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THE BINARY SYSTEM X OPHIUCHI*

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ABSTRACT

A study of all available observations of the X Oph visual binary system (K1 III + M6e) is described. It is concluded that the mass of the long-period variable component is close to $1\mathfrak{M}_{\odot}$, compared to the mass of about $16\mathfrak{M}_{\odot}$ predicted by the mass-luminosity relation. This smaller mass leads to a pulsation constant, $P\sqrt{\bar{\rho}}$ similar to that observed for other types of variables and in close agreement with theory. The age of X Oph is about 5×10^9 years, and hence probably all long-period variables are old. Evidence is given to support the suggestion that long-period variables evolve from early F dwarfs. It is shown that the binary nature of X Oph has not in any way affected its evolution and that it is not necessary, therefore, for a star to belong to a binary system in order to become a long-period variable.

I. INTRODUCTION

Despite the importance of the quantity in theoretical studies, there does not at present exist a well-determined mass for any long-period variable. The reason is simply found in the fact that only three of these objects are known to belong to binary systems, and none of the three can be considered entirely suited to a determination of mass.

The case of Mira is rendered uncertain by the nature of the companion, a faint, peculiar Be star, itself variable. The fact that the companion is often invisible makes the apparent visual orbit ambiguous, and, although Parenago (1950*a*) concluded that the orbital period was only 14 years, more recent work by Deutsch (1957) makes it seem likely that the period is of the order of thousands of years. The masses implied by these two orbits differ by a factor of the order of a thousand.

R Aqr, another of the three known long-period variable binaries, appears the least likely to yield an accurate mass. Its companion has been described by Merrill (1940) as an "atypical planetary nebula," and any interpretation of the spectral variations in terms of orbital motion appears very uncertain.

X Oph, the third of the three systems, is perfectly normal, in that the companion is an ordinary K giant, and the only drawback to a good determination of the masses is that the orbital period is probably of the order of 500 years, of which less than 60 years has so far been observed.

Masses of long-period variables have generally been taken to be those given by the mass-luminosity relation, which predicts a mass in the neighborhood of $16\mathfrak{M}_{\odot}$ for a M6e variable, although in the light of modern theories of stellar evolution there is no justifiable reason for accepting this figure, and, indeed, there exist reasons—such as the pulsation constant for these stars—for believing it incorrect.

The conclusion from the foregoing is that the masses of long-period variables are almost totally unknown, and, while it is fully realized that an accurate mass from X Oph is as yet unobtainable, the intention of this study is to obtain a value accurate to at least one order of magnitude.

II. THE VISUAL ORBIT

There exists a total of about 120 nights' observations (see Appendix I) of the system as a visual binary, covering the interval between 1900 and 1957. These data serve to

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define no more than a 50° arc of the apparent orbit, but this arc is known with relatively high precision. The data were treated in the usual way to derive an apparent, and hence "true," orbit, the elements of which are given in Table 1. The representation of the normal points by this orbit is given in Table 2. The usual procedure of making a least-squares adjustment of the orbital elements by differential corrections could not be applied here, because, when the observed arc is so short, the corrections to the elements may be so large as to make it impossible to consider them differential. Instead, an empirical approach was adopted by drawing at random on tracing paper a large number of arbitrary ellipses. These were all tested in various positions against the observed arc, to see whether they fitted it within reasonable error. Four were found to do so, and these led to "true" orbits having widely differing elements. However, since it is the sum of the masses that is of importance here and not the individual elements, it is only the variation in the quantity (a^3/P^2) that matters. It is found that for all five orbits this quantity lies between the values 0.809×10^{-7} and 1.352×10^{-7} sec^3/year^2, i.e., all five orbits

TABLE 1

a..........	0″.34	i.........	51°	Ω.........	139°	T........	1871
e...........	0.30	ω.........	103°	P.........	557 years		

TABLE 2

Date	Nights	$\Delta\theta_{O-C}$	$\Delta\rho_{O-C}$	Date	Nights	$\Delta\theta_{O-C}$	$\Delta\rho_{O-C}$
1906.85.	6	+1°.5	+0″.02	1938.80....	20	−0.1	−0.01
1922 40.....	27	0 0	+ .01	1945.69.. .	23	+2 4	+ .01
1932.18. . .	11	−1.0	−0.01	1956 75..	19	−0.7	0.00

predict the same total mass to better than a factor of 2, despite their widely differing elements. It may be shown that

$$\frac{a^3}{P^2}=\frac{\dot{A}^2}{\pi^2}\, a \cos^2 i\,(1-e^2)\,,$$

where A is the areal constant in the apparent orbit. This is a constant for all five of the above orbits and is the same for the real orbit also. Hence, in this case, the sum of the masses depends principally on the semi major axis of the orbit and to a lesser extent on i and e. In the selection of the empirical orbits, particular care was directed to finding the largest and smallest orbits that could reasonably be fitted to the observed arc, and it is highly improbable that the real orbit lies significantly outside this range. It is therefore almost equally improbable that the real total mass of the system is not contained within the range given by the empirical orbits.

III. RADIAL VELOCITIES

Although the small separation of the two components of X Oph make it practically impossible to resolve these at the slit of most spectrographs, it is still possible to obtain the spectra of the individual stars through the fortunate fact that the variable has a very large amplitude in blue light and the relative luminosities of the two stars are such that when the variable is at maximum light, it completely dominates the spectrum, while the situation is reversed at minimum light.

There exist radial-velocity data for two epochs—an extensive series of observations by Merrill (1923) around 1921 and several 100-inch coudé (10 A/mm) spectrograms obtained by Dr. Robert P. Kraft while a guest investigator at Mount Wilson in 1956 and 1957. A discussion of the reduction of these velocities to the Mount Wilson system is given in Appendix II. The final velocities, on this system, are given in Table 3. Those for the variable are reduced to zero phase, i.e., maximum visual light.

The probable errors shown in Table 3 are the formal ones found by combining the probable errors of the various measurements listed in Appendix II. Experience often shows that these formal errors are too small, and a more realistic idea of the true probable errors is obtained by comparing the formal probable errors of the measurements in Tables 9 and 10 with the actual corrections to the catalogue velocities. On this basis, more likely values of the probable errors of the final velocities would be obtained by doubling those listed in Table 3.

TABLE 3

Epoch	M Star (km/sec)	K Star (km/sec)	Epoch	M Star (km/sec)	K Star (km/sec)
1921.3.......	−70.0±0.5	−71.1±1.0	1957.6.......	−67.6±0.5	−72.9±0.3

These velocities are not precise enough to distinguish among the five empirical visual orbits but may be used to find the mass ratio of the system. O. C. Wilson (1941) has given the following simple formula for the mass ratio of a double-line spectroscopic binary:

$$k = \frac{v_2 - v_1}{u_1 - u_2},$$

where u and v are the velocities of the two stars at epochs 1 and 2. This formula is independent of any possible constant correction between the zero-phase velocity and the "gamma velocity" of the variable. Substituting the data of Table 3, we obtain (on the basis of twice the probable errors listed in Table 3)

$$K = 0.75 \pm 0.98 \text{ (p.e.)} ,$$

the variable being the less massive of the two. This probable error is again formal and is strongly dependent on the values of the velocities. Actually, by twice applying their probable errors to the velocities, one is able to obtain a range in k from 0 to 11, although these extremes are the least likely, of course. It is best to conclude, perhaps, that the mass ratio is probably of the order of unity.

IV. THE PARALLAX OF THE SYSTEM

Of various determinations of the trigonometric parallax listed for X Oph, the least unlikely has the value 0″.000 ± 0″.007, the only useful conclusion from which is that the system is probably more distant than, say, 100 parsecs.

An attempt was made to obtain the absolute magnitude of the K giant by the method recently discovered by Wilson and Bappu (1957), but no emission attributable to the K star could be found in the H or K lines of its spectrum.

The absolute magnitude of the K giant was finally established by the method of spectrophotometric analysis developed by Oke (1957). Details of this are given in Appendix III. The result was

$$M_v = +1.1 \pm 0.2 ,$$

which places the star at the peak of Sandage's (1957*a*) luminosity function for such objects.

The spectrophotometric analysis indicated a spectral type of about K1, which, together with the absolute magnitude derived, places the star very close to the M67 sequence in the H-R diagram (see Fig. 1).

The absolute visual magnitude of the variable at maximum light, taken from the work of Wilson and Merrill (1942), is −0.3. This result is statistical, and these authors found a range in $M_{v\max}$ for stars of this period of about 0.8 mag. A likely error, for this absolute magnitude, therefore, is presumably ±0.4 mag.

Since the two stars are too close to be separated in a photometer, the measured apparent magnitude is always the combined magnitude. The mean light-curve (Campbell 1955) has an amplitude of 1.73 mag. in the yellow. An observation by Dr. Halton C. Arp (private communication) shows that the magnitude at minimum is $V = 8.44$. Hence the mean magnitude at maximum is $V = 6.71$.

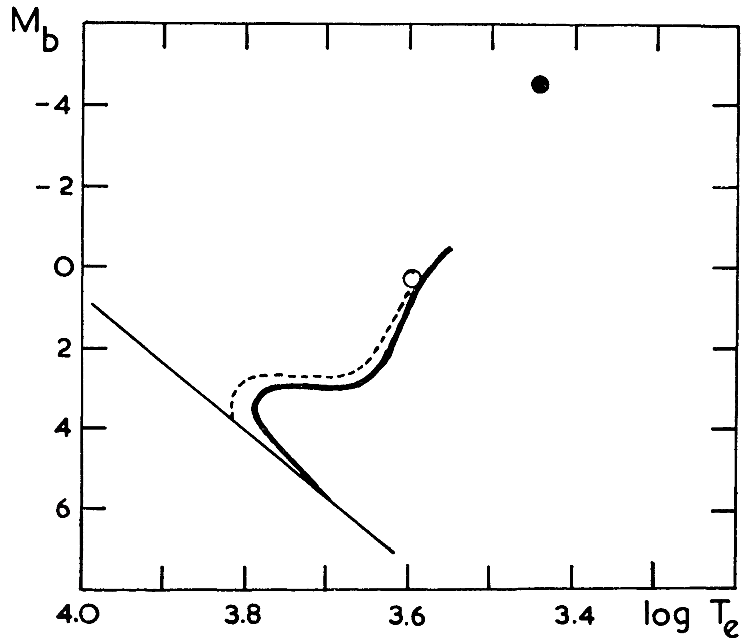

FIG. 1.—The open circle represents the present position of the K star in the $M_{bol}-\log T_e$ plane; the dotted line its supposed course of evolution from the original main sequence (*straight line*). The solid curved line is the M67 sequence. The filled circle represents the approximate "average" position of the long-period variable on the diagram, this being uncertain because of the great uncertainty in the bolometric correction. (Adapted from a diagram by Sandage.)

Pettit and Nicholson (1933) have estimated from a discussion of the variable's heat index that its apparent visual magnitude at minimum light is about 11.5, which is consistent with the average amplitude of variables of this period. The calculation of the apparent magnitude of the K star alone is insensitive to the exact value of this amplitude, provided that the latter is of the order of $4\frac{1}{2}$ mag. or more. Assuming a value of 4.8 mag., one obtains

$$V = 8.51 \text{ (K star alone)}, \qquad V = 7.12 \text{ (variable at maximum alone)}.$$

Employing these apparent magnitudes and the absolute magnitudes quoted above, one obtains the following apparent distance moduli:

$$m - M = 7.4 \text{ (K star)}, \qquad m - M = 7.4 \text{ (variable)}.$$

The correction for interstellar absorption has been estimated as follows. Hiltner (1956) has listed the spectral types, colors, and magnitudes of nine stars which are within 10° of X Oph (Nos. 792–800 in his catalogue). Three further stars within $\frac{1}{2}°$ were selected by me, and their spectral types were obtained with the Newtonian spectrograph of the Goethe Link Observatory. Their magnitudes and colors were kindly measured by Dr. Halton C. Arp at Mount Wilson Observatory. These results are presented in Table 4. For these and Hiltner's stars the average absorption per kiloparsec has been plotted as a function of the true distance modulus in Figure 3. I interpret this figure as follows. In this direction ($l = 7°$, $b = +6°$) the edge of our spiral arm lies at a distance of 1 kpc, out to which the absorption averages 2.3 mag/kpc and beyond which the average absorption is about 0.2 mag/kpc. The solid curve in Figure 3 is calculated on this model and appears to fit the observations satisfactorily. Clearly, X Oph, with an apparent

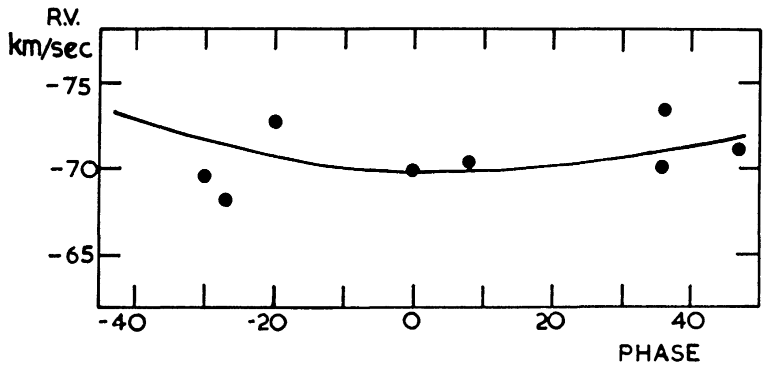

FIG. 2.—The radial velocity (from absorption lines) of the long-period variable as a function of phase (measured in days from maximum visual light). The filled circles are first-epoch measures by Merrill, while the curve is tranposed from the appropriate portion of the velocity-curve for Mira as given by Joy.

TABLE 4

Star HD	Sp. Type	V	$B-V$	A_v	$(m-M)_0$	A_v (kpc)
172401.........	K0 III	6.98	1 07	0.3	5 6	2 3
172522.........	A2 III	6.91	0.22	.6	7.0	2 4
172588.........	F0 II–III	7.22	0 35	0.5	6.8	2.2

modulus of 7.4, lies within our arm, and the absorption to it may be taken as averaging 2.3 mag/kpc. Recourse has now been had to the formula of Parenago (1945), using, however, the above value for the average absorption parameter:

$$A(r, b) = \frac{a_0\beta\left[1 - \exp\left(r \sin \frac{b}{\beta}\right)\right]}{\sin b},$$

where $\beta = 100$ pc; $r =$ the true distance of the star; $b =$ the galactic latitude of the star (absolute value); and $a_0 = 0.0023$ mag/pc. By iteration, one finds, for X Oph,

$$A(r, b) = 0.53 \text{ mag}.$$

This is supported by the color index of the system at minimum light, which, to within a few hundredths of a magnitude, is the apparent color index of the K1 III stars. This is

$$B - V = +1.25,$$

and, taking the intrinsic color as being about +1.10 (Johnson and Morgan 1953), the excess is

$$E_{B-V} = 0.15 ;$$

whence

$$A_v = 3\, E_{B-V} = 0.45 .$$

Adopting $A_v = 0.5$, we find the true distance modulus of X Oph to be $m - M = 6.9 \pm 0.3$, or $r = 240 \pm 35$ pc.

Taking $a^3/P^2 = 1.12 \times 10^{-7}$ from the adopted visual orbit and the above value for r, one obtains

$$\mathfrak{M}_M + \mathfrak{M}_K = \frac{a^3 r^3}{P^2} = 1.54\, \mathfrak{M}_\odot .$$

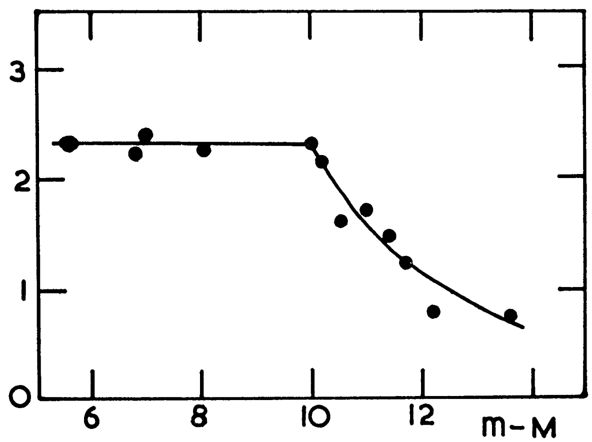

FIG. 3.—The total visual absorption in magnitudes per kiloparsec as a function of true distance modulus for stars near X Oph.

For the sake of formality and as a basis for later discussion, we complete the calculation of the masses using the mass ratio given above, although the latter is only poorly determined. Thus

$$\frac{\mathfrak{M}_M}{\mathfrak{M}_K} = 0.75 .$$

Hence

$$\mathfrak{M}_K = (0.88 \pm 0.64)\mathfrak{M}_\odot , \qquad \mathfrak{M}_M = (0.66 \pm 0.97)\mathfrak{M}_\odot .$$

The probable errors are based on an assumed probable error of 0.20×10^{-7} for (a^3/P^2), this being estimated from the variation in the latter quantity among the five empirical visual orbits.

These masses are too low, since the K giant star presumably has a mass greater than $1\mathfrak{M}_\odot$, in order to have evolved away from the main sequence. The error results from the great uncertainty in both the visual orbit and the mass ratio. However, we may obtain an independent idea of the mass of the K star from the fact that it is very similar to stars in M67. According to evolutionary tracks computed by Sandage (1957*b*), such a star will have originated from an F5 V star of mass $1.2\, \mathfrak{M}_\odot$ (see Fig. 1). Then, if one chooses to ignore the visual orbit entirely and use the radial-velocity data and a mass of 1.2 for the K giant, the mass of the long-period variable becomes

$$\mathfrak{M}_M = (0.90 \pm 1.19)\mathfrak{M}_\odot .$$

On the other hand, if one chooses to dispense with the radial velocities, the mass found from the visual orbit and the evolutionary tracks is

$$\mathfrak{M}_M = (0.30 \pm 0.73)\mathfrak{M}_\odot .$$

These results are consistent to within the one order of magnitude proposed initially. This value is to be compared with the mass of about $16\mathfrak{M}_\odot$ predicted by the mass-luminosity relation and generally quoted in reference books (e.g., Allen 1955). For purposes of discussion I have adopted a value of $0.8\mathfrak{M}_\odot$, although it must be borne in mind that the actual value is quite uncertain.

V. DISCUSSION

One of the most outstanding discrepancies of the long-period variables has been the very high values of their pulsation constants, $K = P\sqrt{\bar{\rho}}$. Abt's (1957) survey of variables in the upper part of the H-R diagram has shown that practically all variables have a value of K within a factor of 2 or 3 of that for the classical cepheids; yet, if the long-period variables have a mass of about $16\mathfrak{M}_\odot$, their value of K exceeds that of the cepheids by a factor of about 9.

In practice, K is usually computed from the equation

$$K = P\left(\frac{\mathfrak{M}}{\mathfrak{M}_\odot}\right)^{1/2}\left(\frac{R}{R_\odot}\right)^{-3/2}$$

or

$$K = P\left(\frac{\mathfrak{M}}{\mathfrak{M}_\odot}\right)^{1/2}\left(\frac{L}{L_\odot}\right)^{-3/4}\left(\frac{T_e}{T_{e\odot}}\right)^{3},$$

in which the bolometric correction is involved. This quantity, poorly determined for most stars, is virtually unknown for the long-period variables (principally because of heavy blanketing by the TiO bands), and one finds values differing by almost a magnitude quoted in the literature. The effective temperature (entering as the third power) is also doubtful. However, Mira, which is a good match to X Oph in period and spectrum and presumably therefore in size, has had its radius measured with the interferometer. Pease (1925) found an angular diameter of $0''.056$ and stated that this may be too large because of seeing effects but that this error is probably less than 10 per cent. This may be compared with a mean diameter of $0''.052$ found by Pettit and Nicholson (1933) with a totally different method. If one adopts a mean diameter of $0''.054$ and a parallax of $0''.020$ (Scott 1942), the mean linear radius is found to be $300\,R_\odot$, which is probably reliable to about 20 per cent. (The various bolometric corrections imply a radius between 250 and 400 solar radii.) Then with $P = 337^{d}$ and $\mathfrak{M} = 0.8\mathfrak{M}_\odot$, one finds

$$K = 0.058 \text{ day}$$

and

$$\frac{K}{K_{\text{cep}}} = 2.2,$$

which is in accordance with other types of variables.

Epstein (1950) has found from studies of giant star models that, theoretically, K should lie close to 0.041 day. However, his assumption that the envelopes of such stars are in radiative equilibrium has been replaced (Ledoux, Simon, and Bielaire 1955) by the more likely one that the envelopes are in convective equilibrium, in which case the theoretical value of K becomes 0.058 day. While the exact agreement with theory is probably fortuitous, a mass of $0.8\mathfrak{M}_\odot$ leads to a very much better agreement than a mass of $16\mathfrak{M}_\odot$.

It has already been pointed out that the K star lies close to the M67 sequence in the

H-R diagram and presumably, therefore, has an age similar to M67—5×10^9 years by current estimates. X Oph, then, appears to be an old population I object, and, since it is typical of long-period variables (a discussion of the effect of its binary nature follows), it may be presumed that long-period variables in general are old objects, a not unreasonable conclusion since a few (type II) long-period variables are found in globular clusters and none are found in galactic clusters. Hence, the presence of Tc in some of these stars (Merrill 1952) indicates that element formation can take place long after these stars are "born."

If the variable now has a mass of $1\mathfrak{M}_\odot$ or less, it must have lost, or be in the process of losing, mass in order to have evolved away from the main sequence to its present position in the H-R diagram in 5×10^9 years. This is substantiated by observational evidence (e.g., Deutsch 1956) that secular mass loss may be common among long-period variables, although no quantitative estimates of the rate of mass loss seem to be available. However, we may proceed in the following speculative manner, presuming as a basis for calculation that the adopted orbit is actually correct. Then the following quantities pertain: the average orbital velocity of the K star around the variable = 4.5 km/sec, and the escape velocity at the surface of the variable = 31 km/sec. This figure does admittedly assume that the surface gravity is that given by the mass and radius alone, which may be incorrect. However, if anything, the actual surface gravity (and hence escape velocity) will be less than the dynamical surface gravity, in which case the calculations to follow are not materially affected. In any case, observations by Merrill and Greenstein (1958) have shown that matter possibly is being ejected from R And at a velocity of 23 km/sec, so that the above estimate is probably not far wrong. Then the velocity of the escaping material at a distance r (the average orbital separation of the two stars) is

$$v_r = \left(\frac{2G\mathfrak{M}}{r}\right)^{1/2} = 4.0 \text{ km/sec},$$

and the relative velocity of the K star and circumstellar material is

$$v = 6.0 \text{ km/sec}.$$

If matter is ejected in a spherically symmetrical manner, then there exists the continuity equation

$$4\pi r^2 \rho(r)\, v_r = \text{rate of mass loss} = \dot{\mathfrak{M}}_M;$$

whence

$$\rho(r) = 1.26 \times 10^{-37} \dot{\mathfrak{M}}_M.$$

The rate of accretion of the circumstellar material by the K star will depend on whether the process is adiabatic or isothermal, i.e., whether $v \ll c$ or whether $v \gg c$, where c is the velocity of sound in the circumstellar material:

$$c = \left(\frac{\gamma kT}{\mu H}\right)^{1/2},$$

where the symbols have their usual astrophysical meanings. Take γ and μ to have about the same numerical value. The kinetic temperature of the material will have decreased from its value at the surface of the variable, but this will tend to be compensated by the heating effect of the approaching K star. If $T = 2000°$ K, then $c = 4$ km/sec, while if $T = 200°$ K, $c = 1$ km/sec. Take $c = 3$ km/sec. Thus the relative velocity of the K star is of the same order of magnitude as the velocity of sound, in which case the rate of accretion (McCrea 1953) is approximately

$$\dot{\mathfrak{M}}_K = \frac{\alpha 2\pi \rho G^2 \mathfrak{M}_K^2}{(v^2 + c^2)^{3/2}}, \qquad \text{where } 1 < \alpha < 2.$$

Substituting for ρ from above, and taking $\alpha \sim 1.5$, $\mathfrak{M}_K = 1.2\mathfrak{M}_\odot$, we find

$$\dot{\mathfrak{M}}_K / \dot{\mathfrak{M}}_M = 0.1 \; .$$

Since the K star appears completely normal in every way, we must assume that it cannot have accreted a total of more than, say, $0.1\mathfrak{M}_\odot$. Hence the variable cannot have lost more than about $1\mathfrak{M}_\odot$ in its lifetime, i.e., its original mass cannot have been more than about $1.8\mathfrak{M}_\odot$, in which case its main-sequence progenitor must have been a star of type later than A8. On the other hand, since the long-period variable appears to be in a more advanced state of evolution than its companion and since the two are coeval, the mass of the variable must originally have been greater than that of its companion, i.e., the variable's progenitor must have been of earlier spectral type than that of the K star, which was stated above to have been an F5 star. Hence the implication is that long-period variables originate from stars on the main sequence between spectral types A8 and F5. Support for this suggestion is lent by kinematic considerations. Strömberg and Merrill (1924) list, for the components of the velocity ellipsoid of late-type long-period variables, $\sigma_1 = 17$ km/sec, $\sigma_2 = 21$ km/sec, $\sigma_3 = 36$ km/sec, while Table 5 is extracted from one by Parenago (1950*b*). It will be seen that the components for the stars in the range dF5–dF7 are in good agreement with those for the long-period variables.

TABLE 5

Sp. Type	σ_1 (km/sec)	σ_2 (km/sec)	σ_3 (km/sec)	Sp. Type	σ_1 (km/sec)	σ_2 (km/sec)	σ_3 (km/sec)
A9–dF1......	9.5	12.8	23.9	dF5–dF7....	16.7	21.4	31.8
dF2–dF4......	11 7	17.0	26.8	dF8–dG2....	22 6	27.5	46.0

It remains to ascertain whether or not the binary nature of X Oph has ever affected its evolution in any serious manner (as, for example, in the case of β Lyrae) and thus made it atypical of long-period variables.

The change in the orbital elements of a binary due to one of the components losing mass has been treated by Huang (1956). He gives the equations

$$\frac{\delta a}{a} = -\frac{\delta' \mathfrak{M}_1}{\mathfrak{M}_1 + \mathfrak{M}_2},$$

$$\frac{\delta P}{P} = -\frac{2\,\delta\,\mathfrak{M}_1}{\mathfrak{M}_1 + \mathfrak{M}_2}.$$

From the discussion given above, assume that the long-period variable originally had a mass of about $1.3\mathfrak{M}_\odot$. Then the total mass was originally $2.5\mathfrak{M}_\odot$, while at present it is $2.0\mathfrak{M}_\odot$. Take $(\mathfrak{M}_1 + \mathfrak{M}_2) = 2.3\mathfrak{M}_\odot$. Then, substituting in the above equations, we find

$$\delta a = -19 \text{ a.u.}, \qquad \delta P = -240 \text{ years} .$$

Hence the original values of these elements were roughly

$$a_0 = 64 \text{ a.u.}, \qquad P_0 = 310 \text{ years} .$$

Actually, since the K star in its orbit is moving through the resisting circumstellar material, there is a tendency for the separation to decrease, which will offset the increasing separation given above. Hence the original value of a may have been much greater than 64 a.u. Therefore, since the radius of the larger star (the variable) is only about 1.4 a.u. and the masses of the two stars each less than $2\mathfrak{M}_\odot$, the stars have never been able to interact to the extent of objects like β Lyrae. Thus the binary nature of X Oph has in no way affected the nature of the long-period variable, and, as a corollary, it follows that, whatever condition it is that causes a star to become a long-period variable, it is not that of being a binary star.

As an example of the dependence of these conclusions on the mass of the variable (which is really quite uncertain), suppose the mass is $3\mathfrak{M}_\odot$. Then the pulsation constant K becomes 0.112 day, and $K/K_{\text{cep}} = 4.2$. The escape velocity becomes 60 km/sec, and the range of possible progenitors is expanded to B3 V–F5 V. The original separation of the stars becomes about 40 a.u., so the conclusion that the stars have never seriously interacted remains unchanged.

It will be noted that if the mass of the long-period variable is $3\mathfrak{M}_\odot$ instead of $0.8\mathfrak{M}_\odot$, there is a marked deterioration in the agreement of the pulsation constant both with theory and with other types of variables. Such slight evidence as there is does not support an escape velocity as high as 60 km/sec. The kinematic arguments, which have been used with some success by Sandage (1957*a*) to discuss the origins of K giants, definitely do not support a mass as high as $3\mathfrak{M}_\odot$—indeed, they imply a mass less than $0.8\mathfrak{M}_\odot$. Furthermore, a mass of $3\mathfrak{M}_\odot$ is incompatible with Deutsch's (1957) conclusions regarding Mira. Thus, while the formal solution for the mass found here appears quite uncertain, these remarks do seem to bear out the suggestion that the true mass is less than $1\mathfrak{M}_\odot$.

The people who have assisted in this work are so manifold that they must, of necessity, be thanked only collectively but it is of especial pleasure to thank Dr. Robert P. Kraft for having suggested the problem and for numerous valuable discussions in the course of the work. I am indebted also to Drs. H. A. Abt and G. van Biesbroeck for reading and commenting on a first draft of the manuscript.

APPENDIX I

Visual Observations

The observations of the system as a visual binary used in this work are essentially those recorded in the Lick Card Catalogue of Visual Binaries and supplied to me by Dr. Hamilton Jeffers, of the Lick Observatory. Table 6 lists these data (with the precession correction applied).

These data were smoothed in the usual way by plotting each of θ and ρ against time, and the curves so obtained were adjusted until the quantity $[\rho^2(d\theta/dt)]$ became independent of the time, in accordance with the law of areas. In this process it became obvious that there is an error in the position angle found by Baize in 1952.70, and this measurement was rejected from the analysis. In general, weights were assigned the points proportional to the number of nights of observation each contained; but where large deviations occurred, account was taken of the total number of observations of X Oph made by that observer. Thus the value of ρ found by van den Bos in 1957.61 is considerably larger than might be expected from the preceding ones and is given half-weight, despite the five nights of observing, since it is the only measurement contributed by van den Bos and the deviation may well arise from the personal equation. The smoothing of the data gave rise to a plotting list, which is given in Table 7. These points were plotted on a sheet representing the plane of the sky, and an ellipse was drawn through them with the aid of a thread and two pins.

TABLE 6

Date	θ	ρ	Nights	Observer
1900.47.........	194°9	0″22	3	Hussey
1907.66.........	188.9	.15	1	Bowyer
1912.43.........	173 4	.23	2	Doolittle
1920.66.........	170.9	.22	5:	vB
1921.46.........	171 4	.24	6:	vB
1922.51.........	170.1	.23	6	vB
1922 69.........	170.8	.27	2	Aitken
1923.55.........	166.9	.25	4	vB
1924.50.........	165.1	.26	4	vB
1930.56.........	162.1	.25	5	vB
1933.53.........	161.6	.25	6	vB
1936.20.........	160.1	.26	3	vB
1937.78.........	159.1	.25	7	vB
1939.77.........	157.8	.25	7	vB
1941.53.........	154.0	.35	3	Jef
1943.48.........	160.6	.27	1	vB
1943.79.........	156 4	.32	1	vB
1944 36.........	154.3	.29	3	Voute
1945.31.........	157.0	.30	7	vB
1945.32.........	153.0	.33	1	vB
1945.44.........	156.2	.27	1	vB
1945.70.........	158.9	.28	3	Jef
1946.48.........	153.6	.29	1	vB
1946.51........	158.7	.31	1	vB
1946 53.........	160.8	.30	1	vB
1948.68.........	153.0	.29	3	vB
1952.70.........	136.3	.32	8	BAZ
1955.62.........	148.1	.30	4	Jef
1955.74........	146.1	.32	5	BAZ
1956 39...... .	153 4	.28	1	vB
1957.54.........	148.27	.29	5	Jef
1957.60.........	144.3	.34	4	vB
1957.61.........	147.1	0.38	5	v.d. Bos

TABLE 7

Date	θ	ρ	Date	θ	ρ
1900...........	194°6	0″178	1930...........	163°5	0″252
1905...........	187.7	.189	1935...........	160.0	.263
1910...........	181.6	.201	1940...........	156.8	.274
1915...........	176.2	.214	1945...........	153.8	.284
1920...........	171.5	.227	1950...........	151.0	.294
1925...........	167.3	0.240	1955...........	148.4	0.304

APPENDIX II

Radial Velocities

The radial velocities presented in Table 3 are arrived at as follows, discussing, first, the possible corrections which may arise because of the intrinsic nature of the long-period variable. Since X Oph is an almost perfect match for Mira in spectrum and period, I have adopted Joy's extensive work on the latter star as a basis for corrections to the X Oph velocities. The emission-line velocities have not been considered here, since they are strongly phase-dependent and somewhat erratic. The absorption-line velocities are probably also phase-dependent, but not nearly as much as the emission-line velocities. Merrill's (1923) absorption-line velocities are shown plotted in Figure 2, and to them has been fitted the appropriate portion of the velocity-curve for Mira as given by Joy (1926). The zero-phase velocity for this epoch is then -69.7 ± 0.5 km/sec. Joy (1954) has also shown that there is a correlation between the magnitude of Mira at maximum visual light and its radial velocity at this phase. The correlation appears linear and of slope

$$(v_1 - v_2) = -3.1(m_1 - m_2) .$$

However, for X Oph, the magnitude at maximum seems always to have been the same (Campbell 1955) to within a couple of tenths of a magnitude, and a correction for this effect has not been attempted.

TABLE 8

Plate	Merrill (km/sec)	Fernie (km/sec)	F−M (km/sec)	Plate	Merrill (km/sec)	Fernie (km/sec)	F−M (km/sec)
γ 9171.	−78.6	−78.7	−0.1	γ 10232.	−88.0	−87.6	+0.4

TABLE 9

Star	Plate	Measured R.V. (km/sec)	Catalogue R.V. (km/sec)	Cat.−Meas. (km/sec)	Weight
τ Boo.	7829	−15.1±1 0	−15.6	−0 5	1.00
α Com.	6834	−16.0±1.1	−17.7	−1 7	0 83
9 Dra.	9151	−31.1±1.8	−30.4	+0.7	0 31
	Weighted mean correction (Cat.−Meas.) = −0.8 km/sec				

The mean of the K-star velocities listed by Merrill (1923) is -70.8 ± 1.0 km/sec.

Corrections for systematic differences between spectrographs and personal equation in measuring spectrograms were arrived at by having on hand a number of Merrill's original spectrograms and also spectrograms of standard stars made with the various spectrographs used. These spectrograms were on loan to Dr. Robert P. Kraft from the Mount Wilson plate files. Spectrograms of X Oph taken by Merrill and measured by me gave the results shown in Table 8 for the difference between our personal equations. Spectrograms of standard stars made with this same spectrograph at about the same time as those in Table 8 gave the correction to the Mount Wilson system as in Table 9.

Second-epoch radial-velocity data were obtained as follows. While a guest investigator at Mount Wilson in June, 1957, Dr. Robert P. Kraft obtained a 100-inch coudé (10 A/mm) spectrogram of the K star in X Oph, which was then predominating the spectrum. At the same time, Kraft obtained a number of spectrograms of standard K-giant stars. My measurement of the X Oph plate showed a velocity for the K star of -72.93 ± 0.24 km/sec, while my measurement of the standard stars gave the corrections to the Mount Wilson system shown in Table 10.

Spectrograms (18 A/mm) of the variable at maximum (M star predominating, spectrum phase + 16 days) were obtained by Dr. Helmut Abt with the 82-inch telescope of the McDonald Observatory in November, 1957. Measurement of these showed a radial velocity of -69.90 ± 0.46 km/sec. Abt (private communication) informs me that the correction for this spectrograph to the Lick Mills system of velocities is

$$\text{Lick} - \text{McDonald} = + 1.21 \text{ km/sec.}$$

From the introduction to the Mount Wilson Radial Velocity Catalogue (1953) one finds

$$\text{Lick} - \text{Mount Wilson} = - 0.92 \text{ km/sec}$$

for M stars. Hence

$$\text{Mount Wilson} - \text{McDonald} = + 2.13 \text{ km/sec},$$

which is the correction to be applied to the measured velocity of the variable. A further correction (= +0.2 km/sec) for phase was applied from Figure 2.

The sum of these corrections applied to the measured velocities give the velocities shown in Table 3 of the text.

TABLE 10

Star	Plate	Measured R.V. (km/sec)	Catalogue R.V. (km/sec)	Cat.−Meas. (km/sec)	Weight
α Boo.......	Ce 11150	− 5.78±0.24	− 5.2	+0.6	17
η Cyg	Ce 11168	−26.06±0 21	−26.5	−0.4	23
	Weighted mean correction (Cat.−Meas.) = +0.04				

APPENDIX III

Determination of M_v of K Star

The details of the method employed for finding the absolute visual magnitude of the K companion were those described by Oke (1957). The principle of the method is to select a number of standard stars of known absolute magnitude (the range in the latter bracketing the expected M_v of the unknown star) and select in the spectra of these stars pairs of lines, the ratio of intensities of which are luminosity-sensitive. These ratios are measured quantitatively by the usual techniques of spectrophotometric analysis, and for each pair of lines a plot is made of the ratio (or some mathematical function thereof) against absolute magnitude, each standard star contributing a point to each plot. Each plot is then entered with the ratio for the unknown star, and the absolute magnitude is read out. (There is a slight departure from Oke's method here.) One obtains as many determinations of the latter as one has pairs of lines, and, since in practice each plot may show considerable scatter, it is best to use a number of pairs and take a mean of the various determinations of M_v which result.

The thirteen standard stars used in this work are listed in Table 11. All these stars are within 100 pc of the sun, and M_v is computed from trigonometric parallaxes. It was found that some plots showed more scatter than others, and for this reason a weight based on this scatter was assigned each determination of M_v. Thus the mean of the determinations is a weighted mean. The results are shown in Table 12.

Oke further extended his method to the determination of spectral type by the same procedure, but using pairs of lines which are sensitive to spectral type rather than luminosity. Regrettably, most such pairs involve hydrogen or iron lines, many of which appear missing or severely multilated in the X Oph spectrum because these lines appear in emission in the underlying spectrum of the long-period variable. However, a single determination was obtained from the ratios of the average intensities in the G band, λ 4920/λ 4300. The calibration was obtained from the eight stars in Table 11 for which color indices are available. It is assumed that these observed colors are intrinsic colors, since all these stars are within 100 pc of the sun. The result for X Oph was a spectral type of K1, and the probable error in this, estimated from the scatter in the plot, is about one spectral subclass.

TABLE 11

Star	Sp. Type	M_v	$B-V$
τ Psc	K0 III–IV	+1.0	....
γ Com	K1 III–IV	−0.4	....
57 Vir	K1 IV	+2.8	....
α Boo	K2 III	0.0	1.23
γ Lib	G8 III–IV	+1.3	....
κ Oph	K2 III	+0.6	1.17
HD 154733	K4 III	+1.0	1.30
α Sct	K3 III	+0.4	1.34
HD 173780	K3 III	+1.2	1.22
γ Aql	K3 II	−2.1	1.51
η Cyg	K0 III	+0.2	1.04
λ Peg	G8 II–III	+0.7	....
14 And	K0 III	+0.9	1.02

TABLE 12

Pair of Lines (λ)	M_v	Weight	Pair of Lines (λ)	M_v	Weight
4129.70/4127.79	+1.3	5	4161.2/4149.39	+0.2	4
4152.18/4154.48	+1.2	5	4305.92/4181.76	+1.7	2
4156.79/4154.48	+0.6	4	4215.50/4181.76	+2.4	2
4177.58/4181.76	+3.6	4	4161.09/4149.39	+0.7	2
4196.64/4198.32	+1.0	4			
4215.50/4250.80	−0.3	5		Weighted mean:	
4461.65/4455.87	+0.8	3		+1.13±0.21 (p.e.)	
4305.92/4198.32	+1.4	2			

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