# A Spectroscopic study of THE RR LyRAE STARS* 

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#### Abstract

The possibility that the RR Lyrae stars do not constitute a homogeneous spectroscopic group has been investigated by surveying the spectra of more than one hundred RR Lyrae stars at very low dispersion. ( $430 \mathrm{~A} / \mathrm{mm}$ at $\mathrm{H} \gamma$ ).

During the quarter of the light-cycle preceding minimum light, all the Bailey type a variables have hydrogen lines of similar strength, the spectral types ranging from F4 to F6. However, the K line of Ca II differs in strength from star to star by an amount that corresponds to about one spectral class, the spectra ranging in appearance from those of normal F-type stars to those of extreme F-type subdwarfs. Intermediate cases are common, and no separation into discrete spectroscopic groups is indicated by the material of this study. The Bailey type c variables have systematically earlier spectral types at minimum light, but they show qualitatively the same spread in spectroscopic peculiarity.

The difference, $\Delta S$, between the spectral types derived from the hydrogen lines and from the K line of Ca II at minimum light has been adopted as a parameter to describe the extent of the weak-line characteristic; high-dispersion studies indicate that this characteristic is due to low metal abundances. $\Delta S$ increases systematically with increasing period, $P$, for both the Bailey type c and the Bailey type a variables with $P<0$ d 75 , the spread in $P$ for a given $\Delta S$ among the latter corresponding approximately to the spreads observed in individual globular clusters. A third $P$ versus $\Delta S$ sequence appears to exist among the variables with $P>0 \mathrm{~d} 75$.

The period-frequency distributions of the strong- and weak-line variables indicate that the RR Lyrae star population near the sun differs from those found in globular clusters and far from the galactic plane, in that it possesses a strong-line (small $\Delta S$ ) component. The concentration of strong-line variables to the plane is confirmed by the intercomparison of the period-frequency distributions of various regions of the Galaxy. In addition, the strong-line variables appear to be relatively less concentrated toward the galactic center.

The solar motion and mean peculiar radial velocity of the weak-line variables resemble those derived for the globular clusters; those for the strong-line stars are intermediate between the values derived for halo and spiral-arm objects.


## I. INTRODUCTION

During the last fifteen years evidence has accumulated that casts doubt upon the uniformity of the physical and kinematic properties of the intrinsic variable stars, with periods less than about 1 day. We shall refer to these objects as "RR Lyrae stars" and throughout this paper shall draw no distinction between the Bailey types a and b, referring to them collectively as "type a." Several investigators (Eggen 1952; Smith 1955a; Woltjer 1956) of the short-period end of the RR Lyrae domain have presented evidence which indicates that the variable stars with periods less than 0.2 day constitute a low-luminosity group, with galactic rotation properties intermediate between those of the halo and spiral-arm populations. There is no comparable discussion for stars in the region of the long-period boundary ( $P \sim 1^{\mathrm{d}}$ ), and, as shown later in this paper, there are indications that departures from uniformity among intrinsic variable stars occur there as well. Even if we ignore the period boundaries of the RR Lyrae class and consider only the Bailey type a's near the center of the domain ( $0^{\mathrm{d}} 4<P<0 \mathrm{~d} 7$ ) whose membership in the RR Lyrae class should be unimpeachable, the accepted criteria (period and shape of light-curve) do not serve to isolate a physically and kinematically homogeneous group of stars. Münch and Terrazas (1946) in a survey of eleven stars found two-SW And ( $P=0$ d 442 ) and AR Per $(P=0.425)$-which differ markedly from RR Lyrae in that their metallic-line spectra are developed to an extent much more in keeping with the spectral type indicated by the strength

[^0]of their hydrogen lines than is the case for RR Lyrae itself. Three years later, Kukarkin (1949) showed that the Bailey type a's with periods between 0.425 and 0.435 day are concentrated at lower galactic latitudes than the class as a whole, and he stressed the fact, recognized previously, that such variables have not been found in globular clusters. Shortly thereafter, Struve $(1950,1951)$ pointed out that there is a systematic increase in radial velocities of the RR Lyrae stars with period. Iwanowska (1953), in a precursor of the present study, found three more stars of "late type" and suggested that they might represent a short-period extension of the classical cepheid sequence, which in our Galaxy has a range of periods from about 1.5 to 50 days.

The above-mentioned investigations indicate the need for a re-examination of the group characteristics of the RR Lyrae class of variable stars. As a first step in this direction, a low-dispersion survey of the spectra of the brighter RR Lyrae stars has been carried out. A simple, exploratory system of spectroscopic subclassification has been devised from the examination of several hundred spectrograms of more than one hundred RR Lyrae stars. Other observational data have then been used to intercompare several properties of the subtypes defined by this scheme.

## II. OBSERVATIONAL MATERIAL AND PROCEDURE

The survey was made with the nebular spectrograph of the Crossley reflector of the Lick Observatory. The linear dispersion of the spectrograph is $430 \mathrm{~A} / \mathrm{mm}$ at $\mathrm{H} \gamma$. The slit-width ( 0.12 mm ) employed corresponds to $27 \mu$ at the photographic plate. Under these conditions, the spectrum of a star of $m_{\mathrm{pg}}=13.0$, widened 0.4 mm , and well exposed between $\lambda \lambda 3800$ and 4400 could be recorded on baked Kodak II $a$-O plates in a little less than 1 hour. Since preliminary observations indicated that near minimum light the changes in the spectra of RR Lyrae stars are small over this interval of time the faint limit of the survey was arbitrarily set near $m_{\mathrm{pg}}=13.0$. Some special instances in which it was necessary to observe fainter stars will be discussed individually.

The program was limited, with few exceptions, to objects for which finding charts have been published. Special emphasis was placed on the following groups: (1) stars for which radial-velocity and proper-motion data are available; (2) Bailey type a's with $P<0 \mathrm{~d} 44$; (3) variables with $P>0 \mathrm{~d} 7$; (4) stars with unusual light-curves or other characteristics of particular interest. These considerations, in addition to the limiting magnitude of the survey, make it clear that the material is affected by strong selection effects. The distribution over the sky is also biased, since at Mount Hamilton the north galactic pole is high in the winter sky, while the south galactic pole and galactic center region are low in the summer sky (maximum altitude $\sim 25^{\circ}$ ). Several observing seasons would be required to give these three important regions coverage comparable to that obtained for the rest of the sky.

During the course of a light-cycle, spectral variations of about one spectral class are common for the RR Lyrae stars. Therefore, to intercompare spectra meaningfully, either each star should be observed at many phases, or all of them must be observed at some single reference phase. The amount of observing time available and the lack of accurate light-elements are practical considerations that preclude both these extremes, and so a compromise was adopted. When recently observed times of maximum light were available, an attempt was made to obtain spectrograms at maximum and minimum and at one or more points on the descending branch of the light-curve. In other cases, series of spectrograms distributed in phase were obtained, and approximate phases were estimated by using the times of maximum inferred from the appearance of the spectra and the periods published in the General Catalogue of Variable Stars. (Kukarkin and Parenago 1948). It has been assumed that, to within the uncertainties inherent in the procedure, the earliest spectral type coincides with light-maximum. This assumption is justified by the results shown in Figure 2. In many instances, recent times of maximum were discovered in the literature after the observations had been obtained,
so that, even though the placement of these observations may not be ideal, the phases are known with some precision. Conditions imposed by the moon, weather, observing schedule, and the co-ordinates of a given star made it necessary in many cases to spread observations over several months to obtain the desired coverage. For various reasons some stars were observed only once, as the alternative to passing them by altogether. Kodak $\mathrm{II} a-\mathrm{O}$ plates, baked for 3 days at $50^{\circ} \mathrm{C}$, were used throughout. The plates were developed in D19 for $4 \frac{1}{2}$ minutes at $65^{\circ} \mathrm{F}$.

## a) The Reduction of the Observations

The spectral-type interval which includes the RR Lyrae stars, A through F, is dominated by the lines of the Balmer series and by the H and K resonance lines of Ca II. With the dispersion and resolution employed in this investigation, these are the only absorption lines visible in A-type spectra. Between F0 and F2 in the revised Yerkes main sequence (Johnson and Morgan 1953) a feature appears in the G-band region and grows in strength until it reaches approximate equality with $\mathrm{H} \gamma$ at F 7 or F 8 . Normal stars may be classified at this dispersion by using the ratio of Ca II $\lambda 3933$ (K) to $\mathrm{H}+\mathrm{H} \epsilon$ or to $\mathrm{H} \delta$ from A 0 to F 2 , and by the ratio of the G band to $\mathrm{H} \gamma$ from F2 to G0. The RR Lyrae stars cannot, however, be classified by use of these criteria.

TABLE 1
MK Standards (Luminosity Class V)


During much of a typical light-cycle the G band is not visible, and in the spectra of some variables it does not appear at this dispersion. Furthermore, the well-known discrepancy between the metallic-line and hydrogen-line types in the spectra of most RR Lyrae stars renders line ratios involving $\lambda 3933$ and the Balmer lines useless. There is thus no recourse but to use the absolute strengths of individual lines, which is a procedure accompanied by numerous pitfalls. Small differences from plate to plate, in emulsion grain and fogging, affect the appearance of a single absorption line more than the ratio of intensities of two lines. Such effects may be minimized, but they cannot be eliminated entirely. Variations in the density of the photographic image produce even larger, spurious differences in line strengths, a bothersome problem in the observation of variable stars with ranges of a magnitude or more, particularly when the ranges and phases are not accurately known. The problem is further aggravated in some cases by errors in published magnitudes. The result of all these difficulties is a much larger spread in densities than is desirable for a spectral classification program. Since the accuracy of classification by absolute strengths of individual features is seriously impaired if the densities of the standard spectra do not match those of the objects to be classified, many spectrograms with a wide range in densities were obtained for each standard star.

## b) The Classification of the Spectrograms

Spectrograms of the luminosity class V standards listed in Table 1 were used for the classification of all the spectrograms of the variables. The main sequence defined by these standards has the advantage that nearly all types from A0 to G0 are available.

There are, on the other hand, no MK luminosity class III standards between F4 and G0, a spectral-type interval frequently used in the classification of the spectra of RR Lyrae stars. An intercomparison of Crossley spectrograms of F-type MK standards of luminosity classes $I b$, II, III, and V with respect to the hydrogen lines and the K line of Ca ir confirmed the general absence of luminosity effects in these lines, as is indicated also by the equivalent widths in F-type stars tabulated by Greenstein (1948). It is the opinion of the writer, therefore, that if luminosity class III standards had been used for classification of the RR Lyrae stars, the gap in the class III sequence would have introduced uncertainties into the results larger than those which could have arisen from the use of an inappropriate luminosity class.

All spectrograms were classified on a Hartmann spectrocomparator. Hydrogenand K-line types were assigned separately by a simple process of interpolation among the standards. These results are given in Table 2. Strictly speaking, they are not spectral types. Although they will be referred to as such throughout this paper, they are, in fact, nothing more than visual estimates of line intensities expressed in terms of spectral class.

An estimate of the uncertainties introduced by the accumulation of sources of error has been obtained from those cases in which two or more spectrograms for one star in a small phase interval ( 0 P 03 at maximum, 0 P. 06 at minimum) are available. Apart from systematic errors that may be present, a spectrogram with no apparent defects appears to yield a type based on a single line that is good to one- or two-tenths of a spectral class; the accuracy varies somewhat with spectral type, since, even along the main sequence, the standards are not everywhere equally spaced with respect to the criteria used in this study. Errors in types derived from poor spectra may be considerably larger. The relative weights to be attached to the types in Table 2 are roughly indicated by the letters $g$, $f$, and $p$, denoting good, fair, and poor quality spectrograms, respectively.

## III. THE SPECTRA OF THE RR LYRAE STARS

## a) General Description

For the great majority of the Bailey type a's the hydrogen type ranges from about A7 at maximum to F 5 at minimum light. The dispersion at minimum is comparable to the errors of classification, and there is little indication of dependence on either period or light-amplitude, although some stars depart systematically from the mean. At maximum light the earliest types are associated with large light-amplitudes. Apart from this amplitude dependence, the hydrogen types of the Bailey type a's are all similar to one another.

The hydrogen lines, however, present a false impression of uniformity. At any given phase a large range in the strength of the K line may be found among different Bailey type a's. This diversity is most pronounced at minimum light, where it corresponds to approximately one spectral class.
"Strong-line" objects, such as SW And and AN Ser, have the hydrogen and Ca ir lines of normal middle F-type stars. The G band appears in normal strength, and at some phases the blend of metallic lines longward of $\lambda 4172$ suggests a luminosity class above the main sequence. On the other hand, stars such as X Ari and SV Eri have K-line types not later than A5 or A7, and the G band is marginally visible or absent from their spectra throughout the light-cycle. In Figure 1, $a$, spectra near minimum light of several stars, ordered in a sequence of increasing K-line strength, are reproduced with two MK standards. In addition to illustrating the range in appearance described above, the figure shows that stars exist in the interval between the extreme examples. Spectra of AT And and BH Peg, obtained at phases that either coincide or interlace with those of X Ari and SV Eri, always have stronger K lines. They could not, on the other hand, be confused with SW And at any phase. Because of this intrinsic range

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| Star |  | $\mathrm{JD}_{6}$ | Type |  |  |  | Star |  | $\mathrm{JD}$ | Type |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2436000+ | Phase | H | CaII | Qual. |  |  | 2436000+ | Phase | H | CaII | Qual. |
| SW | And | 167.620 | ${ }_{0} \mathrm{P}_{43}$ | F6 | F6 | g | BR | Aqr | 433.960 | 0.92 | A8 | A6 | g |
|  |  | . 630 | . 46 | F6 | F6 | g |  |  | 434.834 | . 74 | F5 | F2 | g |
|  |  | . 672 | . 55 | F6 | F6 | g |  |  |  |  |  |  |  |
|  |  | . 712 | . 64 | F6 | F5 | g | BS | Aqr | 455.912 | *. 00 | A7 | A9 | g |
|  |  | . 754 | . 74 | F6 | F5 | g |  |  | 495.742 | . 34 | F1 | F1 | g |
|  |  | 436.775 | . 00 | A8 | A9 | g |  |  |  |  |  |  |  |
|  |  | 437.739 | . 18 | F3 | F4 | g | CP | Aqr | 429.946 | . 22 | F1 | F1 | g |
|  |  | 482.746 | . 94 | A7 | A5 | f |  |  | 433.819 | . 58 | F5 | F2 | p |
|  |  | $.751$ | . 95 | A7 | A6 | f |  |  | 434.789 | . 67 | F5 | F2 | f |
|  |  |  |  |  |  |  |  |  | . 919 | . 95 | A7 | A5 | f |
| XX | And | 408.989 | . 06 | A8 | A2 | g |  |  |  |  |  |  |  |
|  |  | 410.993 | . 83 | F6 | A7 | g | CY | Aqr | 410.840 | . 99 | A2 | A2 | g |
|  |  | 482.785 | . 17 | F0 | A4 | g |  |  | . 848 | . 12 | A7 | A7 | g |
|  |  |  |  |  |  |  |  |  | . 861 | . 33 | A7 | A7 | f |
| AC | And | 420.833 |  | F8 | F8 | g |  |  | 419.976 | . 75 | A8 | A4 | f |
|  |  | . 886 |  | F8 | F8 | g |  |  | . 983 | . 86 | A4 | A4 | f |
|  |  | . 949 |  | F7 | F8 | g |  |  |  |  |  |  |  |
|  |  | 421.845 |  | F5 | F7 | g | AA | Aql | 421.696 | . 06 | A5 | A4 | g |
|  |  | . 916 |  | F6 | F7 | g |  |  | . 793 | . 33 | F2 | F2 | f |
|  |  | 436.015 |  | F5 | F8 | g |  |  | . 885 | . 58 | F5 | F6 | g |
|  |  | . 694 |  | F5 | F6 | g |  |  | 429.896 | . 73 | F5 | F5 | g |
|  |  | . 765 |  | A9 | A9 | g |  |  |  |  |  |  |  |
|  |  | 437.712 |  | F6 | F7 | g | V341 | Aql | 396.979 | . 09 | A6 | A3 | p |
|  |  | 439.011 |  | F4 | F5 | g |  |  | 408.793 | . 53 | F5 | F2 | g |
|  |  | 440.719 |  | F7 | F9 | g |  |  | . 953 | . 81 | F4 | F2 | f |
|  |  | . 843 |  | F6 | F8 | g |  |  |  |  |  |  |  |
|  |  | 441.015 |  | A9 | A9 | g | V706 | Aql | 463.773 |  | F0 | F0 | g |
|  |  | . 876 |  | F5 | F8 | g |  |  |  |  |  |  |  |
|  |  | 442.017 |  | F6 | F8 | g | X | Ari | 263.671 | . 98 | A8 | A1 | g |
|  |  |  |  |  |  |  |  |  | 427.007 | . 83 | F2 | A4 | g |
| AT | And | 434.974 | . 55 | F5 | F2 | g |  |  | 428.008 | . 37 | F4 | A3 | g |
|  |  | 436.751 | . 43 | F7 | F3 | g |  |  | 482.856 | . 61 | F5 | A3 | f |
|  |  | 437.728 | . 01 | F0 | A7 | g |  |  | . 866 | . 62 | F4 | A4 | g |
|  |  | $481.711$ | . 30 | F5 | A9 | $\mathrm{g}$ |  |  |  |  |  |  |  |
|  |  | . 820 | . 48 | F6 | F3 | g | TZ | Aur | $467.006$ | $.67$ | F4 | F2 |  |
|  |  |  |  |  |  |  |  |  | $469.023$ | . 82 | F7 | F5 | f |
| SW | Aqr | 427.853 | . 84 | F3 | A9 | g |  |  | . 988 | . 28 | F2 | F1 | g |
|  |  | . 942 | . 04 | A6 | A2 | f |  |  | 471.024 | . 92 | A9 | A5 | g |
|  |  | . 950 | . 06 | A4 | A1 | g |  |  |  |  |  |  |  |
|  |  | 482.824 | . 53 | F5 | A9 | g | RS | Boo | 223.052 | . 56 | F5 | F3 | g |
|  |  |  |  |  |  | 8 |  |  | . 066 | . 60 | F5 | F3 | f |
| SX | Aqr | 019.868 | . 60 | F6 | A7 | g |  |  | 229.038 | . 42 | F5 | F3 | g |
|  |  |  |  |  |  |  |  |  | 234.986 | . 18 | A9 | A5 | g |
| TZ | Aqr | 433.788 | . 78 | F6 | F1 | p |  |  | 236.037 | . 97 | A7 | A7 | f |
|  |  | . 918 | . 01 | A6 | A4 | g |  |  |  |  |  |  |  |
|  |  | 434.878 | . 69 | F5 | F1 | g | SW | Boo | 321.979 |  | F6 | A9 | p |
| BN | Aqr | 438.920 | . 80 | F3 | A9 | g | TV | Boo | $212.953$ | $.95$ | A7 | A0 | g |
|  |  |  |  |  |  |  |  |  | $.969$ | . 02 | A7 | A0 | g |
| BO | Aqr |  |  | A8 | A3 |  |  |  | 213.001 | . 10 | A9 | A0 | g |
|  |  | $438.869$ | . 66 | A9 | A2 | f |  |  | 214.946 | . 32 | A8 | A0 | g |
|  |  | 440.932 | . 63 | F1 | A7 | f |  |  | . 969 | . 40 | A9 | A2 | g |
|  |  |  |  |  |  |  |  |  | . 996 | . 48 | A9 | A0 | f |
| BR | Aqr | 195.633 | . 35 | F5 | F2 | g |  |  | 215.022 | . 57 | F2 | A0 | f |
|  |  | 212.617 | . 59 | F6 | F2 | f |  |  | 234.892 | . 14 | A7 | A0 | f |
|  |  | 433.847 | 0.69 | F4 | F1 | f |  |  | . 905 | 0.18 | A8 | A0 | g |

TABLE 2 (Continued)

| Star |  | $\mathrm{JD}_{\oplus}$ | Type |  |  |  | Star |  | $\mathrm{JD}_{\oplus}$ | Type |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2436000+ | Phase | H | CaII | Qual. |  |  | 2436000+ | Phase | H | CaII | Qual. |
|  | Boo | 254.915 | *0. ${ }^{\text {P }} 00$ | F0 | A2 | p | RZ | Cep | 383.831 | ${ }_{0} \mathrm{P}_{49}$ | A9 | A4 | f |
|  |  | 263.890 | . 79 | A8 | A0 | f |  |  | . 937 | . 84 | A7 | A0 | g |
|  |  | 306.888 | . 86 | $\dot{\text { A } 8}$ | A2 | f |  |  | 440.828 | . 14 | A9 | A2 | g |
|  | Boo | 377.743 |  | A9 | F1 | g | DX | Cep | 463.850 | *. 00 | A7 | A0 | f |
|  |  |  |  |  |  |  |  |  | 466.865 | . 73 | F6 | A9 | $g$. |
| SS | Cnc | 143.945 | . 72 | F5 | F3 | p |  |  |  |  |  |  |  |
|  |  | 144.048 | . 00 | A4 | A2 | f | RR | Cei | 142.731 | . 77 | F5 | F0 | p |
|  |  | . 057 | . 02 | A4 | A1 | f |  |  | . 805 | . 90 | F5 | A4 | p |
|  |  | 173.001 | . 82 | F5 | F2 | g |  |  | . 841 | . 96 | A7 | A2 | f |
|  |  | . 056 | . 97 | A5 | A2 | p |  |  | . 860 | . 00 | A7 | A2 | f |
|  |  | . 069 | . 00 | A7 | A3 | g |  |  | 441.931 | . 79 | F5 | A9 | g |
|  |  |  |  |  |  |  |  |  | . 936 | . 80 | F5 | F0 | g |
| TT | Cnc | 186.870 | . 23 | F4 | A6 | p |  |  |  |  |  |  |  |
|  |  | 187.020 | . 50 | F6 | A6 | p | RU | Cet | 466.963 |  | F3 | A4 | g |
|  |  | 223.912 | . 97 | A9 | A2 | g |  |  |  |  |  |  |  |
|  |  | 470.012 | . 74 | F4 | A7 | f | RZ | Cet | 433.940 | . 86 | A6 | A2 | g |
|  |  |  |  |  |  |  |  |  | 437.947 | . 71 | F3 | A9 | g |
| VZ | Cnc | 313.660 | . 64 | F4 | F5 | g |  |  | 466.912 | . 44 | F4 | F0 | g |
|  |  | . 664 | . 67 | F4 | F4 | g |  |  |  |  |  |  |  |
|  |  | . 740 | . 09 | A7 | A9 | f | UU | Cet | 166.625 | *. 00 | A7 | A3 | p |
|  |  | 318.728 | . 06 | A9 | A9 | g |  |  | 426.953 | . 49 | F4 | A9 | p |
|  |  | 325.728 | . 30 | F1 | F3 | g |  |  | 463.958 | . 58 | F4 | A9 | f |
|  |  |  |  |  |  |  |  |  | 465.917 | . 82 | F4 | F0 | f |
| W | CVn | 212.943 | . 41 | F6 | F0 | p |  |  | 495.851 | . 20 |  |  | f |
|  |  | . 964 | . 45 | F7 | A8 | f |  |  |  |  |  |  |  |
|  |  | 214.958 | . 06 | F0 | A5 | g | S | Com | 229.008 | . 29 | F4 | A7 | g |
|  |  | . 985 | . 11 | F2 | A6 | g |  |  | 235.046 | . 58 | F4 | A5 | f |
|  |  | 215.031 | . 20 | F4 | A7 | f |  |  | 263.034 | . 30 | F4 | A7 | g |
|  |  | 234.948 | . 29 | F4 | F0 | g |  |  | 280.958 | . 85 | F4 | A7 | f |
|  |  | 235.976 | . 16 | F2 | A7 | g |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | U | Com | 229.065 | *. 00 | F0 | A5 | g |
| Z | CVn | 358.729 |  | A8 | A3 | p |  |  | 235.075 | . 53 | A8 | A2 | g |
|  |  | 376.728 |  | F5 | A7 | p |  |  | 263.001 | . 93 | A8 | A4 | g |
| RR | CVn | 235.014 | . 08 | A7 | A2 | g | RY | Com | 339.821 | . 41 | F5 | F1 | p |
|  |  | 236.066 | . 97 | A7 | A0 | g |  |  | 557.025 | . 59 | F2 | F0 | f |
|  |  | 329.736 | . 65 | F4 | $\dot{\text { A }} 7$ | p |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | ST | Com | 337.893 | . 71 | F5 | F0 | g |
| X | CMi | 195.814 | *. 00 | F5 | F3 | f |  |  |  |  |  |  |  |
|  |  | . 935 | . 33 | F6 | F2 | g | W | Crt | 313.700 | *. 00 | F3 | F1 | g |
|  |  | 222.826 | . 30 | F6 | F3 | p |  |  | . 808 | . 27 | F5 | F2 | f |
| RV | Cap | 368.953 | *. 00 | F4 | A8 | f | UY | Cyg | 395.870 | . 06 | A8 | A7 | f |
|  |  | 395.973 | . 36 | A7 | A4 | f |  |  | . 947 | . 19 | A9 | F1 | g |
|  |  | 429.869 | . 07 | F3 | A8 | g |  |  | 397.915 | . 70 | F5 | F2 | g |
|  |  |  |  |  |  |  |  |  | 408.762 | . 05 | A8 | A8 | g |
| YZ | Cap | 438.825 |  | A4 | A4 | g |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | XZ | Cyg | 172.693 | . 38 | F4 | A5 | f |
| PW | Cas | 430.926 | . 11 | F8 | F8 | f |  |  | . 700 | . 39 | F5 | A9 | g |
|  |  | 435.962 | . 40 | G0 | G0 | g |  |  | . 710 | . 41 | F5 | A6 | f |
|  |  | 437.898 | . 82 | F6 | F8 | g |  |  | 349.847 | . 05 | A7 | A2 | g |
|  |  |  |  |  |  |  |  |  | 379.785 | . 22 | A9 | A7 | g |
| V342 Cas |  | 455.828 | . 03 | G0 | G0 | g |  |  | 383.844 | . 92 | A9 | A4 | g |
|  |  | 456.888 | . 37 | F8 | F8 | g |  |  | . 861 | . 95 | A7 | A2 | g |
|  |  | 457.928 | . 68 | F6 | F8 | g |  |  | . 879 | . 99 | A7 | A0 | g |
|  |  | 466.815 | 0.89 | G0 | G0 | g |  |  | . 892 | 0.02 | A6 | A2 | g |


| Star |  | $\mathrm{JD}_{\oplus}$ | Type |  |  |  | $\mathrm{JD}_{\oplus}$ |  |  |  | Type |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2436000+ | Phase | H | CaII | Qual. | St | ar | 2436000+ | Phase | H | CaII | Qual. |
| XZ | Cyg | 383.908 | ${ }_{0} \mathrm{P}_{05}$ | A6 | A2 | g | XZ | Dra | 390.953 | ${ }_{0} \mathrm{P}_{85}$ | F2 | F1 | g |
|  |  | . 953 | . 15 | A8 | A5 | g | AE | Dra | 481.646 | * . 00 | A7 | A5 | g |
|  |  | 440.700 | . 77 | F4 | A8 | g |  |  | 482.647 | . 66 | F6 | F1 | $f$ |
|  |  | 480.764 | . 64 | F6 | A9 | g |  |  |  |  |  |  |  |
| DM | Cyg |  |  |  |  |  | RX | Eri | 211.716 | . 33 | F4 | A9 | f |
|  |  | 186.617 | . 90 | F6 | F5 | p |  |  | 212.720 | . 04 | A9 | A2 | g |
|  |  | . 642 | . 96 | F6 | F2 | f |  |  | . 748 | . 09 | A9 | A2 | g |
|  |  | . 662 | . 01 | A9 | A5 | f |  |  | . 770 | . 12 | A8 | A3 | f |
|  |  | 195.678 | . 48 | F6 | F3 | p |  |  | . 803 | . 18 | F2 | A5 | p |
|  |  | 427.981 | . 77 | F6 | F6 | g |  |  | 457.962 | . 65 | F5 | A9 | f |
| KP | Cyg |  |  |  |  |  |  |  | 470.955 | . 78 | F6 | A7 | g |
|  |  | 401.888 | . 00 | A8 | A8 | g |  |  |  |  |  |  |  |
|  |  | 402.940 | . 23 | F3 | F4 | g | SV | Eri | 167. 739 | *. 00 | F4 | A5 | g |
|  |  | 410.933 | . 56 | F7 | F7 | g |  |  | 211.633 | . 50 | A9 | A2 | g |
| BX | Del |  |  |  |  |  |  |  | . 644 | . 51 | A9 | A2 | g |
|  |  | 433.879 | . 43 | F8 | F8 | p |  |  | . 653 | . 52 | A8 | A2 | g |
|  |  | 438.800 | . 94 | F0 | F2 | g |  |  | 212.692 | . 98 | F5 | A4 | g |
|  |  | 480.735 | . 35 | F7 | F8 | g |  |  | . 700 | . 99 | F5 | A5 | g |
| DX | Del |  |  |  |  |  |  |  | 455.939 | . 78 | F0 | A4 | g |
|  |  | 368. 974 | . 86 | F5 | F3 | g |  |  | 466.936 | . 19 | F5 | A6 | g |
|  |  | 395.988 | . 02 | A9 | A9 | f |  |  |  |  |  |  |  |
|  |  | 419.831 | . 46 | F5 | F3 | g | BB | Eri | 495.892 | . 69 | F4 | A7 | g |
|  |  | . 902 | . 62 | F6 | F3 | g |  |  |  |  |  |  |  |
|  |  | . 953 | . 72 | F6 | F4 | g | RR | Gem | 195. 751 | *. 00 | A9 | A5 | g |
| FM | Del |  |  |  |  |  |  |  | $.776$ | . 06 | A9 | A7 | f |
|  |  | 449.693 | . 48 | F8 | F8 | f |  |  | . 896 | . 37 | F5 | A9 | g |
|  |  | 455. 776 | . 10 | F5 | F5 | g |  |  | . 970 | . 55 | F6 | F2 | g |
|  |  | 464.780 | . 40 | F5 | F5 | f |  |  | 196.035 | . 72 | F6 | F4 | g |
|  |  | 482.722 | . 90 | F8 | F7 | f |  |  | 222.883 | . 30 | F4 | F2 | g |
| RW | Dra | 390.785 | . 12 | A7 | A0 | f | TW | Her | 019.788 | . 58 | F6 | F2 | f |
|  |  | 391.872 | . 58 | F5 | F2 | f |  |  | . 807 | . 63 | F5 | F2 | g |
|  |  | . 890 | . 62 | F5 | F2 | f |  |  | . 829 | . 68 | F5 | F3 | g |
|  |  | 410.736 | . 17 | A7 | A7 | f |  |  | 020.752 | . 00 | A7 | A2 | g |
|  |  | $420.758$ | . 80 | F2 | A7 | g |  |  | . 762 | . 02 | A4 | A2 | g |
|  |  | 482.686 | . 63 | F3 | F0 | g |  |  | . 771 | . 04 | A7 | A2 | g |
| SU | Dra |  | . 47 | F7 | A7 | f | VX | Her | $264.063$ | . 98 | A7 | A0 | f |
|  |  | $234.810$ | . 35 | F6 | A6 | g |  |  | 281.053 | . 29 | F4 | A4 | p |
|  |  | . 813 | . 35 | F6 | A4 | g |  |  | 419.804 | . 99 | A4 | A0 | g |
|  |  | 235.998 | . 14 | F2 | A4 | f |  |  | 463.690 | . 36 | F3 | A9 | g |
|  |  |  |  |  |  |  |  |  | 464.707 | . 60 | F4 | A8 | g |
| SW | Dra | 358. 710 | . 44 | F5 | F2 | g |  |  | . 724 | . 64 | F4 | A9 | g |
|  |  | 408.715 | . 22 | F2 | A9 | g |  |  |  |  |  |  | 8 |
|  |  | 410.706 | . 72 | F6 | F3 | g | VZ | Her | 336.897 | . 49 | F4 | F0 | g |
|  |  | . 714 | . 73 | F6 | F3 | g |  |  | . 985 | . 69 | F4 | F0 | g |
|  |  |  |  |  |  |  |  |  | 402.761 | . 07 | A6 | A2 | f |
| XZ | Dra | 379. 752 | . 34 | F4 | F3 | g |  |  | 463.670 | . 40 | F4 | A9 | g |
|  |  | . 759 | . 36 | F4 | F3 | g |  |  | 464.682 | . 70 | F4 | A9 | g |
|  |  | 383.854 | . 95 | A7 | A3 | g |  |  | . 743 | . 84 | F4 | F0 | f |
|  |  | . 918 | . 08 | A9 | A7 | g |  |  |  |  |  |  |  |
|  |  | . 961 | . 17 | A9 | F2 | g | AR | Her | 376.820 | *. 00 | F3 | A7 | g |
|  |  | 390.833 | . 60 | F4 | F1 | g |  |  | 390.734 | . 60 | A8 | A6 | g |
|  |  | . 854 | . 64 | F4 | F3 | g |  |  | . 800 | . 74 | F2 | A8 | g |
|  |  | . 877 | . 69 | F5 | F0 | g |  |  | 410.781 | . 25 | A7 | A0 | f |
|  |  | . 899 | . 74 | F5 | F3 | g |  |  |  |  |  |  |  |
|  |  | . 923 | 0.78 | F6 | F2 | g | BD | Her | 438.778 | 0.12 | F2 | F3 | g |


| Star |  | $\mathrm{JD}_{\oplus}$ | Type |  |  |  | $J D_{\oplus}$ |  |  |  | Type |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2436000+ | Phase | H | CaII | Qual. |  |  | 2436000+ | Phase | H | CaII | Qual. |
| BD | Her | 452.804 | $0 . \mathrm{P}_{72}$ | F6 | F4 | g | RR | Lyr | 379.775 | ${ }_{0} \mathrm{P}_{95}$ | F2 | A5 | f |
|  |  |  |  |  |  |  |  |  | 383.725 | . 92 | F5 | A7 | g |
| BL | Her | 390.709 | * . 00 | F0 | F1 | g |  |  | . 760 | . 98 | F1 | A6 | g |
|  |  | 391.840 | . 86 | A8 | A8 | g |  |  | . 770 | . 00 | F2 | A6 | g |
|  |  | 408.732 | . 78 | A8 | F0 | g |  |  | . 781 | . 02 | A8 | A5 | g |
|  |  | . 737 | . 79 | A8 | F0 | g |  |  | . 807 | . 06 | A8 | A5 | g |
|  |  | 410.800 | . 37 | F7 | F8 | g |  |  | . 872 | . 18 | F1 | A7 | g |
|  |  | 464.661 | . 56 | F7 | F8 | g |  |  | . 945 | . 30 | F4 | A8 | g |
|  |  |  |  |  | A8 |  |  |  | 386.721 | . 20 | A9 | A6 | g |
| CE | Her | 336.946 | . 63 | F5 |  | f |  |  | . 794 | . 33 | F2 | A7 | g |
|  |  |  |  |  |  |  |  |  | . 856 | . 44 | F4 | A9 | g |
| OX | Her | 373.922 | *. 00 | A8 | A4 | g |  |  | . 862 | . 45 | F4 | A8 | g |
|  |  | 397.733 | . 44 | F5 | A9 | p |  |  | . 869 | . 46 | F4 | A9 | g |
|  |  | 435.704 | . 58 | F5 | A9 | f |  |  | . 876 | . 48 | F6 | A8 | g |
|  |  |  |  |  |  |  |  |  | . 883 | . 49 | F5 | A7 | g |
| CZ | Lac | 429.984 | *. 00 | F5 | F3 | g |  |  | . 890 | . 50 | F6 | A8 | g |
|  |  | 430.994 | . 34 | A8 | A7 | f |  |  | . 897 | . 51 | F7 | F1 | g |
|  |  | 434.992 | . 59 | F1 | F5 | g |  |  | . 903 | . 52 | F6 | A9 | g |
|  |  | 437.803 | . 10 | F6 | F4 | g |  |  | . 910 | . 54 | F7 | A9 | g |
|  |  | 481.685 | . 65 | F3 | F5 | g |  |  | . 917 | . 55 | F6 | F0 | g |
|  |  | . 785 | . 88 | F4 | F5 | g |  |  | . 924 | . 56 | F6 | F1 | g |
|  |  |  |  |  |  |  |  |  | . 931 | . 57 | F6 | F0 | g |
| RR | Leo | 196.060 | * . 00 | A7 | A0 | p |  |  | . 938 | . 59 | F6 | F1 | g |
|  |  | 215.867 | . 78 | F4 | A6 | f |  |  | . 945 | . 60 | F6 | F0 | g |
|  |  | . 915 | . 89 | F4 | A4 | g |  |  | . 952 | . 61 | F7 | F0 | g |
|  |  | . 926 | . 92 | A8 | A0 | g |  |  | . 959 | . 62 | F6 | F0 | g |
|  |  | 223.958 | . 67 | F5 | A7 | g |  |  | $.965$ | . 63 | F6 | A8 | g |
|  |  |  |  |  |  | 8 |  |  | 440.808 | . 62 | F4 | A8 | f |
| RX | Leo | 339. 775 |  | F2 | A7 | g |  |  | . 856 | . 71 | F4 | A8 | f |
| SS | Leo | 264.021 | . 20 | A9 | A2 | g | RZ | Lyr | 368.897 | . 71 | F4 | A5 | f |
|  |  | 312.860 | . 17 | A7 | A5 | g |  |  | 397. 771 | . 19 | A9 | A4 | g |
|  |  | 313.897 | . 83 | F6 | A8 | f |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | BQ | Lyr | 430.833 | *. 00 | F0 | F0 | g |
| ST | Leo | 223.011 | . 37 | F4 | A8 | g |  |  | 435.860 | . 56 | F3 | F3 | f |
|  |  | . 988 | . 41 | F5 | A8 | f |  |  |  |  |  |  |  |
|  |  | 262.967 | . 96 | A7 | A3 | f | CX | Lyr | 457.708 |  | F4 | A7 | p |
| TV | Leo | 222.975 |  | F6 | A5 | p | EZ | Lyr | 348.939 | . 97 | A7 | A2 | f |
|  |  | 263.992 |  | F6 | A5 | f |  |  | 349.925 | . 84 | F4 | A7 | f |
|  |  | 329.695 |  | F4 | A4 | g |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | IO | Lyr | 377.954 | *. 00 | F1 | A7 |  |
| V | LMi | 339.731 | . 25 | F1 | A7 | g |  |  | 397.828 | . 44 | F5 | F2 | f |
|  |  |  |  |  |  |  |  |  | 419.757 | . 43 | F6 | F3 | $f$ |
| U | Lep | 212.736 | *. 00 | A7 | A2 | g |  |  |  |  |  |  |  |
|  |  | 222.669 | . 08 | A8 | A0 | f | KX | Lyr | 376.844 | *. 00 | F5 | F5 | g |
|  |  | 466.028 | . 60 | F6 | A7 | g |  |  | 377.973 | . 56 | F2 | F1 | g |
|  |  |  |  |  |  |  |  |  | 395.901 | . 22 | A7 | A7 | f |
| TV | Lib | 263.060 | . 33 | F4 | F1 | p |  |  | 420.799 | . 69 | F2 | F3 | g |
|  |  | 281.028 | . 98 | A7 | A2 | p |  |  | . 867 | . 85 | F4 | F5 | g |
|  |  | 313.860 | . 74 | F5 | F3 | f |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | LX | Lyr | 429.710 | *. 00 | A9 | A4 | f |
| Y | Lyr | 426.831 | . 58 | F5 | F4 | f |  |  | 455.719 | . 68 | F6 | A8 | p |
|  |  | 480.672 | . 68 | F5 | F4 | g |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | BE | Mon | 222.793 |  | F4 | F5 |  |
| RR | Lyr | 172.672 | . 58 | F6 | F0 | f |  |  | . 917 |  | F4 | F6 | f |
|  |  | 349.833 | 0.13 | A9 | A4 | g |  |  | 223.878 |  | G0 |  | p |

TABLE 2 (Continued)

| Star | $\mathrm{JD}_{\oplus}$ | Type |  |  |  | $\mathrm{JD}_{\oplus}$ |  |  | Type |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2436000+ | Phase | H | CaII | Qual. | Star | 2436000+ | Phase | H | CaII | Qual. |
| BE Mon | 223.933 |  | F8 | F8 | f | CG Peg | 368.925 | ${ }^{0} \mathrm{P}_{00}$ | F6 | F3 | f |
|  | 464.033 |  | F8 | F8 | g |  | 373.952 | . 76 | F3 | F3 | g |
|  | 466.010 |  | F6 | F8 | g |  | 397.946 | . 12 | F6 | F4 | g |
|  | 467.031 |  | F7 | F7 | g |  | 426.994 | . 31 | F6 | F5 | g |
|  | 469.042 |  | F6 | F6 | g |  | 429.795 | . 30 | F6 | F4 | g |
|  | 470.032 |  | G0 | G0 | f |  | . 922 | . 58 | A7 | A7 | g |
| ST Oph | 401.811 | ${ }_{0} \mathrm{P}_{03}$ | A4 | A0 | g | DH Peg | 410.783 | . 58 | A7 | A7 | f |
|  | 402.870 | . 38 | F4 | A4 | p |  | 419.855 | . 09 | A4 | A2 | f |
|  | 419.717 | . 79 | F4 | A8 | g |  | . 938 | . 42 | A7 | A6 | g |
| V413 Oph | 339.912 |  | F6 | F3 | g | ET Peg | 452.738 | . 81 | F6 | F1 | g |
|  |  |  |  |  |  |  | 455.878 | . 08 | A8 | A5 | g |
| V445 Oph | 020.715 | *. 00 | F6 | F5 | g |  | 456.934 | . 18 | A9 | A 7 | g |
|  | . 805 | . 23 | F0 | A9 | f |  | 463.913 | . 45 | F5 | F2 | g |
|  | . 813 | . 25 | A9 | F0 | g |  |  |  |  |  |  |
|  |  |  |  |  |  | AR Per | 167.921 | . 01 | A9 | A6 | f |
| V452 Oph | 373.877 |  | F5 | F0 | f |  | 168.000 | . 19 | F3 | F1 | f |
|  |  |  |  |  |  |  | 172.642 | . 10 | A9 | F0 | g |
| V453 Oph | 447.665 | *. 00 | F7 | F3 | f |  | 435.010 | . 64 | F5 | F6 | g |
|  | 452.675 | . 16 | F8 | F1 | g |  | 470.920 | . 03 | A8 | A6 | g |
|  | 455.672 | . 25 | F9 | F4 | g |  |  |  |  |  |  |
|  | 463.713 | . 53 | F6 | A9 | g | RU Psc | 419.924 | . 67 | F3 | A5 | g |
|  |  |  |  |  |  |  | 441.849 | . 83 | F3 | A 7 | g |
| V455 Oph | 337.979 |  | A7 | A2 | g |  | . 944 | . 07 | A8 | A2 | g |
|  |  |  |  |  |  |  | 442.007 | . 24 | A7 | A2 | g |
| V716 Oph | 420.716 | *. 00 | A9 | A5 | g |  |  |  |  |  |  |
|  | 426.724 | . 38 | F5 | A7 | g | RY Psc | 212.642 | *. 00 | F0 | A2 | g |
|  |  |  |  |  |  |  | 426.893 | . 46 | F4 | A8 | g |
| S3914 Ori | $456.977$ | . 56 | F8 | G0 | g |  | 495.804 | . 54 | F6 | A 7 | g |
|  | $457.987$ | . 78 | G0 | G0 | g |  |  |  |  |  |  |
|  | 464.003 | . 00 | G0 | G0 | g | SS Psc | 185.812 | . 13 | A8 | A9 | f |
|  | 465.971 | . 36 | F5 | F5 | f |  | 211.613 | . 78 | A9 | A7 | g |
|  | 469.952 | . 14 | F9 | G0 | f |  | 212.671 | . 46 | F2 | F0 | g |
|  | 470.984 | . 37 | F6 | F6 | g |  | 215.660 | . 84 | A8 | A7 | g |
|  |  |  |  |  |  |  | . 738 | . 11 | A9 | A7 | g |
| VV Peg | 166.737 | . 30 | A9 | A3 | f |  | 482.766 | . 96 | A7 | A7 | g |
|  | 429.756 | . 85 | F4 | A5 | f |  |  |  |  |  |  |
|  |  |  |  |  |  | BB Pup | 529.993 | *. 00 | F5 | F2 | p |
| AE Peg | 427.907 | *. 00 | F4 | A7 | f |  | 556.968 | . 18 | F1 | A8 | g |
|  | 441.964 | . 30 | A7 | A2 | g |  |  |  |  |  |  |
|  | . 988 | . 35 | A6 | A0 | f | V440 Sgr | 421.818 | *. 00 | F5 | F0 | g |
|  |  |  |  |  |  |  | 426.774 | . 38 | A7 | A6 | g |
| AO Peg | 402.815 | *. 00 | A7 | A5 | g |  |  |  |  |  |  |
|  | 421.744 | . 59 | F5 | F4 | g | V1211Sgr | 433.725 | *. 00 | F0 | A6 | f |
|  | 437.989 | . 28 | A9 | F0 | f |  | 434.727 | . 16 | F5 | F0 | p |
|  |  |  |  |  |  |  | 435.772 | . 36 | F4 | F1 | f |
| AV Peg | $142.765$ | . 69 | F6 | F6 | $\mathrm{p}$ |  | 450.698 | . 58 | F6 | F3 | f |
|  | $166.646$ | . 87 | F4 | F5 | $\mathrm{f}$ |  |  |  |  |  |  |
|  | . 658 | . 90 | A9 | A8 | g | VY Ser | 306.918 | . 20 | F2 | A5 | g |
|  | . 666 | . 92 | A7 | A7 | f |  | 313.012 | . 74 | F6 | A7 | f |
|  | 426.980 | . 76 | F5 | F6 | g |  | 314.012 | . 14 | F4 | A7 | g |
| BH Peg | 373.969 | *. 00 | F4 | A7 | g | AN Ser | 337.949 | . 10 | A9 | F3 | g |
|  | 395.930 | . 26 | F5 | F1 | g |  | 339.859 | . 76 | F5 | F5 | g |
|  | 397.969 | . 44 | F6 | F0 | g |  | 348.850 | . 98 | A7 | A8 | g |
|  | 426.933 | 0.62 | A8 | A6 | g |  | . 906 | 0.09 | F0 | F2 | g |

TABLE 2 (Continued)

| Star |  | $\mathrm{JD}_{\oplus}$ | Type |  |  |  | $\mathrm{JD}_{\oplus}$ |  |  | Type |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2436000+ | Phase | H | CaII | Qual. |  |  | 2436000+ | Phase | H | CaII | Qual. |
| AP | Ser | 254.967 | ${ }_{*}{ }^{\text {P }}$. 00 | F3 | A5 | g | RV | UMa | 215.058 | ${ }^{*} \mathrm{P}_{0}{ }_{00}$ | F5 | A5 | p |
|  |  | 264.049 | . 67 | F0 | A4 | g |  |  | . 066 | . 02 | F4 | A5 | f |
|  |  | 312.954 | . 88 | A7 | A4 | f |  |  | 234.853 | . 29 | F4 | A6 | f |
| AR | Ser | 313.926 |  | F3 | A5 | f | SX | UMa | 318.750 | . 68 | A9 | A3 | g |
| AT | Ser | 312.904 | . 09 | A9 | A3 | g | TU | UMa | 359.706 | . 68 | F5 | A8 | g |
|  |  | . 980 | . 19 | F0 | A3 | g |  |  | 379.725 | . 58 | F5 | F0 | g |
|  |  | 313.997 | . 55 | F5 | A6 | g |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | UU | Vir | 262.917 | . 21 | F2 | A7 | g |
| AV | Ser | 337.923 |  | F2 | A6 | g |  |  | 263.830 | . 14 | A9 | A6 | g |
|  |  |  |  |  |  |  |  |  | . 959 | . 41 | F5 | F3 | g |
| AW | Ser | 339.972 |  | F6 | A9 | p | XX | Vir |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | 313.963 |  | F6 | A7 | f |
| BF | Ser | 391.776 | *0.00 | A7 | A0 | f |  |  |  |  |  |  |  |
|  |  | 401.764 | . 57 | F3 | A7 | f | AU | Vir | 349.878 |  | A7 | A0 | p |
| CS | Ser | 391.804 |  | F1 | A5 | f | AV | Vir | 280.992 | *. 00 | F6 | A9 | p |
|  |  |  |  |  |  |  |  |  | 282.022 | . 57 | A9 | A4 | g |
| T | Sex | 168.070 | . 11 | A8 | A7 |  |  |  | 313.768 | . 89 | F6 | F0 | g |
|  |  | 215.898 | . 43 | A7 | A2 | f |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | BN | Vul | 396.956 | . 14 | A8 | A6 | f |
| U | Tri | 185. 764 | . 06 | A8 | A2 | f |  |  | 397.883 | . 70 | F4 | A8 | g |
|  |  | 186.708 | . 17 | A9 | A7 | g |  |  | 430.712 | 0.96 | A6 | A0 | g |
|  |  | 195.852 | 0.61 | F5 | F3 | f |  |  |  |  |  |  |  |

* Phase reckoned from arbitrary initial epoch, usually the date of the first spectroscopic observation.


Fig. 1.-Reproductions of spectra of typical RR Lyrae stars. The top and bottom spectrograms in each of the three panels are main-sequence standards. a, Spectra near minimum light of Bailey type a's with $P<0$ d 75 arranged in order of K-line strength. The range in K-line type amounts to about one spectral class; the hydrogen-line types are all near F5. The weakest-lined variables bear a strong superficial resemblance to F-type subdwarfs (Nos. 3,5 , and 8 ), while the strong-line variables differ little from MK standards at $430 \mathrm{~A} / \mathrm{mm}$. $b$, Spectra near minimum light of Bailey type a's with $P>0 \mathrm{~d} 75$. Variables 2,3 , and 4 show a range in peculiarity comparable to that found in Fig. 1, $a$. The emission feature in 4 is $\lambda 3727$ of $[\mathrm{O}$ п1 $]$ due to field emission nebulosity. The emission feature redward of $\mathrm{H} \gamma$ in several spectrograms of Fig. 1, b, is $\lambda 4358$ of Hg I due to city lights; $\lambda 4047 \mathrm{Hg}$ I also appears in several spectra. The spectra of variables 5, 6 , and 7 , all with periods near 0.8 day, indicate that normal spectra may be common among field variables with this period. $c$, The variation of spectral type with phase for XZ Dra. The table below gives the star names, phases, and spectral types for the spectrograms reproduced in the figure.
in K-line types, which is comparable to the variation with phase for typical stars, it is not possible to discuss intrinsic differences among the less observed stars until the phase variations, such as those shown in Figure 1, $c$, for XZ Dra, have been removed. This will be done in the next section. Before leaving this general discussion, however, we wish to point out that the dispersion in spectroscopic peculiarities discussed here has a parallel among stars near the sun. It also occurs among the F-type "subdwarfs," as Greenstein (1956) and the Burbidges (1956) have pointed out. Spectra of three subdwarfs, $\mathrm{BD}+26^{\circ} 3578$, HD 140283 , and $\mathrm{BD}+17^{\circ} 4708$, are included among the RR Lyrae spectra in Figure 1, a. The range in types assigned to them by Roman (1954, 1955)-sdF4, sdF5, and sdF6, respectively-which is comparable to the range in the hydrogen-line types we assign to these stars, is too small to account for the difference between their K lines.

There remains one possible source of dispersion which, of necessity, has been ignored-long-period variations and/or non-repetitive behavior. To have taken account of spectrum variations simultaneously in the fundamental and secondary light-periods would have placed undue constraints on a survey-type observing program. On occasion, however, two spectrograms obtained at the same short-period phase yield spectral types that differ by amounts exceeding the estimated classification error. The matter could be settled by intensive observation of a smaller number of stars.

## b) The Variation of Spectral Type with Phase

Investigation of the variation of spectrum with phase makes it possible (a) to determine the sensitivity of type to phase, (b) to choose the most appropriate phase for the intercomparison of stars, and (c) to devise a technique for the intercomparison of stars not observed at the same phase. Because of the small number of spectrograms obtained for each star, it was expedient to combine observations of many stars into mean spectrum versus phase diagrams, while the large spread in spectroscopic appearance indicated the need for a preliminary organization of the stars according to K-line strength. This procedure, incidentally, provides a simple test of the reality of the small differences in K-line types at minimum suggested in the previous section. If they are real, then the

IDENTIFICATION OF STAR NAMES, PHASES, AND SPECTRAL TYPES IN FIGURE 1

spectrum versus phase diagrams constructed for stars grouped by this criterion should differ systematically throughout the course of a light-cycle. Should the spread be due primarily to photographic effects, non-repetitive behavior, and classification errors the diagrams constructed for stars grouped in this manner should not differ significantly at other phases. The diagrams have been constructed for stars having K-line types near minimum in the intervals A4-A7, A9-F1, and F4-F6 (top, middle, bottom curves, respectively, in Fig. 2, a). Only stars have been used for which times of maximum light (zero phase in Fig. 3) observed later than JD 2434000 are available. The majority of the times of maximum have been taken from two lists by Alania (1954, 1956), augmented by observations of Kron (1958), Pohl (1955), and Stehmann (1958). Individual types have been grouped into mean points of four observations each.

The hydrogen-line curves of the three groups are very similar to one another as can be seen from the combined plot, Figure 2, $b$, of hydrogen-line mean points for the three groups. The hydrogen-line types decline rapidly after maximum light, reach their minimum values near phase $0 . P 5$, and remain practically constant thereafter. A small secondary maximum during the second half of the light-cycle may be present,


Fig. 2.-Spectral types versus phase diagrams. $a$, K-line type versus phase for the three groups described in the text. $b$, Hydrogen-line types versus phase for stars of all three groups. $c, \Delta S$ versus phase for the three groups described in the text. Each point represents the mean of four observations.
possibly associated with the bumps in the light-curves occasionally found before mimimum. The K-line curves are similar in form to those of the hydrogen lines. Of primary interest is the fact that the separation (Fig. 2, a) forced at minimum light by the way in which the stars were grouped persists at other phases. The smaller amplitude of the curve for the A4-A7 group may be due to the paucity of maximum observations for these stars, to the generally smaller light-amplitudes for members of this group, and to the fact that the marginal appearance of the K line between A0 and A1 does not permit earlier classifications to be made. The comparison of the three K-line curves suggests that, for good-quality spectra at least, differences in the K-line types of the order of 2 or 3 tenths of a spectral class are probably real, i.e., we have not seriously underestimated the classification error.

## c) Choice of Reference Phase

It is evident from Figure 2, $a$ and $b$, that the star-to-star spectroscopic differences seen at very low dispersion are (a) primarily restricted to the appearance of the K line and (b) best detected during the half-cycle preceding minimum light. Because, in addition, the variation with phase is smallest during the second half of the light-cycle, we have chosen the phase interval from 0.5 to 0.8 as the one in which to intercompare stars. Errors in the type and phase determinations do not warrant a more precise specification of the reference phase. Since the range in K-line types among the stars in this sample is considerably larger than the estimated classification errors, the material has been treated as if the K line exhibits a continuous range in strength. In the discussion to follow, it should be borne in mind that the classification errors in many instances exceed the smallest unit of subdivision (one-tenth of a spectral type).

## d) A Parameter To Describe the Spectra

The K-line type at minimum light does not by itself provide a satisfactory description of the weak-line characteristic of the RR Lyrae stars. It fails, for example, to provide an adequate comparison between the Bailey type a's and c's, since the latter are systematically earlier in type at minimum light. Even among the type a's there are stars with stronger than average hydrogen lines at minimum, e.g., VX, VZ, and AR Her. To minimize possible temperature effects suggested by these circumstances, we use, instead, the difference between the hydrogen- and K-line types reckoned in units of tenths of spectral class. In symbols,

$$
\Delta S=10[\mathrm{Sp}(\mathrm{H})-\mathrm{Sp}(\mathrm{Ca} \text { II })]
$$

The run of $\Delta S$ with phase constructed from smoothed curves drawn through the mean points given in Figure 2, $a$ and $b$, is shown in Figure 2, $c$.

When spectrograms in the appropriate phase interval are available, $\Delta S$ may be obtained directly from the spectral types. In those instances (common when only one plate is available) where no spectrogram at minimum was obtained, an estimate of $\Delta S$ at minimum can be made from Figure 2, $b$ and $c$. The phase of the observation can be estimated by entering Figure 2, $b$, with the observed hydrogen-line type. Interpolation of the observed $\Delta S$ value among the curves in Figure 2, $c$, at this phase and extrapolation to the phase interval $0^{P} 5-0{ }^{P} \cdot 8$ gives an estimate of $\Delta S$ at minimum. Because of the poor resolution afforded by this procedure near maximum light, no attempt was made to estimate $\Delta S$ at minimum from spectrograms of Bailey type a's with hydrogenline types earlier than F 0 . The adopted $\Delta S$ values (henceforth understood to refer to minimum light), together with the latest observed spectral types and other data to be used in later sections, are tabulated in Table 3. Values of $\Delta S$ in parentheses were obtained by the method described above, and a colon indicates a poor spectrogram. Figure 3 is a plot of $\Delta S$ as a function of the K-line type for those stars for which a good minimum

TABLE 3
SUMMARY OF OBSERVATIONAL DATA ON RR LYRAE STARS

| No. | Star | Period (days) | $\mathrm{m}_{\mathrm{pg}}{ }^{\dagger}$ |  | Spectral Types at Minimum Light |  |  | Rad.Vel. <br> (km/sec) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | SW And | 0.442 | 8.8 | 10.4 | F6 | F6 | 0 | - 22 |
| 2 | XX And | . 723 | 10.5 | 11.5 | F6 | A7 | 9 | - 15 |
| 3* | AC And |  | 10.6 | 11.6 |  |  |  | - 70 |
| 4 | AT And | . 617 | 10.9 | 11.7 | F5 | F2 | 3 | -250 |
| 5 | SW Aqr | . 459 | 10.6 | 12.4 | F4 | A9 | 5 | - 5 |
| 6 | SX Aqr | . 536 | 10.9 | 12.6 | F6 | A7 | 9 | -220 |
| 7 | TZ Aqr | . 571 | 11.9 | 12.6 | F6 | F1 | 5 | + 25 |
| 8 | BN Aqr | . 470 | 11.3 | 13.2 |  |  | (4) |  |
| 9 | BO Aqr | . 694 | 11.5 | 12.7 |  |  | (6) | - 55 |
| 10 | BR Aqr | . 482 | 11.0 | 12.2 | F5 | F2 | 3 |  |
| 11 | BS Aqr | . 198 | 8.8 | 9.2 | F1 | F1 | 0 | + 50 |
| 12 | CP Aqr | . 463 | 10.8 | 12.1 v | F5 | F2 | 3 |  |
| 13 | CY Aqr | . 061 | 10.5 | 11.4 | A7 | A5 | 2 | - 25 |
| 14 | AA Aql | . 362 | 10.8 | 12.1 | F5 | F5 | 0 | - 85 |
| 15 | V341 Aql | . 578 | 10.4 | 11.4 v | F5 | F2 | 3 | -135 |
| 16 | V706 Aql | . 377 | 12.2 | 13.6 |  |  | (0) |  |
| 17 | X Ari | . 651 | 9.0 | 9.9 | F4 | A4 | 10 | - 40 |
| 18 | TZ Aur | . 392 | 11.0 | 12.3 v | F6 | F4 | 2 | - 10 |
| 19 | RS Boo | . 377 | 9.7 | 11.2 | F5 | F3 | 2 | - 10 |
| 20 | SW Boo | . 514 | 11.5 | 12.9 |  |  | (7) | + 15 |
| 21 | TV Boo | . 313 | 10.1 | 11.3 | A9 | A1 | 8 | -85 |
| 22 | UY Boo | . 651 | 10.6 | 12.0 |  |  | (10) | +140 |
| 23 | YZ Boo | . 104 | 10.3 | 10.8 v |  |  | (0) |  |
| 24 | SS Cnc | . 367 | 11.2 | 12.9 | F5 | F3 | 2 | + 5 |
| 25 | TT Cnc | . 563 | 10.8 | 11.9 | F4 | A7 | 7 | + 55 |
| 26 | VZ Cnc | . 178 | 7.2 | 7.9 | F4 | F4 | 0 | + 25 |
| 27 | W CVn | . 552 | 10.0 | 10.9 | F6 | A9 | 7 | + 20 |
| 28 | Z CVn | . 654 | 11.0 | 12.0 |  |  | (8) | 0 |
| 29 | RR CVn | . 559 | 12.5 | 14.1 | F4: | A7: | 7: | - 10 |
| 30 | X CMi | . 364 | 12.5 | 13.6 | F6 | F3 | 3 |  |
| 31 | RV Cap | . 448 | 10.7 | 11.6 | F4 | A8 | 6 | -110 |
| 32 | YZ Cap | . 273 | 11.0 | 11.7 | A4 | A4 | 0 | - 75 |
| 33 | PW Cas | . 801 | 13.2 | 14.8 | G0 | G0 | 0 |  |
| 34 | V342 Cas | . 793 | 12.7 | 13.3 | G0 | G0 | 0 |  |
| 35 | RZ Cep | . 309 | 9.2 | 9.8 | A9 | A4 | 5 | 0 |
| 36 | DX Cep | . 526 | 11.4 | 12.7 v | F6 | A9 | 7 |  |
| 37 | RR Cet | . 553 | 9.2 | 10.3 | F5 | F0 | 5 | - 95 |
| 38 | RU Cet | . 586 | 11.2 | 12.1 v |  |  | (9) |  |
| 39 | RZ Cet | . 511 |  |  | F4 | F0 | 4 | 0 |
| 40 | UU Cet | . 606 | 10.5 | 11.5 | F4 | F0 | 4 |  |
| 41 | S Com | . 587 | 10.5 | 12.1 | F4 | A7 | 7 | - 35 |
| 42 | U Com | . 293 | 11.7 | 12.4 | F0 | A5 | 5 | + 15 |
| 43 | RY Com | . 469 | 11.1 | 12.4 | F5: | F2: | 3 : | - 30 |
| 44 | ST Com | . 599 | 10.7 | 12.0 | F5 | F0 | 5 | -100 |
| 45 | W Crt | . 412 | 10.5 | 12.2 | F5 | F2 | 3 | + 65 |
| 46 | UY Cyg | . 561 | 10.7 | 11.8 | F5 | F2 | 3 | - 5 |
| 47 | Xz Cyg | . 467 | 8.7 | 10.4 | F5 | A9 | 6 | -160 |
| 48 | DM Cyg | . 420 | 10.4 | 11.7 | F6 | F6 | 0 | - 50 |
| 49 | KP Cyg | 0.856 | 12.5 | 14.1 | F7 | F7 | 0 |  |
| 50 | BX Del | 1.092 | 11.6 | 12.6 | F7 | F8 | 0 | + 30 |

TABLE 3 (Continued)

| No. | Star | Period (days) | $\mathrm{m}_{\mathrm{pg}}{ }^{\dagger}$ |  | Spect Mini H | $\begin{gathered} \text { Types } \\ \text { n Ligh } \\ \text { CaII } \end{gathered}$ | $\Delta \mathrm{S}$ | Rad.Vel. (km/sec) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 51 | DX Del | 0.473 | 9.4 | 10.1 | F6 | F4 | 2 | - 45 |
| 52 | FM Del | . 796 | 12.3 | 13.1 | F8 | F8 | 0 |  |
| 53 | RW Dra | . 443 | 10.4 | 12.4 | F4 | F1 | 3 | -125 |
| 54 | SU Dra | . 660 | 9.4 | 10.5 | F6 | A6 | 10 | -185 |
| 55 | SW Dra | . 570 | 9.8 | 11.0 | F6 | F3 | 3 | - 30 |
| 56 | XZ Dra | . 476 | 9.2 | 10.4 v | F5 | F2 | 3 | - 25 |
| 57 | AE Dra | . 603 | 11.5 | 12.5 | F6: | F1: | (4) |  |
| 58 | RX Eri | . 587 | 8.4 | 9.4 | F6 | A7 | 9 | + 70 |
| 59 | SV Eri | . 714 | 9.3 | 9.9 | F5 | A6 | 9 | - 5 |
| 60 | BB Eri | . 570 | 11.1 | 12.2 v | F6 | A8 | 8 | +240 |
| 61 | RR Gem | . 397 | 10.8 | 12.3 | F6 | F3 | 3 | + 94 |
| 62 | TW Her | . 400 | 10.6 | 12.2 | F5 | F3 | 2 | - 15 |
| 63 | VX Her | . 455 | 9.8 | 11.0 | F4 | A9 | 5 | -375 |
| 64 | VZ Her | . 440 | 10.1 | 12.2 | F4 | F0 | 4 | -120 |
| 65 | AR Her | . 470 | 10.3 | 12.1 | F3 | A7 | 6 | -335 |
| 66 | BD Her | 0.474 | 11.6 | 12.4 v | F6 | F4 | 2 |  |
| 67 | CE Her | 1.209 | 11.2 | 12.7 | F5 | A8 | 7 | -235 |
| 68 | OX Her | 0.757 | 12.4 | 13.0 | F5: | A9: | 6 : |  |
| 69 | CZ Lac | . 432 | 11.1 | 12.2 | F5 | F4 | 1 | -120 |
| 70 | RR Leo | . 452 | 9.7 | 11.4 | F5 | A7 | 8 | + 65 |
| 71 | RX Leo | . 653 | 11.4 | 12.5 |  |  | (5) | -105 |
| 72 | SS Leo | . 626 | 9.9 | 11.1 | F6 | A8 | ) | +145 |
| 73 | ST Leo | . 478 | 10.6 | 12.0 v | F5 | A8 | 7 | +170 |
| 74* | TV Leo | . 402 ? | 10.1 | 11.2 | F5: | A5: | 10: | - 55 |
| 75 | $\checkmark$ LMi | . 544 | 10.8 | 12.0 |  |  | (4) | -210 |
| 76 | U Lep | . 581 | 10.1 | 10.8 | F6 | A7 | ) | +110 |
| 77 | TV Lib | . 270 | 10.8 | 12.3 | F5 | F3 | 2 | - 60 |
| 78 | Y Lyr | . 503 | 11.9 | 13.3 v | F5 | F4 | 1 | -110 |
| 79 | RR Lyr | . 567 | 6.9 | 8.0 | F6 | F0 | 6 | - 70 |
| 80 | RZ Lyr | . 511 | 10.0 | 12.2 | F4 | A5 | 9 | -230 |
| 81 | BQ Lyr | . 435 | 13.2 | 14.1 |  |  | (0): |  |
| 82 | CX Lyr | . 617 | 11.7 | 12.7 v |  |  | (7): |  |
| 83 | EZ Lyr | . 525 | 10.7 | 11.8 v | F4 | A7 | 7 | - 75 |
| 84 | 10 Lyr | . 577 | 11.0 | 11.8 | F6 | F3 | 3 |  |
| 85 | KX Lyr | . 441 | 10.6 | 11.2 | F5 | F5 | 0 | - 60 |
| 86 | LX Lyr | . 545 | 11.8 | 12.4 | F6: | A8: | $8:$ |  |
| 87* | BE Mon | . 421 ? | 10.5 | 11.5 | G0 | G0 | 0 |  |
| 88 | ST Oph | . 450 | 11.0 | 12.3 v | F4 | A8 | 6 | - 45 |
| 89 | V413 Oph | . 310 | 11.8 | 13.6 | F6 | F3 | 3 |  |
| 90 | V445 Oph | . 397 | 10.6 | 11.6 | F6 | F5 | 1 | - 15 |
| 91 | V452 Oph | . 557 | 11.5 | 12.5 | F5: | F0: | 5 : |  |
| 92* | V453 Oph | . 971 | 10.6 | 11.4 v | F8 | F4 | 4 | - 95 |
| 93 | V455 Oph | 0.453 | 11.6 | 12.5 |  |  |  |  |
| 94 | V716 Oph | 1.116 | 11.2 | 13.0 | F5 | A7 | 7 | -230 |
| 95 | S3914 Ori | 0.833 | 12.1 | 13.3 | G0 | G0 | 0 |  |
| 96 | VV Peg | . 488 | 10.8 | 12.1 v | F4 | A5 | 9 | + 10 |
| 97 | AE Peg | . 497 | 11.4 | 13.3 | F4 | A7 | 7 |  |
| 98 | AO Peg | . 547 | 12.2 | 13.1 v | F5 | F4 | 1 | +115 |
| 99 | AV Peg | . 390 | 9.9 | 11.3 | F4 | F5 | 0 | -100 |
| 100 | BH Peg | 0.641 | 10.3 | 11.0 | F6 | F1 | 5 | -260 |


| No. | Star |  | $\begin{aligned} & \text { Period } \\ & \text { (days) } \end{aligned}$ | $\mathrm{m}_{\mathrm{pg}}{ }^{\dagger}$ |  | Spectral Types at Minimum Light |  |  | Rad.Vel. (km/sec) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 101 | CG | Peg | 0.467 | 10.7 | 11.7 | F6 | F4 | 2 | + 5 |
| 102 | DH | Peg | . 256 | 9.2 | 10.2 | A7 | A7 | 0 | - 55 |
| 103 | ET |  | . 960 | 12.8 | 14.0 | F6 | F2 | 4 |  |
| 104 | AR | Per | . 426 | 10.2 | 11.2 | F5 | F6 | 0 | - 10 |
| 105 | RU | Psc | . 390 | 9.8 | 10.5 | F3 | A6 | 7 | -115 |
| 106 | RY | Psc | . 530 | 11.5 | 12.3 | F5 | A8 | 7 | + 25 |
| 107 | SS | Psc | . 288 | 11.0 | 12.0 | F2 | F0 | 2 | + 5 |
| 108 | BB | Pup | . 480 | 10.0 | 10.9v | F5: | F2: | 3: | +255 |
| 109 | V440 | Sgr | . 477 | 9.2 | 10.5 | F5 | F0 | 5 | - 50 |
| 110 | V1211 | Sgr | . 867 | 12.7 | 13.5 | F5: | F2: | 3: |  |
| 111 | VY | Ser | . 714 | 9.5 | 10.7 | F6 | A7 | 9 | + 5 |
| 112 | AN | Ser | . 522 | 10.4 | 11.8 | F5 | F5 | 0 | - 60 |
| 113 | AP | Ser | . 254 | 10.4 | 10.9 | F3: | A5: | 8 : | - 40 |
| 114 | AR | Ser | . 330 | 10.5 | 11.5 | F3: | A5: | 8: | +100 |
| 115 | AT | Ser | . 747 | 10.5 | 11.8 | F5 | A6 | 9 | - 70 |
| 116 | AV | Ser | . 488 | 10.3 | 11.8 v |  |  | (6) | - 55 |
| 117 | AW | Ser | 0.597 | 12.1 | 12.6 v | F6: | A9: | 7 : |  |
| 118 | BF | Ser | 1.165 | 11.1 | 12.7 | F3 | A7 | 6 | -175 |
| 119 | CS | Ser | 0.527 |  |  |  |  | (6) |  |
| 120 | T | Sex | . 325 | 9.9 | 10.6v |  |  |  | + 10 |
| 121 | U | Tri | . 447 | 12.1 | 13.0 v | F5 | F3 | 2 | - 60 |
| 122 | RV | UMa | . 468 | 10.0 | 11.1 | F4: | A6: | $8:$ | -180 |
| 123 | SX | UMa | . 307 | 10.6 | 11.2 v |  |  | 6 : | -135 |
| 124 | TU | UMa | . 558 | 9.3 | 10.3 | F5 | A9 | 6 | +105 |
| 125 | UU | Vir | 0.476 | 9.8 | 10.7 | F5 | F3 | 2 | - 15 |
| 126 | XX | Vir | 1.348 | 11.4 | 12.7 | F6: | A7: | 9: | - 55 |
| 127 | AU | Vir | 0.343 | 11.0 | 11.6 v |  |  | 7: | +105 |
| 128 | AV | Vir | . 657 | 11.3 | 12.1 | F6 | F0 | 6 | + 35 |
| 129 | BN | Vul | 0.594 | 11.0 | 12.2 | F4 | A8 | 6 | -235 |

* No. 3, AC And. Two periods, 0.525 and 0.711 , are associated with the light variation. The $\Delta \mathrm{S}$ value is small, possibly negative.
No. 74, TV Leo. The period is questioned in the General Catalogue.
No. 87, BE Mon. The period is questioned in the General Catalogue.
No. 92, V453 Oph. The light-curve has two maxima separated by 0.135 (Seliwanow, 1936).
$\dagger \mathrm{A} \underline{\mathrm{v}}$ in the fifth column indicates that the magnitudes in the fourth and fifth columns are visual.
observation was obtained. It simultaneously illustrates the range in peculiarity and the dispersion in hydrogen-line types encountered at minimum light. The two parallel lines correspond to families of spectra generated by holding the hydrogen-line type fixed at F4 and at F6 while allowing the K-line type to assume all values. The three approximately horizontal lines roughly indicate the paths that Bailey a's with $\Delta S$ values of 1,5 , and 9 follow during the course of a typical light-cycle. The Bailey c's and variables with periods less than 0.2 day, described in a later section, are plotted as open circles and crosses, respectively. The triangles represent the three subdwarfs HD140283, $\mathrm{BD}+17^{\circ} 4708$, and $\mathrm{BD}+26^{\circ} 3578$.


Fig. 3.-The correlation of $\Delta S$ with K-line type at minimum light. Filled and open circles represent Bailey type a and c stars, respectively. Crosses denote variables with $P<0$ d 2 . Triangles represent F-type subdwarfs. The straight lines are described in the text.

## e) Interpretation of the Spectral Types

To interpret the spectral types derived from low-dispersion spectrograms, we borrow the conclusions of high-dispersion studies of the subdwarfs and of RR Lyrae itself. Chamberlain and Aller (1951), using a model atmosphere that reproduced the observed profiles of the Balmer lines, concluded that HD 140283 is an F-type star and that the weakness of its metallic lines is an abundance phenomenon. Santirocco (1958), using an approach rather different from that used by Chamberlain and Aller, has recently drawn similar conclusions about RR Lyrae. Greenstein (1956), in discussing a temperature sequence for the subdwarfs based on metallic-line ratios, has commented that the weakness of the lines tends to result in spuriously early spectral types.

In accord with these investigations we shall regard the hydrogen-line type primarily as a temperature indicator and $\Delta S$ primarily as a metal-abundance indicator. The results of the present study then suggest that the RR Lyrae variables form a family of F-type stars with a large, possibly continuous, range in metal abundances.

## f) The Bailey Type c's and Variables with $P<0 \mathrm{~d} 2$

Emphasis has been placed primarily on the Bailey type a's and b's in this study. However, twelve Bailey type c's and four variables with periods less than 0.2 day were
included in the observing program. Eight of these twelve type c's form a relatively compact group, with spectra that resemble the weak-line type a variables at or a little below maximum light. The hydrogen- and K-line types range from A7 to F3 and from A0 to A7, respectively. The remaining variables differ from these as follows:

TV Boo has systematically the weakest K line of any star in the survey. Of the nine Crossley spectrograms obtained, the K line is marginally visible on only one. A $130-\mathrm{A} / \mathrm{mm}$ spectrogram obtained with the 36 -inch refractor does not show the rotational broadening to be expected if the star is an early-type eclipsing system. The K line resembles that of a B9 dwarf.

DH Peg, SS Psc, and YZ Cap show little or no evidence of the weak-line characteristic. SS Psc, the latest type Bailey type c observed, is spectroscopically similar at $430 \mathrm{~A} / \mathrm{mm}$ to UZ Leo, an eclipsing binary formerly regarded as an RR Lyrae star (Smith 1955b). A comparison of $130-\mathrm{A} / \mathrm{mm}$ spectra of these two stars shows, however, that SS Psc, like TV Boo, has much narrower lines; the RR Lyrae classification is probably correct. A single spectrogram of YZ Cap resembles that of a normal middle A-type star.


Fig. 4.-The correlation of $\Delta S$ with period. The symbols have the same meanings as in Fig. 3
YZ Boo, VZ Cnc, and BS Aqr resemble SS Psc, the spectral types oscillating with relatively small ranges near F0; and CY Aqr is systematically about half a spectral class earlier. None of these stars appears peculiar at the low dispersion used here.

## IV. THE RELATION OF $\Delta S$ TO OTHER OBSERVABLE QUANTITIES

The preceding sections have been confined to the description of low-dispersion spectra of the RR Lyrae stars and to the definition of $\Delta S$, which describes the principal differences found among them. The diversity of spectroscopic characteristics encountered in what is generally regarded as a single family of pulsating stars prompts a search for further indications of heterogeneity.

## a) The Period versus $\Delta S$ Correlation

The regression of $\Delta S$ on $P$, the fundamental light-period, is shown in Figure 4. Filled and open circles denote Bailey type a's and c's, respectively; crosses represent variables with $P<0 \mathrm{~d} 2$. Only stars for which observations at minimum were obtained are included. Restricting attention to the type a's with $P<0^{d} 75$, we notice a systematic increase in $\Delta S$ values with increasing period, accompanied by a large spread in period and a steep rise in $\Delta S$ values near 0.44 day. Longward of this period the entire range of observed $\Delta S$ values may occur, while shortward of it only one value greater than 3
has been found (TV Leo, $P=0.40$ ). The period of this star is questioned in the General Catalogue and, for this reason, has been omitted from Figure 4. It appears that the great majority, if not all, of the Bailey type a's with $P<0$ d.44 are members of a relatively homogeneous spectroscopic group, not found in any of the globular clusters studied to date, which are to be identified with the low-latitude variables investigated by Kukarkin. This small $\Delta S$ group, it should be noticed, includes members with longer periods as well: AN Ser ( $P=0$ d522), DX Del ( 0 d.473), and SW And ( $0^{\mathrm{d}} 442$ ), the prototype, are good examples. Only two program stars serve to delimit the shortward extent of the weak-line domain. $\mathrm{CZ} \mathrm{Lac}(0 \mathrm{~d} 432, \Delta S=1)$ is typical of the strongest-lined RR Lyrae stars; BQ Lyr ( 0 d 435 ) is so faint at minimum light that the narrow spectrograms taken are of inferior quality, but it is fairly certain that this star resembles the small, not the large, $\Delta S$ objects. Between 0.44 and 0.48 day, the entire range in


Fig. 5.-Period versus amplitude diagram for Bailey type a stars with $P<0 \mathrm{~d} 75$ that were observed in the survey. Open circles, crosses, and filled circles represent stars in the $\Delta S$ groups $0-2,3-4$, and $5-10$, respectively.
spectroscopic appearance is found. KX Lyr ( 0 d44) and DX Del ( 0.473 ) are spectroscopically similar to SW And, which also lies in that period interval. At the other extreme, RR Leo ( 0 d $452, \Delta S=8$ ) and ST Leo ( $0 d 478, \Delta S=7$ ) can unambiguously be placed in the upper portion of the diagram. Several of the stars with $\Delta S$ values between 4 and 6 in this period interval differ from the members of both extremes, in that their hydrogen lines seem to be systematically stronger at minimum light than those of all the other Bailey type a's; examples are VZ and AR Her. The K-line types of these variables are not later than F0, but their hydrogen-line types, which are all earlier than F5, result in relatively small $\Delta S$ values. Whatever the reason for their appearance, there is no possibility of confusing the spectra of these stars with that of SW And. We mention in passing that in the period interval 0.44-0.48 day several stars with strongly variable light-amplitudes have been found. The long-period variations in the maximum and minimum brightness of RV UMa, reported by Balazs and Detre (1957), are so large that detectable spectroscopic effects might be expected to accompany them. The possibility that unrecognized complications in the spectra of such stars are responsible, in part, for the dispersion in $\Delta S$ near 0.45 day cannot be excluded.

## b) Period versus Amplitude Relations

On the assumption that spectroscopic similarity as described by $\Delta S$ implies physical homogeneity, the sample has been divided into two groups, the division point being $\Delta S=3$, the largest $\Delta S$ value found for Bailey type a's with $P<0$ d44. To reduce contamination caused by classification errors, however, stars with $\Delta S=3$ and 4 have been plotted separately in Figure 5 and have been omitted from the discussion in the following section. This procedure should isolate the bulk of the stars with essentially normal spectra from the weak-line RR Lyrae stars. A period versus amplitude diagram for these $\Delta S$ groups is plotted in Figure 5. The amplitudes have been taken from the second edition of the General Catalogue. Where necessary, visual amplitudes have been converted to photographic amplitudes by adding to the former a standard color amplitude of

TABLE 4
Period-Frequency Distributions of Bailey Type a Variables

| Period Range $\Delta \mathrm{P}$ <br> (day) | Observed Numbers of Variables with |  | Population Brighter than$m_{\mathrm{pg}}=13.0$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\Delta S \leq 2$ | $\Delta S \geq 5$ | Total No. | Numbers Expected |  |
|  |  |  |  | $\Delta S \leq 2$ | $\Delta S \geq 5$ |
| 0 24-0.28. | 1 | 0 | 2 | 2 | 0 |
| . 28-. 32 . | 1 | 0 | 2 | 2 | 0 |
| . $32-.36$. | 0 | 0 | 8 | 8 | 0 |
| . $36-.40$. | 8 | 0 | 14 | 14 | 0 |
| . $40-.44$ | 4 | 0 | 18 | 18 | 0 |
| .44- 48. | 7 | 10 | 46 | 19 | 27 |
| .48-. 52 | 1 | 5 | 31 | 5 | 26 |
| . $52-.56$. | 2 | 8 | 45 | 9 | 36 |
| . $56-.60$. | 0 | 10 | 38 | 0 | 38 |
| .60-. 64 . | 0 | 3 | 23 | 0 | 23 |
| . $64-.68$. | 0 | 7 | 21 | 0 | 21 |
| .68- . 72 . | 0 | 3 | 6 | 0 | 6 |
| .72-. 76 . | 0 | 2 | 6 | 0 | 6 |
| 0.24-0.76. | 24 | 48 | 260 | 77 | 183 |

0.3 mag. Open and filled circles represent the groups $\Delta S=0-2$ and $5-10$, respectively, while crosses represent the borderline group with $\Delta S=3-4$. The two groups appear to define two sequences well separated in amplitude at periods greater than $P=0.44$. Future searches for strong-line variables longward of $P=0.44$ may be able to take advantage of this separation. Nevertheless, there appear to be genuine exceptions. AN Ser ( $0.522, \Delta m_{\mathrm{pg}}=1.4, \Delta S=0$ ) definitely has a small $\Delta S$, and the light-curve by Prager (1950) appears to be well determined.

## c) Period-Frequency Distributions

The observed period-frequency distributions of the two groups defined in the preceding section must be adjusted because they are biased by the selection effects enumerated in Section II. The percentages of stars in the $\Delta S$ groups 0-2 and 5-10, found in successive period intervals of 0.04 day in Table 3, were used to estimate the numbers of both kinds of stars in the population of variables from which the spectroscopic sample was drawn. In form the apparent-magnitude distribution of the sample matches that of the
known type a variables fairly well to median $m_{\mathrm{pg}}=12$. Between $m_{\mathrm{pg}}=12.0$ and $m_{\mathrm{pg}}=$ 13.0, however, the number of known variables increases, while the number of stars in the sample distribution falls off. Since the period-frequency distributions for the variables brighter than $m_{\mathrm{pg}}=12.0$ or $m_{\mathrm{pg}}=13.0$ do not differ markedly, the results are insensitive to the adopted apparent-magnitude limit of the population, which here has been taken to be $m_{\mathrm{pg}}=13.0$. The period-frequency distributions for this population and for the two $\Delta S$ groups inferred from it are given in Table 4 and Figure 6. Thus approximately 25 per cent of the Bailey type a's brighter than $m_{\mathrm{pg}}=13.0$ appear to


Fig. 6.-Period-frequency distributions of Bailey type a stars. $a$, All variables brighter than $m_{\mathrm{pg}}=$ 13.0. $b$, The distribution of variables with $\Delta S=5-10$ inferred by the procedure described in the text. $c$, The inferred distribution of variables with $\Delta S=0-2 . d$, The residual distribution formed by removal of the high-latitude distribution of Fig. 8, b, from Fig. 6, a, as described in the text. The black areas represent the Bailey type a variables observed in the survey and used in the discussion.
belong to the small $\Delta S$ group with a period-frequency distribution that is roughly symmetric about a maximum in the vicinity of 0.43 day. If the small amplitudes of the variables in the $\Delta S=0-2$ group with periods greater than 0.45 day have operated selectively against their discovery, the fraction of strong-line stars with periods greater than 0.45 day should be increased and the frequency maximum moved longward. The distribution for the $\Delta S=5-10$ group, it may be noted, resembles a mixture of Oosterhoff's type I and type II cluster-variable distributions (Oosterhoff 1939), for which the mean periods of Bailey type a's are near $P=0 \mathrm{~d} 54$ and 0 d 64 , respectively. Examples of cluster-variable distributions of both types are shown in Figure 7.

It is of some interest to compare the period distribution of the strong-line group with the residual distribution formed by removal of a "halo" distribution from the variables brighter than $m_{\mathrm{pg}}=13.0$. Advantage may be taken of the fact that, for periods greater than 0.60 day, strong-line variables are either rare or non-existent. Accordingly, the distribution of the high-latitude group of Figure 8, $b$, has been scaled so that the number


Fig. 7.-Period-frequency distributions of variables in seven globular clusters. $a, b$, and $c$ are the Oosterhoff type I clusters NGC 6121 (M4), 5904 (M5), and $3201 ; e, f$, and $g$ are the type II clusters NGC 5024 (M53), 7078 (M15), and 6656 (M22). $d$ is NGC 6981 (M72); on the basis of the mean period of its Bailey type a variables it is a type I cluster. However, the short-period wing of its type a distribution suggests that it may be an intermediate case.
of stars in it with $P>0.60$ equals the number of stars brighter than $m_{\mathrm{pg}}=13.0$ in this period interval. The residual distribution at shorter periods is shown in Figure $6, d$. The similarity between $c$ and $d$ of Figure 6 with respect to both the forms of the distributions and the inferred numbers of stars suggests that it is the presence of strongline stars near the sun that is responsible for the difference between the period-frequency distributions of the high-latitude stars and those brighter than $m_{\mathrm{pg}}=13.0$.

The conversion of these results into relative space densities is subject to two corrections, neither of which can be applied with confidence. The small $\Delta S$ group shows a concentration to lower galactic latitudes, relative to the other group, the mean latitudes (weighted by sec $b$ ) being $31^{\circ}$ and $58^{\circ}$, respectively. Therefore, the number of small $\Delta S$ objects found to any apparent magnitude is probably reduced relative to the large $\Delta S$ group due to absorption at low latitudes, and thus the estimate of their numbers should be increased. If there is, in addition, a luminosity difference between the two groups, an adjustment must be made to take into account the relative volumes of space surveyed.

There is no clear-cut way to effect a further subdivision of the interval $\Delta S>4$. The tendency for the largest $\Delta S$ values to occur at long periods is suggestive of the spectroscopic differences between Oosterhoff type I and type II clusters implied by recent work of Morgan (1956) and Deutsch (1958) and pointed out previously by Arp


Fig. 8.-Regional period-frequency distributions of Bailey type a's. $a, 308^{\circ}<l<346^{\circ},|b|<15^{\circ}$, $m_{\mathrm{pg}}$ fainter than 15.0. $b,|b|>45^{\circ}, m_{\mathrm{pg}}$ fainter than $13.0 . c, 12^{\circ}<l<282^{\circ},|b|<20^{\circ}$, all $m_{\mathrm{pg}}$. The histograms at left show the numbers of stars omitted as Bailey type c's.
(1955). Deutsch has referred briefly to the spectra of red giants in several clusters as "strong-" or "weak-lined," this description unambiguously separating the clusters according to Oosterhoff types. Morgan, on the other hand, has concluded that, while the principal contributors to the integrated spectra of the type II clusters M15, M53, and M92 must be stars like HD 140283, the spectrum of $\omega$ Cen, also a type II cluster, more resembles the type I clusters M3 and M5, which have relatively prominent metallic-line spectra. Unless $\omega$ Cen is anomalous (its variable-star population, for example, is unusually large for a type II cluster), the correlation of Oosterhoff type with the weak-line characteristic cannot be regarded as perfect. Single Crossley spectrograms of eight cluster-type variables in M4, M5 (type I), and M22 (type II) were obtained in an attempt to determine directly whether or not a combination of type I and II cluster variables could explain the spread in $\Delta S$ values within the $\Delta S=5-10$ group. In spite of care in avoiding the central regions of the clusters and in orienting the slit to avoid resolvable stars, the spectrograms are badly contaminated by the unresolved background and/or scattered light from the cluster centers. All that can be determined is that no startling differences are visible at $430 \mathrm{~A} / \mathrm{mm}$ from cluster to cluster or between the cluster variables and the weak-line RR Lyrae stars near the sun.

It is by no means certain, however, that a close correspondence between field and cluster variables exists, even after removal of the strong-line group. In fact, van den Bergh (1957) has recently shown that in some regions of the Galaxy such a correspondence is difficult to maintain. Figure 8 contains period-frequency diagrams for the Bailey type a's in three regions of the Galaxy based on material in the first edition of the General Catalogue of Variable Stars and its supplements. The variables in the nuclear region surrounding NGC 6522 have not been included, in view of recent doubts as to the correctness of the published periods, expressed by Pavlovskaya (1957) and others. For $P<0 d 44$, stars have been included among the Bailey type a's if, when no rise time is published, their ranges exceed 1 mag. or if, for ranges less than or equal to 1 mag., their rise times are less than $0^{\mathrm{P}} 3$. The diagram for $|b|>45^{\circ}$ and $m_{\mathrm{pg}}$ fainter than 13.0 (Fig. 8, b), which is restricted to stars located at distances of approximately 2 or more kiloparsecs above the galactic plane, may be interpreted as a composite of the variables from type I and type II clusters. However, in the region $308^{\circ}<l<346^{\circ}$ and $|b|<15^{\circ}$ and for $m_{\mathrm{pg}}$ fainter than 15.0 (galactic plane, toward center) (Fig. $8, a)$ the frequency maximum occurs between 0.44 and 0.48 day. No known cluster, or combination of them, can at present duplicate this distribution. Removal of the small number of strong-line stars implied by the population shortward of 0.44 day will not change the distribution markedly. In view of the large size of the sample and the absence of any obvious selection effects, it is difficult not to accept the difference between this distribution and those of the cluster and the high-latitude stars as real. There is thus a second possible explanation of the spread $4<\Delta S<10$. The intermediate $\Delta S$ group, which contains numerous stars with periods between 0.44 and 0.48 day, may be composed of representatives of the field population found toward the galactic center, while true cluster-type variables either are intermingled among them or are restricted to the upper portion of the diagram.

The region $12^{\circ}<l<282^{\circ}$ and $|b|<20^{\circ}$ (galactic plane, excluding center) (Fig. 8, c) contains the bulk of the strong-line Bailey type a's. Although their $z$-distribution cannot be established until the question of their luminosities is settled, it is apparent that they form a relatively flattened system.

## d) The Concentrations of $\Delta S$ Groups to the Galactic Center

The regional distributions indicate in a general way that the period-frequency distributions of the RR Lyrae stars differ from place to place in the Galaxy. The strong-line stars, which indicate a departure from the concept of "pure population II" RR Lyrae variables, are clearly more concentrated toward the plane than the rest of the variables,
but the summation over all longitudes from $12^{\circ}$ to $282^{\circ}$, as in Figure 8, $c$, has buried information that might exist on the relative concentration of these objects toward the galactic center. To investigate this matter, advantage may again be taken of the fact that, while only small $\Delta S$ values occur for $P<0.44$, there are few or none of them between 0.60 and 0.70 day (see Fig. 4). In the vicinity of the sun the numbers of Bailey a's in these two period intervals are comparable, so that their ratio in any area of the sky serves as a convenient measure of the relative concentrations of these two groups of objects, subject to adjustment if there is a luminosity difference. Accordingly, the zone $-30^{\circ}<b<30^{\circ}$ was divided into five pairs of longitude intervals symmetric about longitude $327^{\circ}$; the numbers of Bailey type a's with $P<0$ d 44 and 0 d $60<P<$ 0 d 70 were counted; and their ratio is displayed in Table 5 and Figure 9 as a function of $\lambda_{A}$, the angular distance of the area center from the galactic center. The vertical bars are estimates of the standard deviations, $\sigma_{R}$, computed on the assumption that errors in the counts are due to random fluctuations in the areal distributions. No account of completeness has been taken, since the great majority of variables fainter than


Fig. 9.-The ratio, $R$, of the number of Bailey type a variables with $P<0.44$ to the number with 0 d $60<P<0$ d 70 as a function of $\lambda_{A}$, the angular distance of the area center, as described in text, from the galactic center. The vertical bars are estimates of the uncertainty in $R$.

TABLE 5
Longitude Distribution of RR Lyrae Stars
in Two Period Intervals

|  | $\lambda_{A}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $10^{\circ}$ | $30^{\circ}$ | $55^{\circ}$ | $90^{\circ}$ | $155^{\circ}$ |
| Area (sq. deg.) | 1200 | 1200 | 1800 | 2400 | 4200 |
| $n_{1}\left(0\right.$ d $30<P_{a}<0$ d 44$)$ | 41 | 16 | 36 | 10 | 18 |
| $n_{2}\left(0{ }^{\text {d }} 60<P_{a}<0{ }^{\text {d }} 70\right)$ | 128 | 39 | 35 | 10 | 11 |
| $R=n_{1} / n_{2}$. | 032 | 041 | 1.03 | 1.00 | 1.64 |
| $\sigma_{R} \ldots \ldots$. | $\pm 0.06$ | $\pm 0.13$ | $\pm 0.19$ | $\pm 0.43$ | $\pm 0.46$ |

$m_{\mathrm{pg}}=12$ were discovered by intensive searches in small areas of the sky, in each of which the ratio of discovery probablities for the two kinds of objects should be approximately the same.

Crude conclusions can be drawn from the counts if the average interstellar absorption coefficient for $|b|>10^{\circ}$ (where most of the variables have been discovered) and the limiting magnitudes of the surveys do not vary systematically with $\lambda$. Subject to these assumptions, the observed counts may be compared with those computed for objects with some specified galactic density distribution near the plane as a function of $\lambda$ and the distance, $r$, to which we count. The general character of solutions using simple density laws of the form $1 /(\rho+\beta)$ or $1 /\left(\rho^{2}+\beta\right)$, in which $\rho$ is distance from the galactic center and $\beta$ is an arbitrary, positive constant, is a monotonic decline in areal density with increasing $\lambda$, the maximum at $\lambda=0$ diminishing with decreasing $r$. To represent the observed increase in the ratio of areal densities with $\lambda$, either a much weaker galactic concentration or a lower luminosity must be assumed for Bailey type a's with $P<0^{d} 44$. In the absence of a reliable luminosity estimate for the $\Delta S=0-2$ group, we tentatively interpret the counts as indicating a relatively weaker concentration of strong-line stars toward the galactic center.

TABLE 6
Radial-Velocity Observations

| Star | $\mathrm{JD}_{\odot}$ <br> $2436000+$ | $V_{r}$ <br> $(\mathrm{~km} / \mathrm{sec})$ | $\left\langle V_{r}\right\rangle$ <br> $(\mathrm{km} / \mathrm{sec})$ |
| :---: | :---: | :---: | :---: |
| AT And........ | $\left\{\begin{array}{c}480.854 \\ \{480.955\end{array}\right.$ | $-246\}$ | $-257\}$ |
| CZ Lac........ | $\{467.741$ | -250 |  |
| KX Lyr......... | $\{478.817$ | $-126\}$ | -120 |
| $\{388.914$ | $-71\}$ | -60 |  |

e) Variation of Solar Motion with $\Delta S$

A variation of the solar motion derived from stars with different $\Delta S$ values may be expected to accompany the difference in galactic concentrations described in the previous section. This possibility has been investigated by means of the Bailey type a's observed in the survey for which radial velocities exist.

Nearly all the radial velocities are due to Joy (1938, 1950, 1955), augmented by Bonsack's (1957) values for SW And and DX Del. In addition, the writer has determined velocities for AT And, CZ Lac, and KX Lyr from 130-A/mm spectrograms obtained with the two-prism, $3 \frac{1}{2}$-inch camera arrangement of the 36 -inch refractor. Details of these observations are given in Table 6. The corrections to Joy's normal velocities given by Payne-Gaposchkin (1954) have been used. In most cases they are small.

The apex $L=55^{\circ}, B=0^{\circ}$, was assumed, since it corresponds approximately to those derived from various high-velocity groups, including the RR Lyrae stars themselves. The solar motion was then computed by means of least-squares solutions of the simple equation of condition $V_{r}=V_{\odot} \cos A, A$ being the apex distance. All stars with cos $A<0.2$ were rejected. Table 7 gives $V \odot$ computed for each value of $\Delta S$. The numbers of stars, $n$, used in each case are ridiculously small, but they serve to explore the character of the variation. The $\Delta S$ values 0,1 , and 2 all give similar values of $V \odot$, which supports the assumption of the homogeneity of this group made in Section IV. At $\Delta S=3$ the values of $V \odot$ increase steeply, reaching a maximum at 5 or 6 . Consideration of
the errors in $\Delta S(\sim 2)$ shows immediately that the increase in $V \odot$ as $\Delta S$ goes from 2 to 5 could be merely the smearing effect expected if there were a discontinuity in $V_{\odot}$ near $\Delta S=3$. Hence $V \odot$ was computed for stars grouped in the intervals $\Delta S=0-2$, $3-4$, and $5-10$; these values of $V \odot$ and the resulting distributions of peculiar radial speeds, $\left|V_{r p}\right|$, are shown in Table 8 and Figure 10. We are unable to identify the stars in the intermediate group with either of the other two groups from a consideration of their spectra. Five of the eight stars with $\Delta S=3$ are well observed. Adjustments as large as 2 or 3 in $\Delta S$ for UY Cyg, SW Dra, XZ Dra, AT And, and V341 Aql cannot be justified by our spectrograms.

It thus appears, on the basis of the meager data available, that the strongest-lined $(\Delta S<2)$ and the weakest-line $(\Delta S>5)$ RR Lyrae stars have greatly different galacticrotation properties. The weak-line group, which yields a solar motion similar to that derived from the globular clusters, embraces a much larger range in spectroscopic characteristics than does the strong-line group. If there is a continuous transition between

TABLE 7
solar Motion versus $\Delta S$ for RR Lyrae Stars

| $\Delta S$ | $n$ | $\begin{gathered} V_{(\mathrm{km} / \mathrm{sec})} \end{gathered}$ | $\Delta S$ | $n$ | $\begin{gathered} V \odot \\ (\mathrm{~km} / \mathrm{sec}) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 7 | 79 | 6. | 6 | 208 |
| 1 | 3 | 46 | 7 | 6 | 154 |
| 2 | 6 | 34 | 8 | 1 | 206 |
| 3 | 8 | 108 | 9 | 7 | 152 |
| 4 | 2 | 183 | 10. | 1 | 400 |
| 5. | 6 | 224 |  |  |  |

TABLE 8
Solar Motion and Peculiar Motions of $\Delta S$ Groups

| $\Delta S$ | $n$ | $\underset{(\mathrm{km} / \mathrm{sec})}{V_{\odot}}$ | $\begin{gathered} \sigma_{V \odot} \\ (\mathrm{~km} / \mathrm{sec}) \end{gathered}$ | $\begin{gathered} \langle \| V_{r p}\| \rangle \\ (\mathrm{km} / \mathrm{sec}) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0-2 | 16 | 55 | $\pm 20$ | 35 |
| 3-4 | 10 | 115 | $\pm 25$ | 60 |
| 5-10 . | 27 | 185 | $\pm 35$ | 85 |




Fig. 10.-The distributions of peculiar radial velocities, without regard to sign, for $a$, variables with $\Delta S=0-2 ; b$, variables with $\Delta S=5-10$.
the two, it seems to occur within an interval of $\Delta S$ no larger than 2 near $\Delta S=3$ or 4. The possibility that spectroscopically identical objects may belong to subsystems with greatly differing kinematic properties cannot be excluded, UY Cyg ( $\Delta S=3$ ) and AT And $(\Delta S=3)$ being possible examples. Both are near the high-velocity apex, with radial velocities of -5 and $-250 \mathrm{~km} / \mathrm{sec}$, respectively.

## V. THE LONG-PERIOD BOUNDARY OF THE RR LYRAE DOMAIN

We return to an unexpected feature of Figure 4, the break in characteristics between $P=0 \mathrm{~d} 76$ and 0 d 79 . The variables PW Cas ( 0 d 800 ), V342 Cas ( 0 d 793 ), and S3914 Ori ( 0 d833) are unlike any others described previously in this study. They are normal middle F-type stars at maximum light and as late as G0 at minimum, with K lines so strong and hydrogen lines so weak that the system of classification adopted for typical RR Lyrae stars in this study cannot be applied to them successfully. Faint blends of metallic lines, including the luminosity-sensitive feature $\lambda \lambda 4172-4178$ are suggestive of luminosity class II. There are no published light-curves for these three



Fig. 11.-Period-frequency distributions of $a$, field variables and $b$, cluster variables, both with $P>$ 0 d 74 . The dark areas in $a$ represent stars observed in the present survey; those in $b$ represent the contribution of $\omega$ Cen.
stars, but the Bailey type a classification for V342 Cas (Huth 1956) and the large range ( 1.6 mag.) for PW Cas (Wenzel 1956) seem to rule out the possibility of confusion with eclipsing systems. Two other stars with similar periods, FM Del (0d796) and KP Cyg ( 0 d856), differ from the first three in that they have earlier spectra at maximum light. They may be described as slightly later-type versions of SW And. These variables are plotted at $\Delta S=0$ near $P=0 \mathrm{~d} 8$ in Figure 4. All the variables with periods greater than 0.86 day except one resemble weak-line RR Lyrae stars, although one of them, V453 Oph, appears to be of later type at minimum. The exception, BX Del ( $P=1^{\mathrm{d}} 09$, $\Delta S=0$ ) is described by Hoffmeister (1947) as a classical cepheid. Spectra of several stars in this long-period group are reproduced in Figure 1, $b$, arranged according to the strength of the K lines. The similarity of the sequence defined by stars 2, 3, and 4 to that shown in Figure 1, $a$, is evident. The ratio of periods of these two apparent $\Delta S$ sequences is approximately 2 to 1 .

Before discussing these observations, qualitative adjustments of the survey data should be made to take account of selection effects. There are forty-one known field variables with periods between 0.76 and 1.50 day distributed as in Figure 11, a. Of the twenty-three brighter than $m_{\mathrm{pg}}=15$ at minimum, all fourteen north of declination $-25^{\circ}$ have been included in the present survey. Although almost all periods are represented in this sample, all galactic latitudes are not, since all six of the survey stars with
$P<0 \mathrm{~d} 9$ are within $15^{\circ}$ of the galactic equator. Because the regional period-frequency diagrams indicate an absence of strong-line stars far above the plane, it may be expected that, for example, WX Cap ( $P=0.855$ ) and BX Vir ( $P=0.838$ ), fifteenth-magnitude stars with latitudes $-40^{\circ}$ and $+64^{\circ}$, respectively, are weak-line stars. If this presumption is correct, the apparent break near $P=0 \mathrm{~d} 8$ in Figure 4 should be replaced by an overlap extending at least to $P=0 \mathrm{~d} 85$. That it does not extend much farther may be inferred from the period-frequency distribution for all globular clusters shown in Figure 11, $b$. The data were taken from Sawyer's catalogue (1955). At periods longer than 0.78 day this distribution is populated almost exclusively by variables in $\omega$ Cen (dark areas), which contains almost as many variables in the period interval $0.80-1.00$ day as are known in the rest of the Galaxy. The $\omega$ Cen variables represent the end of a continuous distribution extending longward from the frequency maximum. The cluster variables not in $\omega$ Cen, between $P=0.70$ and 0.78 , also represent long-period tails of Oosterhoff type II distributions. The remaining two stars with periods less than 1.0 day are members of the type I clusters M3 and M4. Longward of $P=0 \mathrm{~d} 9$ the cluster distribution hardly looks continuous, particularly when compared with the field distribution. The sizes of the field and cluster samples, which are in the ratio of 3 to 2 , would not seem to be responsible for the difference. To the contrary, Martin's (1938) estimate of the completeness of the variable search in $\omega$ Cen, together with the bias against discovery of field stars with periods near 1 day, suggests that it is the observed field distribution that is relatively underpopulated because of selection effects between $P=0.8$ and 1 d 2 .

It is possible to account for the $P$ versus $\Delta S$ diagram and the period-frequency diagram of the field by supposing that the Bailey type a's in RR Lyrae subfamilies, defined, say, by small intervals in $\Delta S$, are distributed about two "principal" periods, the short-period group of the two being much more heavily populated in all cases. The extent of the frequency minimum between these two period maxima depends on the character of the dispersion about these periods and on the number of stars in the sample, the gap tending to fill in as the sample is enlarged. The systematic increase in period with increasing $\Delta S$ results, for a population such as the general field that contains several of these families, in an overlap of the long- and short-period groups of different $\Delta S$-subfamilies and in the impression, for periods greater than 0.70 day, of a single, continuous period-frequency distribution.

We close this section with remarks about four stars whose relation to the RR Lyrae variables is uncertain.

BL Her ( $P=1$ d307) cannot be classified as an RR Lyrae star. It has often been regarded as a population II cepheid because of the similarity of its light-curve (hump on the descending branch) to those variables with similar periods in $\omega$ Cen and M15 (Payne-Gaposchkin 1955; Gascoigne, Eggen, and Burr 1957). We might therefore expect this star to possess the spectral peculiarities of M15 described by Morgan (1956) and Deutsch (1958) and inferred also from the RR Lyrae period distribution of that cluster. Five Crossley spectrograms, well distributed in phase, show little evidence of peculiarity (No. 9, Fig. 1, b); the spectral types at maximum and minimum, $\Delta S$ value, and strengths of the metallic blends near $\lambda 4077$ and $\lambda 4172$ all resemble those of KP Cyg, described above. Unless our expectation of the spectroscopic properties of variable No. 1 in M15 is incorrect, BL Her provides another illustration of the lack of a close correspondence between the gross features of light-curves and spectra, SW And being the first. Both the moderate galactic latitude $\left(+18^{\circ}\right)$ and the small radial velocity $(+15 \mathrm{~km} / \mathrm{sec})$ of BL Her are consistent with the assignment of this star to a disk population.

BX $\operatorname{Del}\left(P=1^{\text {d }} 09\right)$ is classified as a classical cepheid by Hoffmeister (1947). The lightcurve has a rather sharp maximum, broad minimum, and a rise time of approximately $0^{P}$ ? 3 . Two Crossley spectrograms with a phase difference of 0 ? 4 yield types, based on all features, of F0 and F7, with little or no peculiarity (No. 8, Fig. 1, b). The blend $\lambda 4172$
is strong. The range in type is similar to those of KP Cyg, FM Del, and BL Her, which are considerably earlier at maximum than typical classical cepheids. On the basis of the similarity in spectral types at maximum and minimum to the $P=0 d 8$ variables and the absence of other recognized cepheids of similar period, it is suggested that the star may belong to the short-period group.

BE Mon ( $P=0$ d 42 ) is spectroscopically similar to the $P=0 \mathrm{~d} 8$ variables and is unique among the RR Lyrae stars if the period is correct. The period, however, is in doubt.

AC And, formerly described as an RR Lyrae binary with periods 0.52 and 0.71 day (Florja 1937; Lurie 1950) and more recently as a single star with two strongly interfering periods (Münch 1951), has approximately the range in type and the $\Delta S$ of KP Cyg. It is curious that the only known RR Lyrae star with such chaotic photometric behavior should have associated with it a period that lies in the gap in Figure 4.

## VI. SUMMARY

The RR Lyrae stars have been shown to be spectroscopically heterogeneous with respect to the strength of their metallic absorption lines. The strong- and weak-line extremes have K-line types ( F 6 and A5 at minimum light, respectively) that correspond

TABLE 9
Properties of $\Delta S$ Groups

| $\Delta S$ | Per Cent Vari- <br> ables Expected <br> Brighter than <br> $m_{\mathrm{pz}}=13.0$ | Mean Period <br> Bailey <br> Type a's <br> (day) | Solar <br> Motion <br> $(\mathrm{km} / \mathrm{sec})$ | $\langle \| V_{r p}\| \rangle$ <br> $(\mathrm{km} / \mathrm{sec})$ |
| :---: | :---: | :---: | :---: | :---: |
| $0-2 \ldots . \ldots .$. | 25 | 0.43 | 55 | 35 |
| $5-10 \ldots .$. | 75 | 0.57 | 185 | 85 |

to equivalent widths in the ratio of 3 or 4 to 1 . This implies, under the most naïve assumptions ( $N \propto W^{2}$ ), an order of magnitude variation in metal abundances. The weakline characteristic, described by the parameter $\Delta S$, appears to vary continuously among the RR Lyrae stars to within the uncertainties of the classification technique employed in this study. As a consequence, any subdivision of the sample into discrete groups, while necessary for some purposes, is arbitrary from the spectroscopic point of view. The division point at $\Delta S=3-4$ has been made definite from considerations of the spectra of variables which, for the present at least, are excluded by their periods from identification with variables in globular clusters. Preliminary estimates of a few properties of the groups so defined are presented in Table 9.

In addition to any spectroscopic considerations, the following regularities constitute a strong observational argument in favor of regarding all the RR Lyrae stars as members of a single family of intrinsic variable stars:

1. The mean period of the strong-line type a's is displaced shortward of the mean periods of type a variables in Oosterhoff type I clusters by an amount comparable to the difference between the mean periods of the Bailey a's in type I and type II clusters (see Figs. 6 and 7). Among the field variables, and probably among cluster variables as well, the trend in mean period is accompanied by a change in metallic-line strengths.
2. The period-amplitude relations of the strong-line and weak-line groups (Fig. 5) are similar in form, both to each other and to that of M3 (Roberts and Sandage 1955).
3. The radial-velocity-curves of RR Lyrae (Struve and Blaauw 1948) and SW

And (Bonsack 1957), which are good examples of weak- and strong-line variables, respectively, bear similar phase relations to their respective light-curves.
4. Two interfering periods have been found in the light-variations of both strongand weak-line stars. A summary on this point has been provided by Detre (1955).
5. Bailey type c and the "long"-period ( $P>0 \mathrm{~d} 8$ ) variables described in Section V are present in both the strong- and the weak-line groups.

The character of the Bailey type c component of the strong-line group is uncertain. SS Psc ( $P=0 \mathrm{~d} 288, \Delta S=2$ ) appears to be a genuine strong-line type c variable. However, at very low dispersion no marked spectroscopic difference can be detected between this star and longer-period members of the "dwarf" cepheid sequence (Smith 1955; Woltjer 1956), such as BS Aqr ( $P=0$ d198) and VZ Cnc ( $P=0$ d178). Should SS Psc prove to be photometrically similar to these stars as well, the possibility would exist that the strong-line type c variables form a continuous sequence with the dwarf cepheids, a circumstance which, with the possible exception of Martin's variable in $\omega$ Cen, does not appear to have a parallel among either the cluster-type or the weak-lined field variables.

It is not possible at present to determine whether or not the weak- and strong-line variables have the same luminosity. Only eight stars of the $\Delta S=0-2$ group observed in the present program are included in the compilation of proper motions by Pavlovskaya (1955). Notni (1957) has recently determined mean absolute magnitudes for two groups of RR Lyrae stars, using, as criteria for subdivision, considerations of light-curve shape as a function of period and also radial-velocity and proper-motion properties that essentially divide the variables into high- and low-velocity groups. However, this Group I includes strong- and weak-line stars in comparable numbers, and the relatively high luminosity ( $M_{v}=-0.8$ ) obtained by him for this group cannot be regarded as a determination for the strong-line stars. The small proper motions of the strong-line objects do, however, suggest a difference from the weak-line stars in this sense. A fundamental luminosity determination for stars grouped along the lines indicated in the present study must be postponed until more proper motions are available.

With respect to their kinematic and spectroscopic properties, the RR Lyrae stars cross the recently proposed boundaries between the halo and disk populations (Oort 1958). The values of $V_{\odot}$ and $\langle | V_{r p}| \rangle$ fo the weak-line group are similar to those derived from the globular clusters (Mayall 946 ); the assignment to the halo is unambiguous. The corresponding quantities for the strong-line group indicate a relatively flattened system with intermediate galactic-rotation properties. The spectroscopic differences that accompany this kinematic separation parallel the distinction drawn by Morgan (1956) between the globular clusters of the halo and the disk; the latter, according to Morgan, show little or no evidence of weakness of the metallic lines. The possibility that such clusters contain variables with small $\Delta S$ values can be checked when the periods of the seventeen variables in the vicinity of the disk globulars NGC 6356 and 6712 have been determined (Sawyer 1955).

We wish to re-emphasize the arbitrary character of the division at $\Delta S=3-4$ employed in this discussion. Though it allows the isolation of gross phenomena that accompany spectroscopic differences among the RR Lyrae stars, it is not a division imposed by nature and should be replaced by a classification scheme that takes greater advantage of the dispersion in the spectroscopic properties of these stars.

The RR Lyrae stars present observational difficulties not encountered in non-variable stars. They are, however, much more luminous than the main-sequence F stars, the only other field stars known to possess such large ranges in kinematic and spectroscopic properties. Future investigations of the connection between these properties could profit greatly from a more refined classification system (Strömgren's narrow-band photometry would be desirable) and from a substantial increase in the number of variables with known radial velocities and proper motions.

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