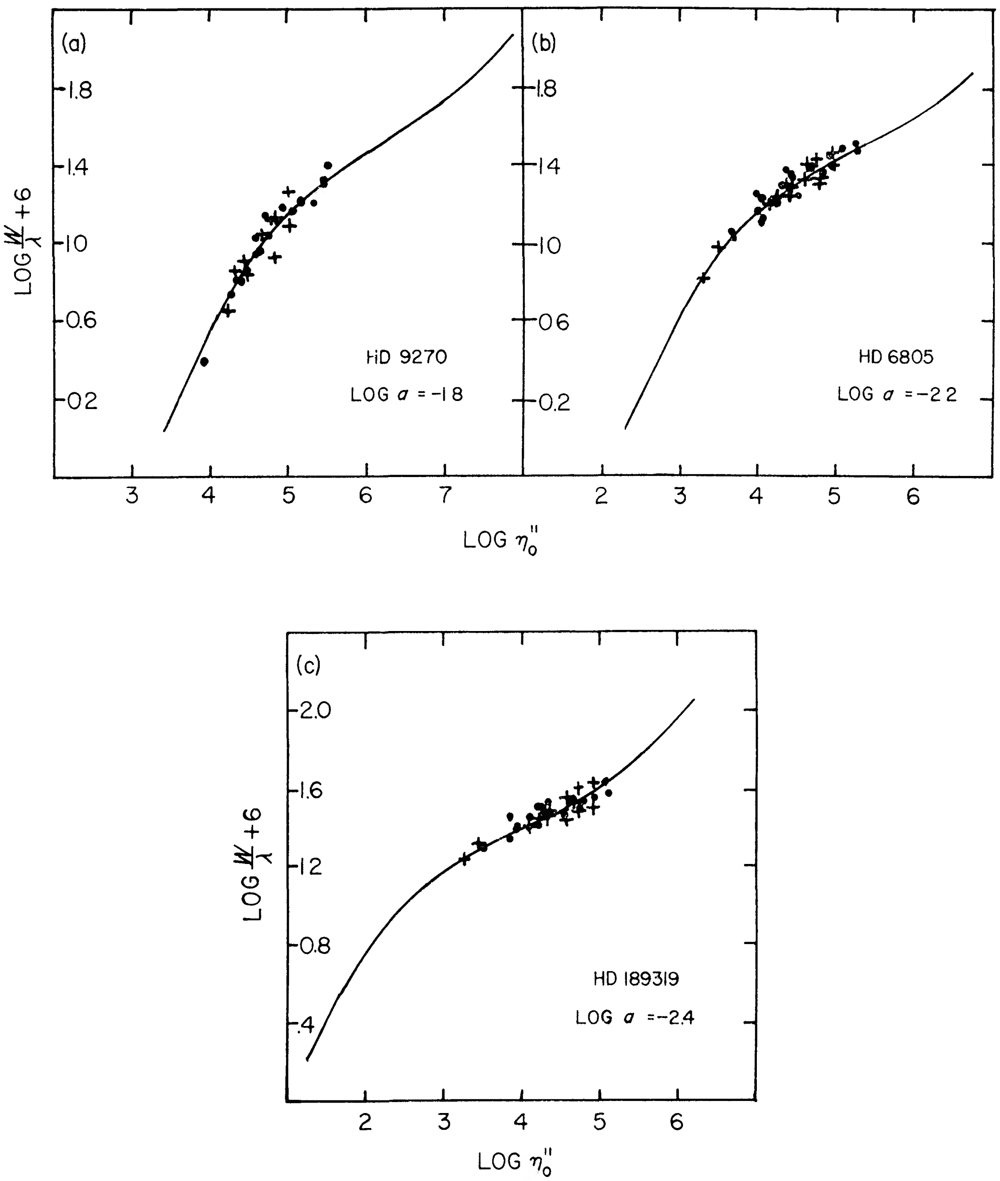

FIG. 2.—Empirical curves of growth (*points*) and adopted theoretical curves of growth for $B_0/B_1 = \frac{1}{3}$. Dots indicate 1.1-ev lines; crosses, 0.3-ev lines. Spectral types: (*a*) G8 III, (*b*) K2 III, and (*c*) K5 III.

physical parameters of α Bootis derived by treating his spectrophotometric observations with both models. The accuracy and convenience of Wrubel's curves recommend them for the type of analysis attempted here. In fitting the theoretical curves to the observations, it was found possible to determine the damping constant as accurately as necessary from the slope of the central part of the curve. The temperature gradient parameter B_0/B_1 was found to have little effect on the results. It was sufficient to compute a mean value of this quantity (using the formula given by Aller 1953, p. 294) for the V I lines and reduce the observations separately with the curves of growth for the two tabulated values nearest the mean. The final abundance was obtained by interpolation between the resulting two values, which were always negligibly different. Figure 2 indicates that the empirical points had a sufficient range in strength to specify their position on the theoretical curve of growth. The error of the fit in log η_0'' probably did not exceed 0.15, even in the coolest stars.

Thus having established a curve of growth, given the equivalent width of a *single* line of Li I, it is possible to correct this value for the difference in thermal motion between lithium and vanadium atoms and apply it to the curve to obtain the corresponding log η_0''. The abundance of Li I relative to V I then follows from the relation

$$\log \frac{Li^0}{V^0} = \log \eta_0'' \text{ (Li I)} + \log \frac{B \text{ (Li I)}}{B \text{ (V I)}} - \log \frac{c/v \text{ (Li)}}{c/v \text{ (V)}} + \log K . \quad (3)$$

Here Li^0 and V^0 represent, respectively, the number per gram of neutral lithium and neutral vanadium atoms; log η_0'' (Li I) is the abscissa of the curve of growth appropriate to the lithium line strength; the B's are the partition functions at the stellar temperatures; c is the velocity of light; the v's are the most probable velocity of small-scale motions; and K is a constant involving the transition probability and wave length of the Li I line, the standard opacity at that wave length, and the calibration of the relative transition probabilities for V I. It is not necessary to evaluate K in determining the variation of the lithium-to-vanadium ratio from star to star.

Unfortunately, the lithium doublet is completely blended, and the equivalent width of a single line is not obtainable. In order to interpret the strength of the doublet, it is necessary to determine whether the absorption coefficients of the two lines are themselves blended by the small-scale motions in the atmosphere or whether the blending is due only to large-scale motions and low spectrographic resolution, which do not affect the absorption coefficient. Using the values of v from the curves of growth, it was found that the absorption coefficients are seriously blended only in the hotter stars, in which the lines are so weak that the cause of blending is irrelevant. Nevertheless, in all stars with sufficiently strong lines, values of the abundance ratio were derived for both cases in order to estimate the maximum possible error. In the case of blended absorption coefficients, it was assumed that the profile of the total coefficient remained symmetrical and had the same height as that of the strong line of the doublet but was sufficiently broad to account for the total area under the absorption-coefficient-curves of both lines. In applying this assumption the velocity v(Li) was simply increased by a factor 1.5 (since the transition probability of the weaker line is half that of the stronger and the total absorption coefficient for the blended doublet is 1.5 times that of the stronger member), and the increased value was used both in correcting the equivalent width and in the appropriate place in equation (3). In applying the alternate assumption, v(Li) was not changed, but rather pairs of values of log η_0'' in which the members differed by log 2 were applied to the curve of growth until a pair was found for which the sum of the corresponding values of W/λ equaled the observed value (corrected for velocity differences). The sum of this pair of η_0'''s was converted to a value of log η_0'', which was then used in equation (3). These two assumptions were applied to all cases in which the equivalent width of the doublet was greater than 40 mA and gave abundance ratios differing up to 30 per cent. Since such strong lines appeared only in cool stars, the value from the "unblended" assumption was always adopted as the final result.

In addition to obtaining the abundance of neutral lithium relative to neutral vanadium from the curve of growth, the equivalent widths of two vanadium lines were applied to give values of the abundance of neutral vanadium relative to itself. These values should, of course, be unity in all stars in the absence of errors and thus served as tests of random and systematic errors in the analysis to this point. The two lines used were the weak line λ 5632.469, which was not used in deriving the curve of growth, and the strong line λ 5727.024, which was. The former is quite similar to the lithium doublet, in that it arises from a state of negligible excitation and is quite weak, even weaker than the lithium doublet in some cases. It should therefore suffer equally with the lithium feature from any peculiar effects which might introduce systematic errors in comparing the Li I doublet with the often much stronger lines of V I. The stronger line has an excitation potential of 1.08 ev and thus serves as a test of the excitation temperature as well as curve-of-growth errors relating to strong lines.

c) The Ionization Correction

Since in these stars both lithium and vanadium are essentially either neutral or singly ionized, the ratio of total abundances is related to the ratio of neutral atoms by

$$\frac{Li}{V}=\frac{Li^0}{V^0}\,\beta\,, \tag{4}$$

where

$$\beta=\left(\frac{V^0}{V^+}+\frac{Li^+/Li^0}{V^+/V^0}\right)\bigg/\left(\frac{V^0}{V^+}+1\right). \tag{5}$$

Here *Li*, *V*, Li^+, and V^+ represent, respectively, the numbers per gram of total lithium, total vanadium, Li II, and V II.

Thus the calculation of the ionization correction β is resolved into a determination of the degree of ionization of vanadium and the calculation of the relative ionization of lithium compared to vanadium. Unfortunately, insufficient lines of V II appear in the spectral region photographed, and the 60-ev excitation potential of the first excited state of Li II precludes the existence of lines of this ion; thus both these quantities must be obtained indirectly.

The degree of ionization of vanadium is determined by using the four lines of Sc I and three lines of Sc II listed in Table 2. In a method similar to that discussed in the previous subsection, it is possible to derive an abundance relative to V I from each scandium line. Subtracting the equation similar to (3) for Sc I from that for Sc II gives

$$\log\frac{Sc^+}{Sc^0}=\Delta\log\eta_0+\log\frac{B\,(\mathrm{Sc\,II})}{B\,(\mathrm{Sc\,I})}-\Delta'\,. \tag{6}$$

Here the first term on the right is the difference of the log η_0'' values for the lines involved, and Δ′ is a constant involving the absolute transition probabilities of the lines. A term in the excitation temperature has been omitted; before using each log η'' in equation (6), it was corrected to a common excitation potential by means of the V I excitation temperature. Note that all the scandium lines except those of multiplet 2 of Sc I have approximately the same excitation potential. Since V I has nearly the same ionization potential as Sc I, the ratio of ionized to neutral vanadium differs from the ratio of equation (6) only in the partition function term. Thus

$$\log\frac{V^+}{V^0}=\Delta\,\langle\log\eta_0\rangle+\log\frac{B\,(\mathrm{V\,II})}{B\,(\mathrm{V\,I})}-\Delta\,. \tag{7}$$

Here $\Delta\langle\log \eta_0\rangle$ now refers to the difference between the mean $\log \eta_0''$ of the three Sc II lines and that of the four Sc I lines. Because the absolute ratio of the two ionization states is needed in computing β, the constant Δ must be evaluated. Accurate theoretical or laboratory determinations of the absolute transition probabilities are not available for this calculation; therefore, it was done by a "semiempirical" method: Unsöld (1948) and Wright (1951*a*) have determined the relative abundances of Sc I, Sc II, V I, and V II in the sun. Applying their results to the left sides of equations (6) and (7) and obtaining $\Delta\langle\log \eta_0\rangle$ from the measurements in the Utrecht *Atlas*, values of Δ were determined. A final value $\Delta = +0.38$, near the mean of the determinations, was adopted. This method of determining the constant minimizes the errors involved in using equation (7), which are introduced by such processes as the use of the V I excitation temperature for both ionization states, since the errors also enter into the determination of the constant and tend to cancel. Note that, although it was not possible to measure all the listed scandium lines in every star, only one value of Δ was needed because the difference between the value of $\log \eta_0''$ from a single line and the mean of all the lines of the same ion listed is, in principle, constant, and these differences could be obtained from the stars in which all lines were observed and applied in the others. Thus the $\Delta\langle\log \eta_0\rangle$ always referred to all seven lines, even though in some cases only six could be observed.

The second term in the numerator of β is obtained simply from the ratio of the Saha ionization equations for the two elements. The quantity then depends only on the ionization temperature:

$$\log \frac{Li^+/Li^0}{V^+/V^0} = -\theta\,[I\,(\text{Li I}) - I\,(\text{V I})] + \log \frac{B\,(\text{Li II})}{B\,(\text{Li I})} - \log \frac{B\,(\text{V II})}{B\,(\text{V I})}\,. \qquad (8)$$

Since the difference in ionization potentials, I, is only 1.34 ev, the quantity is not highly sensitive to the value of θ.

Other investigators (see, e.g., Greenstein 1948; Burbidge and Burbidge 1957) have found that the ionization temperature is closer to the effective temperature than to the excitation temperature. In the present study, two estimates of the ionization temperature were used, both based on the effective temperature but reflecting in different degrees possible differences between stars of similar spectral classifications. For the first estimate, a suggestion of Popper (1959) was followed and a relation established between the $B - V$ color index and the effective temperature for dwarf stars. The colors and spectral classifications were taken from the lists of Johnson and Morgan (1953), Johnson and Harris (1954), and Johnson (1955), and the temperatures were taken from the table of Keenan and Morgan (1951). The relation was well described by the linear equation

$$\theta\,(\text{eff.}) = 0.42 + 0.71\,(B - V)\,. \qquad (9)$$

Following Popper, this relation was applied to the eighteen stars of this study for which $B - V$ colors were available, and the resulting temperatures were used as estimates of the ionization temperature. These eighteen stars were also used to establish a relation between effective and excitation temperature, which is

$$\theta\,(\text{eff.}) = \theta\,(\text{exc.}) - 0.19\,. \qquad (10)$$

The scatter in this relation is approximately ± 0.1. It is apparently independent of luminosity. Equation (10) was applied to all stars to give the second estimate of the temperature to be used in equation (8). For those stars to which equation (9) could not be applied for lack of a $B - V$ determination, a correlation between temperature and spectral classification, based upon the eighteen stars with $B - V$ values and upon the table of Keenan and Morgan, was applied. This correlation did not differ appreciably from the Keenan-Morgan table.

In order to choose or interpolate between the two values of β resulting from the two θ's, the values of θ were used with the value of V^+/V^0 for each star to compute the electron pressure appropriate to each temperature. In many stars the two values of P_e were nearly coincident; these established the run of P_e with temperature and luminosity. Where the two values differed, they were taken to define the possible range in P_e, and a value was chosen from the range according to the star's temperature and luminosity. Instrumental in this choice were twenty-nine values of the absolute visual magnitude based on the width of the K-line emission kindly provided by Dr. O. C. Wilson in advance of publication (these values are more recent than the list given by Wilson and Bappu 1957). In the stars in which an extremely large range of P_e was found (up to a factor of 10^3) the luminosity criterion clearly selected one of the extremes. Since the ratio of the two values of β never exceeded 4, the value of β corresponding to the adopted P_e is not likely to be in error by more than a factor of 2.

d) The Absolute Abundance of Vanadium

Two quantities of interest may be derived from the fit of the theoretical and empirical curves of growth. The first is a quantity which is proportional to the ratio of the absolute abundance of vanadium to the continuous opacity, i.e., "the abundance of vanadium above the photosphere." This quantity, called here γ, is derived by correcting for ionization the analogous quantity for neutral vanadium, γ^0, which is calculated from the equation

$$\log \gamma^0 = \log \eta_0 - \log \eta_0'' + \log B\ (\text{V I}) - \log \frac{c}{v}. \tag{11}$$

The first two terms on the right are the difference between coincident abscissae of the theoretical and empirical curves of growth. Then γ is a measure of the strength of the vanadium lines and of the metallic lines in general.

The absolute abundance of vanadium may be calculated from γ according to the relation

$$\log V = \log \gamma + \log RP_e + \delta. \tag{12}$$

The term RP_e is the ratio of the opacity at the temperature and pressure of the stellar atmosphere to that used in computing the quantities $\log \eta_0'$ (eq. [2]). The final term is a constant involving the calibration of the vanadium transition probabilities and the abundance of hydrogen by mass, as well as some physical constants. Using Allen's (1955) absolute transition probabilities and a hydrogen abundance of 0.70, the value of δ is approximately 22.8. The absolute abundance of vanadium is a quantity particularly useful in testing for systematic effects in the ionization correction β, since it is proportional to the electron pressure, which is computed in a manner similar to one of the terms of β. In addition, any gross true variations in the vanadium abundance will appear.

IV. RESULTS

a) The Table of Numerical Results

The results of this study are presented in Table 3. Here all the values related to each star are collected in a single column headed by the star's HD number. The stars are ordered by spectral classification rather than by number, to group together stars of similar physical properties. The entries in the columns are divided into several groups. The first group gives properties of the stars not determined by the observations of this study and includes the spectral class; the spectroscopic group according to Roman (1952) where available; the absolute visual magnitude according to Wilson or according to Roman's (1952) calibration of the luminosity classes (the latter given in parentheses); and the ionization-temperature parameter based on the $B - V$ color index where avail-

TABLE 3

NUMERICAL RESULTS

HD	(Sun)	9270	62345	104979	219615	90537	98430	168723
Type	G2V	G8III	G8III	G8III	G8III	G8III-IV	G8III-IV	G8IV
Group	---	st-1	st-1	wk-1	CN	st-1	wk-1	wk-1
M_V	+4.7	-0.2	(0.0)	(0.0)	+1.9	(+1.3)	(+1.3)	(+3.5)
θ (ion)	0.88	1.11	1.08	1.04	1.04	1.04	1.04	1.09
θ (exc)	1.11	1.24	1.24	1.24	1.37	1.33	1.52	1.21
v (km/sec)	1.9+	2.7	4.7	3.7	2.8	2.7	2.6	2.4
$\Delta\lambda$ (A)	---	0.34	0.29	0.30	0.27	0.27	0.27	0.28
log V^+/V^o	1.7	1.5	1.2	1.6	1.7	1.0	0.9	1.3
log γ	-7.3	-6.2	-6.7	-6.6	-6.3	-6.5	-6.5	-6.4
log P_e (a)	+1.6	0.0	+0.5	+0.5	+0.3	+1.1	+1.2	+0.4
(b)	+1.3	+0.5	+0.8	+0.4	-0.8	+0.3	-1.0	+0.9
(c)	+1.5	0.0	+0.6	+0.5	+0.3	+0.9	+0.8	+0.6
log V	16.1	15.9	16.1	16.0	16.1	16.6	16.5	16.3
log V^o/V^o (wk)	<0.7	<0.2	<0.2	<0.5	<0.3	<0.3	-0.1	≤0.3
(st)	0.0	0.0	-0.1	-0.1	0.0	-0.2	0.0	-0.1
-log W(Li)/λ	6.26	5.24	5.14	>5.87	5.56	>5.71	>6.00	>6.06
log Li^o/KV^o	3.1	2.9	2.3	<2.6	2.9	<2.2	<1.7	<2.0
log Li/KV (a)	3.9	4.1	3.4	<3.7	3.9	<3.3	<2.8	<3.2
(b)	4.0	4.0	3.3	<3.7	4.1	<3.4	<3.1	<3.1
(c)	4.0	4.1	3.4	<3.7	3.9	<3.3	<2.8	<3.1

+ from Minnaert (1953)

TABLE 3 cont'd

HD	4128	8512	27371	27697	28305	76294	197989	165341
Type	KOIII	KOIII	KOIII	KOIII	KOIII	KOIII	KOIII	KOV
Group	st-1	wk-1	st-1	st-1	wk-1	4150	st-1	---
M_V	+0.8	+1.0	+0.7	+0.7	+0.5	+0.5	+1.3	+5.4
θ (ion)	1.11	1.11	1.12	1.12	1.15	1.11	1.15	0.98
θ (exc)	1.35	1.43	1.16	1.31	1.21	1.42	1.41	1.10
v (km/sec)	2.8	2.9	3.0	4.1	3.2	3.5	3.0	1.8
Δλ (A)	0.32	0.29	0.27	0.29	0.29	0.29	0.33	0.26
log V^+/V^o	1.2	1.2	1.4	1.0	1.4	1.4	1.3	1.1
log γ	-6.1	-6.2	-6.2	-6.6	-6.2	-6.2	-6.3	-7.1
log P_e (a)	+0.3	+0.3	+0.1	+0.4	-0.2	+0.1	-0.1	+1.4
(b)	-0.1	-0.3	+1.2	+0.4	+0.8	-0.8	-0.6	+1.9
(c)	+0.1	+0.3	+0.5	+0.4	+0.1	0.0	-0.1	+1.7
log V	16.3	16.3	16.6	16.0	16.1	16.1	15.9	16.6
log V^o/V^o(wk)	<-0.4	-0.1	+0.5	+0.5	<0.4	<0.0	-0.2	0.3
(st)	-0.2	-0.1	-0.1	-0.3	-0.1	-0.1	-0.1	0.0
-log W(Li)/λ	>5.88	>5.74	5.36	5.71	5.81	≥5.77	>5.97	>5.75
log Li^o/KV^o	<1.8	<2.0	2.7	2.4	2.2	≤2.2	<1.9	<2.3
log Li/KV (a)	<3.0	<3.2	3.9	3.5	3.4	≤3.4	<3.1	<3.3
(b)	<3.0	<3.3	3.7	3.5	3.2	≤3.5	<3.2	<3.2
(c)	<3.0	<3.2	3.8	3.5	3.3	≤3.4	<3.1	<3.2

TABLE 3 cont'd

HD	96833	108381	10476	11909	206778	20468	6805	12533
Type	K1III	K1III-IV	K1V	K1p	K2Ib	K2II	K2III	K2III
Group	wk-1	4150	---	---	---	---	---	---
M_V	+0.7	(+1.3)	+6.5	---	-4.3	(-2.5)	+1.1	-1.6
θ (ion)	1.22	1.14	1.01	1.03	1.50	1.36	1.24	1.24
θ (exc)	1.37	1.44	1.18	1.04	1.61	1.60	1.51	1.67
v (km/sec)	3.0	2.8	2.1	1.8	2.7	3.0	2.4	2.9
Δλ (A)	0.28	0.32	0.26	0.30	0.42	0.30	0.32	0.32
log V^+/V^o	1.0	1.0	1.1	1.5	0.8	1.1	0.9	1.0
log γ	-6.0	-5.7	-6.9	-6.3	-4.9	-5.4	-6.0	-5.3
log P_e (a)	-0.4	+0.3	+1.1	0.60	-2.2	-1.4	-0.4	-0.5
(b)	0.0	-0.5	+1.3	+2.0	-1.6	-1.8	-1.0	-2.3
(c)	-0.2	0.0	+1.2	0.60	-2.0	-1.6	-0.7	-1.6
log V	16.2	16.7	16.4	16.5	15.9	15.8	15.8	15.8
log V^o/V^o(wk)	-0.1	-0.6	<0.4	<0.6	-0.8	-0.1	-0.2	-0.4
(st)	-0.1	0.0	0.0	0.0	+0.1	+0.3	+0.1	0.0
-log W(Li)/λ	4.68	5.22:	>5.99	5.14	≥5.65	4.73	4.80	5.36:
log Li^o/KV^o(ub)	2.9	1.9:	<2.4	3.1	≤0.5	2.2	2.4	1.3:
(bl)	3.0	---	---	3.1	---	2.3	2.4	---
log Li/KV (a)	4.2	3.1:	<3.4	4.2	≤2.2	3.7	3.7	2.6:
(b)	4.1	3.2:	<3.4	3.9	≤2.0	3.8	3.8	2.9:
(c)	4.2	3.1:	<3.4	4.2	≤2.1	3.7	3.8	2.8:

TABLE 3 cont'd

HD	12929	124897	22049	225212	176670	186791	20644	3627
Type	K2III	K2IIIp	K2V	K3Ib	K3II	K3II	K3II-III	K3III
M_V	+1.3	+0.5	+6.5	---	(-2.5)	-2.0	(-1.0)	+0.2
θ (ion)	1.24	1.29	1.06	1.56	1.50	1.50	1.42	1.34
θ (exc)	1.52	1.43	1.25	1.75	1.75	1.74	1.51	1.75
v (km/sec)	2.3	2.5	2.9	2.3	2.8	3.0	3.2	2.5
Δλ (A)	0.26	0.31	0.27	0.40	0.28	0.37	0.32	0.26
log V^+/V^o	1.3	0.8	0.7	0.8	0.6	0.8	1.0	0.4
log γ	-5.6	-6.0	-7.4	-4.2	-5.5	-5.2	-5.6	-5.9
log P_e (a)	-0.7	-0.6	+1.2	-2.6	-2.0	-2.3	-1.8	-0.6
(b)	-1.4	-0.3	+1.2	-2.6	-2.4	-2.6	-1.0	-2.3
(c)	-1.0	-0.5	+1.2	-2.6	-2.2	-2.4	-1.6	-0.6
log V	15.9	16.0	16.0	16.1	15.2	15.3	15.6	16.1
log V^o/V^o(wk)	-0.1	-0.2	<0.5	-1.0	0.0	-0.4	-0.2	-0.4
(st)	+0.4	+0.1	-0.1	0.0	-0.1	+0.4	0.0	0.0
-log W(Li)/λ	>5.75	≥6.38	>5.16	5.52:	4.84	4.57	5.10	5.49
log Li^o/KV^o(ub)	<1.5	≤0.8	<2.3	-0.1:	1.8	2.2	1.8	1.0
(bl)	---	---	---	---	1.8	2.2	1.8	---
log Li/KV (a)	<2.8	≤2.2	<3.3	1.6:	3.4	3.8	3.4	2.3
(b)	<2.9	≤2.1	<3.3	1.6:	3.5	3.8	3.2	2.6
(c)	<2.9	≤2.2	<3.3	1.6:	3.4	3.8	3.4	2.3

TABLE 3 cont'd

HD	81797	98262	35620	30504	201251	50778	69267	90432
Type	K3III	K3III	K3p	K4II	K4II	K4III	K4III	K4III
M_v	-1.0	-0.7	---	(-2.5)	(-2.5)	(-0.1)	+0.2	(-0.1)
θ (ion)	1.34	1.41	---	1.52	1.52	1.45	1.47	1.45
θ (exc)	1.55	1.60	1.57	1.42	1.73	1.61	1.55	1.50
v (km/sec)	2.6	2.8	2.2	2.2	2.5	2.3	2.3	1.9
Δλ (A)	0.28	0.28	0.28	0.30	0.30	0.28	0.27	0.29
log V^+/V^o	0.2	0.5	0.3	0.0	+0.5	-0.1	+0.2	-0.1
log γ	-5.6	-5.7	-5.4	-5.6	-4.9	-5.6	-5.4	-5.4
log P_e (a)	-0.5	-1.2	---	-1.5	-2.2	-1.0	-1.4	-1.0
(b)	-0.5	-1.2	-0.9	+0.6	-2.2	-0.7	-0.6	+0.1
(c)	-0.5	-1.2	-0.9	-1.5	-2.2	-0.8	-1.0	-0.9
log V	16.5	15.9	16.4	15.8	15.9	16.3	16.2	16.5
log V^o/V^o (wk)	-0.2	-0.2	-0.3	-0.1	-0.5	0.0	-0.2	+0.2
(st)	0.0	+0.2	+0.3	+0.1	+0.3	+0.3	+0.1	0.0
-log W(Li)/λ	4.75	5.31	5.43	4.75	≥5.46	>5.71	≥5.62	>5.62
log Li^o/KV^o (ub)	1.8	1.3	0.8:	1.7	≤0.4	<0.4	≤0.6	<0.3
(bl)	1.8	---	---	1.7	---	---	---	---
log Li/KV (a)	3.1	2.8	---	3.1	≤2.0	<1.8	≤2.0	<1.6
(b)	3.1	2.8	2.2:	2.7	≤2.0	<1.7	≤1.8	<1.4
(c)	3.1	2.8	2.2:	3.1	≤2.0	<1.7	≤1.9	<1.6

TABLE 3 cont'd

HD	200905	196321	29139	189319	201091	201092	6860
Type	K5Ib	K5II	K5III	K5III	K5V	K7V	M0III
M_V	-4.0:	(-2.5)	+0.2	-1.0	+6.6	+8.4	-0.8
θ (ion)	1.58	1.55	1.49	1.50	1.27	1.40	1.55
θ (exc)	1.39	1.48	1.73	1.65	1.48	1.52	1.64
v (km/sec)	2.0	2.0	1.7	2.0	1.5	1.3	1.8
Δλ (A)	0.38	0.30	0.32	0.30	0.25	0.23	0.30
log V^+/V^o	+0.4	-0.3	0.0	-0.2	-0.4	-1.0	-0.2
log γ	-4.4	-5.3	-4.5	-5.3	-7.0	-6.5	-4.9
log P_e (a)	-2.4	-1.5	-1.4	-1.2	+0.6	+0.3	-1.6
(b)	+0.4	+0.4	-1.7	-0.9	+0.6	+0.8	-0.8
(c)	-2.4	-1.5	-1.4	-1.2	+0.6	+0.8	-1.2
log V	16.2	16.1	17.0	16.2	16.2	16.9	16.8
log V^o/V^o (wk)	-0.6	-0.2	-0.4	-0.2	≤0.1	-0.7:	+0.2
(st)	0.0	+0.2	+0.1	0.0	0.0	0.0	+0.1
-log W(Li)/λ	4.53	>5.51	>5.18	4.89	≥5.63	5.26	>5.42
log Li^o/KV^o (ub)	1.1	<0.3	<-1.1	1.1	≤1.9	1.7	<0.0
(bl)	1.2	---	---	1.1	---	---	---
log Li/KV (a)	2.8	<1.7	<0.3	2.4	≤2.8	2.4	<1.3
(b)	2.3	<1.3	<0.3	2.3	≤2.8	2.3	<1.2
(c)	2.8	<1.7	<0.3	2.4	≤2.8	2.3	<1.2

able and on the spectral class otherwise. The second group gives physical parameters of the atmosphere derived quite directly from the measurements of the spectrograms; this group contains the excitation-temperature parameter; the most probable velocity of vanadium atoms, including thermal motion and microturbulence; the half-width for weak lines, which includes the effect of macroturbulence (and instrumental broadening); and the degree of ionization of vanadium. The third group gives physical properties largely derived from the previously listed quantities, including the quantity γ; the electron pressure, giving (*a*) an estimate based on the ionization temperature given in the first group, (*b*) an estimate based on the excitation temperature through equation (10), and (*c*) the finally adopted estimate; and the absolute abundance of vanadium in atoms per gram of stellar material. The remaining entries include the abundance of vanadium relative to itself as a test for errors, derived from both the weak line at λ 5632 ("wk") and the strong line at λ 5727 ("st"); the equivalent width of the lithium doublet; the abundance of neutral lithium relative to neutral vanadium divided by the constant K, giving the result for both blended ("bl") and unblended ("ub") absorption coefficients where these differ; and, finally, the abundance of total lithium relative to total vanadium, giving three values labeled *a*, *b*, and *c* with the same significance as in the electron-pres-

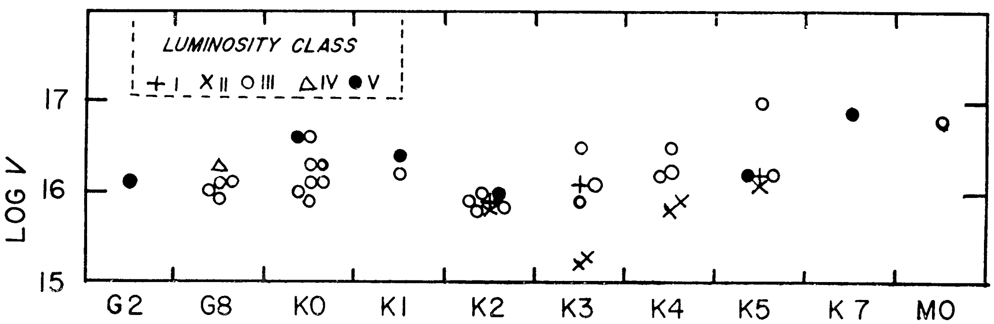

FIG. 3.—Correlation between the absolute abundance of vanadium, spectral type, and luminosity class. Individual star values are plotted.

sure entries. The symbol ($<$) in these columns indicates an upper limit based on a maximum depth estimate for an invisible line, and ($\leq$) indicates an upper limit based on a marginally visible feature. A colon (:) indicates an unusually uncertain value.

The ration of neutral atoms, Li^0/KV^0, is presented because these values are quite well established and are less likely to be seriously modified by future improvements in understanding the properties of the atmospheres than are values of the total abundance ratio. In the cases in which two values of Li^0/KV^0 are presented, the final result was always computed from the value based on the assumption of unblended absorption coefficients. The two values are given to show the maximum possible error from this source.

b) The Properties of the Atmospheres

The tabulated physical parameters of the stellar atmospheres are of considerable interest because no such extensive table for cool stars has been published heretofore. While the accuracy of the values suffers from the limited amount of material entering each one, their comparative magnitudes gain from the homogeneity of the data.

The tabulated values of the absolute abundance of vanadium are satisfactorily constant, in view of the large uncertainties in the electron pressure. All the values are contained in the range of log V from 15.2 to 17.0, and all but seven are in the range from 15.8 to 16.6, inclusive. Figure 3 exhibits the abundance of vanadium as a function of spectral and luminosity class (omitting intermediate classes and peculiar stars) and shows no significant trend. This indicates primarily that the adopted electron pressures are consistent throughout the stars studied and that therefore the similarly derived ionization corrections for the lithium-to-vanadium ratios should be devoid of spurious systematic effects. The agreement of the solar value of log V indicates that no serious differences have arisen from the different source of data.

It is interesting to notice a few of the extreme cases. All the values below log $V = 15.8$ are K3 stars, the two lowest being HD 176670 and HD 186791, both K3 II. In these cases it seems likely that the electron pressures are too low, since the adopted values are almost as low as that for the K3 I*b* star HD 225212, which has a normal log V. A high value is indicated for the K5 III star HD 29139. This star is over 1 mag. fainter than the other K5 III star and therefore should have a higher electron pressure; however, the derived ranges do not permit this. Thus the high value of log V depends on a value of log P_e, which may well be low. The value of γ for this star is quite near that of the supergiant HD 200905, which must have a lower opacity. It would seem possible that the abundance of vanadium is high in HD 29139.

Several of the most luminous stars show peculiar excitation temperatures. The most extreme cases of this are HD 200905 (K5 I*b*), HD 196321 (K5 II), and HD 30504 (K4 II), which have excitation temperatures almost as high as some K0 III stars. These values probably reflect the large departures from thermodynamic equilibrium expected in the tenuous atmospheres of these stars but still adequately describe the populations of the two energy states from which they were derived.

A brief examination of the three stars studied which are called "peculiar" from low-dispersion spectrograms is in order. HD 11909 is classed K1p by Roman, who states that the H-lines and λ 4290 are strong and indicate luminosity class II, while the CN bands and Sr II are weak, indicating class III. The parameters of the atmosphere listed in Table 3, with the exception of θ(exc.) and v suggest that this star should be classified G8 III. The electron pressure adopted for this star is the value derived from the ionization temperature listed near the top of the column, which rests on a color index measured by G. Wallerstein at the Leuschner Observatory and kindly communicated to the writer. This choice makes the star minimally abnormal. The anomalies in θ and v may arise from the binary nature of this star; it is a single-line spectroscopic binary with a rather close companion (Gordon 1946). These effects could not arise from blending with incipiently visible lines of the companion, since the epoch of the spectrograms is at the phase of maximum separation of the lines (at least 0.4 A) and no trace is seen of the lines of the companion.

The other peculiar stars, HD 35620 and HD 124897, do not differ significantly in the parameters derived here from stars of classes K3 III and K2 III, respectively.

The general trends in the electron pressures and the excitation temperatures are according to expectation. Further, the values of the electron pressure for the hotter dwarfs and giants agree well with the values given by Aller (1953, p. 225). Figure 4 gives the relations of the corrected half-width for weak lines $\Delta\lambda'$ and the most probable velocity v to the surface (effective) temperature. The former quantity is equal to the mean $\Delta\lambda$ for weak lines approximately corrected for the instrumental profile by assuming that the true and instrumental half-widths add according to their squares. The points in the figure are averages over a single spectral type and luminosity class and are presented on effective-temperature abscissae because the ordinate quantities may be expected to be correlated with surface temperature. Since the contribution of thermal motions to v is small, the temperature dependence exhibited by this quantity in Figure 4 is due to a correlation of the microturbulent velocity with temperature. There is little correlation of v with luminosity, except to separate the dwarfs from the more luminous stars. On the other hand, $\Delta\lambda'$ is little correlated with temperature but strongly dependent on luminosity. The comparison of the two parts of Figure 4 indicates that large-scale turbulence is the principal line-broadening agent in the giants and supergiants and that these motions are strongly dependent on luminosity but largely independent of temperature. The lack of dependence of v upon luminosity is in contradiction with previous results for hotter stars (see, for example, Greenstein 1948), but, since the lines used here are largely on the central part of the curve of growth and the derived velocities cannot be much affected by errors in the excitation temperatures, it would seem that this discordant result is quite well established. The velocities derived from the V I lines refer to different physical depths in different stars, but the significance of this fact is not clear at present.

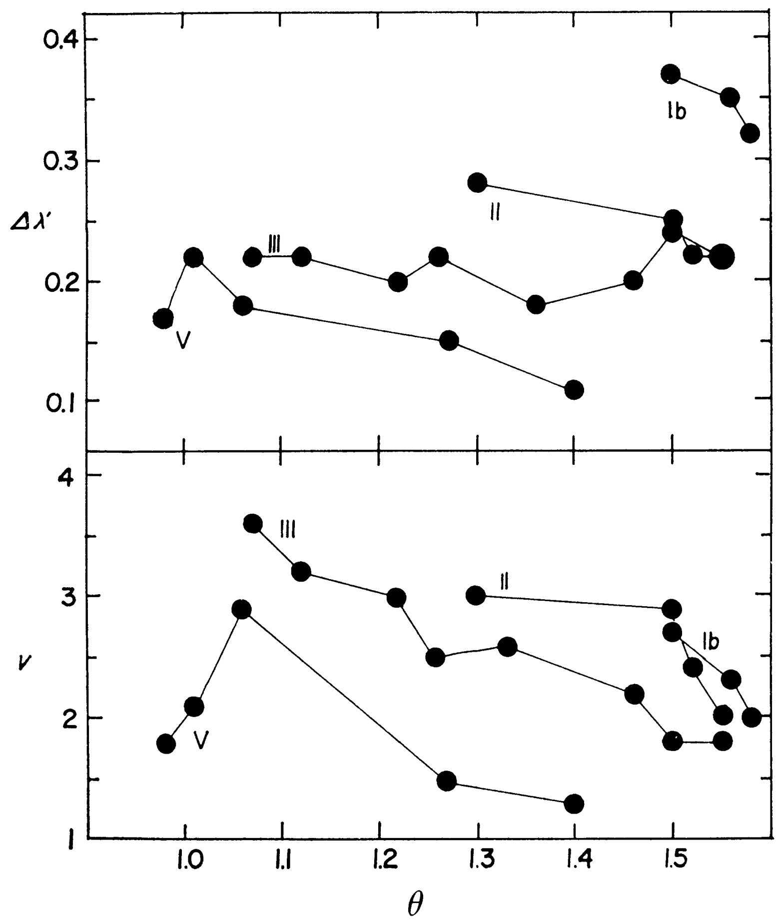

FIG. 4.—Mean half-width corrected for instrumental effects (Δλ′, A) and curve-of-growth velocity parameter (v, km/sec) correlated with effective temperature parameter for four luminosity classes. Points are averages over spectral subclasses.

The sample of stars included in this study which are assigned by Roman to her various spectroscopic groups is too small to permit a definitive test of the correlation of the physical parameters of the atmospheres with these groups. Such an attempt was made with the material available, and, in general, no correlation was found, except that possibly "4150" and "weak CN" groups tend to have lower excitation temperatures and higher degrees of ionization than the other stars. The latter effect could arise from a low electron pressure caused by reduced metal abundances. The reason for the low excitation temperature is not obvious, but the effect has been noted also by Greenstein and Keenan (1958).

c) *Relative Abundances*

The principal results of this paper—the abundance ratios of lithium to vanadium—are each based upon the photometry of only a single spectral feature; therefore, they are much more sensitive to random errors than are the results discussed in the previous subsection. The values of the abundance of neutral vanadium relative to itself (log V^0/V^0) were derived to test both the random and the systematic errors resulting from the photometry and the curve-of-growth analysis.

The values of log V^0/V^0 derived from the weak vanadium line (λ 5632) should exhibit any systematic errors arising from physical stratification in the stellar atmospheres or basic inadequacies of the analysis which affect the ratios of neutral lithium relative to neutral vanadium. This line is too weak to be measured accurately in the G8 and K0 stars; therefore, most of the log V^0/V^0(wk) values listed for these stars are upper limits, and the rest are unreliable. Considering only the values derived for stars cooler than K1, it is seen that they are systematically negative, indicating a possible error in the laboratory transition probability or a depression of the continuum at this wave length, but there is no trend with temperature. However, the values for the three supergiants are all low. A plot was made of log Li/KV versus log V^0/V^0(wk); no correlation was found.

The strong vanadium line (λ 5727) was measurable in all the stars, and a complete set of values of log V^0/V^0(st) has been obtained. These are affected by the choice of excitation temperature because of the 1.08-ev excitation potential of the line and by the choice of damping constants in the cooler stars. Inspection of Table 4 indicates a slight systematic increase in this value with decreasing temperature, amounting to about 0.2 for the entire range of stars. The results of the previous paragraph indicate that this has a negligible effect on the lithium abundance ratios. There is no effect in log V^0/V^0(st) connected with the supergiants.

The vanadium-to-vanadium ratios indicate that no significant systematic error has been introduced into the results by the application of a simple curve-of-growth analysis to the lines of the range of strength and excitation used here. A possible exception to this statement is in the case of the supergiants, where the vanadium ratios indicate that the final results may be low, but probably by a factor less than 5.

The scatter in the vanadium-to-vanadium ratios has been evaluated on the assumption of no trends in either ratio. The usual analysis yielded 0.10 as the probable error of a single measurement of log V^0/V^0(st), including all the stars observed; and 0.12 as the probable error of a single measurement of log V^0/V^0(wk), considering all stars later than K1 except those for which a maximum is indicated and HD 201092, for which the exposure was too weak in this region to give a reliable result. These probable errors, which include all sources of error affecting log Li^0/KV^0, are negligible compared to the range of 4 in log Li/KV.

Thus these test quantities indicate that there is no spurious trend in the ratio of neutral lithium to neutral vanadium and that the random errors in this quantity are small compared to the range in the final ratios of total abundances. Further, the calculation of the absolute abundance of vanadium discussed in the previous subsection indicates that there is no false trend in the ionization correction.

It is interesting, but not essential to the interpretation of the results, to evaluate the constant K which appears in the abundance ratios. This constant may be calculated from Allen's (1955) values for the absolute transition probabilities of the V I lines and the Li I lines, and from the opacity α (eq. [2]) at λ 6708 as given by the tables of Chandrasekhar and Breen. The resulting value is 2.0×10^{-8}.

An alternate procedure is to use the ratio of lithium to calcium in the sun determined by Greenstein and Richardson (1951) and the ratio of calcium to vanadium given by Unsöld (1948) to determine a ratio of lithium to vanadium devoid of an undetermined constant. The comparison of this ratio with that in Table 3 gives a value of K equal to 1.1×10^{-8}.

In view of the various evaluations of the transition probabilities and analyses of the solar atmosphere involved in these two values, the agreement is quite satisfactory. It reflects favorably upon the results obtained here for the sun and upon the procedure used in general.

d) Discussion of Errors

Before concluding this presentation of the numerical results, it is necessary to estimate the probable error of the finally adopted abundance ratio, considering all the likely sources of error. This estimate was made by subjecting the various initial and intermediate quantities in the reductions for a typical star to perturbations equal to the probable errors of these quantities. The perturbations were then propagated separately through the reduction procedure, and the increment in the result was determined for each source of error. The sum of the squares of these increments was taken as the square of the probable error of the result. The star used in the analysis was HD 96833, a K1 III star with a strong Li I feature.

Errors in the equivalent width of the Li I feature, in the excitation temperature, and in the two parameters derived in fitting the theoretical curve of growth, $\log \eta_0'' - \log \eta_0$ and $\log v$, all contribute significantly to the error in the ratio of neutral atoms, Li^0/KV^0. Using estimates of these contributing errors approximately equal to their averages over all the stars, the total probable error in $\log Li^0/KV^0$ was found to be 0.15. This is in excellent agreement with the estimate of 0.12 derived for the probable error of the analogous quantity $\log V^0/V^0$ (wk) which includes essentially the same errors but could be obtained without considering them individually.

The error in the ionization correction β is due entirely to the error in the estimate of the ionization temperature; inaccuracy in V^+/V^0 contributes negligibly in the test star. In cooler stars the importance of the error in V^+/V^0 grows in relation to that in the temperature. The value of this ratio depends in part on the ionization constant Δ, which rests on Unsöld's and Wright's estimates of absolute transition probabilities. The accuracy of Δ is difficult to assess, but an error of 1.0 in this quantity (a factor of 10 in V^+/V^0) would change β by a factor of only 2 in the coolest star studied. The error in the ionization temperature is also difficult to estimate, since the value used rests on the assumed similarity of the ionization and effective temperatures. The error was taken to be equal to that in the excitation temperature, which is probably a generous estimate, since the adopted value of β is a mean chosen with the aid of additional astrophysical data. In the test star, the error in $\log \beta$ was estimated to be 0.20.

The net error in the final abundance ratio, $\log Li/KV$, was then estimated to be 0.25. While this estimate is rather uncertain, it is safe to assert that differences of 0.5 in the logarithms (factors of 3 in the ratios) are real.

In this connection it is interesting to observe that the two independently derived abundance ratios from the two spectrograms of HD 9270 differed by only 30 per cent. In HD 69267, where the intermediate quantities differed generally by more than the average errors and only an upper limit could be estimated for the abundance, the two independently derived ratios differed by 62 per cent. These latter values depended strongly on the amount of grain "noise" in the tracings at λ 6708, since this was used to estimate the maximum equivalent width possible for the Li I doublet. These results indicate that the errors estimated above are quite reasonable.

V. DISCUSSION

a) The Abundance of Lithium

The values of the quantity $\log Li/KV$ derived in this study are presented graphically in Figure 5, where they are plotted as functions of spectral class. The values derived from measurable lines are indicated by circles, and triangles are used to represent upper limits. Approximate luminosity classes are indicated by the degree of filling of these symbols. The value from the star HD 11909 has been presented with the G8 stars, since this classification is more harmonious with the derived atmospheric parameters than the K1 given by Roman. Note that horizontal displacements within a spectral subclass of the points in the figure have been made arbitrarily to reduce crowding; they have no significance. The figure shows that within any spectral subclass the abundance ratios are highly variable, with a spread of up to a factor of 100 in the cooler stars. Since small abundance ratios are unobservable in the hotter stars, such a range may also be present among these objects. No correlation with luminosity is evident, neither in the gross features of Figure 5 nor in a more detailed comparison of the Wilson M_v's with the abundance ratios (Table 3). Figure 5 also exhibits an envelope limiting the maximum abundance ratio for each class which declines with the surface temperature. Both the appearance of the spectra and the results on the absolute abundance of vanadium (see

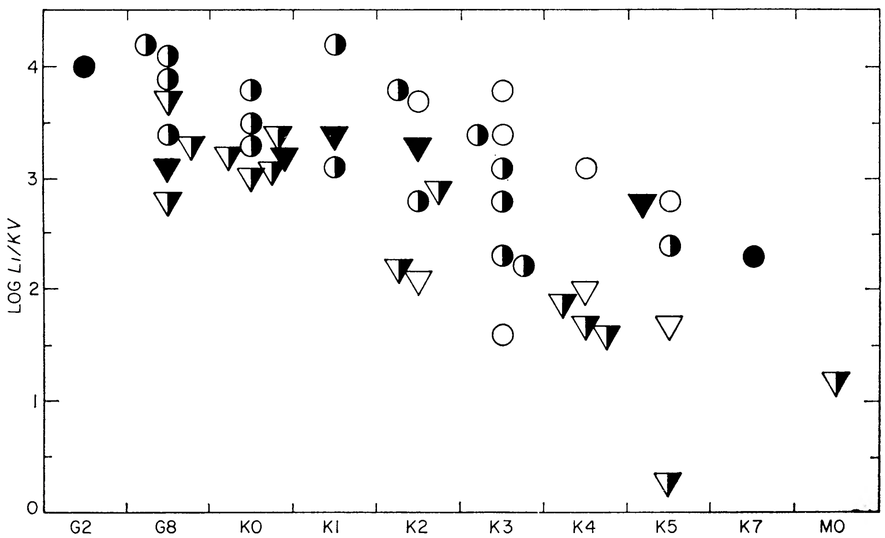

FIG. 5.—Abundance of lithium relative to vanadium, divided by constant K, correlated with spectral type. Circles are values from measured lines; triangles are upper limits. Open figures are stars of luminosity classes I*b* and II; filled figures are classes IV and V; and half-filled figures are intermediate luminosities.

Fig. 3) indicate that these variations are due to changes in the abundance of lithium. It was suggested that the star HD 29139 may have an abundance of vanadium exceeding normal; but if the abundance of vanadium is high by a factor of 10 in this star, the abundance of lithium still does not approach the maximum observed for K5 stars. In general, no further correlation can be established between the abundance of lithium and any of the properties of the atmospheres listed in Table 3.

It is conceivable that the results obtained here do not reflect real changes in the abundance of lithium nuclei at all but, rather, different equilibria between lithium atoms and some unobservable molecular state. It seems a priori unlikely that these equilibria can change enough between stars of similar atmospheric conditions to account for the variations exhibited in Figure 5, but it is nevertheless worthwhile to consider molecules in a little detail.

Each of the molecules H_2, Li_2, and LiH is composed of two atoms with one active electron, joined by a covalent bond. Of the two lithium compounds, LiH will be the most abundant in a star because the partial pressure of H is large compared to that of Li and also because LiH is more strongly bound than Li_2. If LiH is to account for the variations in the atomic lithium abundance observed, the ratio of LiH to Li must reach the order of 10^3 in some stars studied. But since H_2 is more strongly bound than LiH, the ratio of H_2 to H must exceed the ratio of LiH to Li; it is easily calculated that this excess must amount to a factor of the order of 10^2 at 4000°. This implies an enormous depletion of atomic hydrogen in some stars, which would easily be seen in the spectra and would, for example, cause large variations in the absolute abundance of vanadium plotted in Figure 3. It is unlikely that any significant amount of H_2, and therefore of LiH, exists in the stars studied.

The effects of oxides of lithium must also be considered. If these were present, the corresponding compounds of sodium should exist in similar proportions, and the variations in the Li I doublet should be accompanied by variations in the Na I D lines. No such variations were noticed, and, as mentioned previously, McKellar and Stilwell obtained the same result in the carbon stars. Further, it is likely that if oxidation were important, vanadium would be depleted more by the formation of the strongly bound compound VO (which is observed in cooler stars) than would lithium by the formation of an oxide. Thus the abundance of lithium relative to vanadium would be expected to *increase* with decreasing surface temperature rather than the opposite effect observed. In general, it seems possible to neglect the effects of molecule formation.

The data then indicate that stars cooler than the sun generally have less lithium than the sun, and Greenstein and Richardson have shown that the sun has significantly less than the earth. If the terrestrial abundance is typical of the material out of which the solar system has formed, the conclusion is inescapable that the net effect of the processes acting on the material of the solar surface has been to destroy lithium. Greenstein and Richardson suggest that these gases are convected to regions of sufficiently high temperatures to burn the lithium by reactions with protons. But, aside from the specific process, the results of this study indicate that the net effect in cooler stars is also destruction. Certainly it is conceivable that the initial compositions of the stars were different, but there is no reason to suppose that the present temperatures are related to the initial compositions. It seems, then, that basically the stars cooler than the sun destroy lithium and that the efficiency of destruction is highly variable but increases with decreasing temperature.

b) The Production and Destruction of Lithium

It is interesting to examine the various methods that have been proposed by which stars may manufacture and destroy lithium and to note any suggestions that may be made about them on the basis of the present results, although it seems unlikely that any definite conclusions can be reached.

(i) *Processes in stellar atmospheres.*—Fowler, Burbidge, and Burbidge (1955) proposed that Li, Be, and B may be formed in stellar atmospheres with strongly varying magnetic fields. Protons accelerated to high energies (<100 Mev) by such fields in low-density regions high in the atmosphere might produce these light nuclei as results of spallation reactions with heavier nuclei. Much of the affected material would then be injected into the interstellar gas.

Babcock has made an extensive search for stars with magnetic fields but has emphasized stars of type near A. His recent catalogue (1958) also includes cooler stars, but among these he has detected significant fields only in S stars and in a few M stars which show peculiar emission lines. He examined a few of the stars studied here but found no evidence of magnetic fields. Since the sun has complex but weak magnetic fields, it seems likely that some degree of magnetic activity is a property of all stars; but until either theory or observation shows how the activity varies with the other observable properties, it will be impossible to establish a connection with the lithium abundances.

(ii) *Convection.*—It was noted above that Greenstein and Richardson accounted for the small abundance of lithium on the sun relative to the earth on the hypothesis that convective mixing of the surface material to the interior has led to the destruction of the

Li nuclei. Recent calculations indicate that the requirements for a satisfactory model for the sun demand an outer convection zone (Schwarzschild, Howard, and Härm 1957; Weymann 1957). Earlier, Osterbrock (1953) had demonstrated the need for considerable convection zones in the outer parts of K dwarfs. A recent rediscussion of the pertinent observational material by Limber (1958*a*) reduced the discrepancy noted by Osterbrock between his models for the middle and late M dwarfs and the observations, and further theoretical work (Limber 1958*b*) indicates that the cool dwarfs have completely convective interiors. The general trend of the results of these authors is that the cooler the stars, the more of its mass must be included in an outer convection zone. Hoyle and Schwarzschild (1955), in attempting to fit models for the giant stars to the evolutionary tracks deduced from the globular-cluster color-magnitude diagrams, found it necessary to invoke outer convection zones for stars more than 2 mag. above the main sequence. The results of these authors are not sufficient to show how the convection zone changes with temperature among stars of the same luminosity class, but presumably the depth of convection does change with temperature, and it is not unreasonable to assume that the trend is similar to that found in the dwarfs.

Since detailed models for the sun and the K dwarfs are available, it is possible to estimate the mean lifetime of a lithium nucleus at the bottom of the convection zones of these stars. Presumably, the longer the lifetime at the bottom of the convection zone, the more lithium should appear at the surface if convection is primarily responsible for the destruction of this element. Salpeter (1955), using the most recent results on the cross-section for the capture of protons by Li^7, gives parameters which lead to the following equation for the mean lifetime of a Li^7 nucleus:

$$t = 8.3 \times 10^{-12} (x\rho)^{-1} T^{2/3} \exp\left(\frac{84.5}{T^{1/3}}\right) \text{ sec}. \qquad (13)$$

Here T is the temperature in millions of degrees, ρ is the density, and x is the abundance of hydrogen by mass. Using equation (13) and the conditions at the bottom of the convection zones given in the interior models and taking x as 0.70, the lifetimes in Table 4

TABLE 4

LIFETIME OF Li^7 AT BOTTOM OF CONVECTION ZONES

Type	T	ρ	t (years)	Reference
G2 V..........	1 04	0.020	3.1×10^{19}	Weymann (1957)
K1 V..........	2.5	0.545	$1\ 3\times10^{9}$	Osterbrock (1953)
M0 V..........	2.6	1.93	1.5×10^{8}	Osterbrock (1953)

were computed. Here the first column gives the spectral class, the second and third give the temperature and density at the bottom of the convection zone, the fourth gives the mean life of a Li^7 nucleus in years, and the final column gives the reference for the data in the second and third columns. It is clear at once that the lifetime in the sun is too long to account for the destruction of lithium as suggested by Greenstein and Richardson. However, as Hoyle and Schwarzschild have pointed out, the depth of the convection zone is highly sensitive to the efficiency of convection in the lower atmosphere, which cannot be accurately estimated at present. Thus it is quite possible that the values of T and ρ used in Table 4 are considerably in error, but, since the same method was used in all cases for taking the atmosphere into account, it is likely that the errors are all in the same sense, so that the trend exhibited in t is correct. Thus one would expect K dwarfs to have considerably less lithium in their atmospheres than the G dwarfs. The K0, K1, and K2 dwarfs studied have abundances at least five to ten times less than the sun; the K5 dwarf has a deficiency of at least a factor of 15; and the K7 dwarf has a deficiency of about a factor of 40. Clearly, this is in the right sense. Unfortunately, the observations permit no conclusions about possible variations within a spectral class.

If the convection zones of giants vary in a manner similar to those of the dwarfs, a decline in the lithium abundance with surface temperature would be expected in these stars also. Thus it may be possible to account for the general decline of the observed lithium abundance with surface temperature through the destruction of this atom in convection zones which reach deeper into stars of low surface temperature than into the hotter stars.

The large variations in the abundance of lithium among similar stars remain to be explained. It is, of course, possible that the initial abundances of this element differ strongly, in which case the problem is passed to the interstellar medium. It is, however, not difficult to suggest mechanisms which might cause differences in the depth of convection among superficially similar stars. If two stars differ in metal content and therefore in opacity, they will also differ in the relative efficiency of convective and radiative transport of energy in the lower atmosphere.[1] As mentioned above, the efficiency of convection in the atmosphere affects the depth of the convection zone, but changes of opacity which are caused by changes in the metal abundance will not cause gross effects in the spectrum. Further, the semi-empirical evolutionary tracks of Sandage (1957) indicate that two giant stars of equal radius and luminosity may have different masses. The more massive star must have a higher central temperature and therefore, on the average, steeper temperature gradients and probably a different convection zone than the other. Since the spectroscopic appearance depends mostly on radius and luminosity, these stars would appear similar. Thus possibly also the variations in the lithium abundance may be effects of convection.

It is also possible that the proper kind of mixing can increase the surface lithium abundance. Cameron (1955) proposed that the proton-proton chain of hydrogen-burning may end in part by the reactions

$$He^3 (\alpha, \gamma) Be^7 (e^-, \nu) Li^7 .$$

Fowler (1958) has discussed the consequences of this reaction with respect to energy production in stars near solar type, taking into account the newly discovered high cross-section for the first of these reactions. With regard to the present problem, Cameron has noted that the intermediate nucleus, Be^7, is much less likely to capture protons than is Li^7. Thus, if the Be^7 can be removed by convection from the high-temperature region before capturing an electron to form Li^7, the resultant Li^7 might be preserved. However, the time for the electron-capture reaction is not longer than the order of a year, and it seems unlikely that convective motion can be sufficiently rapid to remove the Be^7 to cooler regions from the hydrogen-burning zone in this time. But if this process does operate, what is observed is a balance between the effects of convective destruction and production of lithium.

Thus it seems possible to construct a fairly reasonable, if largely speculative, case linking the abundance of lithium to convection. Further investigation of this possibility is certainly indicated. A statistical study of K dwarfs, in which one of the above causes of variation would be eliminated, is impossible because there are probably fewer than ten such stars bright enough to be observed at the necessary dispersion with existing equipment. However, detailed comparisons of the spectra of stars similar except for lithium abundances might reveal differences in opacities and surface gravities or in other physical or chemical properties which could not be detected in this rough analysis.

[1] I am indebted to Dr. H. L. Helfer for pointing out this possibility.

Further, of course, more refined models of the interiors of both the giants and the dwarfs would be most valuable.

c) *Summary*

The observed variations in the lithium-to-vanadium abundance ratios—which can be ascribed to changes in the lithium abundances—cannot yet be "explained" in any final way. The cool stars apparently destroy lithium by processes which increase in efficiency with decreasing temperature but which are very strongly variable.

It has been suggested that magnetic activity at the surfaces of stars may accelerate protons and, especially in regions of very low density, produce the light elements by spallation reactions. Many of the atoms so produced would be injected into the interstellar medium. Too little is known of the relation of stellar magnetic activity to the other observable properties of the atmosphere to permit any detailed connection with the present results.

A possible connection exists between the depths of surface convection zones in cool stars and the abundance of lithium in the surface. Models of dwarf interiors indicate that the liftime of lithium at the bottom of the convection zones decreases with surface temperature. If reactions in the convected gas destroy the surface lithium, a decrease in the surface abundance with temperature would be expected—and is observed. If a similar effect operates in the giants and if the depth of convection is strongly affected by surface opacity or total mass, it may be possible to account for both the trend and the fluctuations observed. It seems possible to test these ideas observationally.

The results of this study and the preceding discussion suggest a general picture such as the following: the light elements are produced in magnetically induced reactions in the outer atmospheres of stars (whence they are injected into the interstellar gas) and/or in the interstellar medium. The abundances observed in stellar atmospheres are the remains of the initial material after destruction by convective mixing. In this picture the abundance in the hot stars should have suffered no depletion. It therefore becomes of increased interest to search for beryllium in such stars.

This study has not provided any definite answers to the general question of the origin of the light elements, but it has provided useful data and led to suggestions for further investigation. Since it is of the nature of a first survey, such results are quite satisfying.

I am happy to express my gratitude to Dr. Jesse L. Greenstein for suggesting this problem and giving continuing advice and criticism throughout the course of the investigation, and to numerous members of the observatories and California Institute staffs for helpful discussions.

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